University of Anbar

Engineering Thermodynamics II

Lecture Note #04: Mass and Energy Analysis of Control Volumes

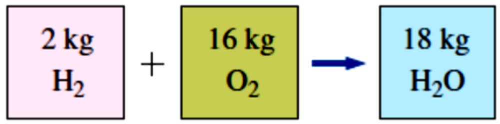
Objective of Lecture Note

- **Develop the conservation of mass principle.**

2

Conservation of Mass

Conservation of mass: Mass, like energy, is a conserved property, and it cannot be created or destroyed during a process.



Mass is conserved even during chemical reactions.

***Closed Systems*: The mass of the system remain constant during a process.**

***Control Volumes*: Mass can cross the boundaries, and so we must keep track of the amount of mass entering and leaving the control volume.**

Mass and Volume Flow Rates

Mass Flow Rate ($\dot{m}$): The amount of mass flowing through a cross section per unit time. The dot over a symbol is used to indicate time rate of change.

$$\dot{m} = \rho V_{avg} A_c \qquad \text{(kg/s)}$$

Average velocity:

$$V_{avg} = \frac{1}{A_c}\int_{A_c} V_n \, dA_c$$

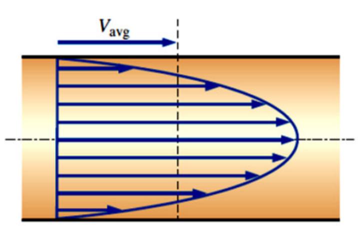
Volume Flow Rate ($\dot{V}$): The volume of the fluid flowing through a cross section per unit time .

Volume flow rate

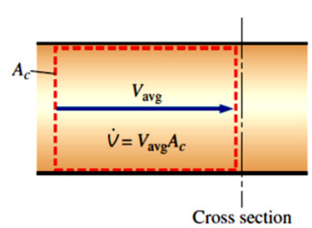
$$\dot{V} = \int_{A_c} V_n \, dA_c = V_{avg} A_c = VA_c \qquad (\text{m}^3/\text{s})$$

Mass flow rate

$$\dot{m} = \rho \dot{V} = \frac{\dot{V}}{v}$$



The average velocity V_{avg} is defined as the average speed through a cross section.



The volume flow rate is the volume of fluid flowing through a cross section per unit time.

Conservation of Mass Principle

The conservation of mass principle for a control volume: The net mass transfer to or from a control volume during a time interval Δt is equal to the net change (increase or decrease) in the total mass within the control volume during Δt.

$$\begin{pmatrix}\text{Total mass entering}\\ \text{the CV during } \Delta t\end{pmatrix} - \begin{pmatrix}\text{Total mass leaving}\\ \text{the CV during } \Delta t\end{pmatrix} = \begin{pmatrix}\text{Net change of mass}\\ \text{within the CV during } \Delta t\end{pmatrix}$$

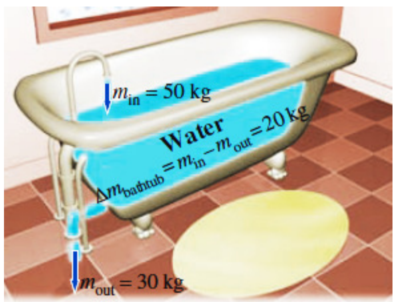
$$m_{in} - m_{out} = \Delta m_{CV} \qquad \text{(kg)}$$

$$\Delta m_{CV} = m_{final} - m_{initial}$$

$$\dot{m}_{in} - \dot{m}_{out} = dm_{CV}/dt \qquad \text{(kg/s)}$$

$\dot{m}_{in}$ and $\dot{m}_{out}$ **The total rates of mass flow into and out of the control volume**

dm_{CV}/dt **The rate of change of mass within the control volume boundaries.**



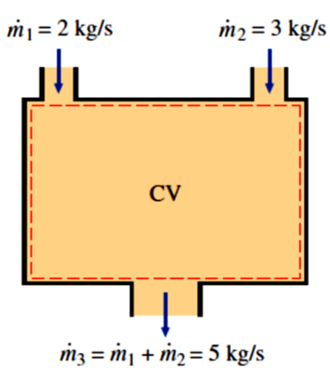
Conservation of mass principle for an ordinary bathtub.

The above equations are often referred to as the mass balance and are applicable to any control volume undergoing any kind of process.

Mass Balance for Steady-Flow Processes

During a steady-flow process, the total amount of mass contained within a control volume does not change with time ($m_{CV} = constant$).

Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it.



Conservation of mass principle for a two-inlet–one-outlet steady-flow system.

For steady-flow processes, we are interested in the amount of mass flowing per unit time, that is, *the mass flow rate.*

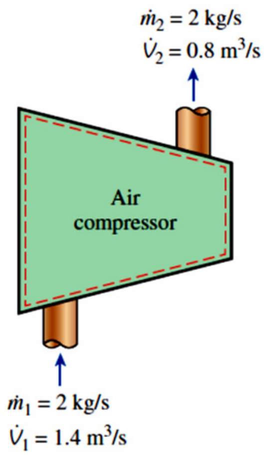
$$\dot{m}_1 = \dot{m}_2 \quad \rightarrow \quad \rho_1 V_1 A_1 = \rho_2 V_2 A_2 \qquad \text{Single stream}$$

$$\sum_{in} \dot{m} = \sum_{out} \dot{m} \qquad \text{(kg/s)} \qquad \text{Multiple inlets and exits}$$

Many engineering devices such as nozzles, diffusers, turbines, compressors, and pumps involve a single stream (only one inlet and one outlet).

Special Case: Steady and Incompressible Flow

The conservation of mass relations can be simplified even further when the fluid is incompressible ($\rho = constant$), which is usually the case for liquids.



During a steady-flow process, volume flow rates are not necessarily conserved although mass flow rates are.

Steady, incompressible flow: $$\sum_{in} \dot{V} = \sum_{out} \dot{V} \quad (\text{m}^3/\text{s})$$

$$\dot{V}_1 = \dot{V}_2 \rightarrow V_1 A_1 = V_2 A_2$$ **Steady, incompressible flow (single stream)**

- **There is no such thing as a "conservation of volume" principle.**
- **However, for steady flow of liquids, the volume flow rates, as well as the mass flow rates, remain constant since liquids are essentially incompressible (constant-density) substances.**

EXAMPLE 5–1 Water Flow through a Garden Hose Nozzle

A garden hose attached with a nozzle is used to fill a 10-gal bucket. The inner diameter of the hose is 2 cm, and it reduces to 0.8 cm at the nozzle exit (Fig. 5–10). If it takes 50 s to fill the bucket with water, determine (*a*) the volume and mass flow rates of water through the hose, and (*b*) the average velocity of water at the nozzle exit.

SOLUTION A garden hose is used to fill a water bucket. The volume and mass flow rates of water and the exit velocity are to be determined.
Assumptions **1** Water is a nearly incompressible substance. **2** Flow through the hose is steady. **3** There is no waste of water by splashing.
Properties We take the density of water to be 1000 kg/m³ = 1 kg/L.
Analysis (*a*) Noting that 10 gal of water are discharged in 50 s, the volume and mass flow rates of water are

$$\dot{V} = \frac{V}{\Delta t} = \frac{10 \text{ gal}}{50 \text{ s}}\left(\frac{3.7854 \text{ L}}{1 \text{ gal}}\right) = \mathbf{0.757 \text{ L/s}}$$

$$\dot{m} = \rho\dot{V} = (1 \text{ kg/L})(0.757 \text{ L/s}) = \mathbf{0.757 \text{ kg/s}}$$

(*b*) The cross-sectional area of the nozzle exit is

$$A_e = \pi r_e^2 = \pi(0.4\ \text{cm})^2 = 0.5027\ \text{cm}^2 = 0.5027 \times 10^{-4}\ \text{m}^2$$

The volume flow rate through the hose and the nozzle is constant. Then the average velocity of water at the nozzle exit becomes

$$V_e = \frac{\dot{V}}{A_e} = \frac{0.757\ \text{L/s}}{0.5027 \times 10^{-4}\ \text{m}^2}\left(\frac{1\ \text{m}^3}{1000\ \text{L}}\right) = \mathbf{15.1\ m/s}$$

Discussion It can be shown that the average velocity in the hose is 2.4 m/s. Therefore, the nozzle increases the water velocity by over six times.

9

10