University of Anbar

Thermodynamics Processes

Lecture Note #01

The Ideal-Gas Equation of State

Objectives of Lecture Note

- Describe the hypothetical substance "ideal gas" and the ideal-gas equation of state.
- Apply the ideal-gas equation of state in the solution of typical problems.

The Ideal-Gas Equation of State

- **Equation of state: Any equation that relates the pressure, temperature, and specific volume of a substance.**
- **The simplest and best-known equation of state for substances in the gas phase is the ideal-gas equation of state. This equation predicts the $P - v - T$ behavior of a gas quite accurately within some properly selected region.**

$$P = R\left(\frac{T}{v}\right) \quad \textbf{Or} \quad Pv = RT$$ **Ideal gas equation of state**

P is the absolute pressure, T is the absolute temperature, and v is the specific volume.

$$R = \frac{R_u}{M} \qquad (\text{kJ/kg}\cdot\text{K or kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})$$

R: gas constant
M: molar mass or molecular weight (kg/kmol)
R_u: universal gas constant KJ/Kmol.K

$$R_u = \begin{cases} 8.31447\ \text{kJ/kmol}\cdot\text{K} \\ 8.31447\ \text{kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K} \\ 0.0831447\ \text{bar}\cdot\text{m}^3/\text{kmol}\cdot\text{K} \\ 1.98588\ \text{Btu/lbmol}\cdot\text{R} \\ 10.7316\ \text{psia}\cdot\text{ft}^3/\text{lbmol}\cdot\text{R} \\ 1545.37\ \text{ft}\cdot\text{lbf/lbmol}\cdot\text{R} \end{cases}$$

Substance	R, kJ/kg·K
Air	0.2870
Helium	2.0769
Argon	0.2081
Nitrogen	0.2968

Different substances have different gas constants.

3

Mass = Molar mass × Mole number

$$m = MN \qquad (\text{kg})$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$ Ideal gas equation at two states for a fixed mass

$$V = mv \longrightarrow PV = mRT$$

$$mR = (MN)R = NR_u \longrightarrow PV = NR_uT$$

$$V = N\bar{v} \longrightarrow P\bar{v} = R_uT$$

Various expressions of ideal gas equation

Real gases behave as an ideal gas at low densities (i.e., low pressure, high temperature).

Per unit mass	Per unit mole
v, m^3/kg	$\bar{v}$, m^3/kmol
u, kJ/kg	$\bar{u}$, kJ/kmol
h, kJ/kg	$\bar{h}$, kJ/kmol

Properties per unit mole are denoted with a bar on the top.

The ideal-gas relation often is not applicable to real gases; thus, care should be exercised when using it.

4

- An ideal gas is an *imaginary* substance that obeys the relation $Pv = RT$.
- At low pressures and high temperatures, the density of a gas decreases, and the gas behaves as an ideal gas under these conditions.
- Many familiar gases such as air, nitrogen, oxygen, hydrogen, helium, argon, neon, krypton, and even heavier gases such as carbon dioxide can be treated as ideal gases.
- Dense gases such as water vapor in steam power plants and refrigerant vapor in refrigerators, however, should not be treated as ideal gases. Instead, the property tables should be used for these substances.

5

EXAMPLE 3–10 Mass of Air in a Room

Determine the mass of the air in a room whose dimensions are 4 m × 5 m × 6 m at 100 kPa and 25°C.

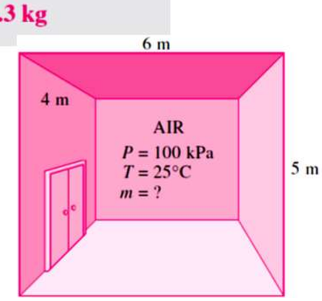
Solution The mass of air in a room is to be determined.

Analysis A sketch of the room is given in Fig. 3–48. Air at specified conditions can be treated as an ideal gas. From Table A–1, the gas constant of air is $R = 0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$, and the absolute temperature is $T = 25°\text{C} + 273 = 298\ \text{K}$. The volume of the room is

$$V = (4\ \text{m})(5\ \text{m})(6\ \text{m}) = 120\ \text{m}^3$$

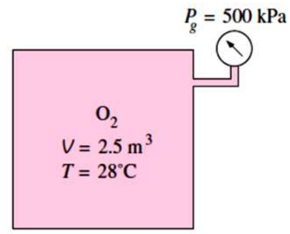
The mass of air in the room is determined from the ideal-gas relation to be

$$m = \frac{PV}{RT} = \frac{(100\ \text{kPa})(120\ \text{m}^3)}{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(298\ \text{K})} = \mathbf{140.3\ kg}$$



Example 2

The pressure gage on a $2.5\ m^3$ oxygen tank reads $500\ kPa$. Determine the amount of oxygen in the tank if the temperature is $28°C$ and the atmospheric pressure is $97\ kPa$.



Assumptions At specified conditions, oxygen behaves as an ideal gas

Properties The gas constant of oxygen is $R = 0.2598$ kPa.m³/kg.K (Table A-1).

Analysis The absolute pressure of O_2 is

$$P = P_g + P_{atm} = 500 + 97 = 597 \text{ kPa}$$

Treating O_2 as an ideal gas, the mass of O_2 in tank is determined to be

$$m = \frac{PV}{RT} = \frac{(597 \text{ kPa})(2.5 \text{ m}^3)}{(0.2598 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(28+273)\text{K}} = \mathbf{19.08\ kg}$$

7

Example 3

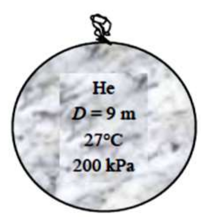
A spherical balloon with a diameter of $9\ m$ is filled with helium at $27\ °C$ and $200\ kPa$. Determine the mole number and the mass of the helium in the balloon.

Assumptions At specified conditions, helium behaves as an ideal gas.

Properties The universal gas constant is $R_u = 8.314$ kPa.m³/kmol.K. The molar mass of helium is 4.0 kg/kmol (Table A-1).

Analysis The volume of the sphere is

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(4.5 \text{ m})^3 = 381.7 \text{ m}^3$$



Assuming ideal gas behavior, the mole numbers of He is determined from

$$N = \frac{PV}{R_u T} = \frac{(200 \text{ kPa})(381.7 \text{ m}^3)}{(8.314 \text{ kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(300 \text{ K})} = \mathbf{30.61\ kmol}$$

Then the mass of He can be determined from

$$m = NM = (30.61 \text{ kmol})(4.0 \text{ kg/kmol}) = \mathbf{123\ kg}$$

8

EXAMPLE 3-10 Temperature Rise of Air in a Tire During a Trip

The gage pressure of an automobile tire is measured to be 210 kPa before a trip and 220 kPa after the trip at a location where the atmospheric pressure is 95 kPa (Fig. 3–44). Assuming the volume of the tire remains constant and the air temperature before the trip is 25°C, determine air temperature in the tire after the trip.

SOLUTION The pressure in an automobile tire is measured before and after a trip. The temperature of air in the tire after the trip is to be determined.

Assumptions **1** The volume of the tire remains constant. **2** Air is an ideal gas.

Properties The local atmospheric pressure is 95 kPa.

Analysis The absolute pressures in the tire before and after the trip are

$$P_1 = P_{\text{gage.1}} + P_{\text{atm}} = 210 + 95 = 305 \text{ kPa}$$
$$P_2 = P_{\text{gage.2}} + P_{\text{atm}} = 220 + 95 = 315 \text{ kPa}$$

Note that air is an ideal gas and the volume is constant, the air temperatures after the trip is determined to be

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \longrightarrow T_2 = \frac{P_2}{P_1}T_1 = \frac{315 \text{ kPa}}{305 \text{ kPa}}(25 + 273 \text{ K}) = 307.8 \text{ K} = \mathbf{34.8^\circ C}$$

Therefore, the absolute temperature of air in the tire will increase by 6.9% during this trip.

Discussion Note that the air temperature has risen nearly 10°C during this trip. This shows the importance of measuring the tire pressures before long trips to avoid errors due to temperature rise of air in tire. Also note that the unit Kelvin is used for temperature in the ideal gas relation.

9

Example 5

The pressure in an automobile tire depends on the temperature of the air in the tire. When the air temperature is $25°C$, the pressure gage reads $210\ kPa$. If the volume of the tire is $0.025\ m^3$. (a) Determine the pressure rise in the tire when the air temperature in the tire rises to $50°C$.

(b) Determine the amount of air that must be bled off to restore pressure to its original value at this temperature.

Assume the atmospheric pressure is 100 kPa.

Properties The gas constant of air is $R = 0.287$ kPa.m³/kg.K (Table A-1).

Analysis Initially, the absolute pressure in the tire is

$$P_1 = P_g + P_{\text{atm}} = 210 + 100 = 310\text{kPa}$$

Treating air as an ideal gas and assuming the volume of the tire to remain constant, the final pressure in the tire can be determined from

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \longrightarrow P_2 = \frac{T_2}{T_1}P_1 = \frac{323 \text{ K}}{298 \text{ K}}(310 \text{ kPa}) = 336 \text{ kPa}$$

Thus the pressure rise is

$$\Delta P = P_2 - P_1 = 336 - 310 = \mathbf{26\ kPa}$$

The amount of air that needs to be bled off to restore pressure to its original value is

$$m_1 = \frac{P_1V}{RT_1} = \frac{(310 \text{ kPa})(0.025 \text{ m}^3)}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(298 \text{ K})} = 0.0906 \text{ kg}$$

$$m_2 = \frac{P_1V}{RT_2} = \frac{(310 \text{ kPa})(0.025 \text{ m}^3)}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(323 \text{ K})} = 0.0836 \text{ kg}$$

$$\Delta m = m_1 - m_2 = 0.0906 - 0.0836 = \mathbf{0.0070\ kg}$$

10

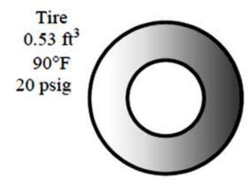
Example 6

The air in an automobile tire with a volume of $0.53\ ft^3$ is at $90°F$ and $20\ psig$. Determine the amount of air that must be added to raise the pressure to the recommended value of $30\ psig$. Assume the atmospheric pressure to be $14.6\ psia$ and the temperature and the volume to remain constant.

Assumptions **1** At specified conditions, air behaves as an ideal gas. **2** The volume of the tire remains constant.

Properties The gas constant of air is $R = 0.3704$ psia.ft³/lbm.R (Table A-1E).

Analysis The initial and final absolute pressures in the tire are



$$P_1 = P_{g1} + P_{atm} = 20 + 14.6 = 34.6 \text{ psia}$$

$$P_2 = P_{g2} + P_{atm} = 30 + 14.6 = 44.6 \text{ psia}$$

Treating air as an ideal gas, the initial mass in the tire is

$$m_1 = \frac{P_1 V}{RT_1} = \frac{(34.6 \text{ psia})(0.53 \text{ ft}^3)}{(0.3704 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R})(550 \text{ R})} = 0.0900 \text{ lbm}$$

Noting that the temperature and the volume of the tire remain constant, the final mass in the tire becomes

$$m_2 = \frac{P_2 V}{RT_2} = \frac{(44.6 \text{ psia})(0.53 \text{ ft}^3)}{(0.3704 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R})(550 \text{ R})} = 0.1160 \text{ lbm}$$

Thus the amount of air that needs to be added is

$$\Delta m = m_2 - m_1 = 0.1160 - 0.0900 = \mathbf{0.0260\ lbm}$$

11

Example 7

A rigid tank contains 20 lbm of air at 20 psia and 70°F. More air is added to the tank until the pressure and temperature rise to 35 psia and 90°F, respectively. Determine the amount of air added to the tank.

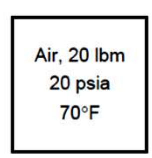
Assumptions **1** At specified conditions, air behaves as an ideal gas. **2** The volume of the tank remains constant.

Properties The gas constant of air is $R = 0.3704$ psia.ft³/lbm.R (Table A-1E).

Analysis Treating air as an ideal gas, the initial volume and the final mass in the tank are determined to be

$$V = \frac{m_1 R T_1}{P_1} = \frac{(20 \text{ lbm})(0.3704 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R})(530 \text{ R})}{20 \text{ psia}} = 196.3 \text{ ft}^3$$

$$m_2 = \frac{P_2 V}{RT_2} = \frac{(35 \text{ psia})(196.3 \text{ ft}^3)}{(0.3704 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R})(550 \text{ R})} = 33.73 \text{ lbm}$$



Thus the amount of air added is

$$\Delta m = m_2 - m_1 = 33.73 - 20.0 = \mathbf{13.73\ lbm}$$

12

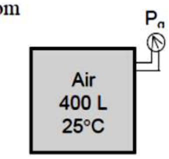
Example 8

A 400-L rigid tank contains 5 kg of air at 25°C. Determine the reading on the pressure gage if the atmospheric pressure is 97 kPa.

Assumptions At specified conditions, air behaves as an ideal gas.

Properties The gas constant of air is $R = 0.287$ kPa.m^3/kg.K (Table A-1).

Analysis Treating air as an ideal gas, the absolute pressure in the tank is determined from



$$P = \frac{mRT}{V} = \frac{(5\text{ kg})(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(298\text{ K})}{0.4\text{ m}^3} = 1069.1\text{ kPa}$$

Thus the gage pressure is

$$P_g = P - P_{\text{atm}} = 1069.1 - 97 = \mathbf{972.1\ \ kPa}$$

Example 9

A $1\ m^3$ tank containing air at 25°C and 500 kPa is connected through a valve to another tank containing 5 kg of air at 35°C and 200 kPa. Now the valve is opened, and the entire system is allowed to reach thermal equilibrium with the surroundings, which are at 20°C. Determine the volume of the second tank and the final equilibrium pressure of air.

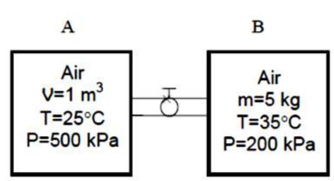
Assumptions At specified conditions, air behaves as an ideal gas.

Properties The gas constant of air is $R = 0.287$ kPa.m^3/kg.K (Table A-1).

Analysis Let's call the first and the second tanks A and B. Treating air as an ideal gas, the volume of the second tank and the mass of air in the first tank are determined to be

$$V_B = \left(\frac{m_1RT_1}{P_1}\right)_B = \frac{(5\text{ kg})(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(308\text{ K})}{200\text{ kPa}} = \mathbf{2.21\ m^3}$$

$$m_A = \left(\frac{P_1V}{RT_1}\right)_A = \frac{(500\text{ kPa})(1.0\text{ m}^3)}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(298\text{ K})} = 5.846\text{ kg}$$



Thus,

$$V = V_A + V_B = 1.0 + 2.21 = 3.21\text{ m}^3$$

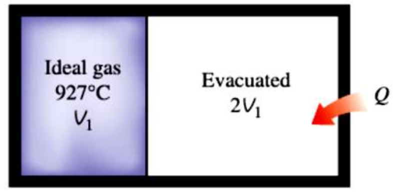
$$m = m_A + m_B = 5.846 + 5.0 = 10.846\text{ kg}$$

Then the final equilibrium pressure becomes

$$P_2 = \frac{mRT_2}{V} = \frac{(10.846\text{ kg})(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\text{ K})}{3.21\text{ m}^3} = \mathbf{284.1\ kPa}$$

Example 10

A rigid tank whose volume is unknown is divided into two parts by a partition. One side of the tank contains an ideal gas at $927°C$. The other side is evacuated and has a volume twice the size of the part containing the gas. The partition is now removed and the gas expands to fill the entire tank. Heat is now applied to the gas until the pressure equals the initial pressure. Determine the final temperature of the gas.



Analysis According to the ideal gas equation of state,

$$P_2 = P_1$$
$$V_2 = V_1 + 2V_1 = 3V_1$$

Applying these,

$$m_1 = m_1$$
$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$
$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$
$$T_2 = T_1 \frac{V_2}{V_1} = T_1 \frac{3V_1}{V_1} = 3T_1 = 3[927 + 273)\ \text{K}] = 3600\ \text{K} = \mathbf{3327°C}$$

15