University of Anbar

Engineering Thermodynamics II

Lecture Note 02: Moving Boundary Work

Objective of Lecture Note

- Examine the moving boundary work or $P\,dV$ work commonly encountered in reciprocating devices such as automotive engines and compressors.

4.1 Moving Boundary Work (PdV)

Quasi-equilibrium process: A process during which the system remains nearly in equilibrium at all times.

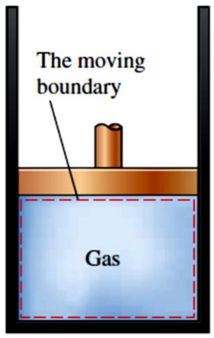
Moving boundary work ($P\,dV$ work): The expansion and compression work in a piston-cylinder device.

$$\delta W_b = F\,ds = PA\,ds = P\,dV$$

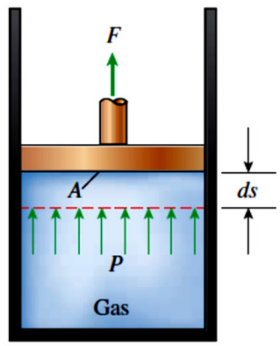
$$W_b = \int_1^2 P\,dV \qquad \text{(kJ)}$$

P is absolute pressure, which is always positive

The work associated with a moving boundary is called ***boundary work.***



W_b is positive → for expansion
W_b is negative → for compression



A gas does a differential amount of work δW_b as it forces the piston to move by a differential amount *ds*.

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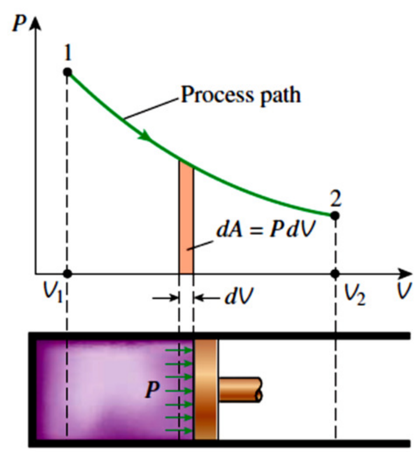
$$\text{Area} = A = \int_1^2 dA = \int_1^2 P\,dV$$

The area under the process curve on a $P-V$ diagram is equal, in magnitude, to the work done during a quasi-equilibrium expansion or compression process of a closed system.



The area under the process curve on a *P-V* diagram represents the boundary work.

(On the $P-v$ diagram, it represents the boundary work done per unit mass)

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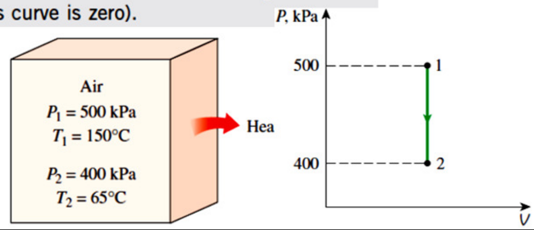
EXAMPLE 4–1 Boundary Work for a Constant-Volume Process

A rigid tank contains air at 500 kPa and 150°C. As a result of heat transfer to the surroundings, the temperature and pressure inside the tank drop to 65°C and 400 kPa, respectively. Determine the boundary work done during this process.

Solution Air in a rigid tank is cooled, and both the pressure and temperature drop. The boundary work done is to be determined.
Analysis A sketch of the system and the *P*-*V* diagram of the process are shown in Fig. 4–6. The boundary work can be determined from Eq. 4–2 to be

$$W_b = \int_1^2 P\, dV^{\nearrow 0} = 0$$

Discussion This is expected since a rigid tank has a constant volume and $dV = 0$ in this equation. Therefore, there is no boundary work done during this process. That is, the boundary work done during a constant-volume process is always zero. This is also evident from the *P*-*V* diagram of the process (the area under the process curve is zero).



EXAMPLE 4–2 Boundary Work for a Constant-Pressure Process

A frictionless piston–cylinder device contains 10 lbm of steam at 60 psia and 320°F. Heat is now transferred to the steam until the temperature reaches 400°F. If the piston is not attached to a shaft and its mass is constant, determine the work done by the steam during this process.

Solution Steam in a piston cylinder device is heated and the temperature rises at constant pressure. The boundary work done is to be determined.
Analysis A sketch of the system and the *P*-*v* diagram of the process are shown in Fig. 4–7.
Assumption The expansion process is quasi-equilibrium.
Analysis Even though it is not explicitly stated, the pressure of the steam within the cylinder remains constant during this process since both the atmospheric pressure and the weight of the piston remain constant. Therefore, this is a constant-pressure process, and, from Eq. 4–2

$$W_b = \int_1^2 P\, dV = P_0 \int_1^2 dV = P_0(V_2 - V_1) \quad \textbf{(4–6)}$$

or

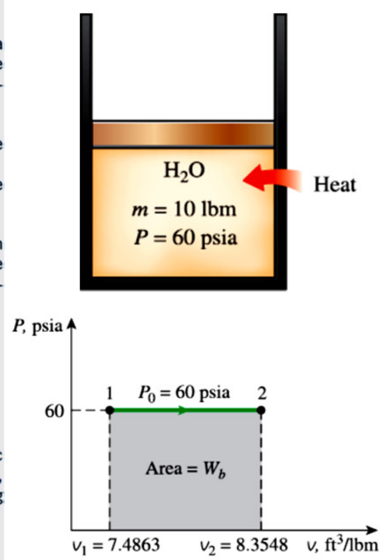
$$W_b = mP_0(v_2 - v_1)$$

since $V = mv$. From the superheated vapor table (Table A–6E), the specific volumes are determined to be $v_1 = 7.4863$ ft³/lbm at state 1 (60 psia, 320°F) and $v_2 = 8.3548$ ft³/lbm at state 2 (60 psia, 400°F). Substituting these values yields

$$W_b = (10\text{ lbm})(60\text{ psia})[(8.3548 - 7.4863)\text{ ft}^3/\text{lbm}]\left(\frac{1\text{ Btu}}{5.404\text{ psia}\cdot\text{ft}^3}\right)$$

$$= \mathbf{96.4\ Btu}$$

Discussion The positive sign indicates that the work is done by the system. That is, the steam used 96.4 Btu of its energy to do this work. The magnitude of this work could also be determined by calculating the area under the process curve on the *P*-*V* diagram, which is simply $P_0\,\Delta V$ for this case.



EXAMPLE 4–3 Isothermal Compression of an Ideal Gas

A piston–cylinder device initially contains 0.4 m^3 of air at 100 kPa and 80°C. The air is now compressed to 0.1 m^3 in such a way that the temperature inside the cylinder remains constant. Determine the work done during this process.

Solution Air in a piston–cylinder device is compressed isothermally. The boundary work done is to be determined.
Analysis A sketch of the system and the *P*-*V* diagram of the process are shown in Fig. 4–8.
Assumptions **1** The compression process is quasi-equilibrium. **2** At specified conditions, air can be considered to be an ideal gas since it is at a high temperature and low pressure relative to its critical-point values.
Analysis For an ideal gas at constant temperature T_0,

$$PV = mRT_0 = C \quad \text{or} \quad P = \frac{C}{V}$$

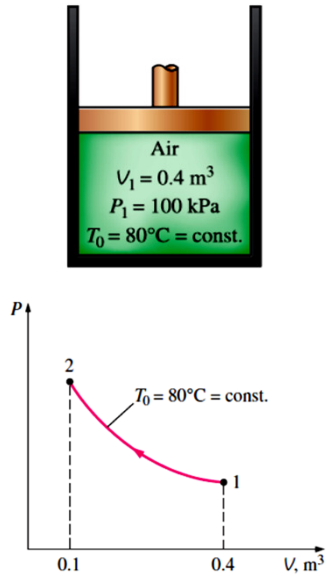
where *C* is a constant. Substituting this into Eq. 4–2, we have

$$W_b = \int_1^2 P\,dV = \int_1^2 \frac{C}{V}dV = C\int_1^2 \frac{dV}{V} = C\ \ln\frac{V_2}{V_1} = P_1V_1\ \ln\frac{V_2}{V_1} \qquad \text{(4–7)}$$

In Eq. 4–7, P_1V_1 can be replaced by P_2V_2 or mRT_0. Also, V_2/V_1 can be replaced by P_1/P_2 for this case since $P_1V_1 = P_2V_2$.
Substituting the numerical values into Eq. 4–7 yields

$$W_b = (100\ \text{kPa})(0.4\ \text{m}^3)\left(\ln\frac{0.1}{0.4}\right)\left(\frac{1\ \text{kJ}}{1\ \text{kPa}\cdot\text{m}^3}\right)$$
$$= \mathbf{-55.5\ kJ}$$

Discussion The negative sign indicates that this work is done on the system (a work input), which is always the case for compression processes.



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Polytropic Processes (General Process)

$P = CV^{-n}$ **Polytropic process: *C*, *n* (polytropic exponent) constants**

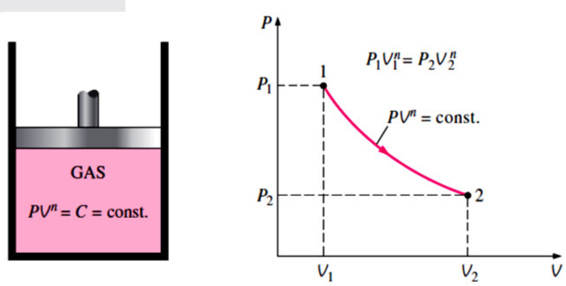
$$W_b = \int_1^2 P\,dV = \int_1^2 CV^{-n}\,dV = C\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} = \frac{P_2V_2 - P_1V_1}{1-n}$$ **Polytropic process $n \neq 1$**

$$W_b = \frac{mR(T_2 - T_1)}{1-n} \qquad n \neq 1 \qquad \text{(kJ)}$$ **Polytropic for ideal gas**

$$W_b = \int_1^2 P\,dV = \int_1^2 CV^{-1}\,dV = PV\ln\left(\frac{V_2}{V_1}\right)$$ **When $n = 1$ (isothermal process)**

$$W_b = \int_1^2 P\,dV = P_0\int_1^2 dV = P_0(V_2 - V_1)$$ **When $n = 0$ Constant pressure process (isobaric process)**

What is the boundary work for a constant-volume process?



Schematic and *P*-*V* diagram for a polytropic process.

(1) General $$W_b = \int_1^2 P\, dV$$

(2) Isobaric process

$$W_b = P_0(V_2 - V_1) \qquad (P_1 = P_2 = P_0 = \text{constant})$$

(3) Polytropic process

$$W_b = \frac{P_2V_2 - P_1V_1}{1-n} \quad (n \neq 1) \qquad (PV^n = \text{constant})$$

(4) Isothernal process of an ideal gas

$$W_b = P_1V_1 \ln\frac{V_2}{V_1}$$

$$= mRT_0 \ln\frac{V_2}{V_1} \qquad (PV = mRT_0 = \text{constant})$$

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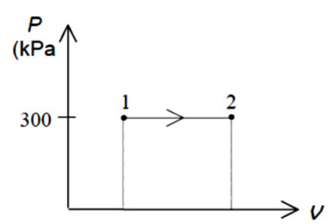
Example 4

A mass of 5 kg of saturated water vapor at 300 kPa is heated at constant pressure (Isobaric Process) until the temperature reaches 200°C. Calculate the work done by the steam during this process and show the process on the $P - v$ diagram.

Properties Noting that the pressure remains constant during this process, the specific volumes at the initial and the final states are (Table A-4 through A-6)

$$\left.\begin{array}{l} P_1 = 300 \text{ kPa} \\ \text{Sat. vapor} \end{array}\right\} v_1 = v_{g@300\text{ kPa}} = 0.60582 \text{ m}^3/\text{kg}$$

$$\left.\begin{array}{l} P_2 = 300 \text{ kPa} \\ T_2 = 200°\text{C} \end{array}\right\} v_2 = 0.71643 \text{ m}^3/\text{kg}$$



Analysis The boundary work is determined from its definition to be

$$W_{b,\text{out}} = \int_1^2 P\,dV = P(V_2 - V_1) = mP(v_2 - v_1)$$

$$= (5\text{ kg})(300\text{ kPa})(0.71643 - 0.60582)\text{ m}^3/\text{kg}\left(\frac{1\text{ kJ}}{1\text{ kPa}\cdot\text{m}^3}\right)$$

$$= \mathbf{165.9\ kJ}$$

Discussion The positive sign indicates that work is done by the system (work output).

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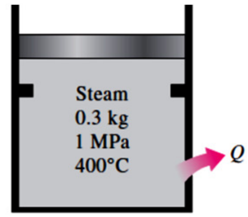
Example 7

A piston–cylinder device with a set of stops initially contains 0.3 kg of steam at 1.0 MPa and 400°C. The location of the stops corresponds to 60 percent of the initial volume. Now the steam is cooled. Determine the compression work if the final state is
(*a*) 1.0 MPa and 250°C (Isobaric Process) and
(*b*) Also determine the temperature at the final state if 500 Kpa..



Analysis (a) The specific volumes for the initial and final states are (Table A-6)

$$\left.\begin{array}{l} P_1 = 1\,\text{MPa} \\ T_1 = 400°\text{C} \end{array}\right\} v_1 = 0.30661\ \text{m}^3/\text{kg} \qquad \left.\begin{array}{l} P_2 = 1\,\text{MPa} \\ T_2 = 250°\text{C} \end{array}\right\} v_2 = 0.23275\ \text{m}^3/\text{kg}$$

Noting that pressure is constant during the process, the boundary work is determined from

$$W_b = mP(v_1 - v_2) = (0.3\ \text{kg})(1000\ \text{kPa})(0.30661 - 0.23275)\text{m}^3/\text{kg} = \mathbf{22.16\ kJ}$$

The temperature at the final state is

$$\left.\begin{array}{l} P_2 = 0.5\,\text{MPa} \\ v_2 = (0.60 \times 0.30661)\ \text{m}^3/\text{kg} \end{array}\right\} T_2 = \mathbf{151.8°C} \quad \text{(Table A-5)}$$

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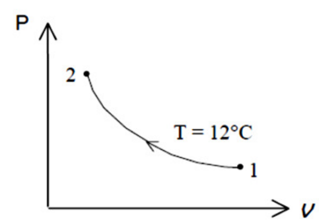
Example 10

A mass of 2.4 kg of air at 150 kPa and 12°C is contained in a gas-tight, frictionless piston–cylinder device. The air is now compressed to a final pressure of 600 kPa. During the process, heat is transferred from the air such that the temperature inside the cylinder remains constant (Isothermal Process). Calculate the work input during this process.

Assumptions **1** The process is quasi-equilibrium. **2** Air is an ideal gas.

Properties The gas constant of air is $R = 0.287$ kJ/kg.K (Table A-1).

Analysis The boundary work is determined from its definition to be



$$W_{b,out} = \int_1^2 P\,dV = P_1 V_1 \ln\frac{V_2}{V_1} = mRT\ln\frac{P_1}{P_2}$$

$$= (2.4\ \text{kg})(0.287\ \text{kJ/kg}\cdot\text{K})(285\ \text{K})\ln\frac{150\ \text{kPa}}{600\ \text{kPa}}$$

$$= \mathbf{-272\ \ kJ}$$

Discussion The negative sign indicates that work is done on the system (work input).

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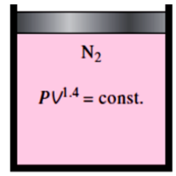
Example 15

A frictionless piston–cylinder device contains 2 kg of nitrogen at 100 kPa and 300 K. Nitrogen is now compressed slowly according to the relation $PV^{1.4} = constant$ constant until it reaches a final temperature of 360 K. Calculate the work input during this process.



Assumptions **1** The process is quasi-equilibrium. **2** Nitrogen is an ideal gas.

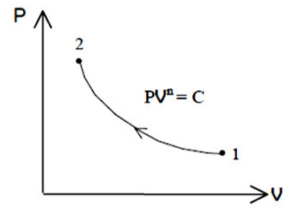
Properties The gas constant for nitrogen is R = 0.2968 kJ/kg.K (Table A-2a)

Analysis The boundary work for this polytropic process can be determined from

$$W_{b,\text{out}} = \int_1^2 P\,dV = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{mR(T_2 - T_1)}{1-n}$$

$$= \frac{(2\text{ kg})(0.2968\text{ kJ/kg}\cdot\text{K})(360-300)\text{K}}{1-1.4}$$

$$= \mathbf{-89.0\ kJ}$$



Discussion The negative sign indicates that work is done on the system (work input).

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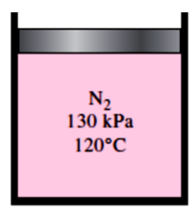
H.W

A piston–cylinder device initially contains 0.25 kg of nitrogen gas at 130 kPa and 120°C. The nitrogen is now expanded isothermally (Isothermal Process) to a pressure of 100 kPa. Determine the boundary work done during this process.



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THE
END
thank you all!
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