

**University of Anbar**

**Engineering Thermodynamics II**

**Lecture Note #03: Energy Balance for Closed Systems**



**Objective of Lecture Note**

- **Develop the general energy balance applied to closed systems.**



## 4.2 Energy Balance For Closed Systems

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{system}}_{\text{Change in internal, kinetic, potential, etc., energies}} \quad (\text{kJ}) \quad \text{Energy balance for any system undergoing any process}$$

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{dE_{system}/dt}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} \quad (\text{kW}) \quad \text{Energy balance in the rate form}$$

The total quantities are related to the quantities per unit time is

$$Q = \dot{Q} \Delta t, \quad W = \dot{W} \Delta t, \quad \text{and} \quad \Delta E = (dE/dt) \Delta t \quad (\text{kJ})$$

$$e_{in} - e_{out} = \Delta e_{system} \quad (\text{kJ/kg}) \quad \text{Energy balance per unit mass basis}$$

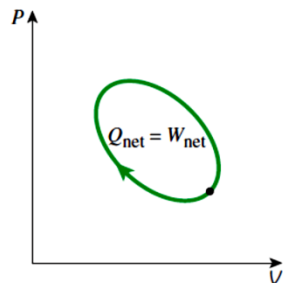
$$\delta E_{in} - \delta E_{out} = dE_{system} \quad \text{or} \quad \delta e_{in} - \delta e_{out} = de_{system} \quad \text{Energy balance in differential form}$$

$$W_{net,out} = Q_{net,in} \quad \text{or} \quad \dot{W}_{net,out} = \dot{Q}_{net,in} \quad \text{Energy balance for a cycle}$$



$$Q_{net,in} - W_{net,out} = \Delta E_{system} \quad \text{or} \quad Q - W = \Delta E \quad \begin{array}{l} Q = Q_{net,in} = Q_{in} - Q_{out} \\ W = W_{net,out} = W_{out} - W_{in} \end{array}$$

Energy balance when sign convention is used (i.e., heat input and work output are positive; heat output and work input are negative).



For a cycle  $\Delta E = 0$ , thus  $Q = W$ .

General  $Q - W = \Delta E$

Stationary systems  $Q - W = \Delta U$

Per unit mass  $q - w = \Delta e$

Differential form  $\delta q - \delta w = de$

Various forms of the first-law relation for **closed systems** when sign convention is used.

The first law cannot be proven mathematically, but no process in nature is known to have violated the first law, and this should be taken as sufficient proof.

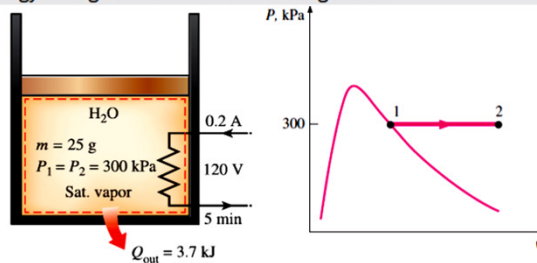


**EXAMPLE 4-5 Electric Heating of a Gas at Constant Pressure**

A piston-cylinder device contains 25 g of saturated water vapor that is maintained at a constant pressure of 300 kPa. A resistance heater within the cylinder is turned on and passes a current of 0.2 A for 5 min from a 120-V source. At the same time, a heat loss of 3.7 kJ occurs. (a) Show that for a closed system the boundary work  $W_b$  and the change in internal energy  $\Delta U$  in the first-law relation can be combined into one term,  $\Delta H$ , for a constant-pressure process. (b) Determine the final temperature of the steam.

**Solution** Saturated water vapor in a piston-cylinder device expands at constant pressure as a result of heating. It is to be shown that  $\Delta U + W_b = \Delta H$ , and the final temperature is to be determined.

**Assumptions** 1 The tank is stationary and thus the kinetic and potential energy changes are zero,  $\Delta KE = \Delta PE = 0$ . Therefore,  $\Delta E = \Delta U$  and internal energy is the only form of energy of the system that may change during this process. 2 Electrical wires constitute a very small part of the system, and thus the energy change of the wires can be neglected.



**Analysis** We take the contents of the cylinder, including the resistance wires, as the system (Fig. 4-13). This is a closed system since no mass crosses the system boundary during the process. We observe that a piston-cylinder device typically involves a moving boundary and thus boundary work  $W_b$ . The pressure remains constant during the process and thus  $P_2 = P_1$ . Also, heat is lost from the system and electrical work  $W_e$  is done on the system.

(a) This part of the solution involves a general analysis for a closed system undergoing a quasi-equilibrium constant-pressure process, and thus we consider a general closed system. We take the direction of heat transfer  $Q$  to be to the system and the work  $W$  to be done by the system. We also express the work as the sum of boundary and other forms of work (such as electrical and shaft). Then the energy balance can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{system}}_{\text{Change in internal, kinetic, potential, etc., energies}}$$

$$Q - W = \Delta U + \Delta KE + \Delta PE$$

$$Q - W_{other} - W_b = U_2 - U_1$$

For a constant-pressure process, the boundary work is given as  $W_b = P_0(V_2 - V_1)$ . Substituting this into the preceding relation gives

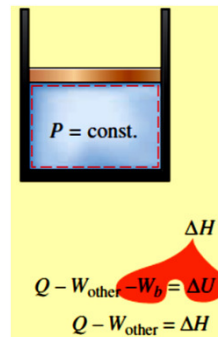
$$Q - W_{other} - P_0(V_2 - V_1) = U_2 - U_1$$

However,

$$P_0 = P_2 = P_1 \rightarrow Q - W_{other} = (U_2 + P_2V_2) - (U_1 + P_1V_1)$$

Also  $H = U + PV$ , and thus

$$Q - W_{other} = H_2 - H_1 \quad (\text{kJ}) \quad (4-18)$$



which is the desired relation (Fig. 4-14). This equation is very convenient to use in the analysis of closed systems undergoing a constant-pressure quasi-equilibrium process since the boundary work is automatically taken care of by the enthalpy terms, and one no longer needs to determine it separately.

(b) The only other form of work in this case is the electrical work, which can be determined from

$$W_e = VI \Delta t = (120 \text{ V})(0.2 \text{ A})(300 \text{ s}) \left( \frac{1 \text{ kJ/s}}{1000 \text{ VA}} \right) = 7.2 \text{ kJ}$$

$$\text{State 1: } \left. \begin{array}{l} P_1 = 300 \text{ kPa} \\ \text{sat. vapor} \end{array} \right\} h_1 = h_g @ 300 \text{ kPa} = 2724.9 \text{ kJ/kg} \quad (\text{Table A-5})$$

The enthalpy at the final state can be determined directly from Eq. 4-18 by expressing heat transfer from the system and work done on the system as negative quantities (since their directions are opposite to the assumed directions). Alternately, we can use the general energy balance relation with the simplification that the boundary work is considered automatically by replacing  $\Delta U$  by  $\Delta H$  for a constant-pressure expansion or compression process:

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

$$W_{e,\text{in}} - Q_{\text{out}} - W_b = \Delta U$$

$$W_{e,\text{in}} - Q_{\text{out}} = \Delta H = m(h_2 - h_1) \quad (\text{since } P = \text{constant})$$

$$7.2 \text{ kJ} - 3.7 \text{ kJ} = (0.025 \text{ kg})(h_2 - 2724.9) \text{ kJ/kg}$$

$$h_2 = 2864.9 \text{ kJ/kg}$$

Now the final state is completely specified since we know both the pressure and the enthalpy. The temperature at this state is

$$\text{State 2: } \left. \begin{array}{l} P_2 = 300 \text{ kPa} \\ h_2 = 2864.9 \text{ kJ/kg} \end{array} \right\} T_2 = 200^\circ\text{C} \quad (\text{Table A-6})$$

Therefore, the steam will be at 200°C at the end of this process.



## Energy balance for a constant-pressure expansion or compression process

General analysis for a closed system undergoing a quasi-equilibrium constant-pressure process.  $Q$  is **to** the system and  $W$  is **from** the system.

For a constant-pressure expansion or compression process:

$$\Delta U + W_b = \Delta H$$

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

$$Q - W = \Delta U + \cancel{\Delta KE}^0 + \cancel{\Delta PE}^0$$

$$Q - W_{\text{other}} - W_b = U_2 - U_1$$

$$Q - W_{\text{other}} - P_0(V_2 - V_1) = U_2 - U_1$$

$$Q - W_{\text{other}} = (U_2 + P_2V_2) - (U_1 + P_1V_1)$$

$$H = U + PV$$

$$Q - W_{\text{other}} = H_2 - H_1$$



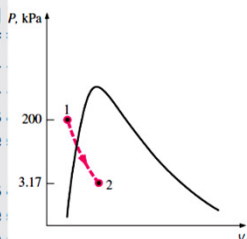
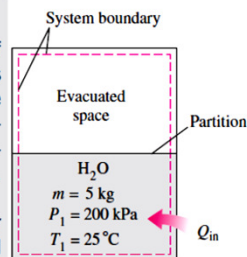
**EXAMPLE 4–6 Unrestrained Expansion of Water**

A rigid tank is divided into two equal parts by a partition. Initially, one side of the tank contains 5 kg of water at 200 kPa and 25°C, and the other side is evacuated. The partition is then removed, and the water expands into the entire tank. The water is allowed to exchange heat with its surroundings until the temperature in the tank returns to the initial value of 25°C. Determine (a) the volume of the tank, (b) the final pressure, and (c) the heat transfer for this process.

**Solution** One half of a rigid tank is filled with liquid water while the other side is evacuated. The partition between the two parts is removed and water is allowed to expand and fill the entire tank while the temperature is maintained constant. The volume of tank, the final pressure, and the heat transfer are to be determined.

**Assumptions** 1 The system is stationary and thus the kinetic and potential energy changes are zero,  $\Delta KE = \Delta PE = 0$  and  $\Delta E = \Delta U$ . 2 The direction of heat transfer is to the system (heat gain,  $Q_{in}$ ). A negative result for  $Q_{in}$  indicates the assumed direction is wrong and thus it is a heat loss. 3 The volume of the rigid tank is constant, and thus there is no energy transfer as boundary work. 4 The water temperature remains constant during the process. 5 There is no electrical, shaft, or any other kind of work involved.

**Analysis** We take the contents of the tank, including the evacuated space, as the system (Fig. 4–15). This is a closed system since no mass crosses the system boundary during the process. We observe that the water fills the entire tank when the partition is removed (possibly as a liquid–vapor mixture).



(a) Initially the water in the tank exists as a compressed liquid since its pressure (200 kPa) is greater than the saturation pressure at 25°C (3.1698 kPa). Approximating the compressed liquid as a saturated liquid at the given temperature, we find

$$v_1 \cong v_f @ 25^\circ\text{C} = 0.001003 \text{ m}^3/\text{kg} \cong 0.001 \text{ m}^3/\text{kg} \quad (\text{Table A-4})$$

Then the initial volume of the water is

$$V_1 = mv_1 = (5 \text{ kg})(0.001 \text{ m}^3/\text{kg}) = 0.005 \text{ m}^3$$

The total volume of the tank is twice this amount:

$$V_{\text{tank}} = (2)(0.005 \text{ m}^3) = \mathbf{0.01 \text{ m}^3}$$

(b) At the final state, the specific volume of the water is

$$v_2 = \frac{V_2}{m} = \frac{0.01 \text{ m}^3}{5 \text{ kg}} = 0.002 \text{ m}^3/\text{kg}$$

which is twice the initial value of the specific volume. This result is expected since the volume doubles while the amount of mass remains constant.

At 25°C:  $v_f = 0.001003 \text{ m}^3/\text{kg}$  and  $v_g = 43.340 \text{ m}^3/\text{kg}$  (Table A-4)

Since  $v_f < v_2 < v_g$ , the water is a saturated liquid–vapor mixture at the final state, and thus the pressure is the saturation pressure at 25°C:

$$P_2 = P_{\text{sat}} @ 25^\circ\text{C} = \mathbf{3.1698 \text{ kPa}} \quad (\text{Table A-4})$$



(c) Under stated assumptions and observations, the energy balance on the system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

$$Q_{in} = \Delta U = m(u_2 - u_1)$$

Notice that even though the water is expanding during this process, the system chosen involves fixed boundaries only (the dashed lines) and therefore the moving boundary work is zero (Fig. 4-16). Then  $W = 0$  since the system does not involve any other forms of work. (Can you reach the same conclusion by choosing the water as our system?) Initially,

$$u_1 \equiv u_f @ 25^\circ\text{C} = 104.83 \text{ kJ/kg}$$

The quality at the final state is determined from the specific volume information:

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.002 - 0.001}{43.34 - 0.001} = 2.3 \times 10^{-5}$$

Then

$$\begin{aligned} u_2 &= u_f + x_2 u_{fg} \\ &= 104.83 \text{ kJ/kg} + (2.3 \times 10^{-5})(2304.3 \text{ kJ/kg}) \\ &= 104.88 \text{ kJ/kg} \end{aligned}$$

Substituting yields

$$Q_{in} = (5 \text{ kg})[(104.88 - 104.83) \text{ kJ/kg}] = \mathbf{0.25 \text{ kJ}}$$

**Discussion** The positive sign indicates that the assumed direction is correct, and heat is transferred to the water.



### Example 2

A  $0.5 - \text{m}^3$  rigid tank contains refrigerant-134a initially at 160 kPa and 40 percent quality. Heat is now transferred to the refrigerant until the pressure reaches 700 kPa. Determine (a) the mass of the refrigerant in the tank and (b) the amount of heat transferred. Also, show the process on a  $P$ - $v$  diagram with respect to saturation lines.

**Assumptions 1** The tank is stationary and thus the kinetic and potential energy changes are zero. **2** There are no work interactions.

**Analysis** (a) We take the tank as the system. This is a closed system since no mass enters or leaves. Noting that the volume of the system is constant and thus there is no boundary work, the energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

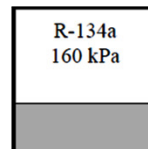
$$Q_{in} = \Delta U = m(u_2 - u_1) \quad (\text{since } W = KE = PE = 0)$$

Using data from the refrigerant tables (Tables A-11 through A-13), the properties of R-134a are determined to be

$$\left. \begin{array}{l} P_1 = 160 \text{ kPa} \\ x_1 = 0.4 \end{array} \right\} \begin{array}{l} v_f = 0.0007437, \quad v_g = 0.12348 \text{ m}^3/\text{kg} \\ u_f = 31.09, \quad u_{fg} = 190.27 \text{ kJ/kg} \end{array}$$

$$v_1 = v_f + x_1 v_{fg} = 0.0007437 + 0.4(0.12348 - 0.0007437) = 0.04984 \text{ m}^3/\text{kg}$$

$$u_1 = u_f + x_1 u_{fg} = 31.09 + 0.4(190.27) = 107.19 \text{ kJ/kg}$$



**Example 2**

Cont..

$$P_2 = 700 \text{ kPa} \left. \vphantom{P_2} \right\} u_2 = 376.99 \text{ kJ/kg (Superheated vapor)}$$

$$(v_2 = v_1)$$

Then the mass of the refrigerant is determined to be

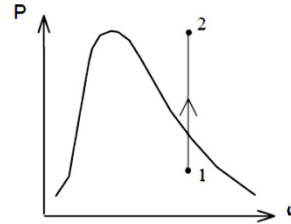
$$m = \frac{V_1}{v_1} = \frac{0.5 \text{ m}^3}{0.04984 \text{ m}^3/\text{kg}} = \mathbf{10.03 \text{ kg}}$$

(b) Then the heat transfer to the tank becomes

$$Q_{in} = m(u_2 - u_1)$$

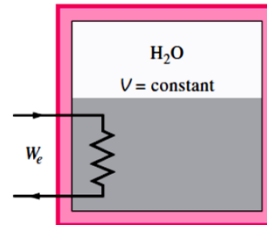
$$= (10.03 \text{ kg})(376.99 - 107.19) \text{ kJ/kg}$$

$$= \mathbf{2707 \text{ kJ}}$$



**Example 3**

A well-insulated rigid tank contains 5 kg of a saturated liquid-vapor mixture of water at 100 kPa. Initially, three-quarters of the mass is in the liquid phase. An electric resistor placed in the tank is connected to a 110-V source, and a current of 8 A flows through the resistor when the switch is turned on. Determine how long it will take to vaporize all the liquid in the tank. Also, show the process on a T-v diagram with respect to saturation lines.



**Assumptions** 1 The tank is stationary and thus the kinetic and potential energy changes are zero. 2 The device is well-insulated and thus heat transfer is negligible. 3 The energy stored in the resistance wires, and the heat transferred to the tank itself is negligible.

**Analysis** We take the contents of the tank as the system. This is a closed system since no mass enters or leaves. Noting that the volume of the system is constant and thus there is no boundary work, the energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{system}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

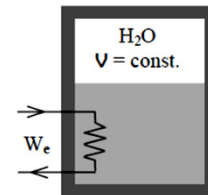
$$W_{e,in} = \Delta U = m(u_2 - u_1) \quad (\text{since } Q = KE = PE = 0)$$

$$VI\Delta t = m(u_2 - u_1)$$

The properties of water are (Tables A-4 through A-6)

$$P_1 = 100 \text{ kPa} \left. \vphantom{P_1} \right\} v_f = 0.001043, \quad v_g = 1.6941 \text{ m}^3/\text{kg}$$

$$x_1 = 0.25 \left. \vphantom{x_1} \right\} u_f = 417.40, \quad u_{fg} = 2088.2 \text{ kJ/kg}$$

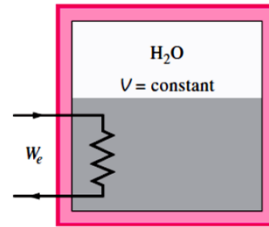


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**Example 3****Cont..**

A well-insulated rigid tank contains 5 kg of a saturated liquid-vapor mixture of water at 100 kPa. Initially, three-quarters of the mass is in the liquid phase. An electric resistor placed in the tank is connected to a 110-V source, and a current of 8 A flows through the resistor when the switch is turned on. Determine how long it will take to vaporize all the liquid in the tank. Also, show the process on a T-v diagram with respect to saturation lines.



$$v_1 = v_f + x_1 v_{fg} = 0.001043 + [0.25 \times (1.6941 - 0.001043)] = 0.42431 \text{ m}^3/\text{kg}$$

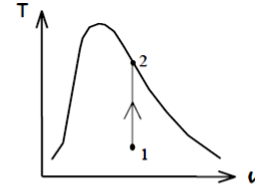
$$u_1 = u_f + x_1 u_{fg} = 417.40 + (0.25 \times 2088.2) = 939.4 \text{ kJ/kg}$$

$$v_2 = v_1 = 0.42431 \text{ m}^3/\text{kg} \left. \vphantom{v_2} \right\} \text{sat. vapor} \quad u_2 = u_g @ 0.42431 \text{ m}^3/\text{kg} = 2556.2 \text{ kJ/kg}$$

Substituting,

$$(110 \text{ V})(8 \text{ A})\Delta t = (5 \text{ kg})(2556.2 - 939.4) \text{ kJ/kg} \left( \frac{1000 \text{ VA}}{1 \text{ kJ/s}} \right)$$

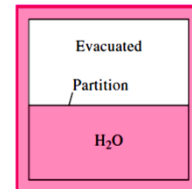
$$\Delta t = 9186 \text{ s} \cong \mathbf{153.1 \text{ min}}$$



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**H.W**

An insulated tank is divided into two parts by a partition. One part of the tank contains 2.5 kg of compressed liquid water at 60°C and 600 kPa while the other part is evacuated. The partition is now removed, and the water expands to fill the entire tank. Determine the final temperature of the water and the volume of the tank for a final pressure of 10 kPa.



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