University of Anbar

Collage of Science

Department of Applied Mathematics

Third Year – First Semester

Lectures in Mathematical analysis

By: Dr. Rifaat Saad Abdul-Jabbar

Lecture No. 4

Functions

Definition: (Function)

Consider two sets A and B, assigning to each element in A to some element in B which denoted by $f(x)$, for $x \in A$. Then f is said to be a *function* from A to B (or mapping). The set A is called *domain* of f , $f(x)$ is called value of x, the set of all values is called *range* of *f*.

Definition:

If f is a function from the set A to the set B, let $E \subset A, f(E)$ is the set of all values of $x \in E$, we call it the *image* of E under f.

Definition:

If there exists a $1-1$ mapping of A *onto* B, we say that A and B can be put in 1-1 correspondence or A and B *have the same cardinal number* or $A \sim B$ (A equivalent to B)

Definition:

For any positive integer n, let $J_n = \{1,2,3, \dots , n\}$, then for any set A, we say that:

(a)- A is *finite* if $A \sim J_n$ for some n. (ϕ is finite)

(b)- A is *countable* if $A \sim J = \mathbb{N} \backslash \{0\}$.

(c)- A is *infinite* if A is not finite.

(d)- A is *uncountable* if A is neither finite nor countable.

(e)- A is *at most countable* if A is finite or countable.

Example

The set of all integers $\mathbb{Z}$ is countable.

Consider the following arrangement:

$$\mathbb{Z} = \{0,1,-1,2,-2, \dots\}$$

$$J = \{1,2,3,4,5, \dots\}$$

Or define the following mapping:

$f : J \to \mathbb{Z}$ as:

$$f(n)=\begin{cases}\frac{n}{2} & n \text{ is even}\\ -\frac{n-1}{2} & n \text{ is odd}\end{cases}$$

Definition: (Sequence)

By a sequence , we mean a function f defined on the set S of all positive integers denoted by $\{x_n\}$, where $x_n = f(n)$, called a terms of the sequence.

Example $\{2n\} = \{2,4,6,8, \dots\}$

Theorem Every infinite subset of a countable set A is countable.

Proof: Suppose $E \subset A$ and E is infinite, arrange the elements x of A in a sequence $\{x_n\}$ of distinct elements, construct a sequence $\{n_k\}$as:

Let n_1 be the smallest positive integer $s.t.\, x_{n_1} \in E$,

Let n_2 be the smallest positive integer greater than n_1 $s.t.\, x_{n_2} \in E$, and so on

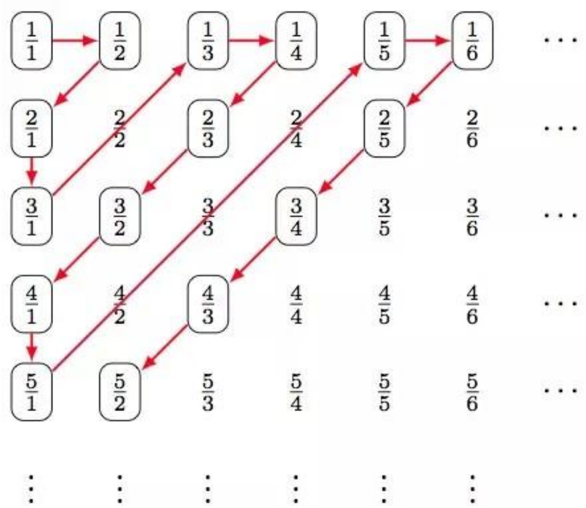
Putting $f(k) = x_{n_k}$, $k = 1,2, \dots$

We obtain a 1-1 correspondence between A and J .

Then E is countable.

Example : $\mathbb{Q}$ is countable.

Consider the following arrangement:



References

1- Principles Of Mathematical Analysis - W.Rudin. https://59clc.files.wordpress.com/2012/08/functional-analysis-_-rudin-2th.pdf