Effective Stress Concept

Topics

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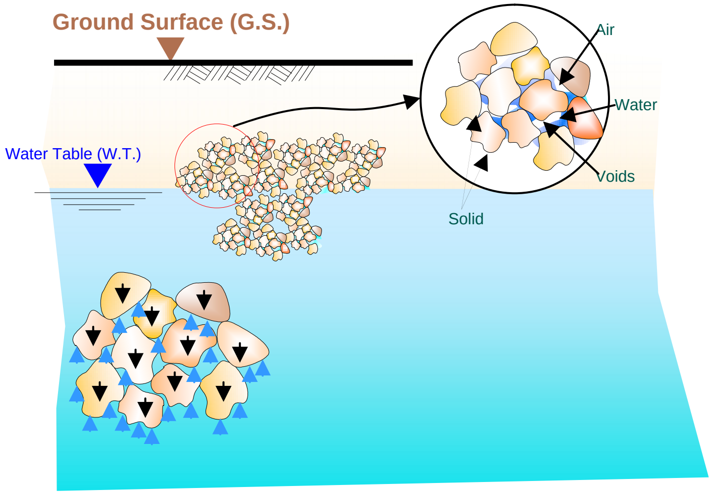
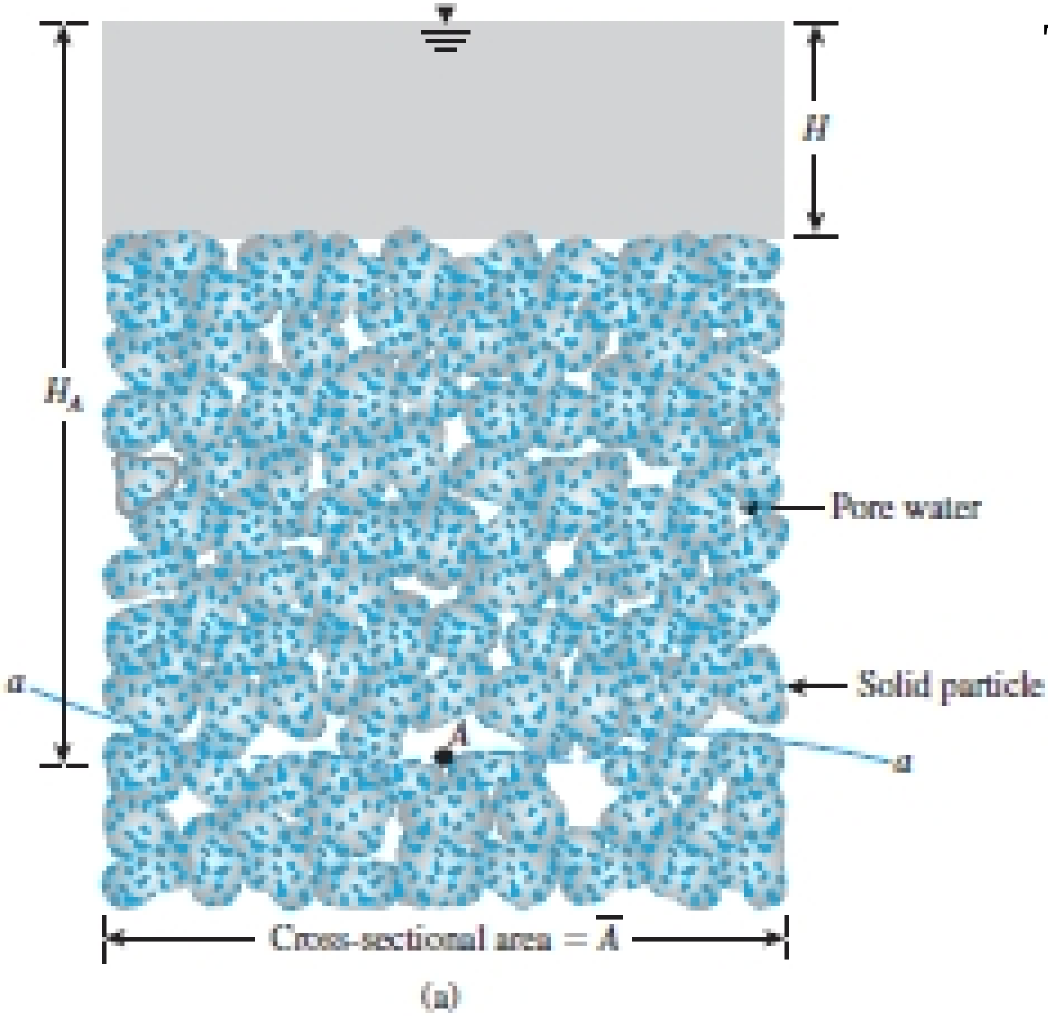
- Effective Stress in Saturated Soil with Seepage
- Seepage Force
- Filter Requirements and Selection of Filter Material
- Capillary Rise in Soil
- Effective Stress in Capillary Zone



● *Effective Stress Concept*

- Soil is a multi phase system
- To perform any kind of analysis - we must understand stress distribution
- The concept of effective stress:
  - The soil is “loaded” (footing for example)
  - The resulting stress is transmitted to the soil mass
  - The soil mass supports those stresses at the point to point contacts of the individual soil grains



The total stress at A is calculated from:

The weight of the soil above A

The weight of the water above A

$\sigma = H\gamma_w + (H_A - H)\,\gamma_{sat}$

σ = Total Stress at A

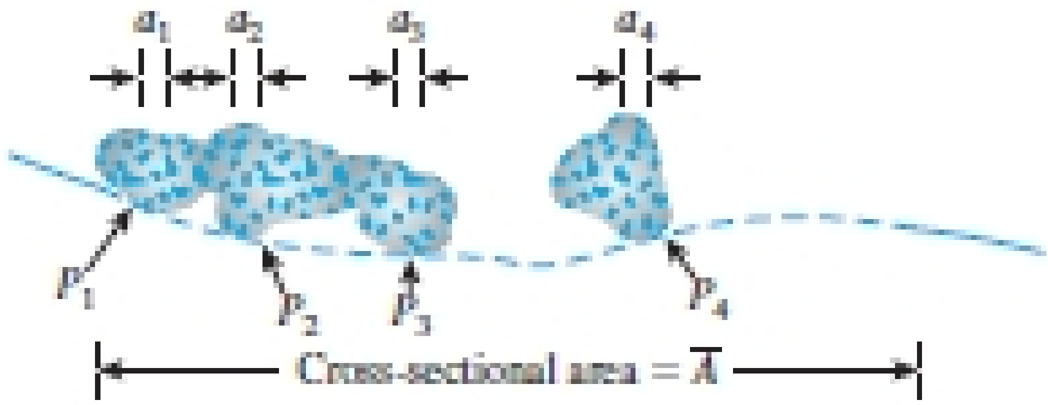
γ_w = Unit Weight of Water

γ_{sat} = Saturated Unit Weight

H_A = Height of A to Top of water

H = Height of water

- **σ** is the stress applied to the soil by its own weight
- As you go deeper in the soil mass, the stress increases
- The soil carries the stress in 2 ways:
    - A portion is carried by the water (acts equally in all directions)
    - A portion is carried by the soil solids at their point of contact.



(b)

$\sigma' = (P_{1v}+P_{2v}+P_{3v} \,.....+P_{nv}) / A$

If $a_s = a_1 + a_2 + a_3 +...a_n$

Then the space occupied by water = $A - a_s$

Assume $u = H_A \gamma_w$ $\qquad$ H_A = Height of water

$\sigma = \sigma' + u(A - a_s) / A$

Since a_s is very small, assume = 0

$\sigma = \sigma' + u$

Recall the following equation:

$\sigma = H\gamma_w + (H_A - H)\, \gamma_{sat}$

Now, $\sigma' = \sigma - u$

Substituting: $\sigma' = [H\gamma_w + (H_A - H)\, \gamma_{sat}] - H_A\, \gamma_w$

Rearranging: $\sigma' = (H_A - H)(\gamma_{sat} - \gamma_w) = H_{soil}\gamma'$

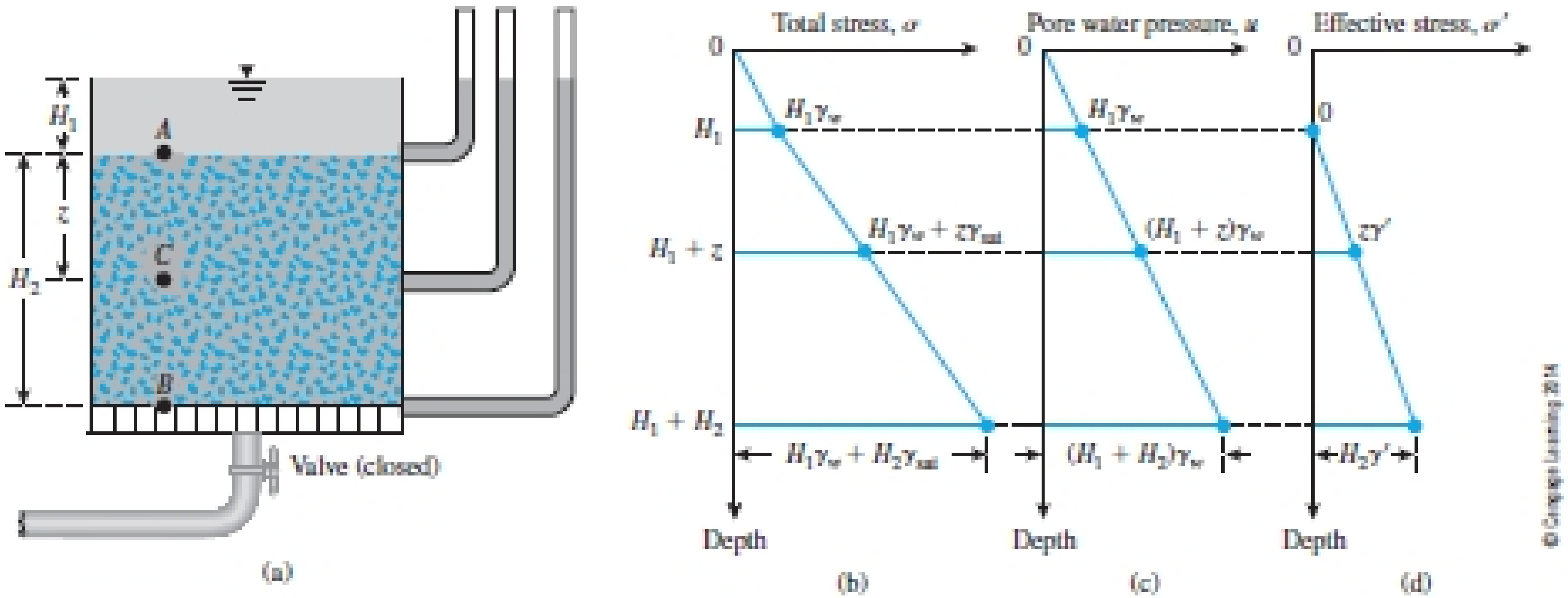
Effective Stress is independent of height of water

In the equation: $\sigma = \sigma' + u$

σ' is the soil skeleton stress

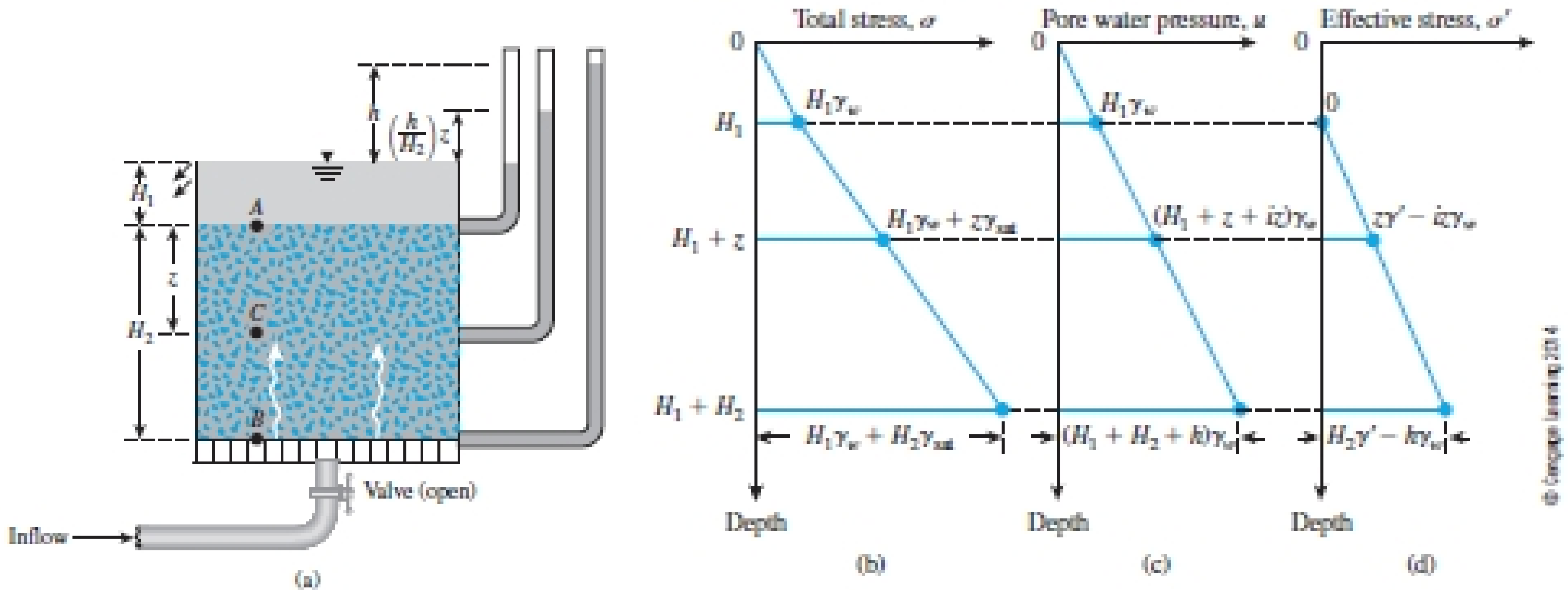
u is the stress in the water, or pore water pressure

● *Effective Stress in Saturated Soil with no Seepage*



●*Effective Stress in Saturated Soil with Seepage*

Upward flow



Note that the h/H_2 is the hydraulic gradient that caused flow therefore,

$$\sigma'_c = z\gamma' - iz\gamma_w$$

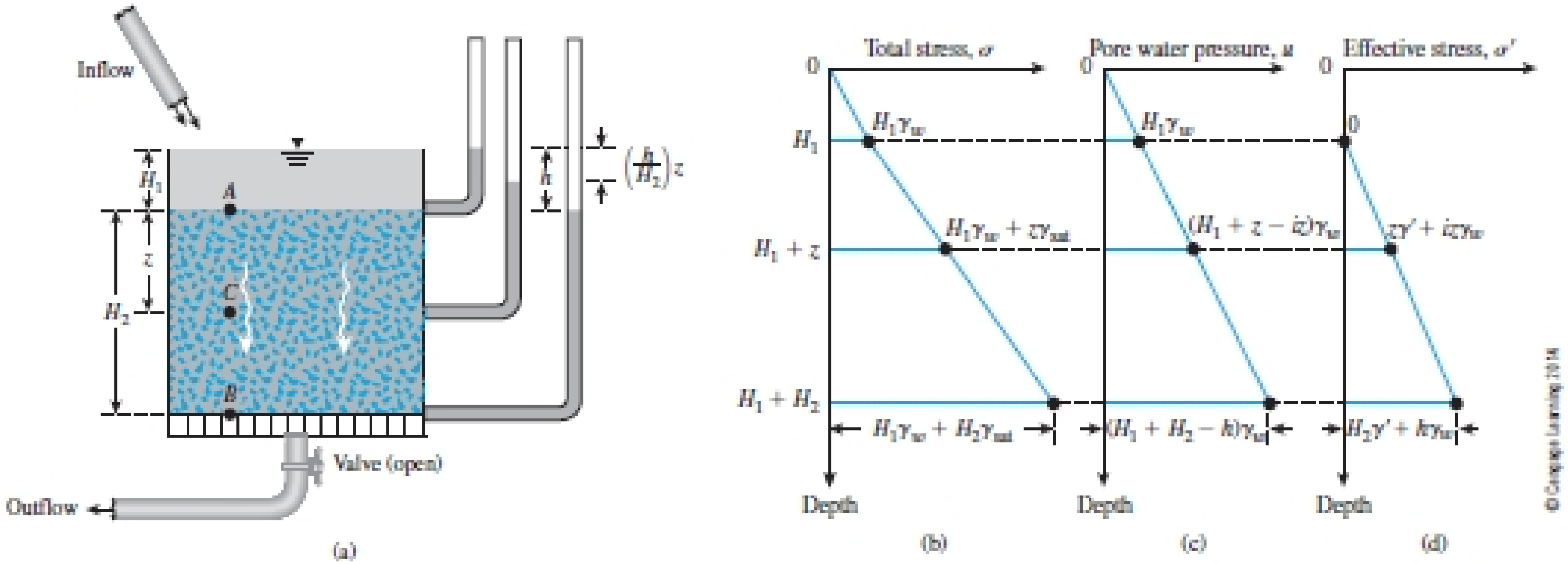
And limiting conditions may occur when $\sigma'_c = z\gamma' - iz\gamma_w = 0$ which lead to

i_{cr} = *critical hydraulic gradient*

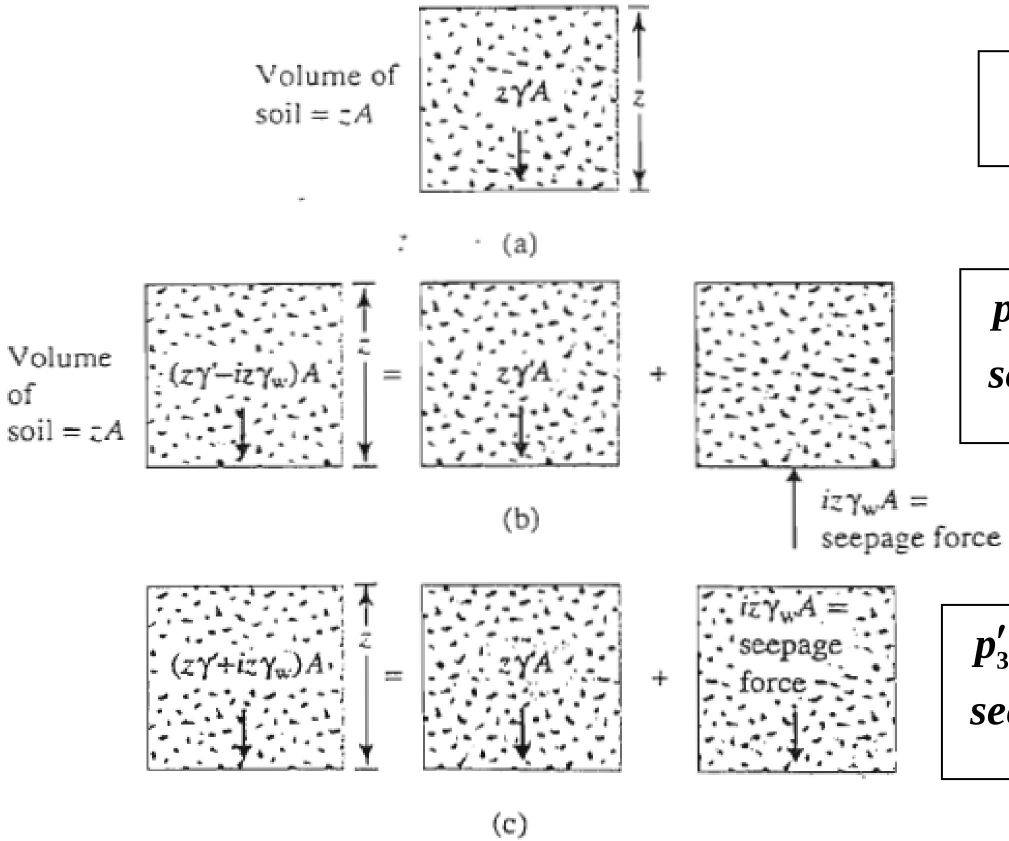
$$i_{cr} = \frac{\gamma'}{\gamma_w}$$

for most soils 0.9-1.1 ith average value of 1

Downward flow



● Seepage Force



$$p_1' = z\gamma$$

$$p_2' = (z\gamma' - iz\gamma_w)A = p_1' + \ seepage \ \ force$$

$$seepage \ \ force = -iz\gamma_w A$$

$$p_3' = (z\gamma' + iz\gamma_w)A = p_3' + seepage \ \ forc$$

$$seepage \ \ force = iz\gamma_w A$$

The volume of the soil contributing to the effective stress force equals zA, so the seepage force per unit volume of the soil is

$$\frac{iz\gamma_w A}{zA} = i\gamma_w$$

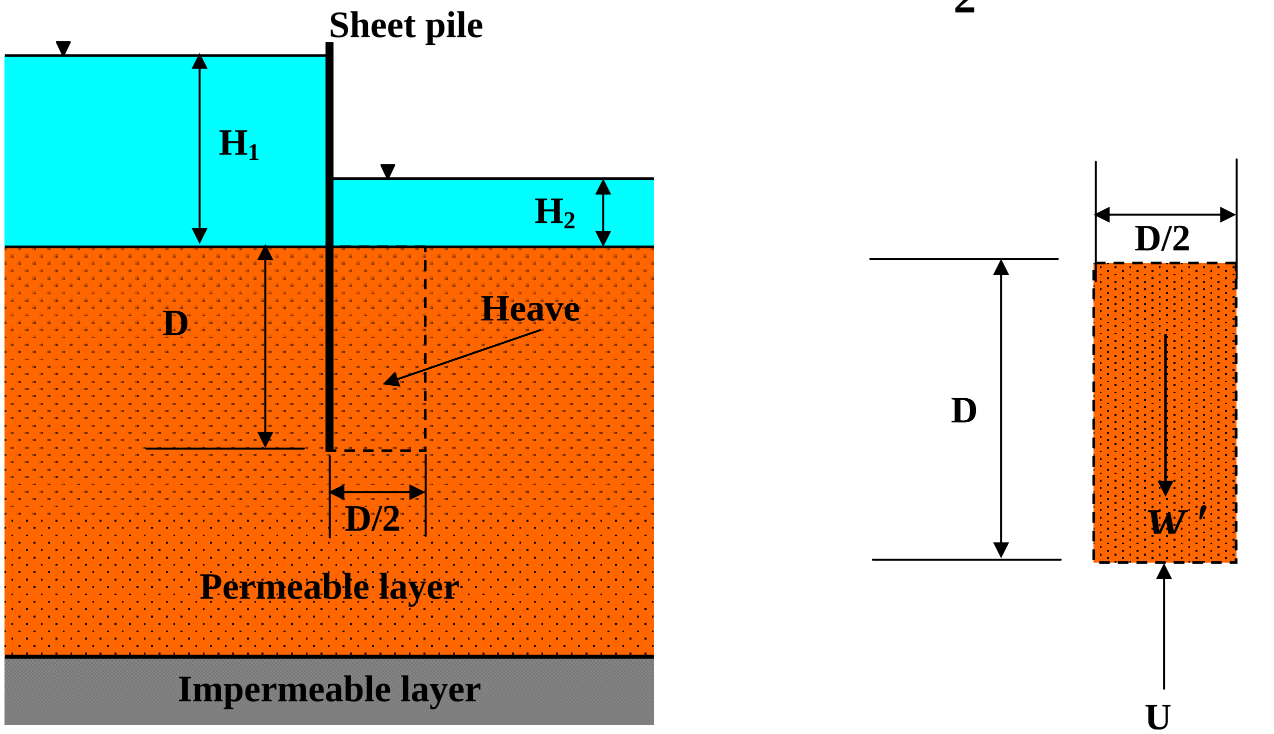
in the direction of seepage (see the fig.)

Therefore, in isotropic soil and in any direction, the force acts in the same direction as the direction of flow. Thus, the flow nets can be used to find the hydraulic gradient at any point to find seepage force at that point. This concept is useful to estimate F.S against heave

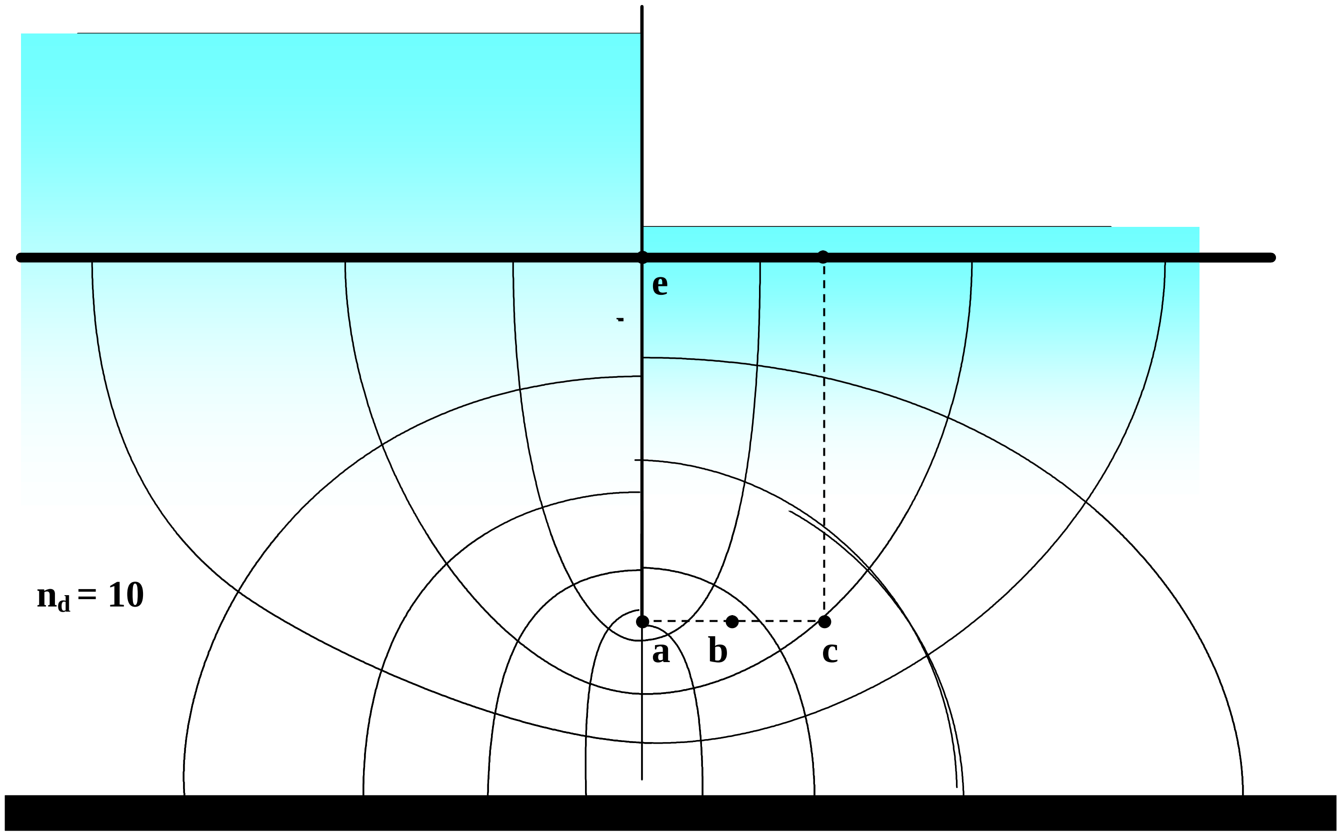
Factor of Safety against heave at the downstream of hydraulic structures

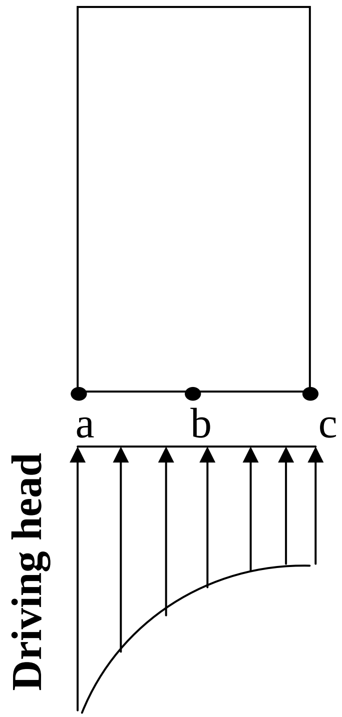
Terzaghi (1922)

$$F.S = \frac{Submerged\ weight}{Uplift\ \ force\ caused\ by\ seepage} = \frac{W'}{U} = \frac{D(D/2)(\gamma_{sat} - \gamma_w)}{soil\ volume\ \ x\ (i_{av}\gamma_w)} = \frac{\frac{1}{2}D^2\gamma'}{\frac{1}{2}D^2 i_{av}\gamma_w} = \frac{\gamma'}{i_{av}\gamma_w}$$



Estimation of i_{av}



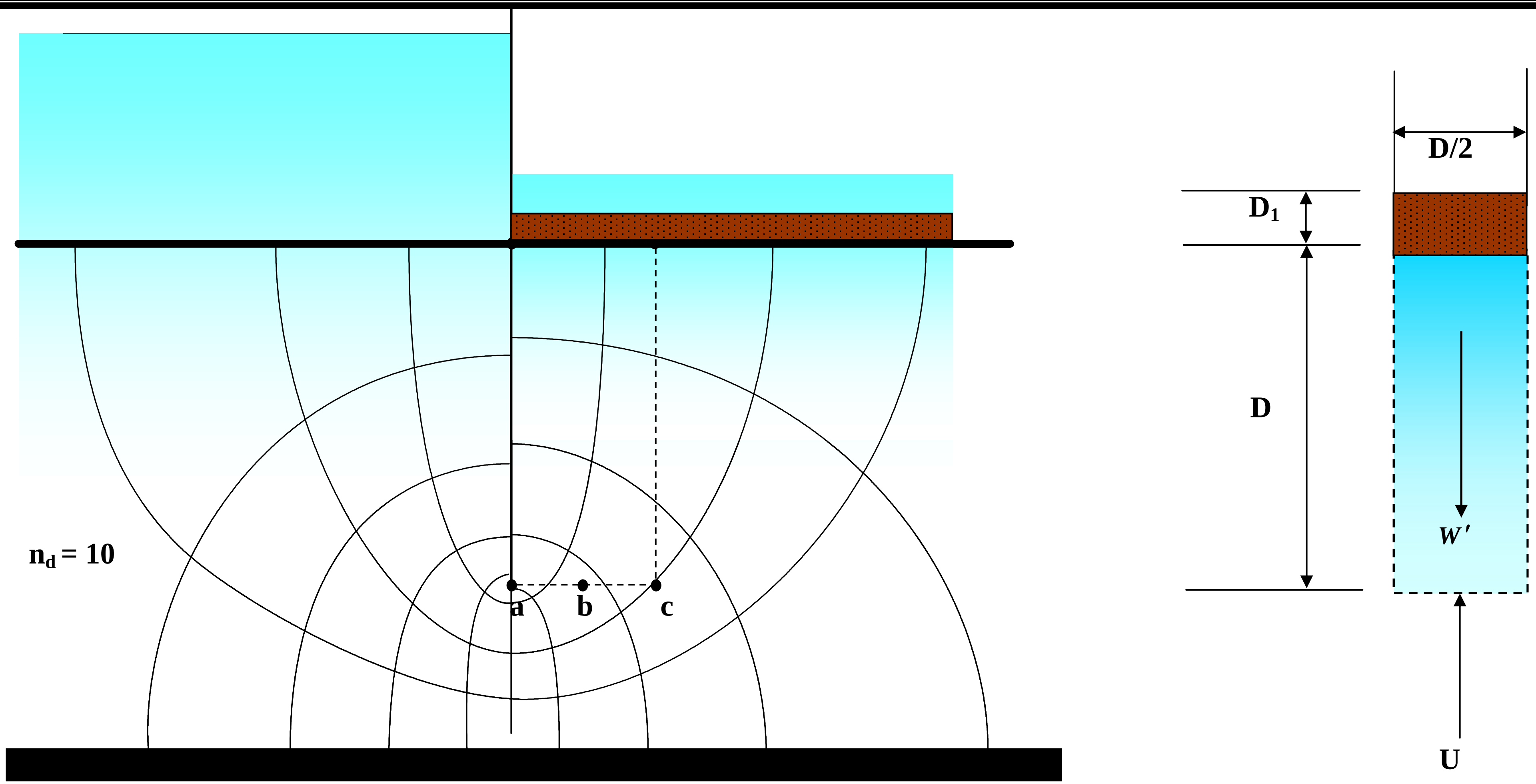


point	driving head
a	$\frac{4}{10} H$
b	$\frac{3.3}{10} H$
c	$\frac{2.5}{10} H$
$h_{av} \quad \frac{(h_a \quad h_c)/2 \quad h_b}{2}$ $i_{av} \quad \frac{h_{av}}{D}$	

• ***Filter Requirements and Selection of Filter Material***

In practice, for the safe of the hydraulic structure, a minimum value of 4 to 5 for F.S against heaving is used, because of the uncertainty in the analysis. One way to increase the F.S is using filter.

Filter:- is a granular material with opening small enough to prevent the movement of the soil particles upon which is placed and, at the same time, is previous enough to offer little resistance to seepage through it.


$n_d = 10$
a
b
c
D/2
D_1
D
W'
U

$$F.S=\frac{W'+W'_F}{U}=\frac{\frac{1}{2}D^2\gamma'+\frac{1}{2}DD_1\gamma'_F}{\frac{1}{2}D^2 i_{av}\gamma_w}=\frac{\gamma'+\left(\frac{D_1}{D}\right)\gamma'_F}{i_{av}\gamma_w}$$

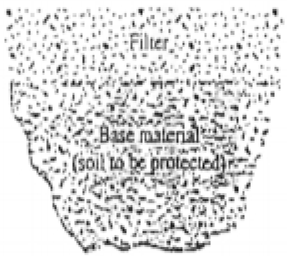
Selection of Filter Material

1. $\dfrac{D_{15(F)}}{D_{85(B)}} < 4$

2. $\dfrac{D_{15(F)}}{D_{15(B)}} > 4$

Here, $D_{15(F)}$, $D_{15(B)}$ = diameters through which 15% of the filter and base material, respectively, will pass

$D_{85(B)}$ = diameter through which 85% of the base material will pass



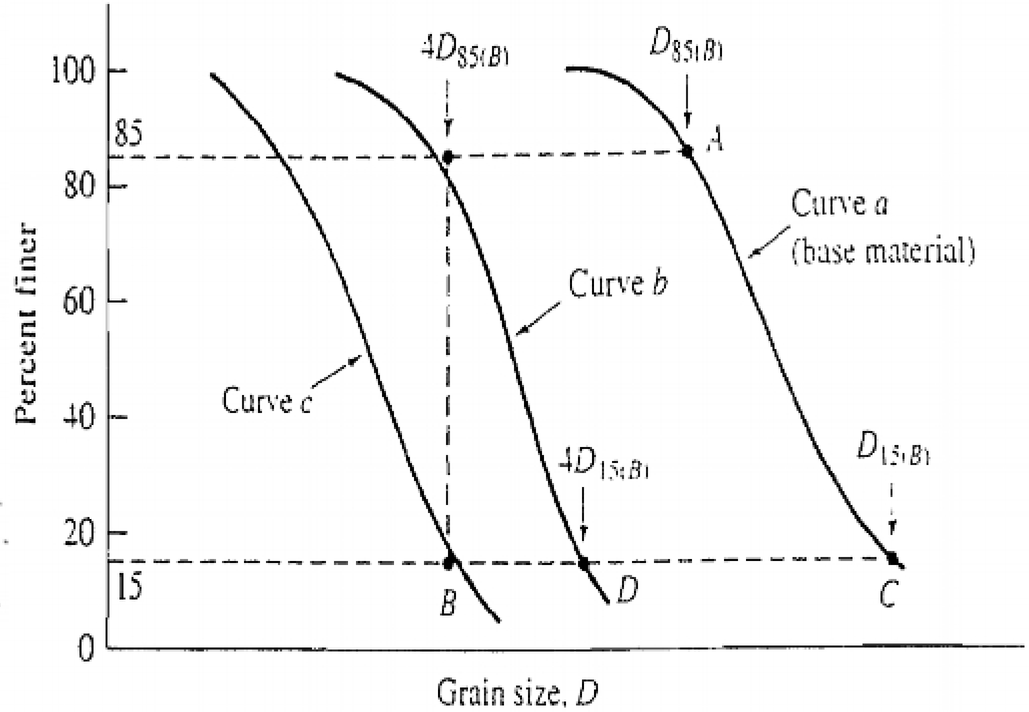


Figure 8.13 Definition of base material and filter material

Capillary Rise
Capillarity rise in Soil

$$\left(\frac{\pi}{4} d^2\right) h_c \gamma_w = \pi d T \cos \alpha$$

$$h_c = \frac{4T \cos \alpha}{d \gamma_w}$$

where T = surface tension (force/length)
α = angle of contact
d = diameter of capillary tube
γ_w = unit weight of water

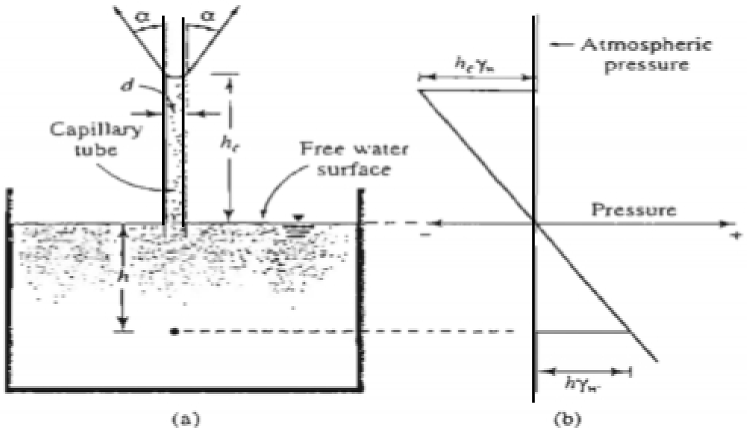


Figure 8.15 (a) Rise of water in the capillary tube; (b) pressure within the height of rise in the capillary tube (atmospheric pressure taken as datum)

For pure water and clean glass $\alpha = 0$

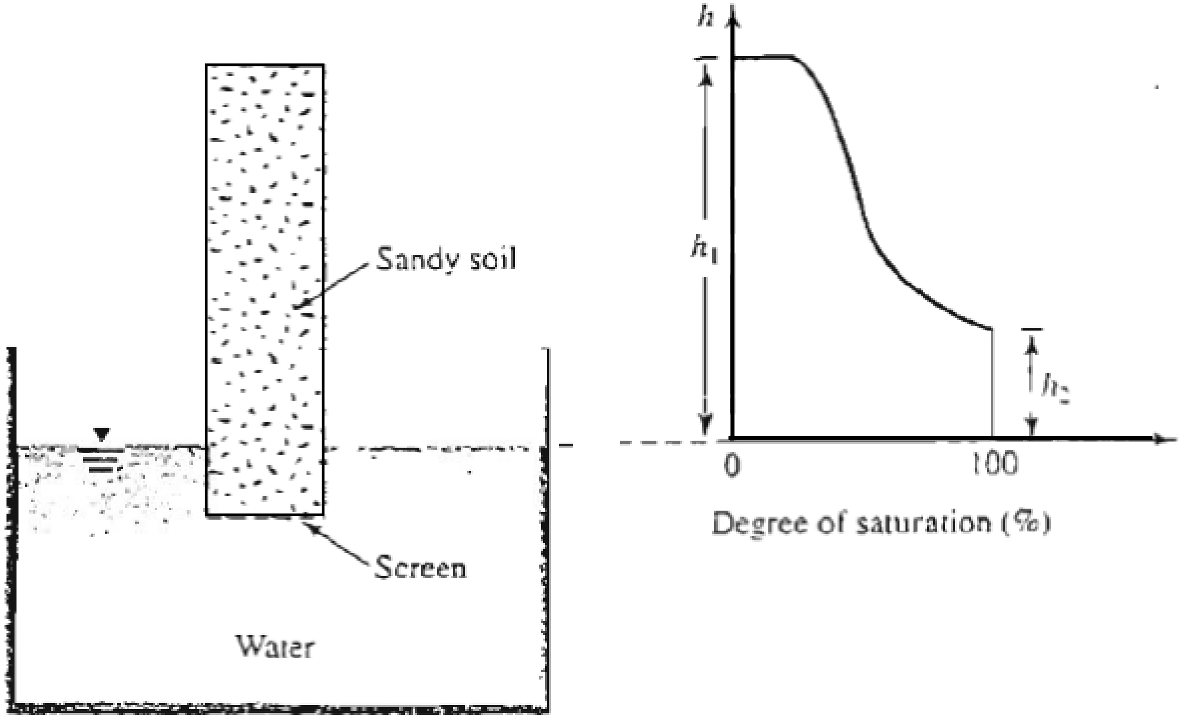
$$\therefore h_c = \frac{4T}{d\gamma_w}$$

For water T = 72 m.N/m

$$h_c \propto \frac{1}{d}$$

the smaller the capillarity tube, the larger capillary rise

For soils, the capillary tubes formed because of the continuity of voids have variable cross sections. The results of the nonuniformity on capillary can be demonstrated as shown in the fig.



(a) (b)

(a) a soil column in contact with water — Variation of ***S*** in the soil column

Capillary effect in sandy soil

Hazen (1930) give a formula to estimate the height of capillary

$$h_c\,(mm) = \frac{C}{eD_{10}}$$

where D_{10} = effective size (mm)
e = void ratio
C = a constant that varies from 10 to 50 mm^2

Approximate Range of Capillary Rise in Soils

Soil type	Range of capillary rise	
	m	ft
Coarse sand	0.1–0.2	0.3–0.6
Fine sand	0.3–1.2	1–4
Silt	0.75–7.5	2.5–25
Clay	7.5–23	25–75

● *Effective Stress in Capillary Zone*

The general relationship of effective stress is

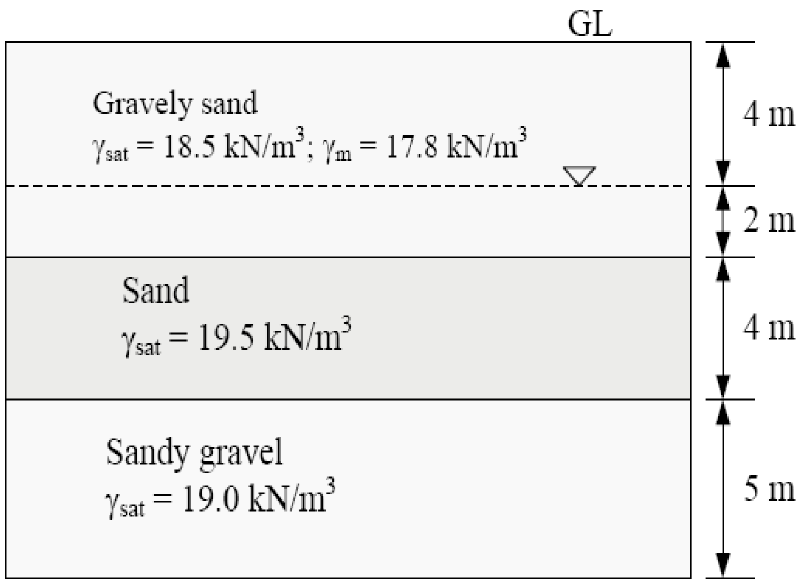
$$\sigma' = \sigma - u$$

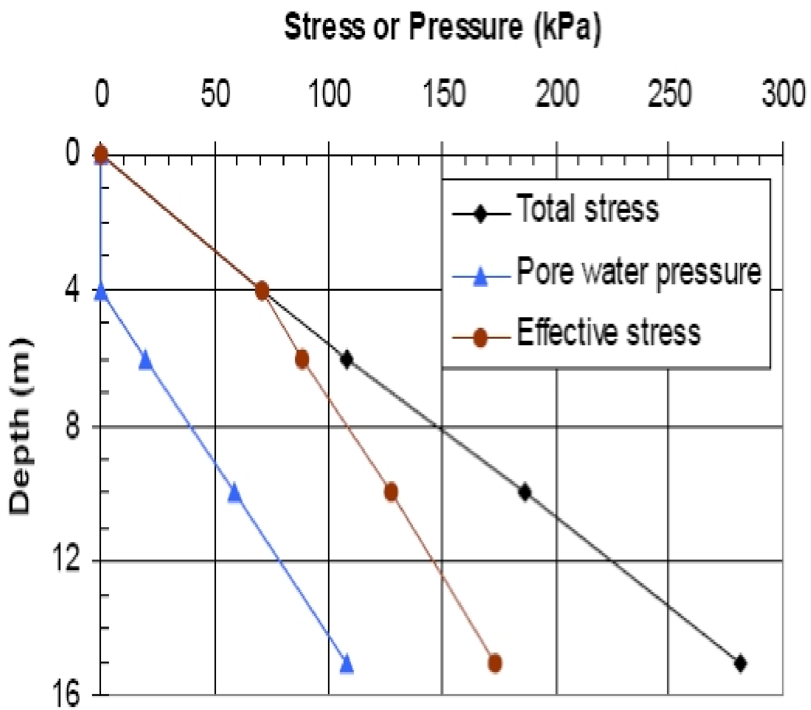
For soil fully saturated by capillary $\quad u = -h_c \gamma_w$

For soil partially saturated by capillary $\quad u = -\left(\frac{S}{100}\right) h_c \gamma_w$

● *Examples*

EXAMPLE1. Plot the variation of total and effective vertical stresses, and pore water pressure with depth for the soil profile shown below in Fig.





Solution:

Within a soil layer, the unit weight is constant, and therefore the stresses vary linearly. Therefore, it is adequate if we compute the values at the layer interfaces and water table location, and join them by straight lines.

At the ground level,

$\sigma_v = 0$; $\sigma_v' = 0$; and $u=0$

At 4 m depth,

$\sigma_v = (4)(17.8) = 71.2$ kPa; $u = 0$

$\therefore \sigma_v' = 71.2$ kPa

At 6 m depth,

$\sigma_v = (4)(17.8) + (2)(18.5) = 108.2$ kPa

$u = (2)(9.81) = 19.6$ kPa

$\therefore \sigma_v' = 108.2 - 19.6 = 88.6$ kPa

At 10 m depth,

$\sigma_v = (4)(17.8) + (2)(18.5) + (4)(19.5) = 186.2$ kPa

$u = (6)(9.81) = 58.9$ kPa

$\therefore \sigma_v' = 186.2 - 58.9 = 127.3$ kPa

At 15 m depth,

$\sigma_v = (4)(17.8) + (2)(18.5) + (4)(19.5) + (5)(19.0) = 281.2$ kPa

$u = (11)(9.81) = 107.9$ kPa

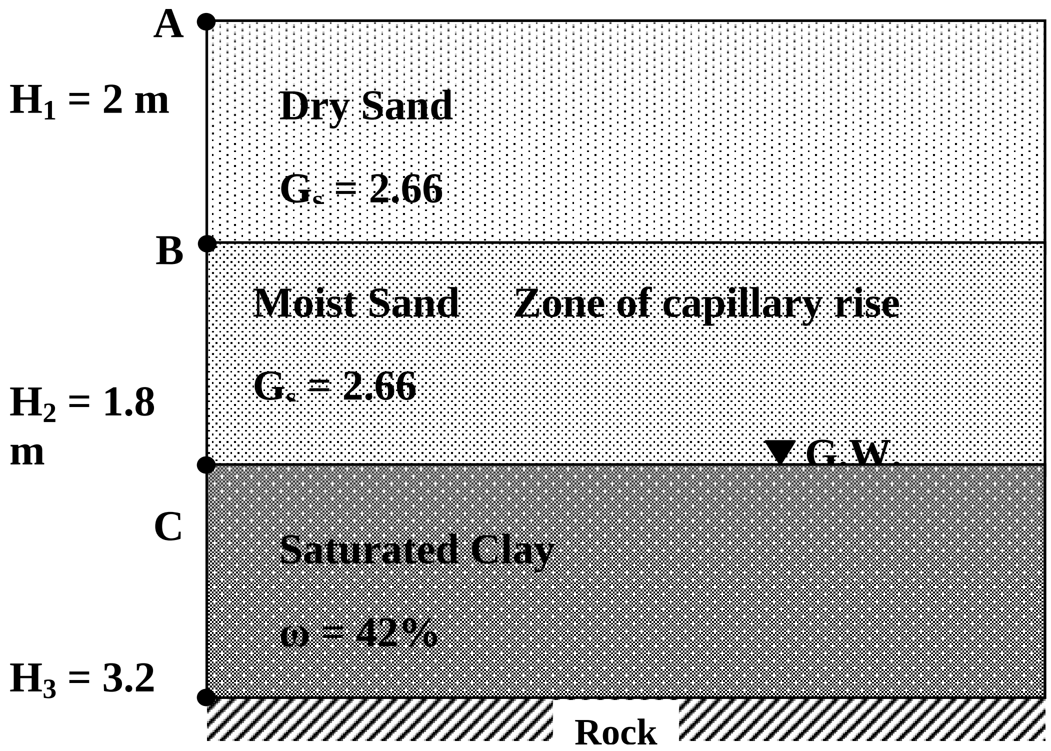
$\therefore \sigma_v' = 281.2 - 107.9 = 173.3$ kPa

The values of σ_v, u and σ'_v computed above are summarized in Table 1.

Table 6.1 Values of σ_v, u and σ'_v in Ex. 1

depth (m)	σ_v (kPa)	u (kPa)	σ_v' (kPa)
0	0	0	0
4	71.2	0	71.2
6	108.2	19.6	88.6
10	186.2	58.9	127.3
15	281.2	107.9	173.3

EXAMPLE2. Plot the variation of total and effective vertical stresses, and pore water pressure with depth for the soil profile shown below in Fig.



Dry sand

$$\gamma_d = \frac{G_s}{1+e}\gamma_w = \frac{2.66}{1+0.55}9.81 = 16.84 \text{ kN/m}^3$$

Moist sand

$$\gamma_t = \frac{G_s + Se}{1+e}\gamma_w = \frac{2.66+0.5*0.55}{1+0.55}9.81 = 18.58 \text{ kN/m}^3$$

$$e = \frac{G_s\omega}{S} = \frac{2.71*0.42}{1} = 1.138$$

Saturated Clay

$$\gamma_{sat} = \frac{G_s + e}{1+e}\gamma_w = \frac{2.66+1.138}{1+1.138}9.81 = 17.66 \text{ kN/m}^3$$

Point	σ_v kN/m^2	u kN/m^2	σ'_v kN/m^2
A	**0**	**0**	**0**
B	**2*16.84=33.68**	**0**	**33.68**
		- S γ_w H_2 = - 0.5*9.81*1.8 = - 8.83	**33.68-(-8.83) = 42.51**
C	**2*16.84+1.8*18.58 = 67.117**	**0**	**67.117**
D	**2*16.84+1.8*18.58+3.2*17.66 =123.68**	**3.2*9.81=31.39**	**123.68-31.39 = 92.24**

The plot is shown below in Fig.

Variation of σ_v, u and σ'_v with depth