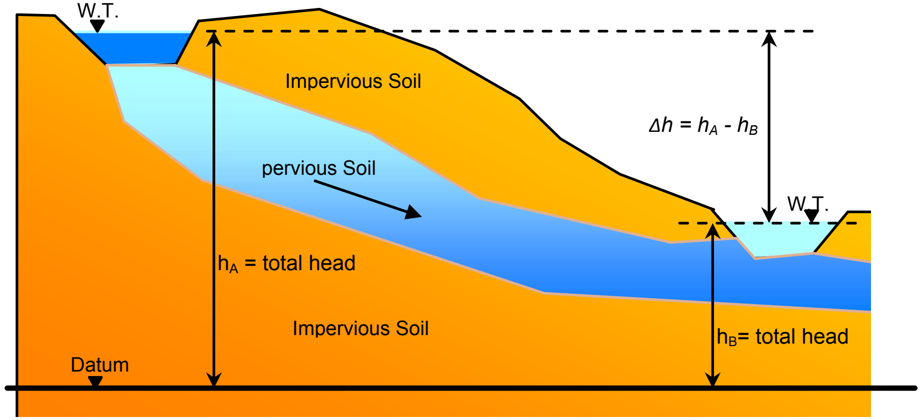
Permeability and Seepage

Topics

1. Permeability

- Overview of Underground Water Flow
- Permeability
- Theory
- Laboratory and Field Tests
- Empirical Correlations
- Equivalent Permeability in Stratified Soil



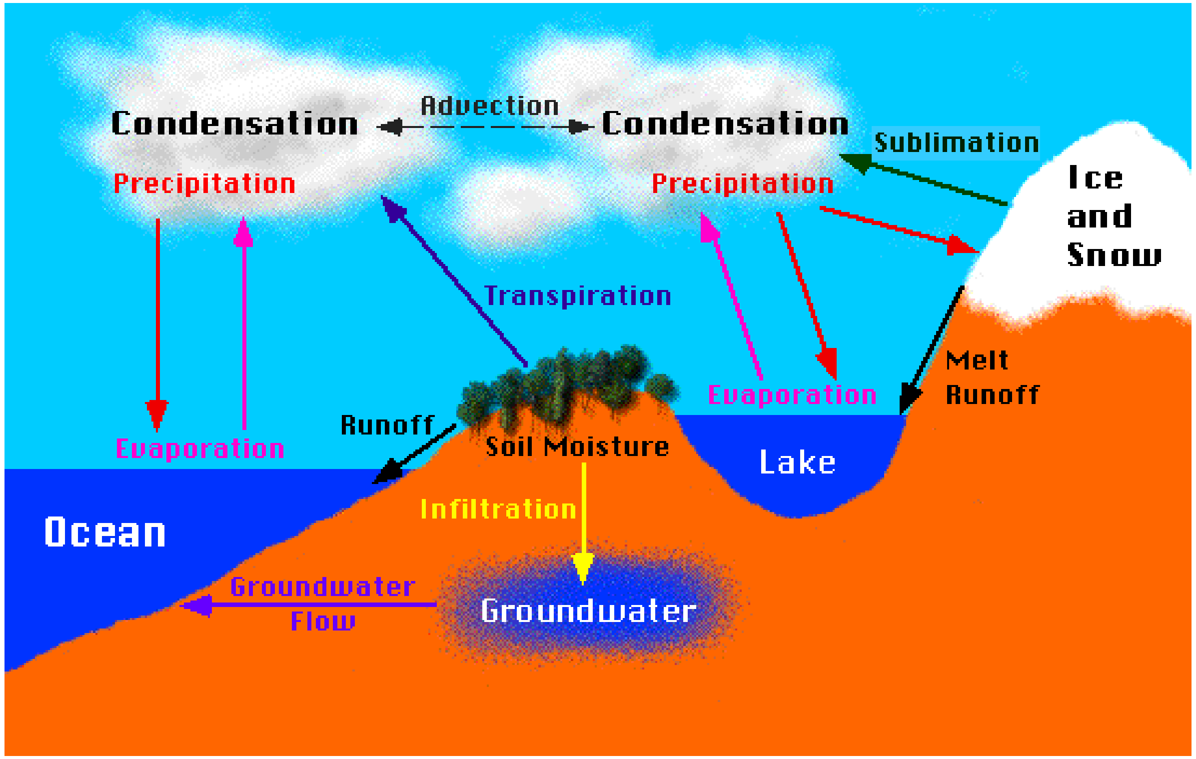
2. Seepage

- Laplace's Equation of Continuity
- Continuity Equation for Solution of Simple Flow Problems
- Flow Nets
- Seepage Calculation
- Seepage pressure and Uplift Pressure

Permeability

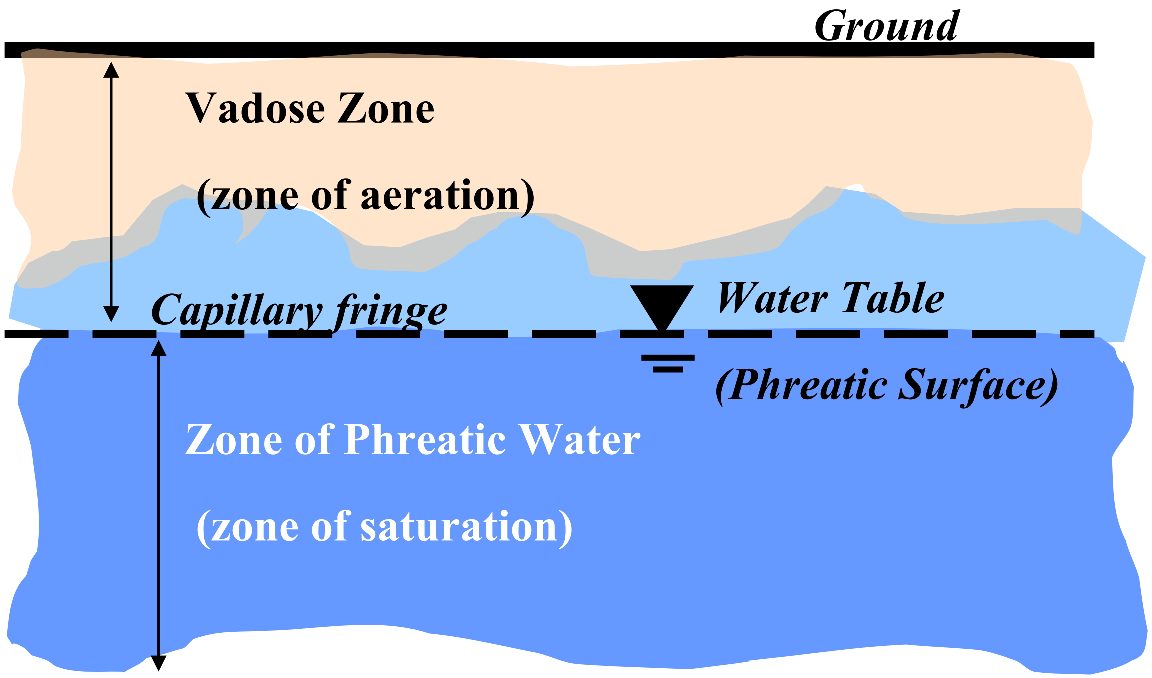
● Overview of Underground Water Flow

Hydrologic Cycle



Aspects of Hydrology

- A relatively small amount of the earth's water (<1%) is contained in the groundwater, but the effects of this water are out of proportion to their amount
- The permeability of soil affects the distribution of water both between the surface and the ground mass and within the ground mass itself



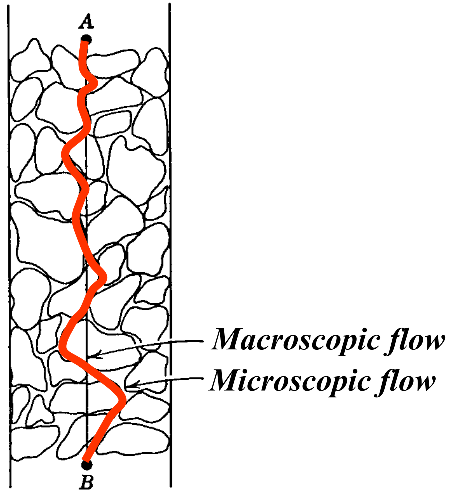
● ***Permeability***

Definition-

The property of soils

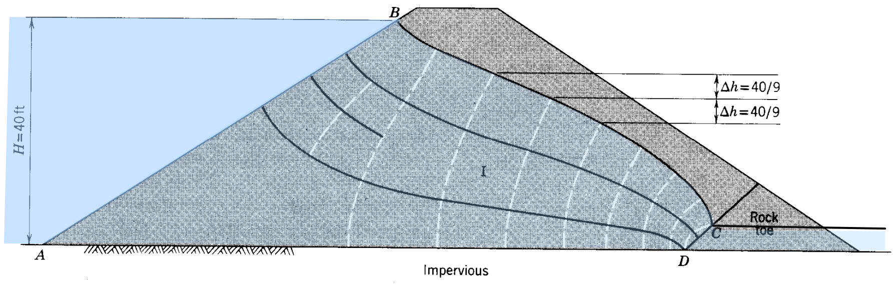
- ♦ allows water to pass through them at some rate.
- ♦ is a product of the granular nature of the soil, although it can be affected by other factors (such as water bonding in clays)
- ♦ Different soils have different permabilities, understanding of which is critical to the use of the soil as a foundation or structural element
- ♦ Soil and rock are porous materials

- Fluid flow takes place through interconnected void spaces between particles and *not through the particles themselves*
- No soil or rock material is strictly "impermeable"

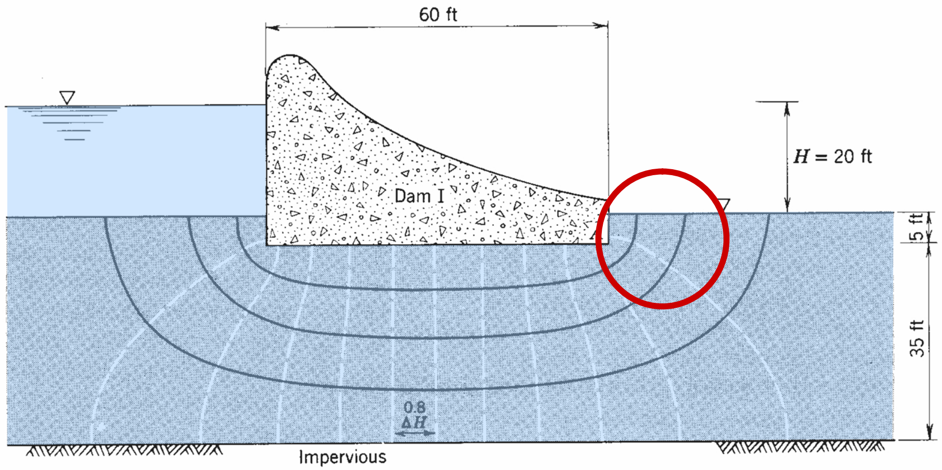


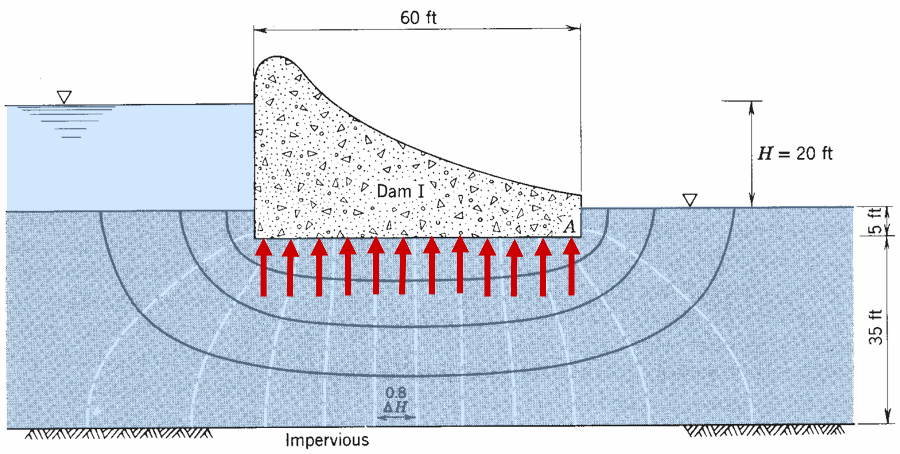
The study of flow of water through porous media is necessary for-

- **Estimation Seepage Loss**
- **Estimation Pore Water Pressures**
- **Evaluation Quicksand Conditions**
- **Dewatering System Design**
- **Drainage System Design**

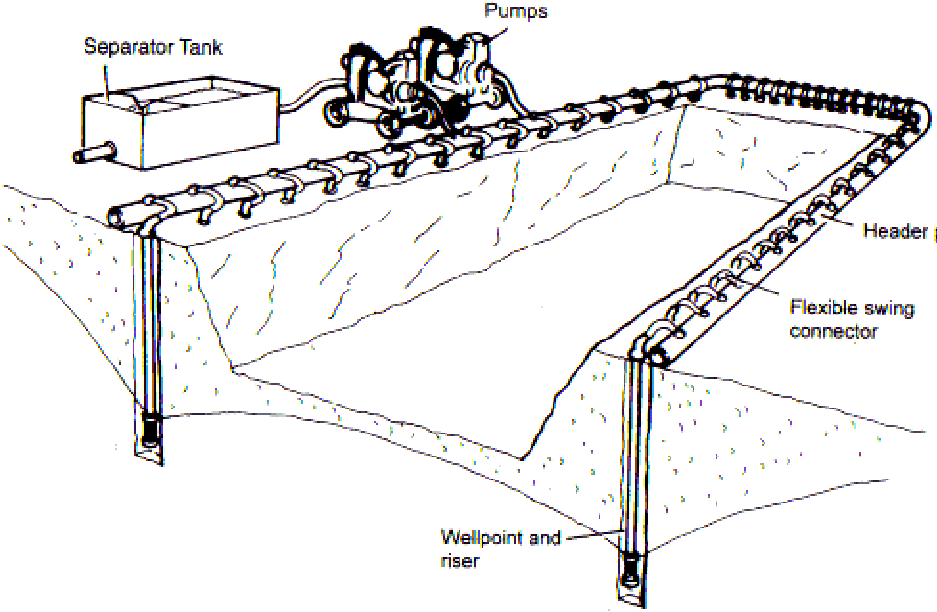


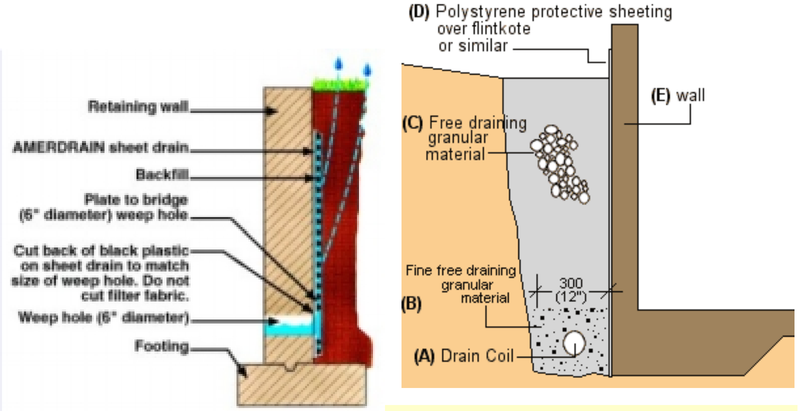
Seepage through the body of the dam



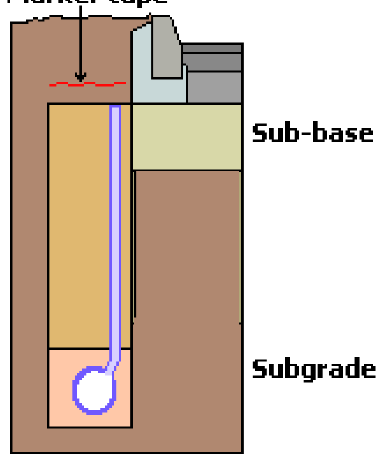


Typical Wellpoint System





Drainage behind Retaining Walls



Pavement Drainage

● ***<u>Theory</u>***

Bernoulli's Law

$$h_t = \frac{p}{\gamma_w} + \cancelto{\approx \text{zero}}{\frac{v^2}{2g}} + Z$$

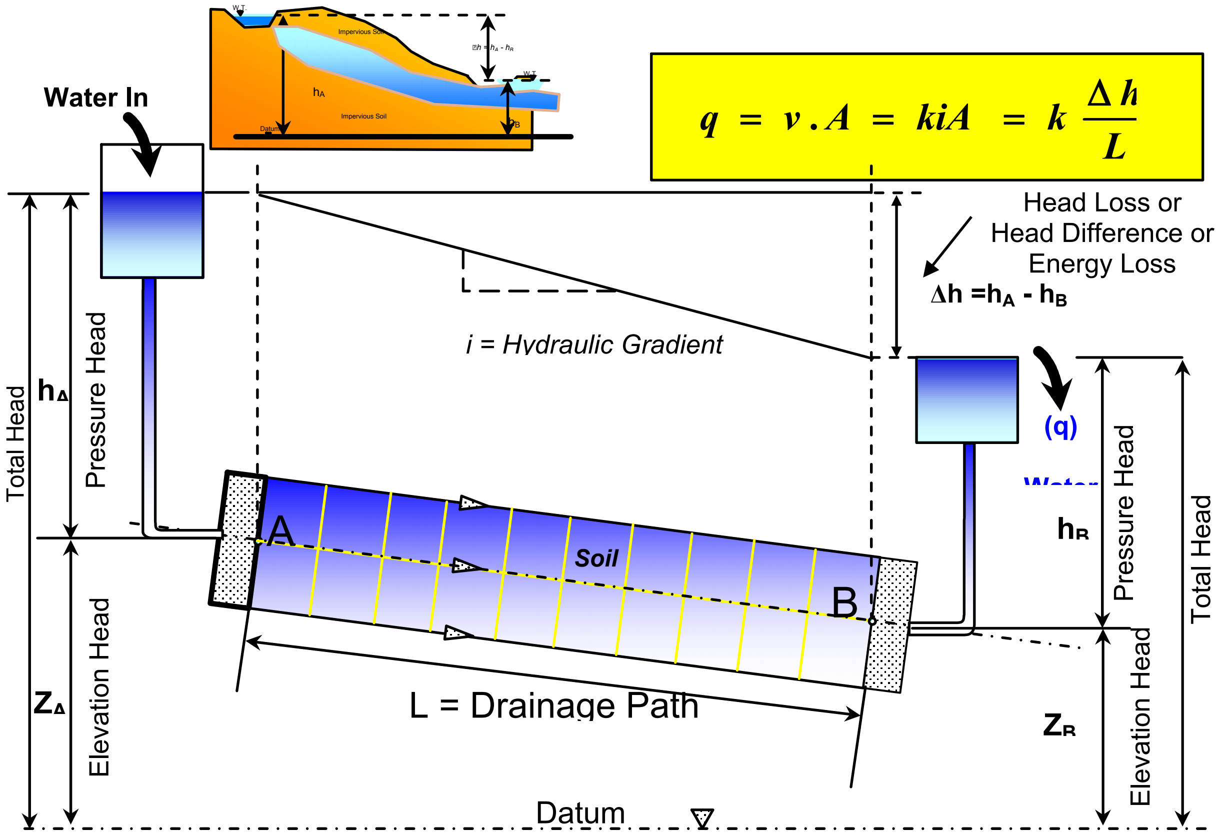
Where

Pressure head (Kinetic component) $= \dfrac{p}{\gamma_w} = h_p$

Velocity head (pressure component) $= \dfrac{v^2}{2g} = h_v$

Elevation head (Gravitational (potential) component) $= Z = h_e$

Head Differential



What causes flow of water through soil?

- ***Answer***:

A difference in ***TOTAL HEAD***

The loss of head between A & B, can be given by

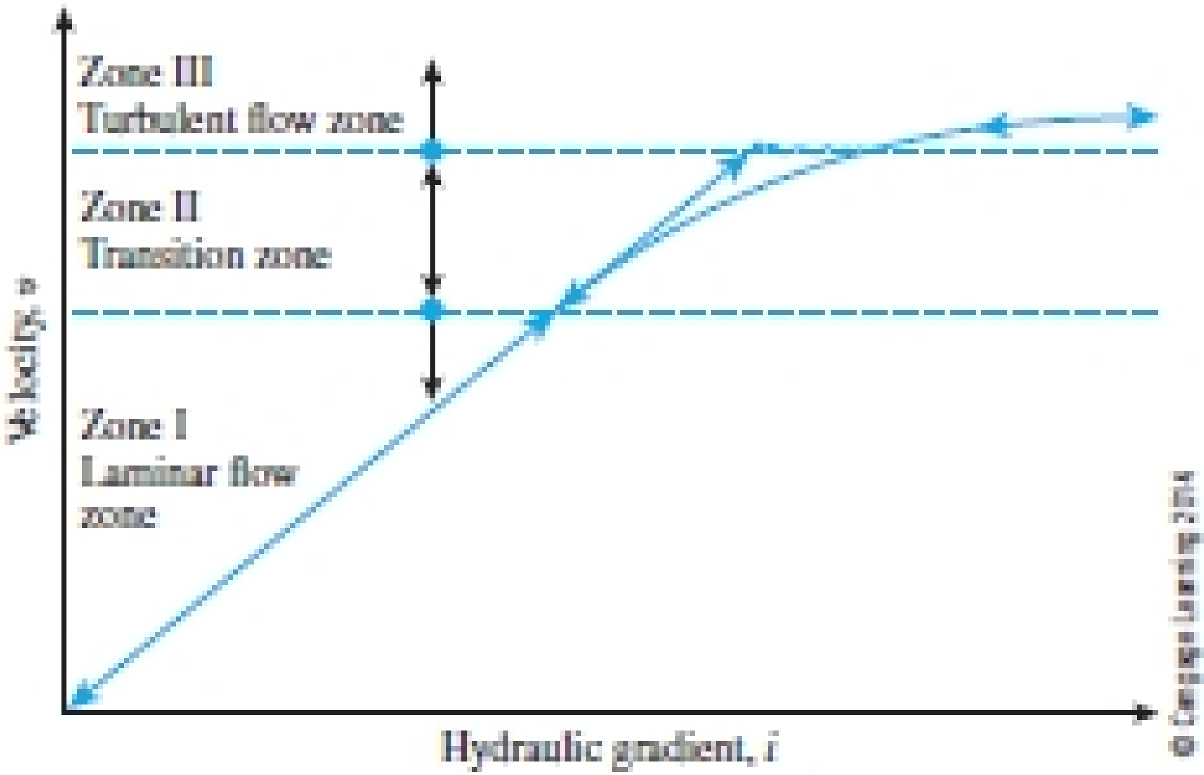
$$\Delta h = h_A - h_B = \left(\frac{P_A}{\gamma_w} + Z_A\right) - \left(\frac{P_B}{\gamma_w} + Z_B\right)$$

Δ h can be expressed in nondimensional form as

Hydraulic gradient

$$i = \frac{\Delta h}{L}$$

In general, the variation of velocity (v) with the hydraulic gradient (i) will be as shown in the figure below



In most soils, the flow of water through the void spaces can be considered laminar and thus
$\boldsymbol{v} \propto \boldsymbol{i}$

Darcy's Law

In 1856, Darcy published a simple equation for discharge velocity of water through saturated soils, which may expressed as

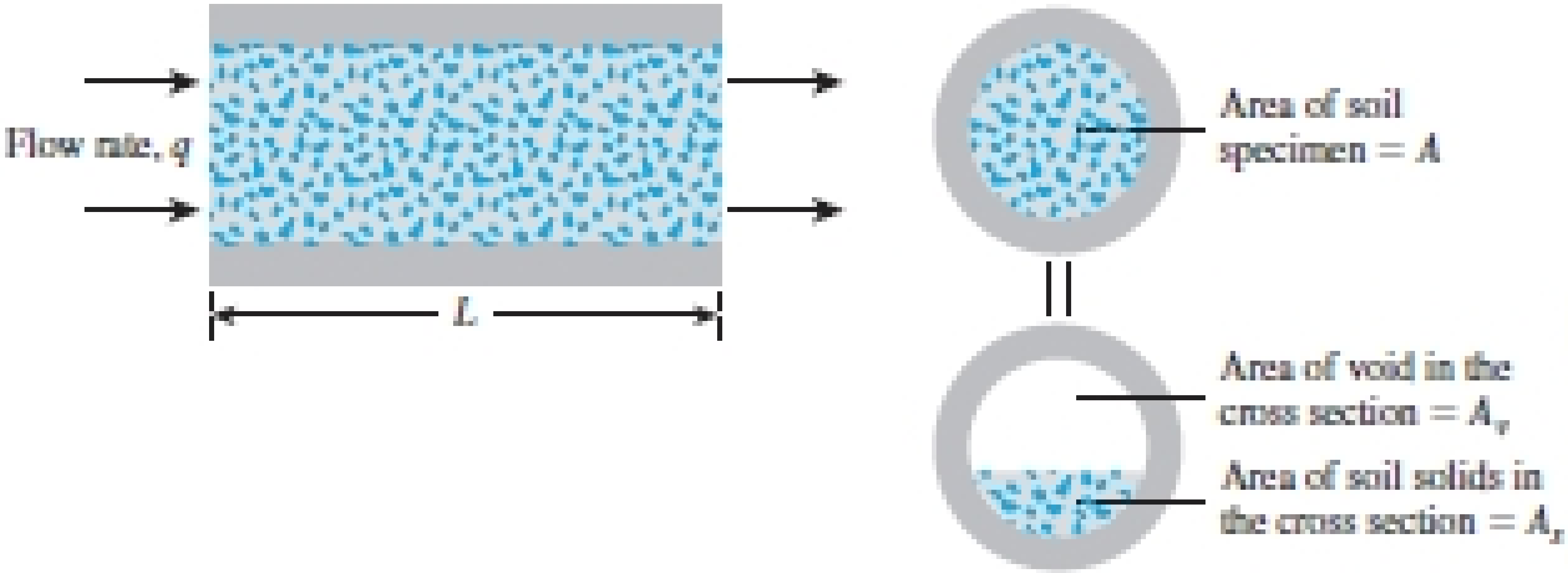
$$v = ki$$

Where

v = discharge velocity = quantity of water flowing in unit time through a unit gross – sectional area of soil at right angles to the direction of flow

k = coefficient of permeability

(*v*) is based on the gross – sectional area of the soil, however the actual velocity of water (seepage velocity, v_s)through the void spaces is higher than v – this can be derived as following:


Flow rate, q
L
Area of soil
specimen = A
Area of void in the
cross section = A_v
Area of soil solids in
the cross section = A_s

If the flow rate is q then

$$q = vA = A_v.v_s$$

$$A = A_v + A_s$$

$$\therefore q = v(A_v + A_s) = A_v.v_s$$

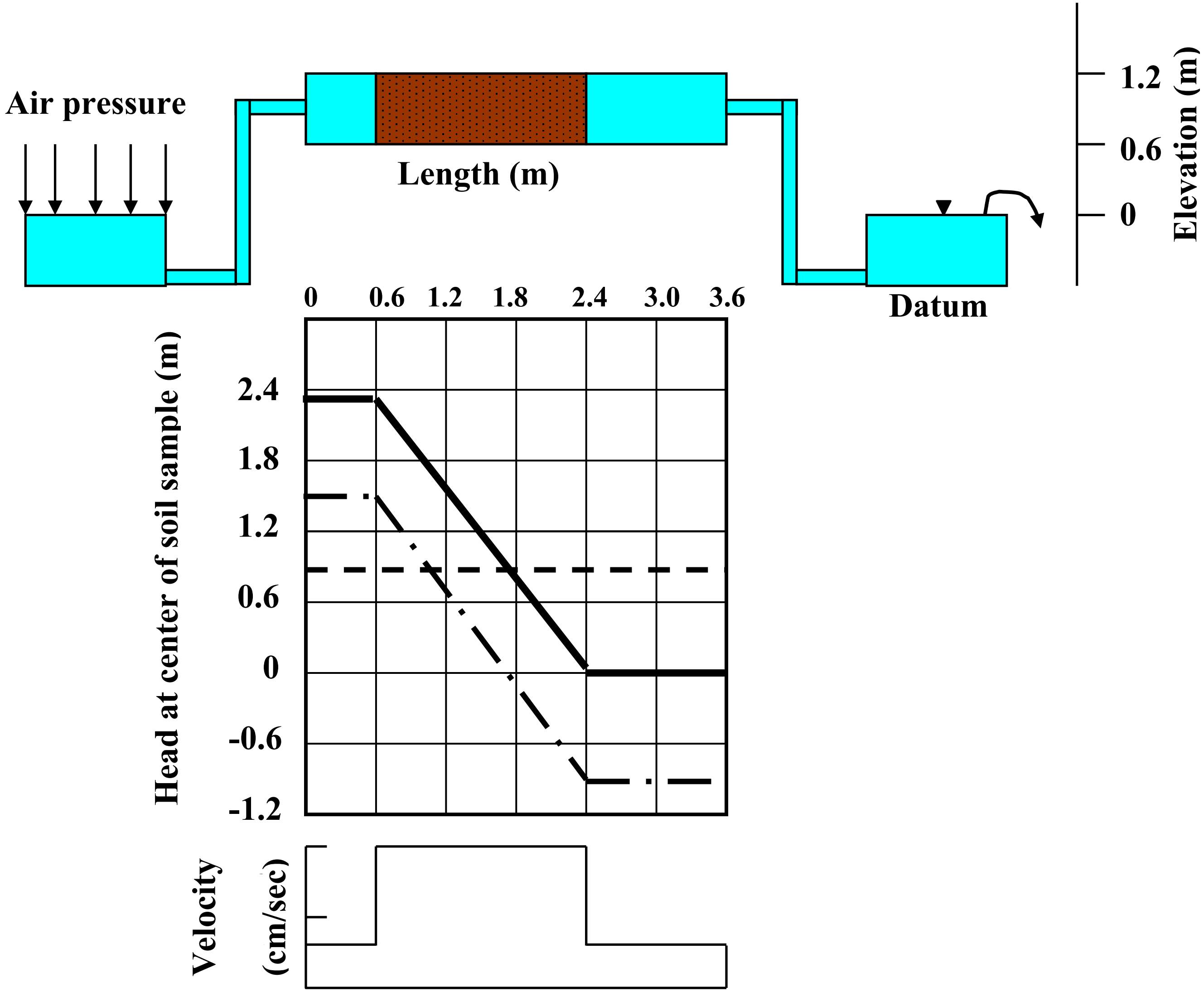
so

$$v_s = \frac{v(A_v + A_s)}{A_v} = \frac{v(A_v + A_s)L}{A_v L} = \left(\frac{v(V_v + V_s)}{V_v}\right) \div Vs$$

$$v_s = v\left[\frac{1 + \frac{V_v}{V_s}}{\frac{V_v}{V_s}}\right] = v\left(\frac{1+e}{e}\right) = \frac{v}{n}$$

$$v_s = \frac{v}{n}$$

Horizontal flow



In this case the air pressure will produce the required head for horizontal flow. Thus

$$\textit{Total head loss} = \frac{23.4}{9.81} = 2.385\ m$$

$$v = k.i = 0.5\frac{2.385}{1.8} = 0.663$$

$$\cdot\ v_s = \frac{v}{n} = \frac{0.663}{0.33} = 2\ cm/\sec$$

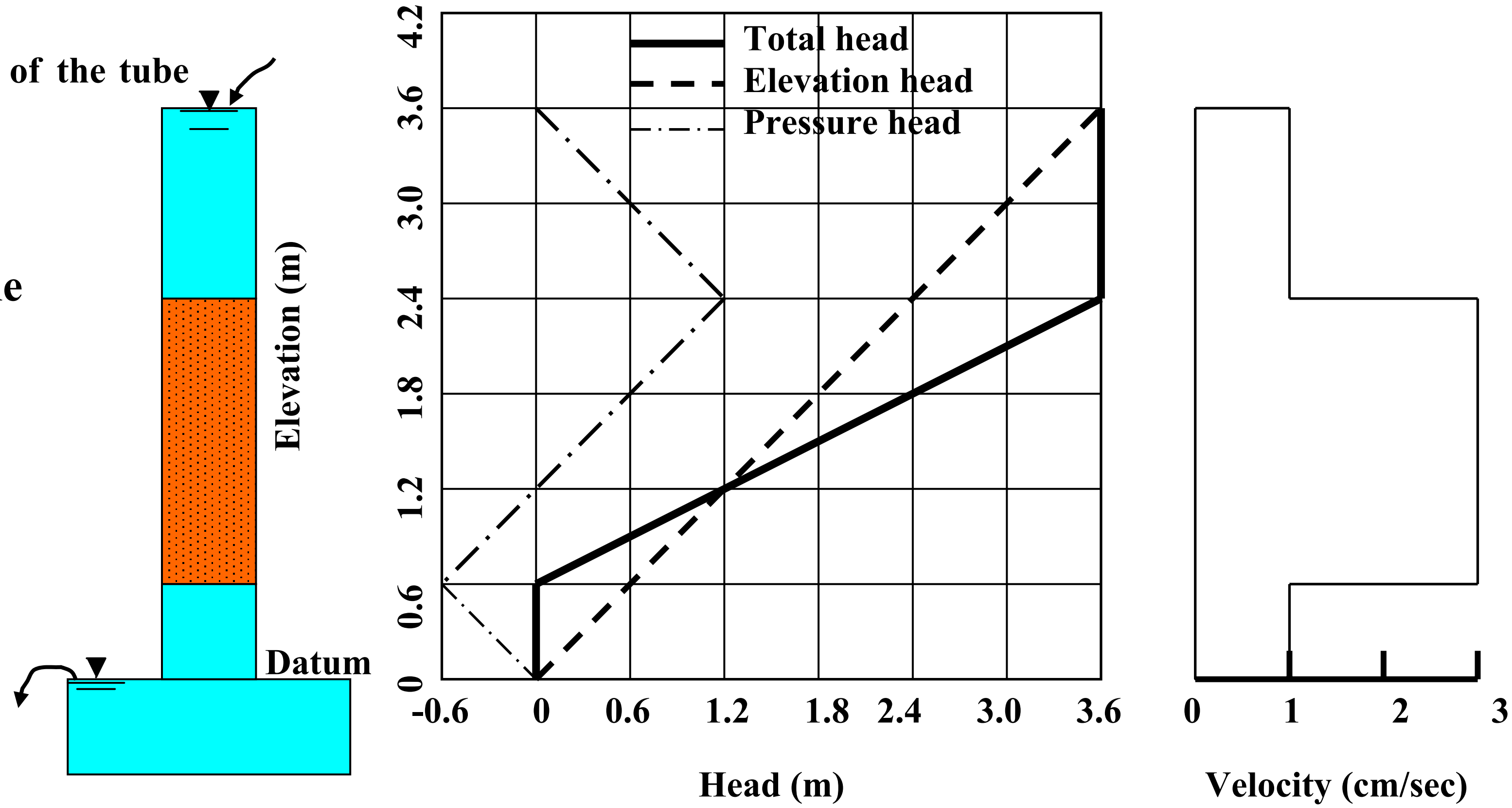
Downward Flow

$$v = k.i = 0.5.\frac{3.6}{1.8} = 1 \quad \text{cm/sec}$$

at the entrance and the exit parts of the tube

$$\therefore v_s = \frac{v}{n} = \frac{1}{0.33} = 3 \quad \text{cm/sec}$$

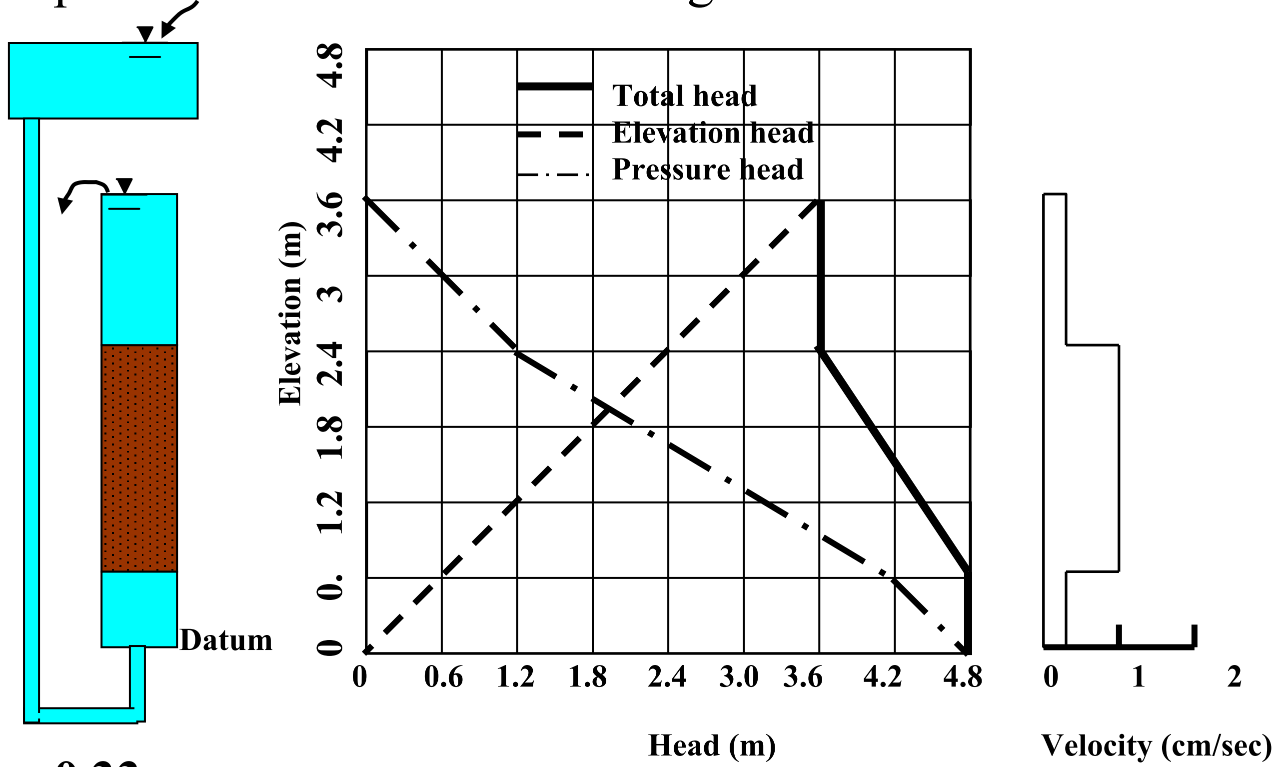
through the soil sample



.

Upward flow

The same tube was tested under upward flow as shown in the figure below



$$v = k.i = 0.5.\frac{1.2}{1.8} = 0.33 \Rightarrow v_s = \frac{v}{n} = \frac{0.33}{0.33} = 1 \quad \text{cm/sec}$$

Hydraulic Conductivity or Coefficient of permeability (k)

- It is defined as the rate of flow per unit area of soil under unit hydraulic gradient, it has the dimensions of velocity (L/T) such (cm/sec or ft/sec).
- It depends on several factors as follows:

1. Shape and size of the soil particles.
2. Distribution of soil particles and pore spaces.
3. Void ratio. Permeability increases with increase of void ratio.
4. Degree of saturation. Permeability increases with increase of degree of saturation.
5. Composition of soil particles.
6. Soil structure

7. Fluid properties. When the properties of fluid (water) affecting the flow are included, we can express k by the relation

$$\boldsymbol{k(cm/s) = \frac{K\rho g}{\mu} = \frac{K\gamma_w}{\mu}}$$

Where K = intrinsic or absolute permeability, cm^2

ρ = mass density of the fluid, g/cm^3

g = acceleration due to gravity, cm/sec^2

μ = absolute viscosity of the fluid, poise [that is, g/(cm.s)]

(***k***) varies widely for different soils, as shown in the table below

Typical values of permeability coefficient (k)	
Soil type	**k (mm/sec)**
Coarse gravel	**10 to 10^{3}**
Fine gravel, coarse and medium sand	**10^{-2} to 10**
Fine sand, loose silt	**10^{-4} to 10^{-2}**
Dense silt, clayey silt	**10^{-5} to 10^{-4}**
Silty clay, clay	**10^{-8} to 10^{-5}**

The coefficient of permeability of soils is generally expressed at a temperature of 20^{o}C. at any other temperature T, the coefficient of permeability can be obtained from eq.(12) as

$$\frac{k_{20}}{k_T} = \frac{(\rho_{20})(\mu_T)}{(\rho_T)(\mu_{20})}$$

Where

k_T , k_{20} = coefficient of permeability at T°C and 20°C, respectively

ρ_T , ρ_{20} = mass density of the fluid at T°C and 20°C, respectively

μ_T , μ_{20} = coefficient of viscosity at T°C and 20°C, respectively

Since the value of ρ_{20} / ρ_T is approximately 1, we can write

$$k_{20} = k_T \frac{\mu_T}{\mu_{20}}$$

Where $$\frac{\mu_T}{\mu_{20}} = f(T) \approx 1.682 - 0.0433T + 0.00046T^2$$

● *Laboratory and Field Tests*

The four most common laboratory methods for determining the permeability coefficient of soils are the following:

1. Constant – head test.
2. Falling – head test.
3. Indirect determination from consolidation test
4. Indirect determination by horizontal capillary test.

Laboratory Tests

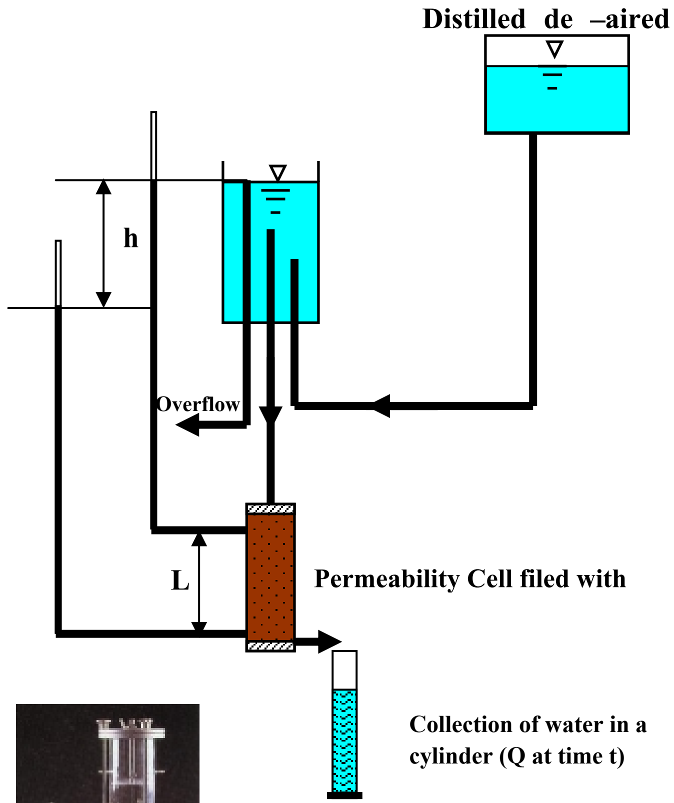
Constant – head test

$$Q = qt = kiAt \Rightarrow k = \frac{QL}{hAt}$$

- **Suitable for cohesionless soils with permeabilities > 10 x10^{-4} cm/sec**

- **The simplest of all methods for determining the coefficient of permeability**

- **This test is performed by measuring**
  - **the quantity of water, Q, flowing through the soil specimen,**
  - **the length of the soil specimen, L,**
  - **the head of water, h, and**



Permeability Cell with

Falling – head test

● Suitable for cohesive soils with permeabilities < 10 x 10^{-4} cm/sec

The rate of flow through the soil is

$$q = kiA = k\frac{h}{L}A = -a\frac{dh}{dt}$$

where h = head difference at any time t

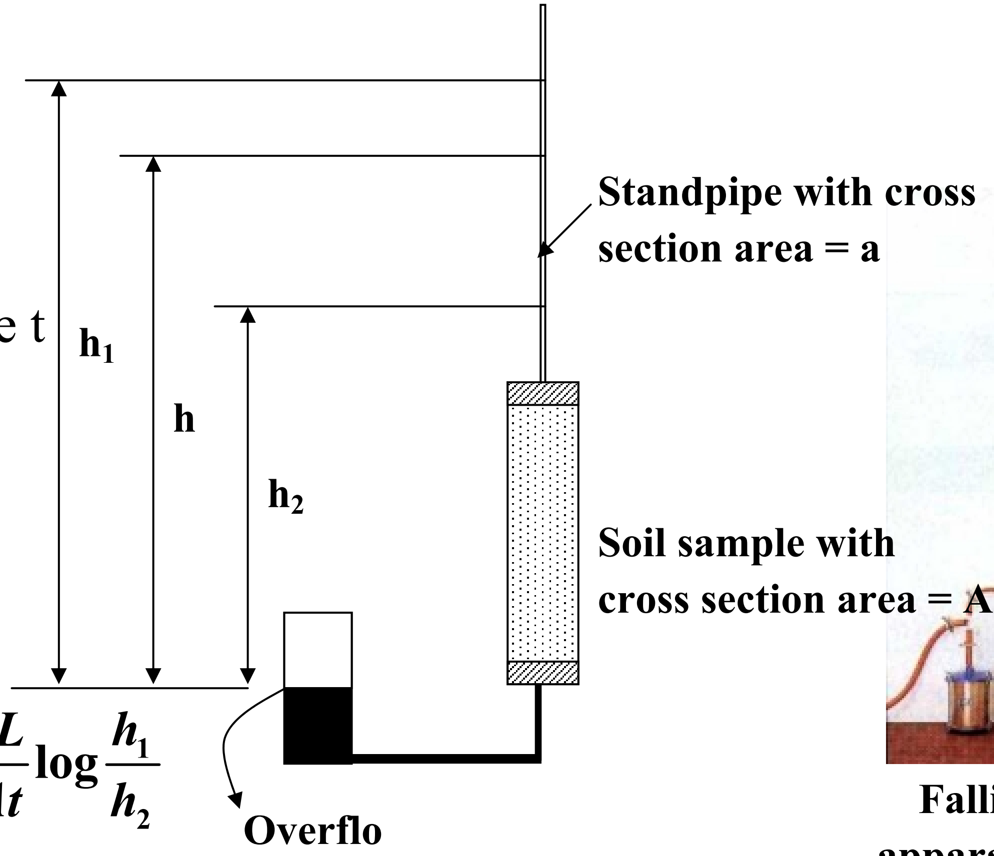
A = area of specimen

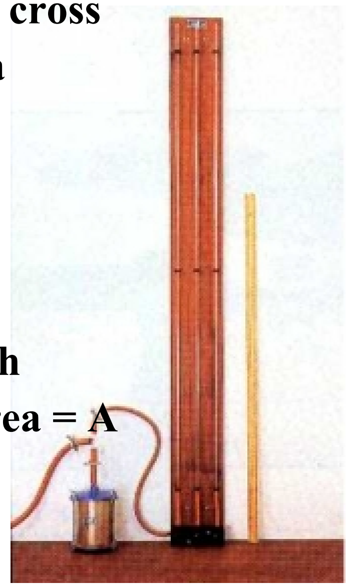
a = area of standpipe

L = length of specimen

From eq.(above),

$$\int_0^t dt = \int_{h_1}^{h_2} \frac{aL}{Ak}\left(-\frac{dh}{h}\right) \qquad k = 2.303\frac{aL}{At}\log\frac{h_1}{h_2}$$





Falling head
apparatus (ELE)

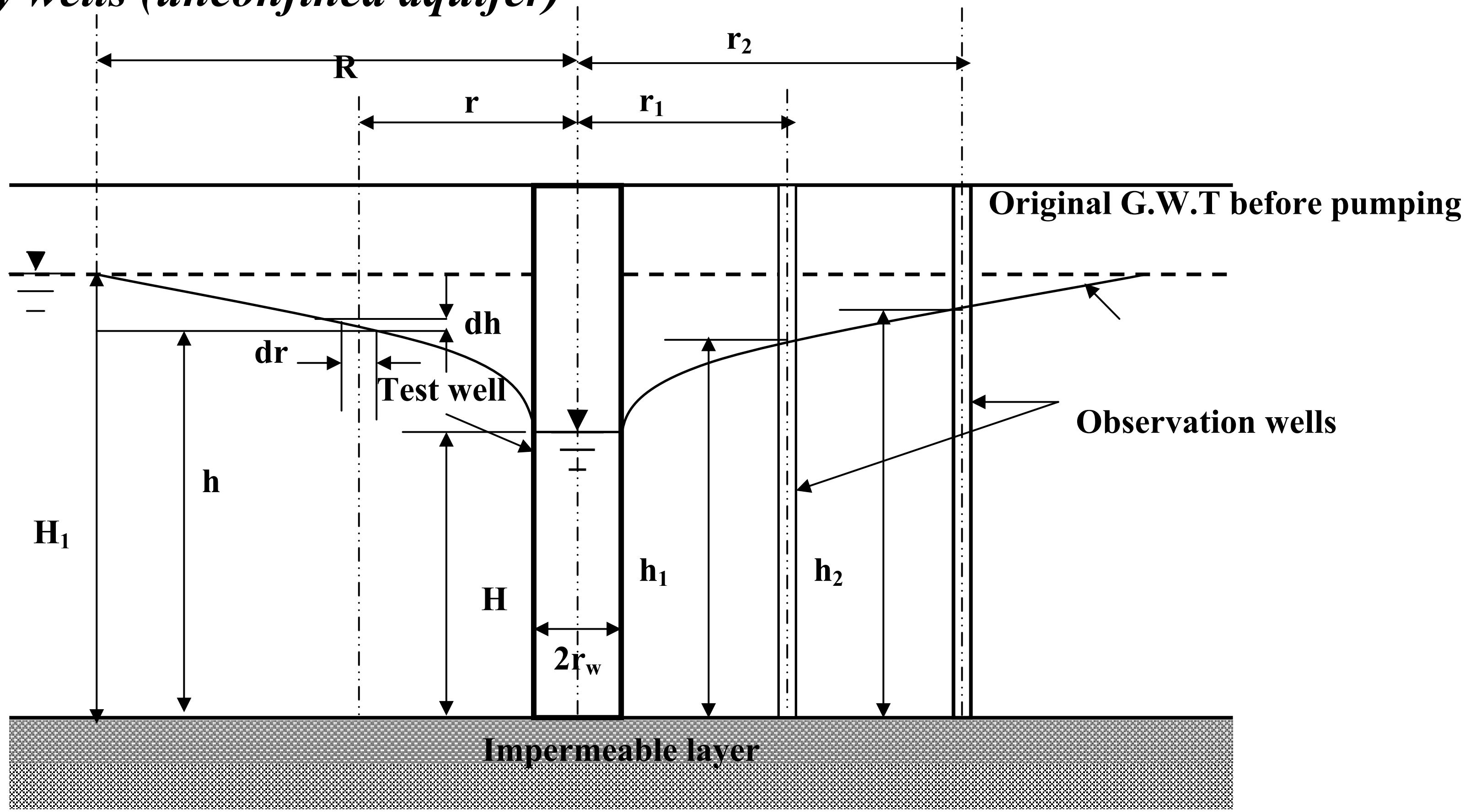
Field tests

There are many useful methods to determine the permeability coefficient in field such as

1. pumping from wells
2. Bore hole test
3. Open – end test
4. Packer test
5. Variable – head tests by means of piezometer observation well

Pumping from wells

♦ ***Gravity wells (unconfined aquifer)***



$$q = kiA$$

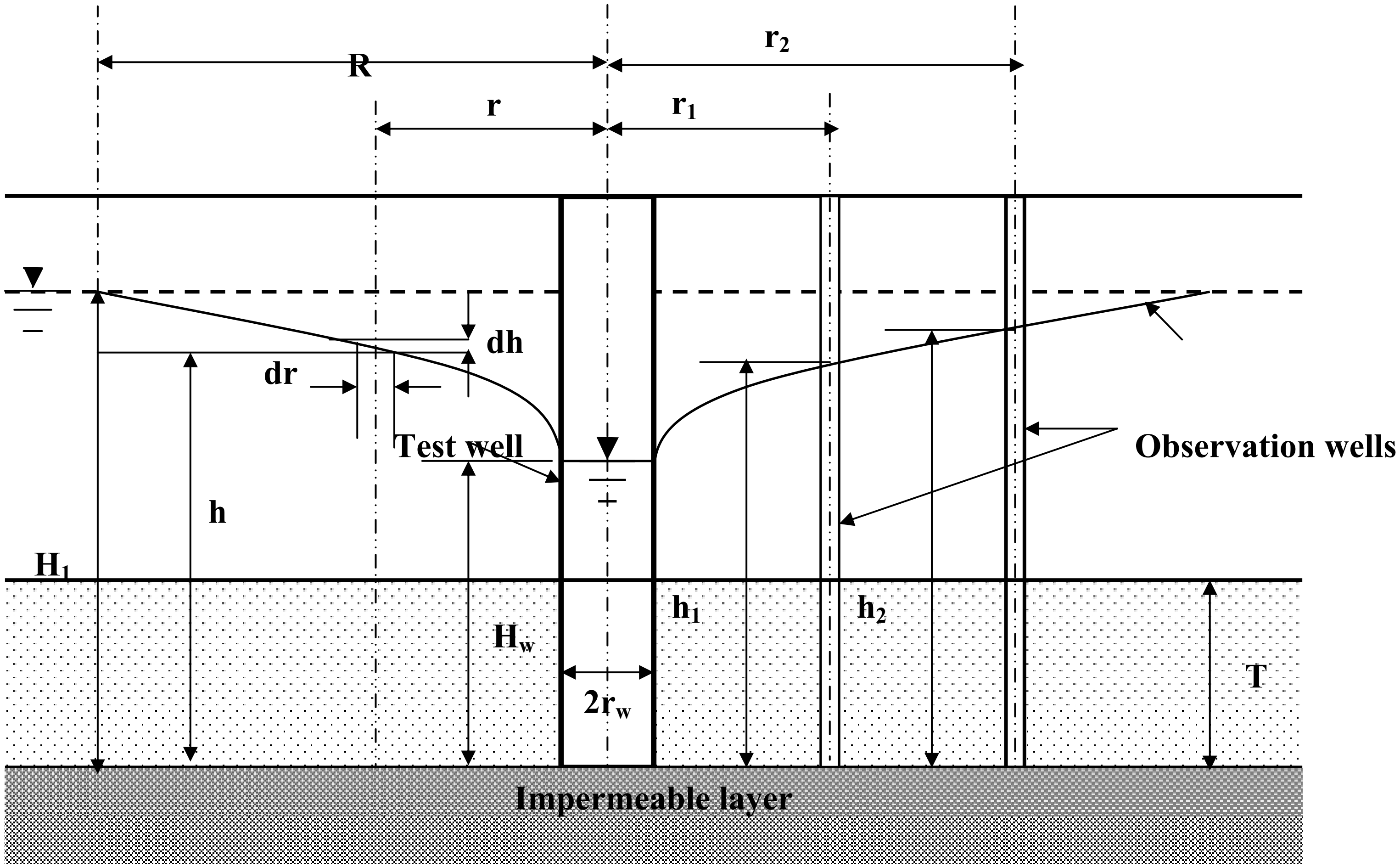
$$q = k\frac{dh}{dr}2\pi hr$$

$$\int_{r_1}^{r_2}\frac{dr}{r} = \frac{2\pi k}{q}\int_{h_1}^{h_2} hdh$$

So

$$k = \frac{2.303q\left[\log\left(r_2 / r_1\right)\right]}{\pi\left(h_2^2 - h_1^2\right)}$$

♦ *Artesian wells (confined aquifer)*



$$q = kiA = k\frac{dh}{dr}2\pi rT$$

$$\int_{r_1}^{r_2}\frac{dr}{r} = \int_{h}^{h_2}\frac{2\pi kT}{q}dh$$

$$k = \frac{q\log(r_2/r_{1)}}{2.727T(h_2 - h_1)}$$

If we substitute $h_1 = H_w$ at $r_1 = r_w$ and $h_2 = H_1$ ar $r_2 = R$ in, we get

$$k = \frac{q\log(R/r_w)}{2.727T(H_1 - H_w)}$$

● ***Empirical Correlations***

Several empirical equations for estimation of the permeability coefficient have been proposed in the past.

Granular Soil

Hazen

$$k\ (\text{cm/sec}) = cD_{10}^2$$

c = a constant that varies from 1.0 to 1.5
D_{10} = the effective size (mm)

Casagrande relation

$k = 1.4e^2 k_{0.85}$ $\quad$ $k_{0.85}$ = ***permeability coefficient at*** $e = 0.85$

From several works of many researechers , one may suggest that

$$k \propto \frac{e^3}{1+e}$$

Cohesive Soil

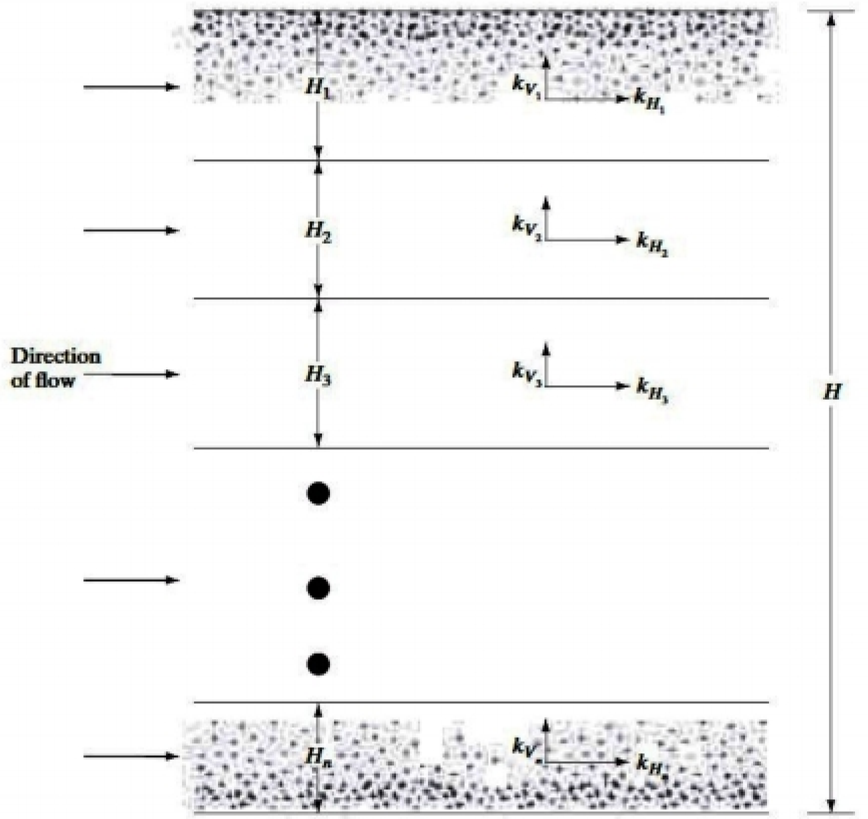
Samarasinghe et. al. (1982)

$$k = C\left(\frac{e^n}{1+e}\right)$$

where C and n are constants to be determined experimentally

Equivalent Permeability in Stratified Soil

Horizontal direction.



Equivalent hydraulic conductivity determination—horizontal flow in stratified soil

$$q = v.1.H = v_1.1.H_1 + v_2.1.H_2 + v_3.1.H_3 + ... + v_n.1.H_n$$

Where v = average discharge velocity

v_1 , v_2 , v_3 , …..v_n = discharge velocities of flow in layers denoted by the subscripts.

From Darcy's law

$$v = k_{H(eq)}.i_{eq}$$

$$v_1 = k_{h1}.i_1$$

$$v_1 = k_{h2}.i_2$$

$$v_1 = k_{h3}.i_3$$

$$\vdots$$

Since $i_{eq} = i_1 = i_2 = i_3 = \cdots = i_n$ then

$$v_1 = k_{hn}.i_n$$

$$k_{H(eq)} = \frac{1}{H}\left(k_{h1}H_1 + k_{h2}H_2 + k_{h3}H_3 + \cdots + k_{hn}H_n\right)$$

Or

$$k_{H(eq)} = \frac{\sum_{i=1}^{n} k_{hi}H_i}{H}$$

Vertical direction

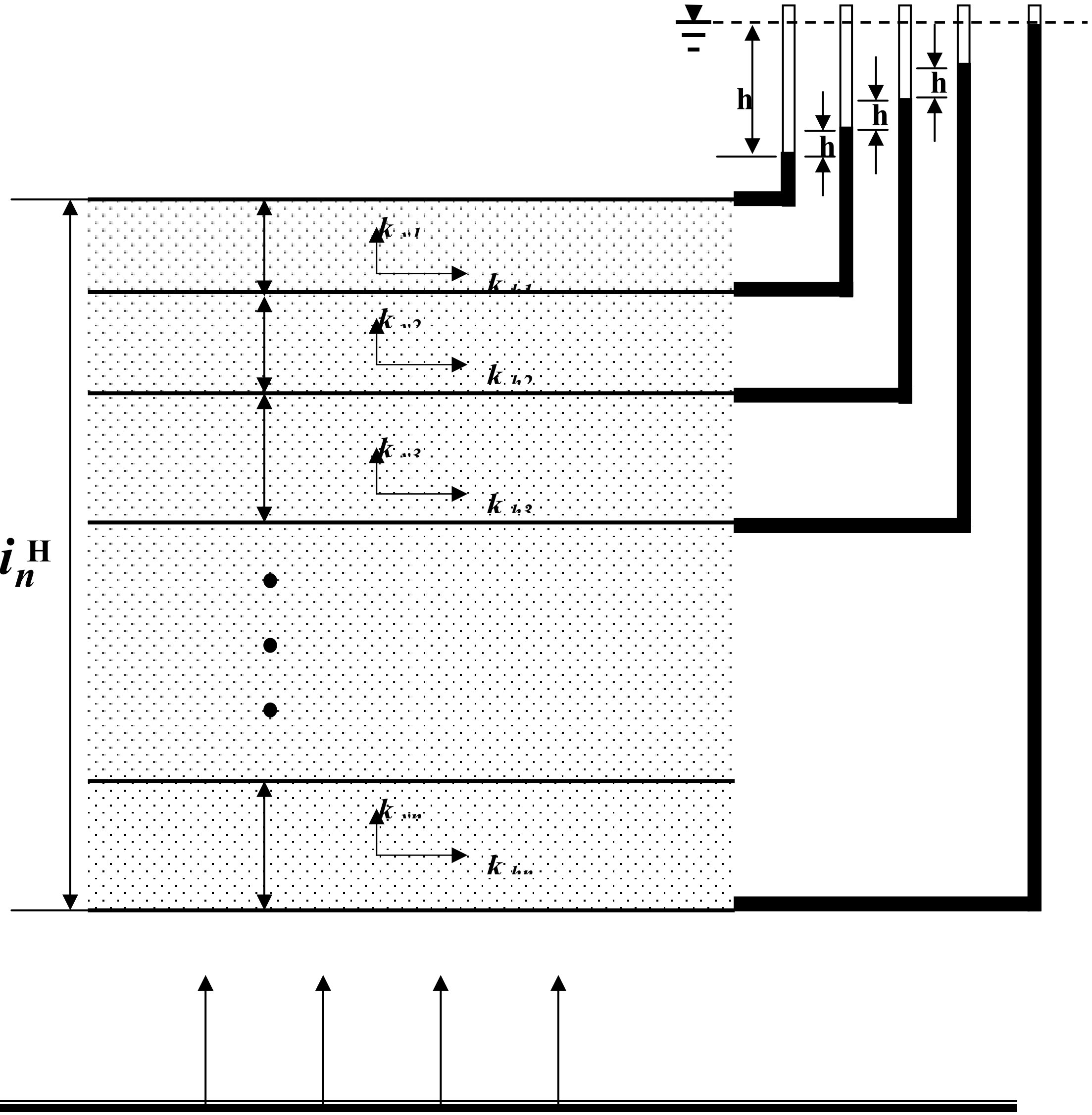
$$v = v_1 = v_2 = v_3 = \cdots = v_n$$

and

$$h = h_1 + h_2 + h_3 + \cdots + h_n$$

using Darcy's law $v = ki$, we can write

$$k_{v(eq)} \cdot \frac{h}{H} = k_{v1}.i_1 = k_{v2}.i_2 = k_{v3}.i_3 = \cdots = k_{vn}.i_n$$



again

$$h = H_1.i_1 + H_2.i_2 + H_3.i_3 + \cdots + H_n.i_n$$

the solutions of these equations gives

Direction of flow

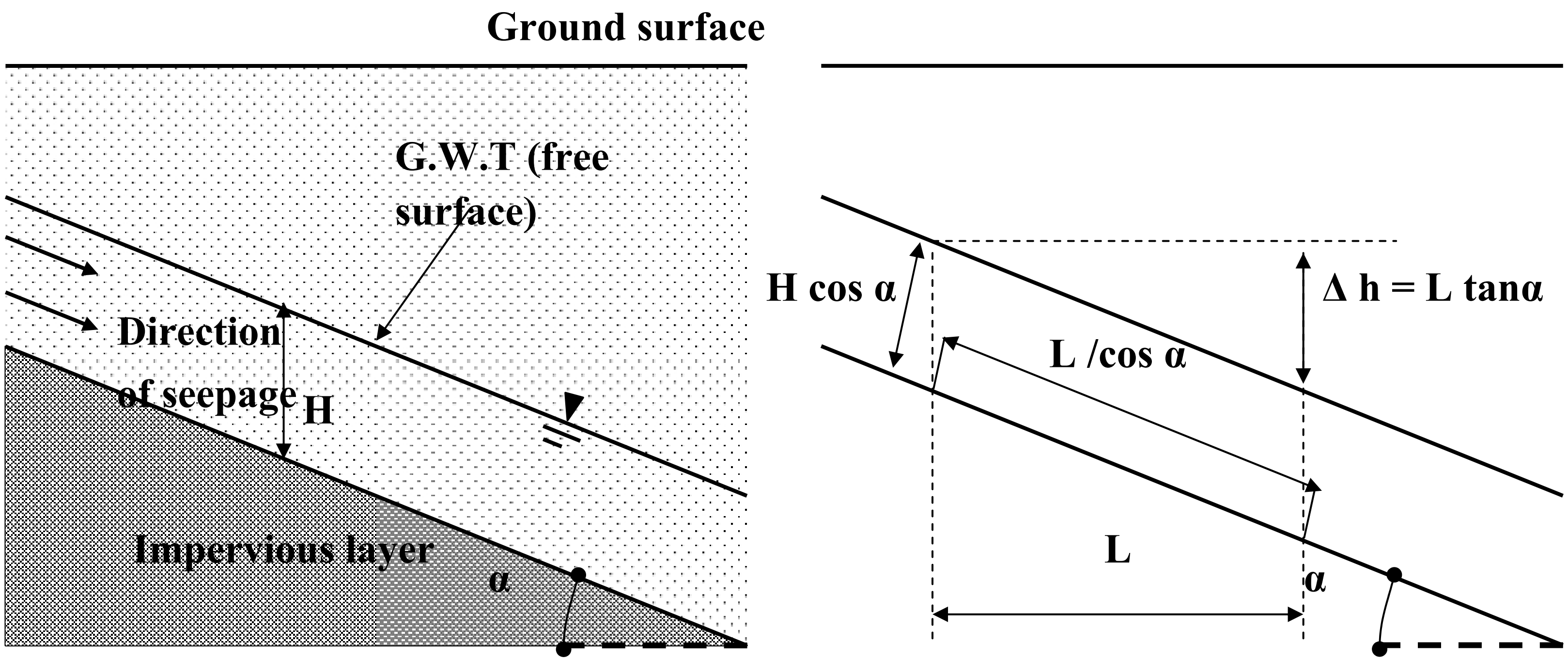
$$k_{v(eq)} = \frac{H}{\left(\frac{H_1}{k_{v1}}\right) + \left(\frac{H_2}{k_{v2}}\right) + \left(\frac{H_3}{k_{v3}}\right) + \cdots + \left(\frac{H_n}{k_{vn}}\right)}$$

or

$$k_{v(eq)} = \frac{H}{\sum_{i=1}^{n} \frac{H_i}{k_{vi}}}$$

Examples

1. An impervious layer as shown in the figure underlies a permeable soil layer. With k = 4.8x10^{-3} cm/sec for the permeable layer, calculate the rate of seepage through it in cm^3/sec/cm length width. Given H = 3 m and α = 5^o .



Solution

From the above figure

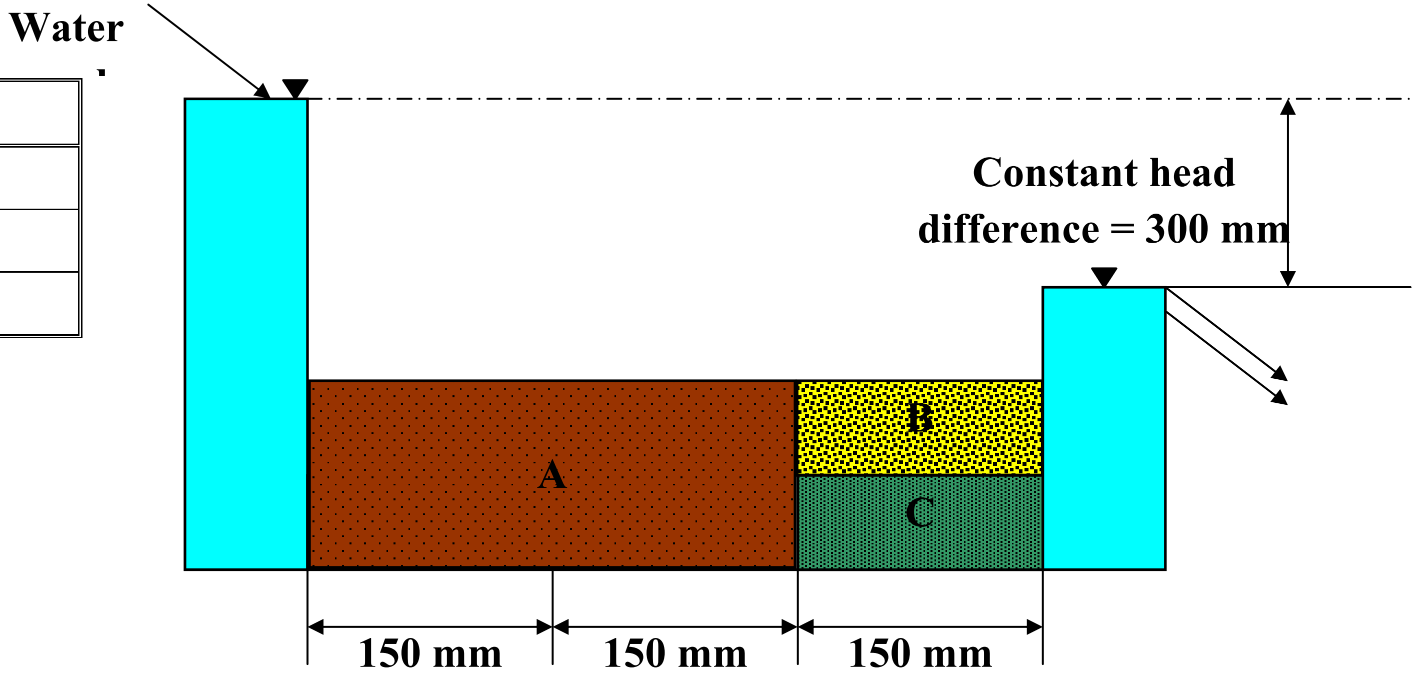
$$i = \frac{headloss}{length} = \frac{L\tan\alpha}{\left(\frac{L}{\cos\alpha}\right)} = \sin\alpha$$

$$q = kiA = (k)(\sin\alpha)(H\cos\alpha.1) = \left(4.8x10^{-4}\right)(\sin 5).(3\cos 5.) = 12.5x10^{-4}$$

$q = 12.5$ cm^3/sec/cm length

2. The following figure shows the layers of soil in a tube 100mmx100mm in cross – section. Water is supplied to maintain a constant head difference of 300 mm across the sample. The permeability coefficient of the soils in the direction of flow through them are as follows: Find the rate of supply.

Soil	*k (cm/sec)*
A	*$1x10^{-2}$*
B	*$3x10^{-3}$*
C	*$5x10^{-4}$*



Solution

For the soil layers B & C (the flow is parallel to the stratification)

$$k_{H(eq)} = \frac{1}{H}\left(k_{h1}H_1 + k_{h2}H_2\right) = \frac{1}{10}(3x10^{-3}(5) + 5x10^{-4}(5)) = 1.75x10^{-3} \quad \text{cm/sec}$$

For the layer A with equivalent layer of B&C

$$\therefore k_{eq} = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2}} = \frac{45}{\frac{30}{1x10^{-2}} + \frac{15}{1.75x10^{-3}}} = 3.8x10^{-3}$$

$$k_{eq} = 0.003888 cm/\sec$$

$$q = k_{eq} i A = 0.003888 \frac{300}{450}(10)^2 = 0.259\ cm^3/\sec$$

3. The permeability coefficient of a sand at a void ratio of 0.55 is 0.1 ft/min. estimate its permeability coefficient at avoid ratio of 0.7. Use Casagrande empirical relationship

<u>Solution</u>

From Casagrande relation $k=1.4e^2k_{0.85} \Rightarrow k \propto e^2$ ***.So***

$$\frac{k_1}{k_2}=\frac{e_1^2}{e_2^2} \Rightarrow \frac{0.1}{k_2}=\frac{(0.55)^2}{(0.7)^2} \Rightarrow k_2=\frac{(0.1)(0.7)^2}{(0.55)^2}=0.16 \text{ ft/min at } e = 0.7$$

4. for normally consolidated clay soil, the following are given:

Void ratio	*k (cm/sec)*
1.1	***$0.302x10^{-7}$***
0.9	***$0.12x10^{-7}$***

Estimate the permeability coefficient of clay at void ratio of 1.2 .
Use Samarasingh et. al. relation.

Solution

Samarasingh et.al. eq.

$$k = C_3 \frac{e^n}{1+e}$$

$$\therefore \frac{k_1}{k_2} = \frac{\left(\dfrac{e_1^n}{1+e_1}\right)}{\left(\dfrac{e_2^n}{1+e}\right)}$$

$$\frac{03.02x10^{-7}}{0.12x10^{-7}} = \frac{\dfrac{(1.1)^n}{1+1.1}}{\dfrac{(0.9)^n}{1+0.9}} \Rightarrow 2.517 = \left(\frac{1.9}{2.1}\right)\left(\frac{1.1}{0.9}\right)^n$$

$$\therefore 2.782 = (1.222)^{n}$$

$$n = \frac{\log(2.782)}{\log(1.222)} = \frac{0.444}{0.087} = 5.1$$

So

$$k = C_3\left(\frac{e^{5.1}}{1+e}\right)$$

To find C_3

$$0.302x10^{-7} = C_3\left[\frac{(1.1)^{5.1}}{1=1.1}\right] = \left(\frac{1.626}{2.1}\right)C_3$$

$$C_3 = \frac{(0.302x10^{-7})(2.1)}{1.626} = 0.39x10^{-7}\,cm/\sec$$

Hence

$$k = (0.39x10^{-7})\left(\frac{e^{5.1}}{1+e}\right)$$

At a void ratio of 1.2

$$k = (0.39x10^{-7})\left(\frac{1.2^{5.1}}{1+1.2}\right) = 0.449x10^{-7}\ \text{cm /sec.}$$

5. pumping test from Gravity well in a permeable layer underlain by an impervious stratum was made. When steady state was reached, the following observations were made q = 100 gpm; h_1 = 20 ft; h_2 = 15 ft; r_1 = 150 ft; and r_2 = 50 ft. Determine the permeability coefficient of the permeable layer.

<u>Solution</u>

Since $$k = \frac{2.303q\log_{10}\left(\frac{r_1}{r_2}\right)}{\pi\left(h_1^2 - h_2^2\right)}$$

Given: q = 100gpm = 13.37 ft^3 / min, so

$$k = \frac{2.303x13.37\log_{10}\left(\frac{150}{50}\right)}{\pi\left(20^2 - 15^2\right)} = 0.0267\,ft/\min \approx 0.027\,ft/\min$$

Seepage

● Laplace's Equation of Continuity

♦ ***Introduction***

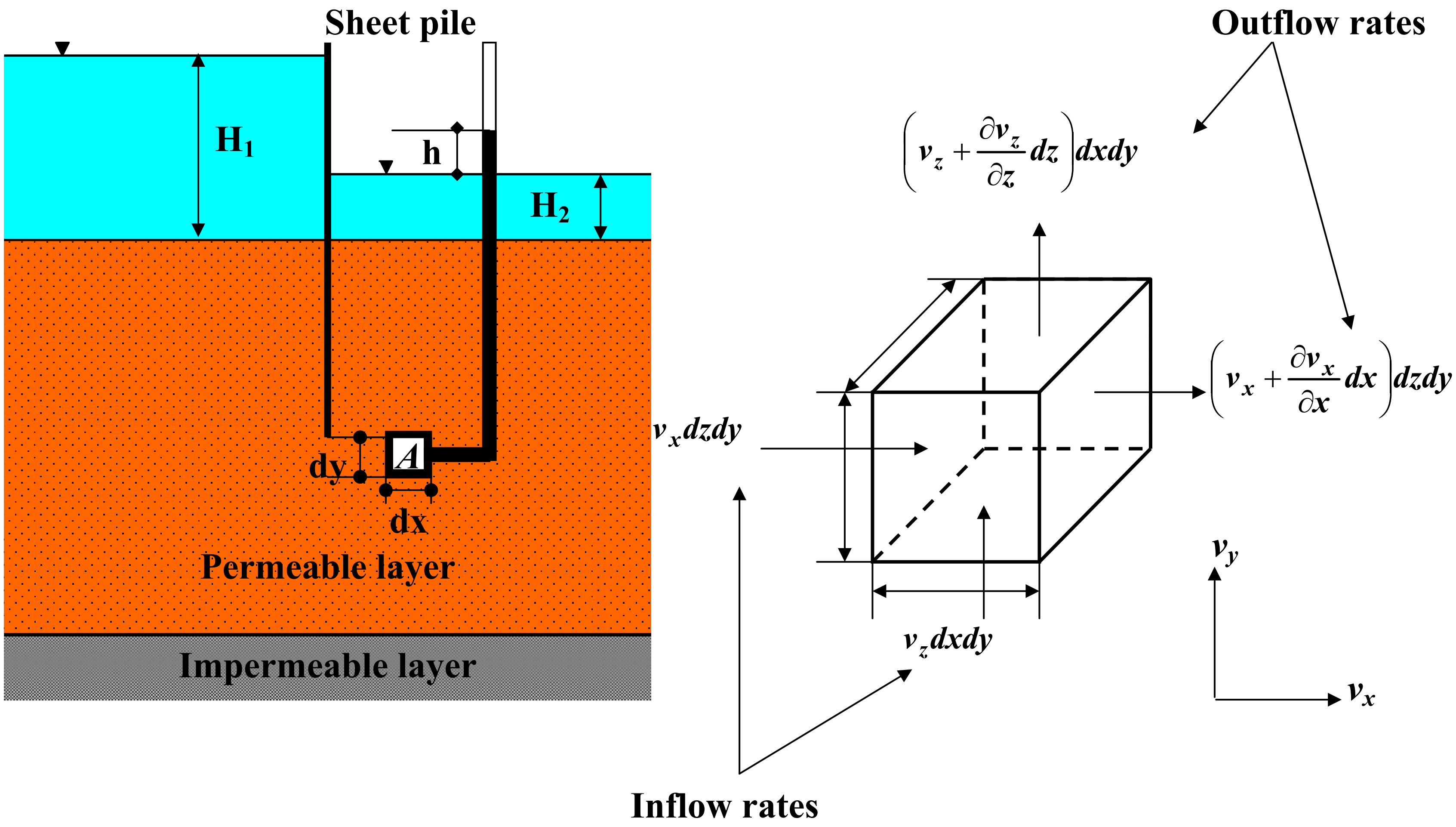
In many instances, the flow of water through soil is not in one direction only, nor is it uniform over the entire area perpendicular to the flow.

In such cases, calculation of ground water flow is generally made by use of graphs referred to as ***flow nets***.

The concept of the flow net is based on ***Laplace's equation of continuity***, which describes the steady flow condition for a given point in the soil mass.

♦ ***Derivation***

To derive the Laplace differential equation of continuity, let us take a single row of sheet piles that have been driven into a permeable soil layer, as shown in the figure below.



Flow at element A

Assumptions:

1. The row of sheet piles is impervious
2. The steady state flow of water from the upstream to the downstream side through the permeable layer is a two – dimensional flow.
3. The water is incompressible
4. No volume change occurs in the soil mass. Thus, the total rate of inflow should be equal to the total rate of outflow

$$\left[\left(v_x + \frac{\partial v_x}{\partial x} dx\right) dz.dy + \left(v_z + \frac{\partial v_z}{\partial z} dz\right) dx.dy\right] - \left[v_x .dz.dy + v_z .dx.dy\right] = 0$$

Or

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0 \quad \ldots\ldots\ldots(1)$$

Using Darcy's law, the discharge velocities can be expressed as

$$v_x = k_x i_x = k_x \frac{\partial h}{\partial x} \text{ and } v_z = k_z i_z = k_z \frac{\partial h}{\partial z} \quad \text{……….(2)}$$

Where k_x, k_z are the permeability coefficients in the horizontal and vertical directions respectively.

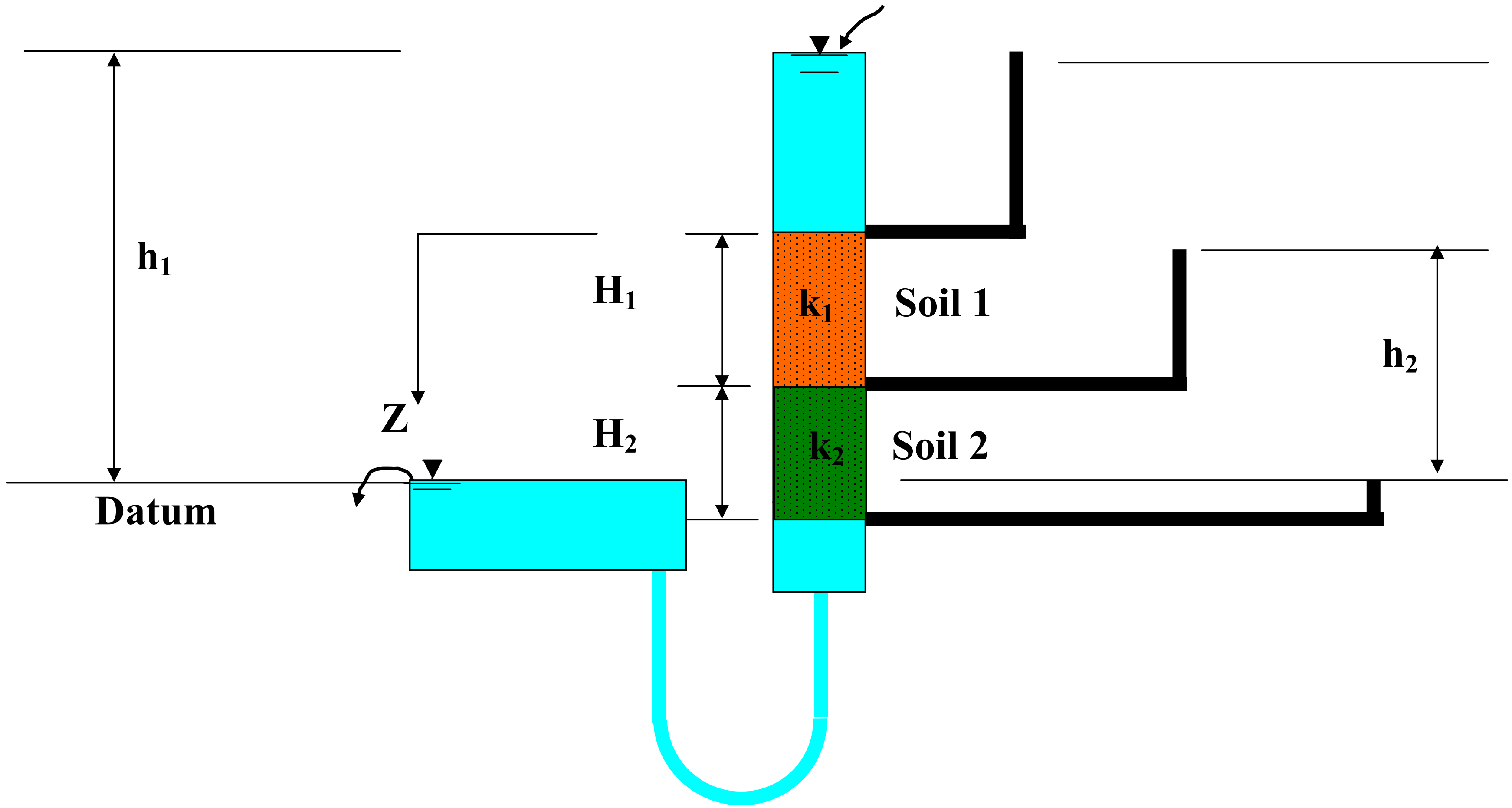
From Eqs. 1 and 2, we can write that

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

If the soil is isotropic with respect to the permeability coefficients – that is, $k_x = k_z$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

- Continuity Equation for Solution of Simple Flow Problems



$$\frac{\partial^2 h}{\partial z^2} = 0 \quad \Rightarrow \quad h = A_1 z + A_2$$

Soil 1

@ z = 0 $\quad$ h = h_1

@ z = H_1 $\quad$ h = h_2

$$h_1 = A_2$$

$$h_2 = A_1 H_1 + h_1 \quad \Rightarrow \quad A_1 = -\frac{(h_1 - h_2)}{H_1}$$

$$\therefore \quad h = -\frac{(h_1 - h_2)}{H_1} z + h_1 \quad for \quad 0 \le z \le H_1$$

Soil 2

@ z = H_1 $\quad$ h = h_2

@ z = H_1+H_2h = 0

$$h_2 = A_1 H_1 + A_2 \qquad \Rightarrow \quad A_2 = h_2 - A_1 H_1$$

$$0 = A_1 (H_1 + H_2) + A_2 \qquad \Rightarrow \quad A_1 = -\frac{h_2}{H_2} \quad and \quad A_2 = h_2 (1 + \frac{H_1}{H_2})$$

$$\therefore \quad h = -\frac{-h_2}{H_2} z + h_2 (1 + \frac{H_1}{H_2}) \quad for \quad H_1 \leq z \leq H_1 + H_2$$

At any given time

$$q_1 = q_2$$

$$k_1 \frac{h_1 - h_2}{H_1} A = k_2 \frac{h_2 - 0}{H_2} A$$

$$h_2 = \frac{h_1 k_1}{H_1 \left(\frac{k_1}{H_1} + \frac{k_2}{H_2} \right)}$$

$$\therefore \quad h = h_1\left(1 - \frac{k_2 z}{k_1 H_2 + k_2 H_1}\right) \qquad for \quad 0 \le z \le H_1$$

$$h = h_1\left[\left(\frac{k_1}{k_1 H_2 + k_2 H_1}\right)\right](H_1 + H_2 - z) \qquad for \quad H_1 \le z \le H_1 + H_2$$

• Flow Nets

The following methods are available for the determination of flow nets:

1. Graphical solution by sketching
2. Mathematical or analytical methods
3. Numerical analysis
4. Models
5. Analogy methods

All the methods are based on Laplace's continuity equation.

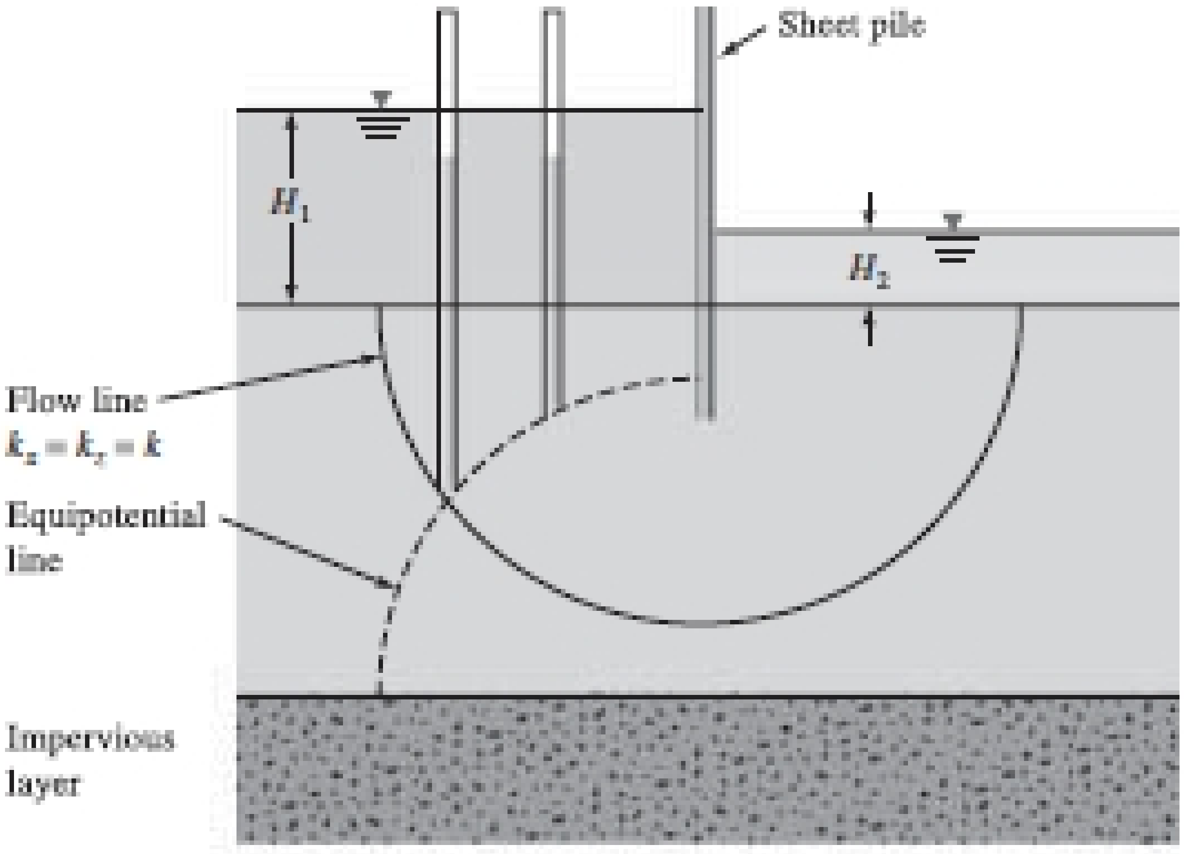
Flow net in isotropic medium

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

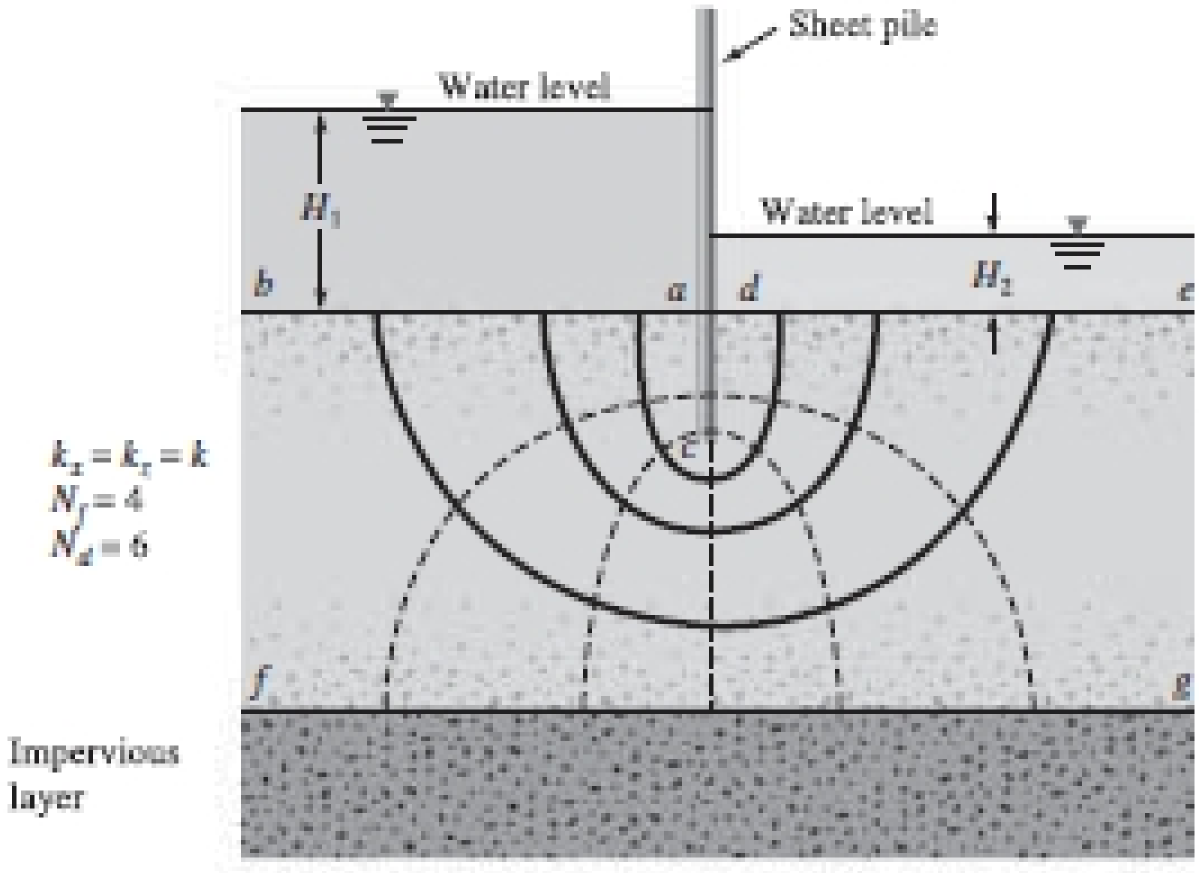
It represents two orthogonal families of curves – that is, the ***flow lines*** and the ***equipotential lines***.

Flow line is a line along which a water particle will travel from upstream to the downstream side in the permeable soil medium.

Equipotential line *is a line along which the potential head at all points is the same.*



(a)


Sheet pile
Water level
Water level
H_1
H_2
b
a
d
e
c
f
g
$k_x = k_z = k$
$N_f = 4$
$N_d = 6$
Impervious layer

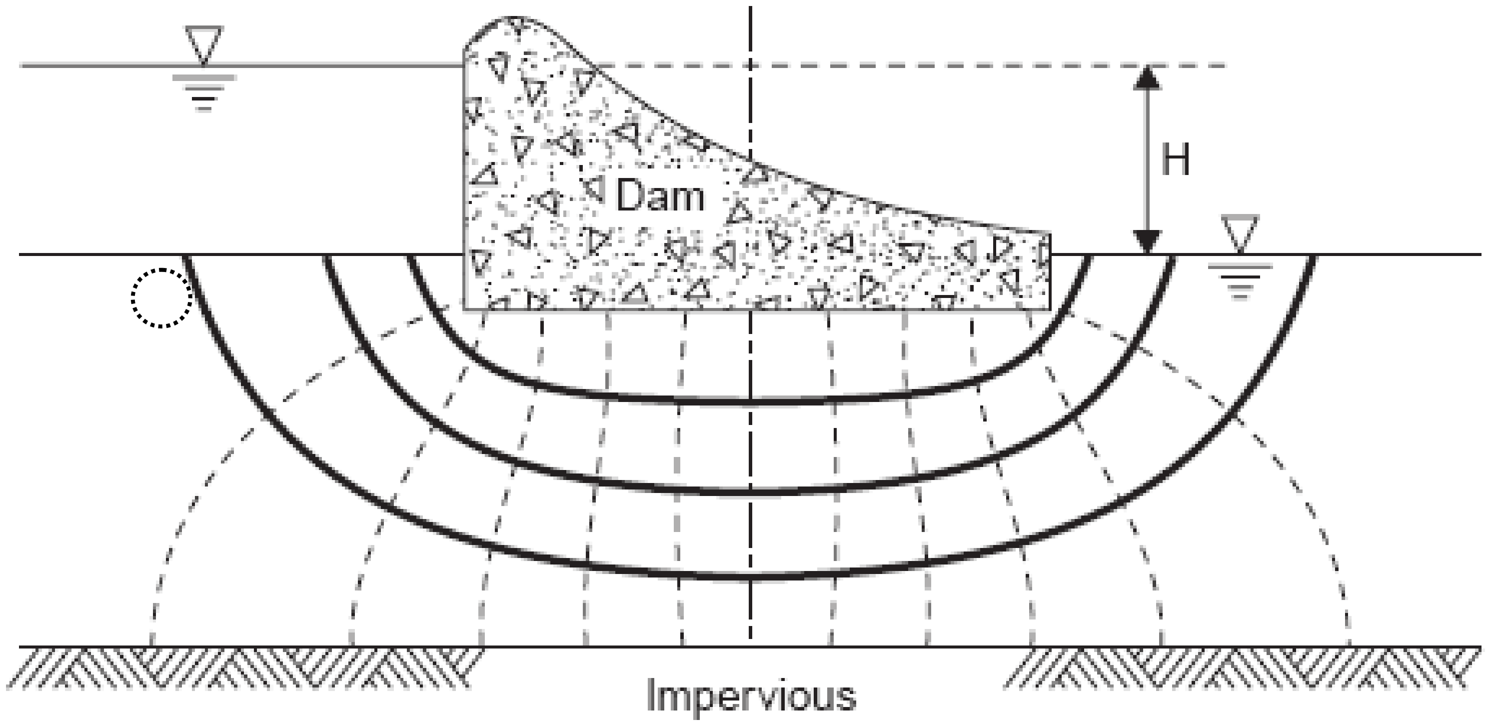
(b)

A combination of number of flow lines and equipotential lines is called a ***flow net***.

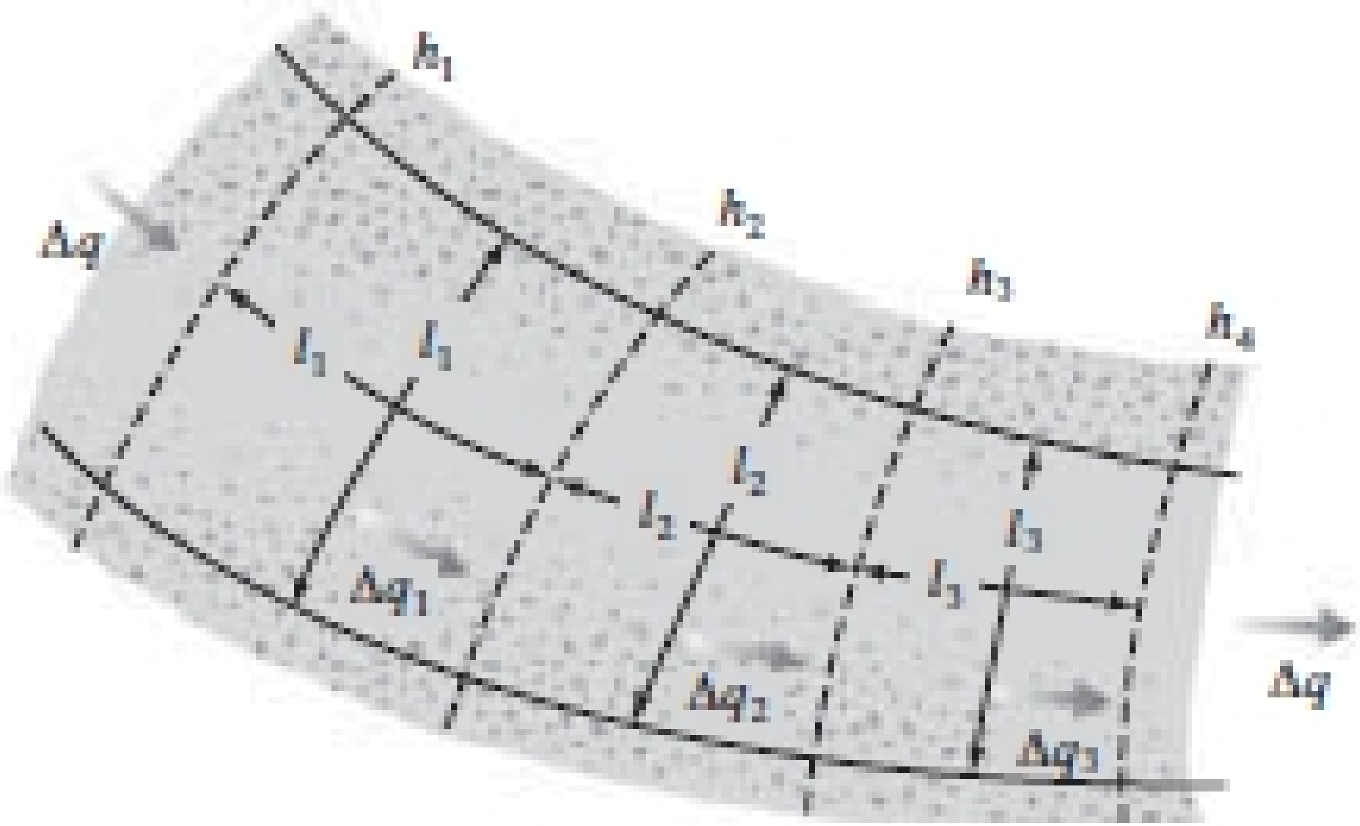
To construct a flow net, the flow and equipotential lines are drawn (see the above figure which is an example of a completed flow net) in such a way that

1. The equipotential lines intersect the flow lines at right angles.
2. The flow elements formed are approximate squares.

The following figure shows another example of a flow net in an isotropic permeable layer.


Dam
H
Impervious

● Seepage Calculation



Let h_1, h_2, h_3, h_4,……,h_n be the Piezometric levels

$$\Delta q_1 = \Delta q_2 = \Delta q_3 = \cdots = \Delta q$$

From Darcy's law, the rate of flow is equal to ***k.i.A*** . Thus

$$\Delta q = k\left(\frac{h_1 - h_2}{l_1}\right)l_1 = k\left(\frac{h_2 - h_3}{l_2}\right)l_2 = k\left(\frac{h_3 - h_4}{l_3}\right)l_3 = \cdots$$

So

$$h_1 - h_2 = h_2 - h_3 = h_3 - h_4 = \cdots = \frac{H}{N_d}$$

potential drop between any adjacent equipotential lines

And

$$\Delta q = k\,\frac{H}{N_d}$$

Where

H = the difference of head between the upstream and downstream sides

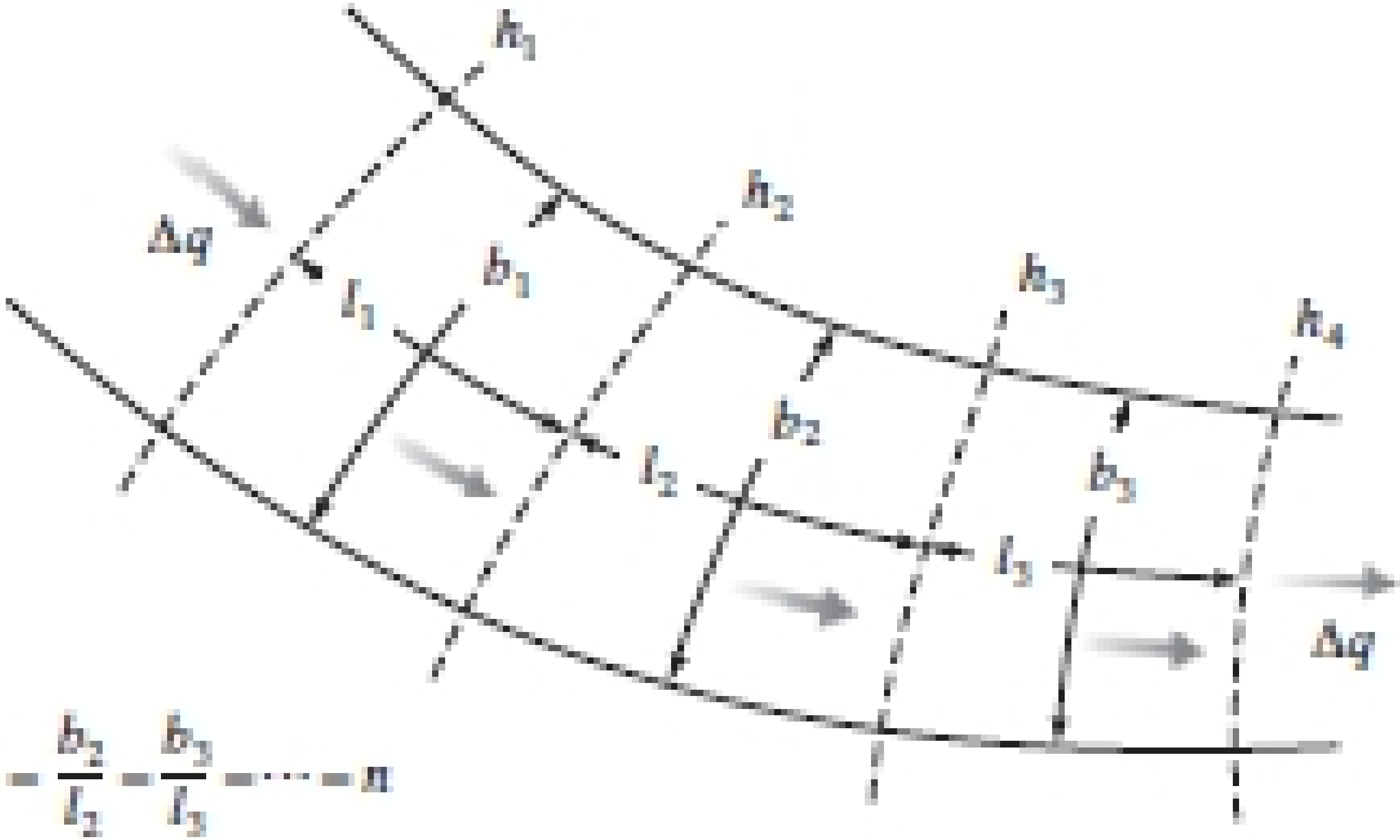
$\mathbf{N_d}$ = number of potential drops

If the number of flow channels in a flow net is equal to N_f , then

$$q = k.H.\frac{N_f}{N_d} = k.H.\oint$$

Where $\oint$ shape factor of the flow net

$$\oint = \frac{N_d}{N_f}$$



$$\frac{b_1}{l_1} = \frac{b_2}{l_2} = \frac{b_3}{l_3} = \cdots = n$$

$$\Delta q = k\left(\frac{h_1 - h_2}{l_1}\right)b_1 = k\left(\frac{h_2 - h_3}{l_2}\right)b_2 = k\left(\frac{h_3 - h_4}{l_3}\right)b_3 = \cdots$$

If $\dfrac{b_1}{l_1} = \dfrac{b_2}{l_2} = \dfrac{b_3}{l_3} = \cdots = n$.

So

$$\Delta q = k.H.\left(\frac{n}{N_d}\right)$$

$$\therefore q = k.H.\left(\frac{N_f}{N_d}\right).n = k.H.\oint.n$$

for square elements n =1

In general the flow nets may contain square and rectangular elements, in that case we can solve the problem by treating each part separately then we get the sum of the parts.

Flow nets in anisotropic meduim

In nature, most soils exhibit some degree of anisotropy. So to account for soil anisotropy with respect to permeability, some modification of the flow net construction is necessary.

The differential equation of continuity for two – dimensional flow in anisotropic soil, where $k_x \neq k_z$, is

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

in that case the equation represents two families of curves that do not meet at 90^o .
However, we can rewrite the preceding equation as

$$\frac{\partial^2 h}{(k_z / k_x)\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

Substituting $x' = \sqrt{k_z / k_x}\,.x$

then

$$\frac{\partial^2 h}{\partial x'^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

To construct the flow net, use the following procedures:

1. Adopt a vertical scale (that is, z – axis) for drawing the cross – section.

2. Adopt a horizontal scale (that is, x – axis) such that horizontal scale = $\sqrt{k_z / k_x} \cdot$ (vertical scale).
3. With scales adopted in steps 1 and 2, plot the vertical section through the permeable layer parallel to the direction of flow.
4. Draw the flow net for the permeable layer on the section obtained from step 3, with flow lines intersecting equipotential lines at right angles and the elements as approximate squares.

Depending on the problem geometry, we can also adopt transformation in the z – axis direction in the same manner describe above by adopting horizontal scale and then vertical scale will equal horizontal scale multiplying by $\sqrt{k_x / k_z}$

i.e. that the continuity equation will be written as follow:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z'^2} = 0 \quad \text{where} \quad z' = \sqrt{k_x / k_z}\,.z$$

The rate of seepage per unit width can be calculated by the following equation

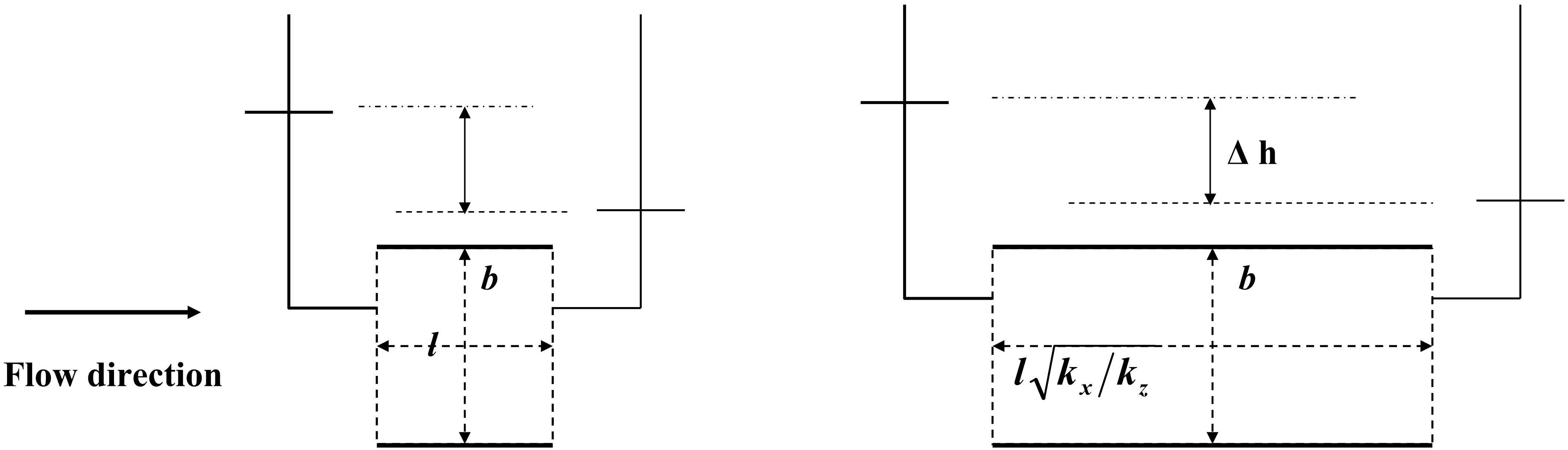
$$q = k_e.H.\oint = \sqrt{k_x.k_z}\,.H.\frac{N_f}{N_d}$$

Where

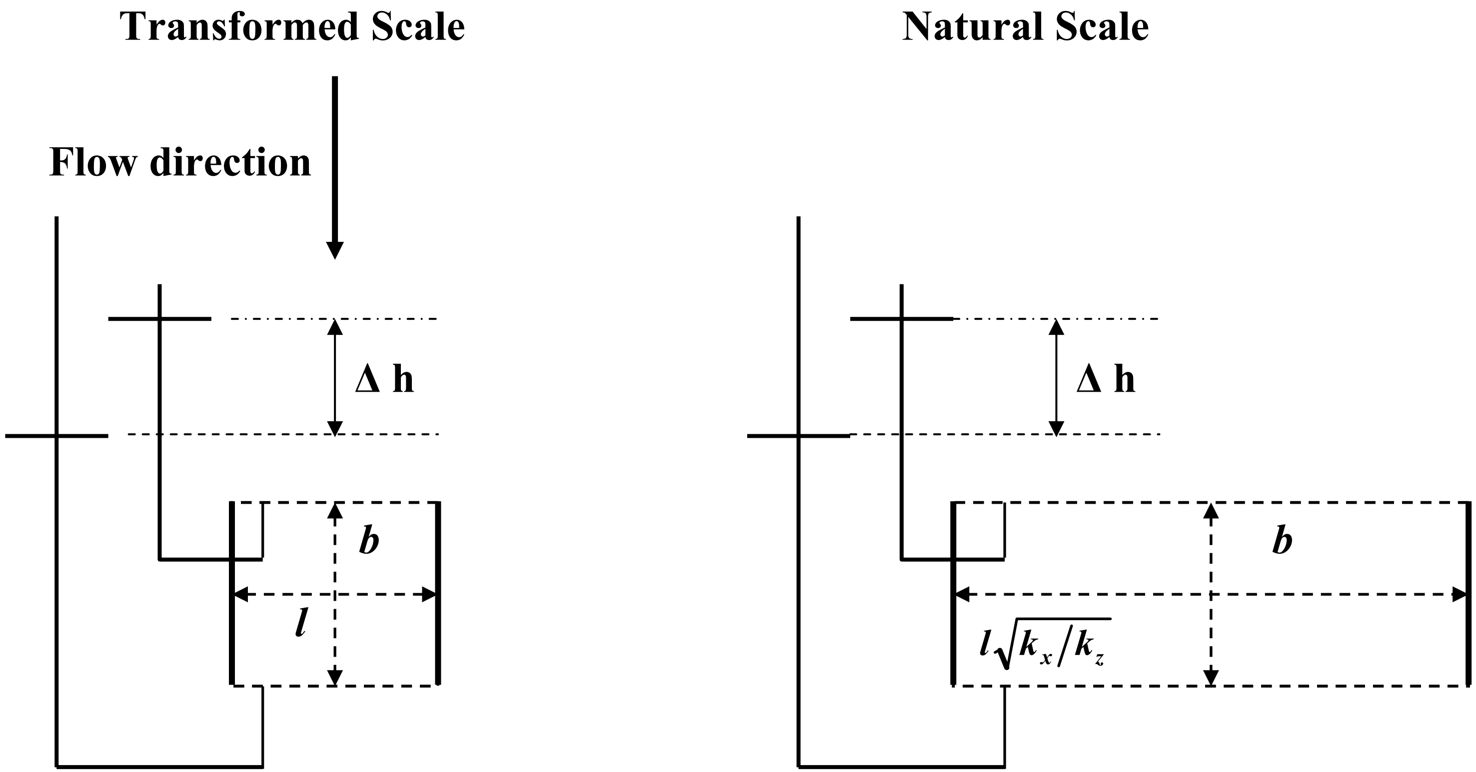
k_e = effective permeability to transform the anisotropic soil to isotropic soil

To prove that $k_e = \sqrt{k_x.k_z}$ whatever is the direction of flow let us consider two elements one from a flow net drawn in natural scale the other one drawn in transformed scale as shown below.

Transformed Scale **Natural Scale**



$$\boxed{k_e \cdot \frac{\Delta h}{l} b(1) = k_x \frac{\Delta h}{l\sqrt{k_x/k_z}} \Rightarrow k_e = \sqrt{k_x . k_z}}$$



$$k_e \frac{\Delta h}{b} l(1) = k_z \frac{\Delta h}{b} .l\sqrt{k_x/k_z} \Rightarrow k_e = \sqrt{k_x .k_z}$$

In the anisotropic soil, the permeability coefficient having a maximum value in the direction of stratification and a minimum value in the direction normal to that of stratification: these directions are devoted by x & z i.e.

$\boldsymbol{k_x} = \boldsymbol{k}_{\max}$ and $\boldsymbol{k_z} = \boldsymbol{k}_{\min}$

From Darcy' s law

$$\boldsymbol{v_x} = \boldsymbol{k_x}.\boldsymbol{i_x} = \boldsymbol{k_x}.\left(-\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{x}}\right)$$

$$\boldsymbol{v_z} = \boldsymbol{k_z}.\boldsymbol{i_z} = \boldsymbol{k_z}.\left(-\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{z}}\right)$$

Also, in any direction S, inclined at angle α to the x – direction

$$v_s = k_s . i_s = k_s . \left(-\frac{\partial h}{\partial s} \right)$$

Now

$$\frac{\partial h}{\partial S} = \frac{\partial h}{\partial x} . \frac{\partial x}{\partial S} + \frac{\partial h}{\partial z} . \frac{\partial z}{\partial S}$$

$$\frac{\partial x}{\partial S} = \cos \alpha$$

$$\frac{\partial z}{\partial S} = \sin \alpha$$

$$\frac{v_s}{k_s} = \frac{v_x}{k_x} \cos \alpha + \frac{v_z}{k_z} \sin \alpha$$

Also

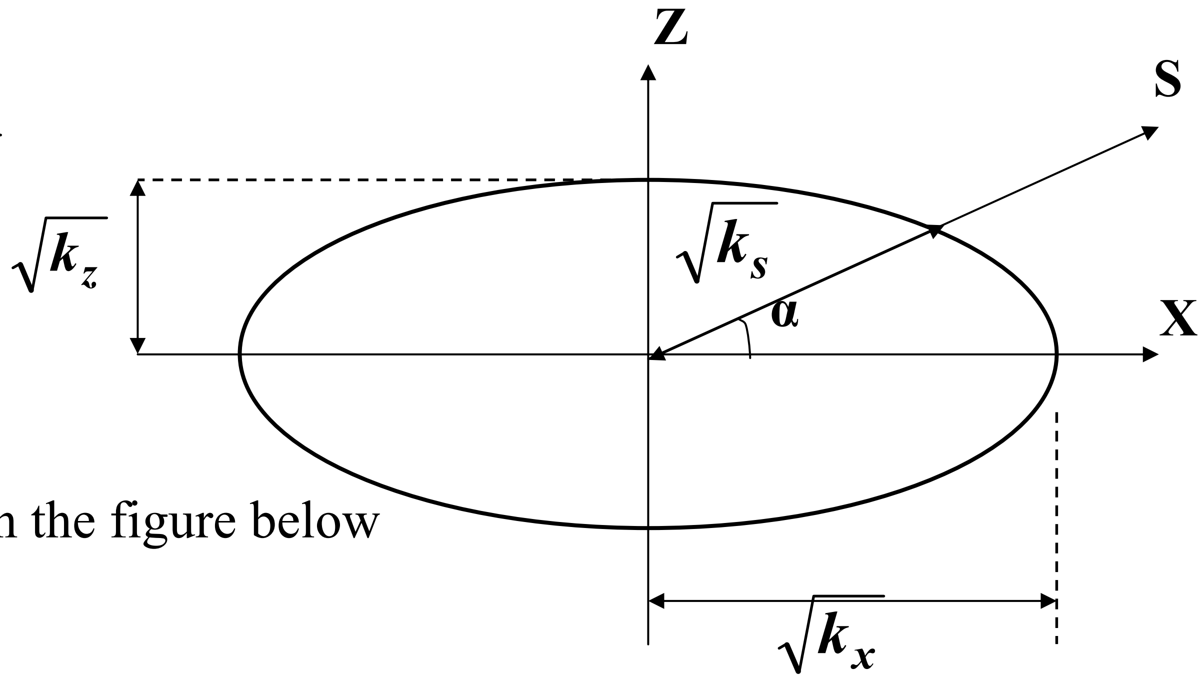
$$v_x = v_s \cos\alpha$$
$$v_z = v_s \sin\alpha$$

$$\therefore \frac{1}{k_s} = \frac{\cos^2\alpha}{k_x} + \frac{\sin^2\alpha}{k_z}$$

Or

$$\frac{S^2}{k_s} = \frac{x^2}{k_x} + \frac{z^2}{k_z}$$

is in the form of the ellipse as shown in the figure below



Permeability Ellipse

Transfer Condition

In case of flow perpendicular to soil strata, the loss of head and rate of flow are influenced primarily by the less pervious soil whereas in the case of flow parallel to the strata, the rate of flow is essential controlled by comparatively more pervious soil.

The following shows a flow channel (part of two – dimensional flow net) going from soil A to soil B with $k_A \neq k_B$ (two layers). Based on the principle of continuity, i.e., the same rate of flow exists in the flow channel in soil A as in soil B, we can derive the relationship between the angles of incident of the flow paths with the boundary for the two flow channels. Not only does the direction of flow change at a boundary between soils with different permeabilities, but also the geometry of the figures in the flow net changes. As can be seen in the figure below, the figures in soil B are not squares as is the case in soil A, but rather rectangles.

$$\Delta q_A = \Delta q_B$$

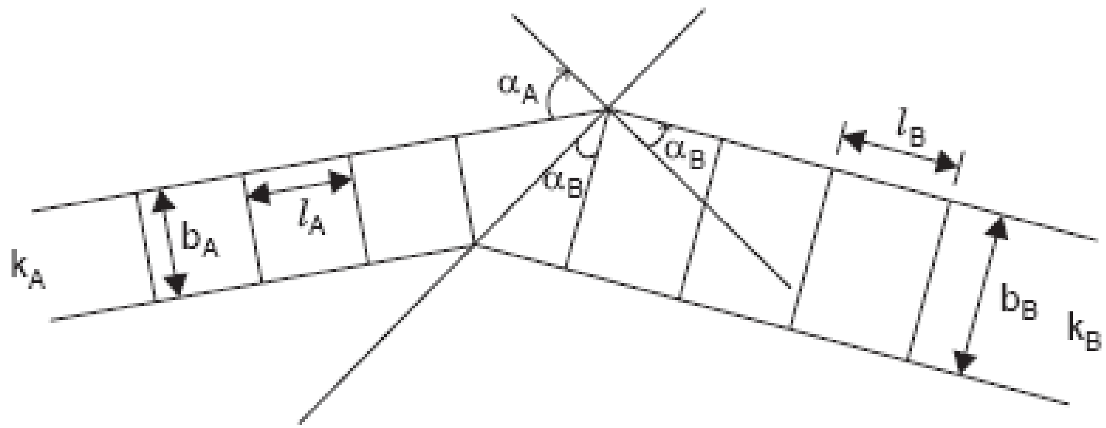
$$\Delta q_A = k_A \frac{\Delta h}{l_A} b_A$$

$$\Delta q_B = k_B \frac{\Delta h}{l_B} b_B$$

$$k_A \frac{\Delta h}{l_A} b_A = k_B \frac{\Delta h}{l_B} b_B$$

$$\frac{l_A}{b_A} = \tan\alpha_A \cdots and \cdots \frac{l_B}{b_B} = \tan\alpha_B$$

$$\frac{k_A}{\tan\alpha_A} = \frac{k_B}{\tan\alpha_B} \Rightarrow \frac{k_A}{k_B} = \frac{\tan\alpha_A}{\tan\alpha_B}$$

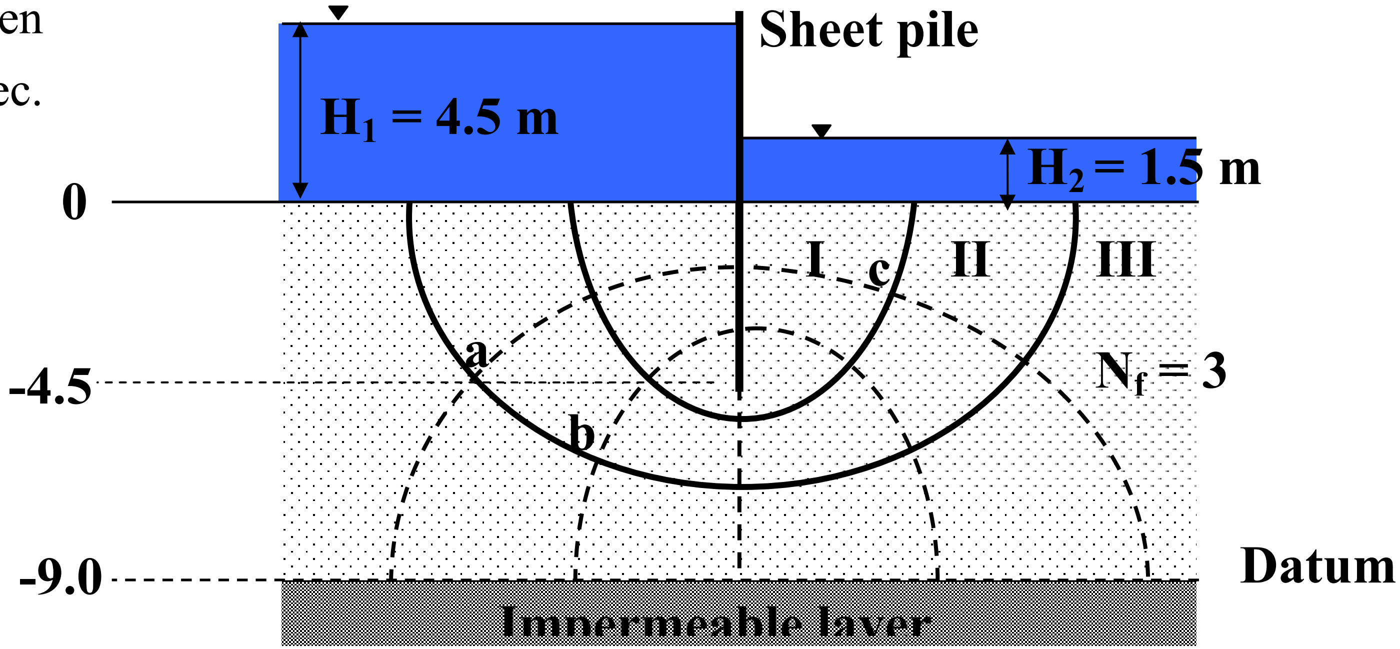


Example

A flow net for flow around single row of sheet piles in a permeable soil layer is shown in the figure. Given $k_x = k_z = k = 5x10^{-3}$ cm/sec.

Determine:

1. How high (above the ground surface) the water will rise if piezometers are placed at points a, b, c, and d.
2. The total rate of seepage through the permeable layer per unit width.
3. The rate of seepage through the flow channel II per unit width (perpendicular to the section shown)



Point	Potential drop, m	Rise above the ground surface, m
A	1x 0.5 = 0.5	4.5 – 0.5 = 4.0
B	2 x 0.5 = 1.0	4.5 – 1.0 = 3.5
C	5 x 0.5 = 2.5	4.5 – 2.5 = 2.0
D	5 x 0.5 = 2.5	4.5 – 2.5 = 2.0

Solution

a. H = 4.5 – 1.5 = 3.0 m So, head loss / drop = $\frac{3}{6}$ = 0.5 m drop

b. $q = k.H.\oint = k.H.\frac{N_f}{N_d} = 0.05x10^{-3}(3.0)\frac{3}{6} = 7.5x10^{-5}$ m^3 / sec / m length

c. $\Delta q = k\frac{H}{N_d} = 0.05x10^{-3}.\frac{3}{6} = 2.5x10^{-5}$ m^3 /sec /m length

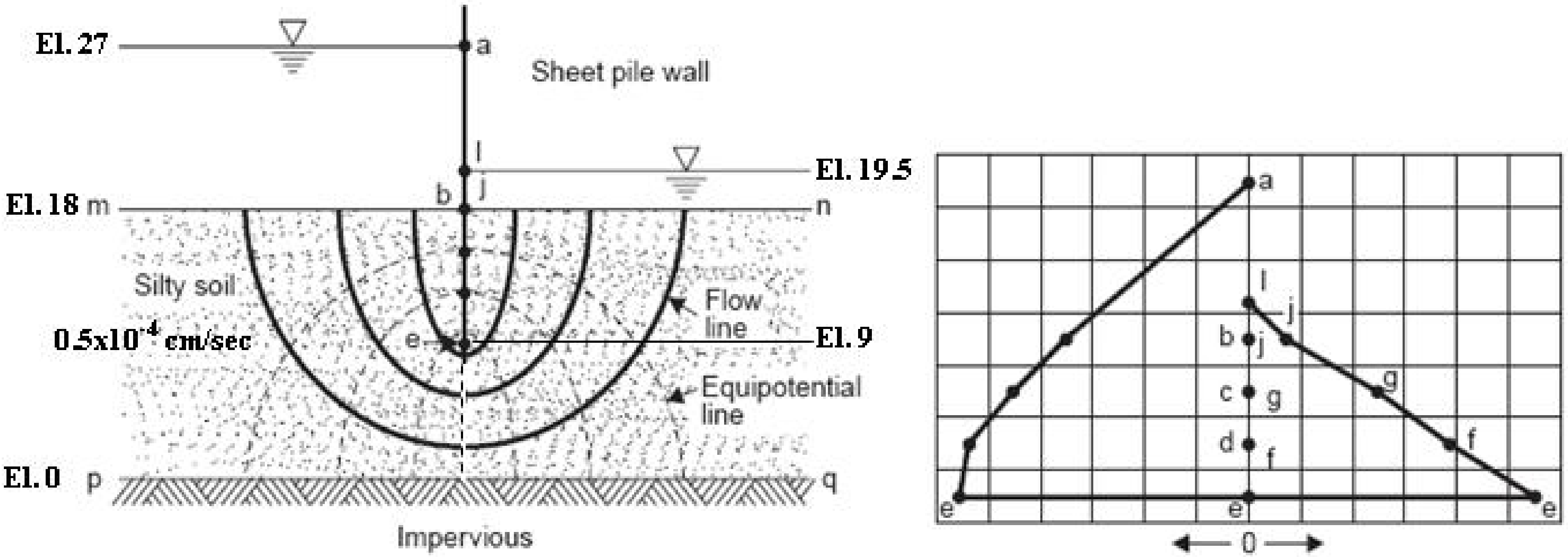
- Seepage pressure and Uplift Pressure

1. Seepage Pressure on Sheet Piles

Example

Given. Flow net in the following figure

Find. Pore pressure at points a to i; quantity of seepage; exit gradient.



(a) Flow net

(b) Water pressure on the wall

The water pressure plot, such shown in the above figure, is useful in the structural design of the wall and in study of water pressure differential tending to cause leakage through the wall.

Total head loss H = 27 – 19.5 =7.5 m Head loss /drop = $7.5/8 = 0.9375$ m

Let γ_w = 10 kN/m^2

Point	h_e , m	h_t , m	h_p , m	Water pressure kN/m^2
a	27	27.0	0	0
b	18	27.0	9.0	90
c	14.7	27 – 1x0.9375 = 26.0625	11.325	113.25
d	11.7	27 – 2x0.9375 = 25.125	13.425	134.25
e	9.0	27 – 4x0.9375 = 23.25	14.25	142.5
f	11.7	27 – 6x0.9375 = 21.375	9.675	96.75
g	14.7	27 – 7x0.9375 = 20.4375	5.7375	57.375
h	18.0	27 – 8x0.9375 = 19.5	1.50	15.0
i	19.5	19.50	0	0

Seepage under wall

$$q = kH\oint = 5x10^{-9}(7.5)\frac{4}{8} = 18.75x10^{-9} \text{ m}^3 \text{/sec / m. length}$$

Exit gradient

$$i = \frac{\Delta h}{l} = \frac{1.25}{3.45} = 0.362$$

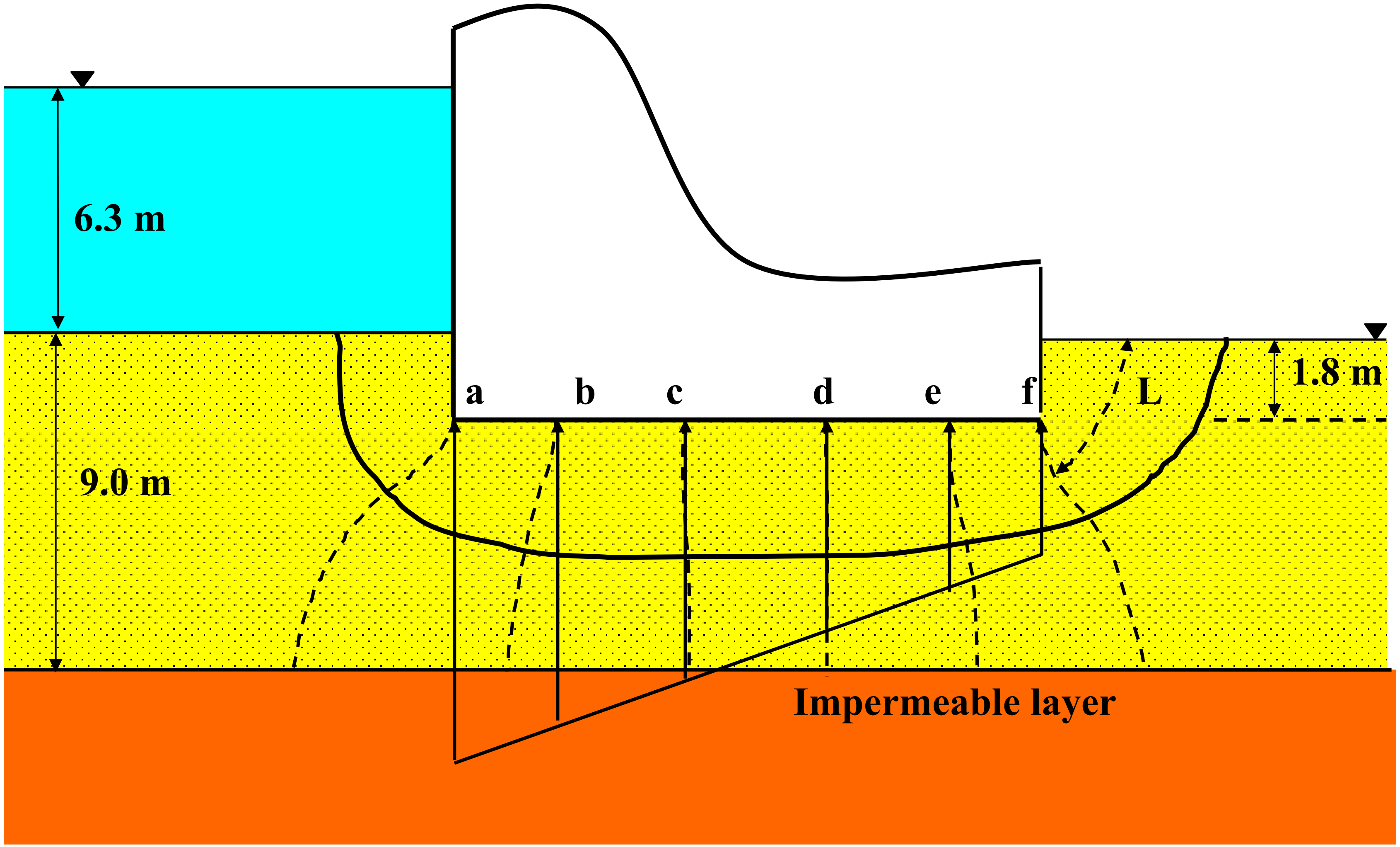
2. Uplift Pressure under Hydraulic structures

Example

The following figure shows a weir, the base of which is 1.8 m below the ground surface. The necessary flow net also been drawn (assuming $k_x = k_z = k$).

H = 6.3 m.

So, the loss of head for each potential drop is $H/7 = 6.3/7 = 0.9$ m.


6.3 m
9.0 m
a
b
c
d
e
f
L
1.8 m
Impermeable layer

The total head at the ground level in the upstream side = 6.3 + 1.8 = 8.1 m

Let γ_w = 10 kN/m^2

Point	Total head, h_t	Pressure head, h_p	Uplift pressure, kN/m^2 $U = h_p \times \gamma_w$
A	$8.1 - 1x0.9 = 7.2$	**7.2**	**72**
B	$8.1 - 2x0.9 = 6.3$	**6.3**	**63**
C	$8.1 - 3x0.9 = 5.4$	**5.4**	**54**
D	$8.1 - 4x0.9 = 4.5$	**4.5**	**45**
E	$8.1 - 5x0.9 = 3.6$	**3.6**	**36**
F	$8.1 - 6x0.9 = 2.7$	**2.7**	**27**

i_{exit} = 0.9 / L

High value of exit gradient will affect the stability of the structure and a factor of safety will be applied. This will discussed later