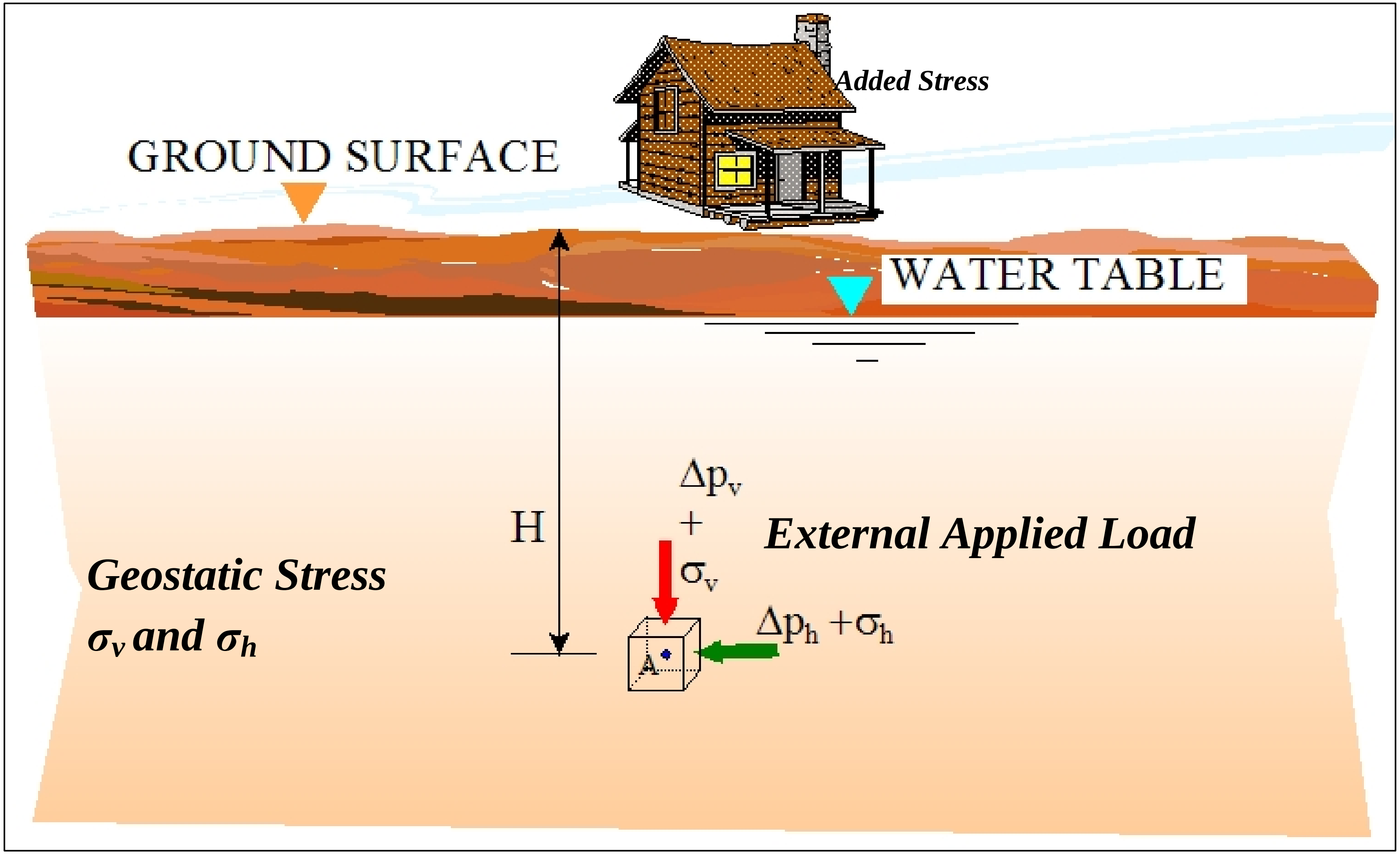
Stresses in a Soil Mass

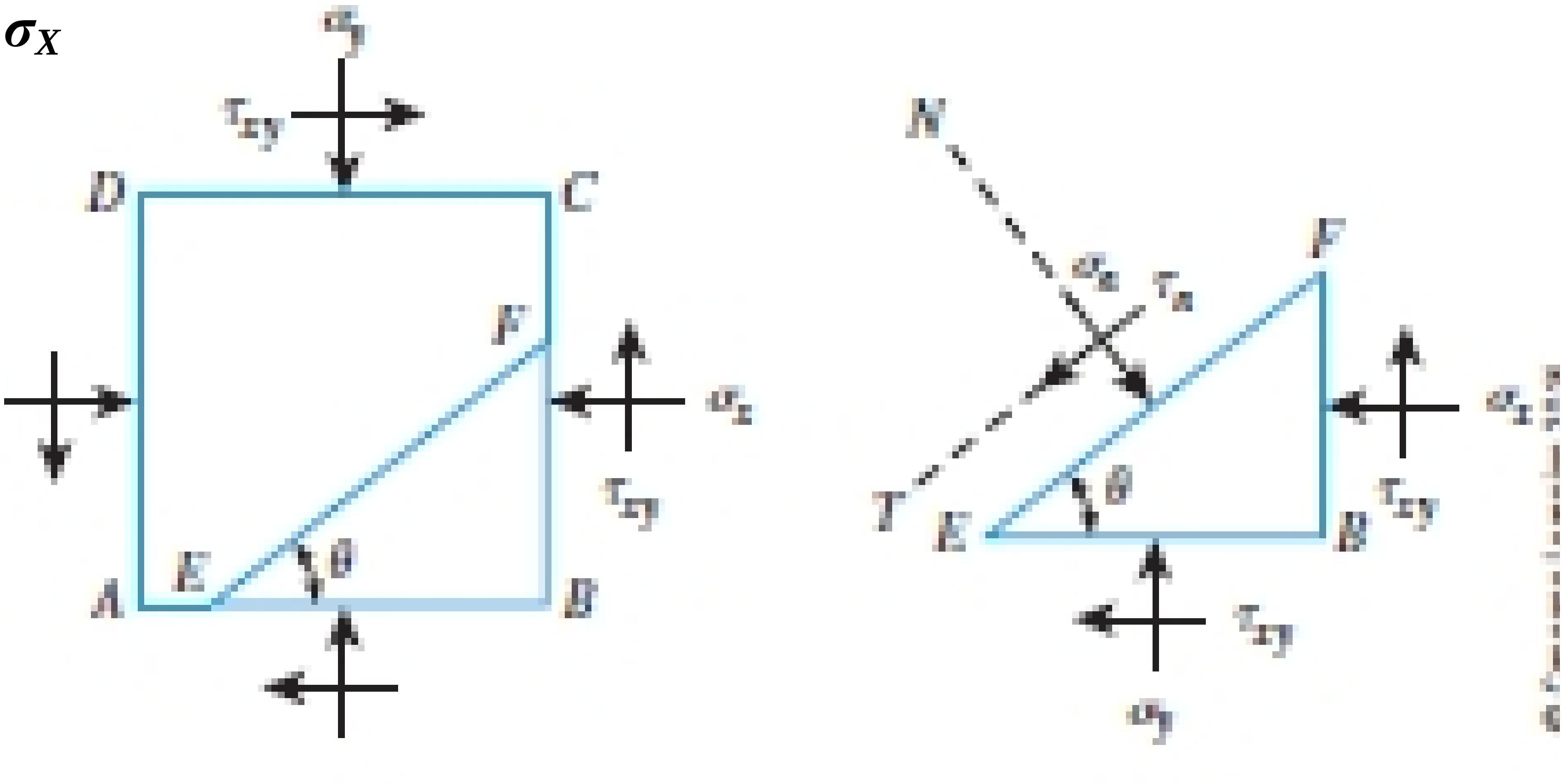
Topics

- **Normal and Shear Stresses on a Plane**
- **Stress distribution in soils**
- **Stress Caused by a Point Load**
- **Vertical Stress Caused by a Line Load**
- **Vertical Stress Caused by a Strip Load**
- **Vertical Stress below the Center of a uniformly Loaded Circular Area**
- **Vertical Stress at any Point below a uniformly Loaded Circular Area**
- **Vertical Stress Caused by a Rectangular Loaded Area**
- **Approximate method**


Added Stress
GROUND SURFACE
WATER TABLE
H
Δp_v
+
σ_v
External Applied Load
Geostatic Stress
σ_v and σ_h
A
Δp_h +σ_h

● Normal and Shear Stresses on a Plane

$\sigma_Y > \sigma_X$



(a)

(b)

From geometry for the free body diagram EBF

$$\overline{EB} = \overline{EF}\cos\theta$$

$$\overline{FB} = \overline{EF}\sin\theta$$

Summing forces in N and T direction, we have

$$\sigma_n(\overline{EF}) = \sigma_x(\overline{EF})\sin^2\theta + \sigma_y(\overline{EF})\cos^2\theta + 2\tau_{xy}(\overline{EF})\sin\theta\cos\theta$$

$$\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \text{..........................(1)}$$

Again

$$\tau_n(\overline{EF}) = -\sigma_x(\overline{EF})\sin\theta\cos\theta + \sigma_y(\overline{EF})\sin\theta\cos\theta - \tau_{xy}(\overline{EF})\cos^2\theta + \tau_{xy}(\overline{EF})\sin^2\theta$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta \qquad \text{..............................(2)}$$

If $\tau_n = 0$ then

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_y - \sigma_x} \qquad \text{..(3)}$$

This eq. gives 2 values of θ that are 90^o apart, this means that there are 2 planes that are right angles to each other on which shear stress = 0, such planes are called *principle planes* and the normal stress that act on the principle planes are to as ***principle stresses.***

To find the principle stress substitute eq.3 into eq.1, we get

$$\sigma_n = \sigma_1 = \frac{\sigma_y + \sigma_x}{2} + \sqrt{\left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau^2{}_{xy}} \qquad \textit{major principle stress}$$

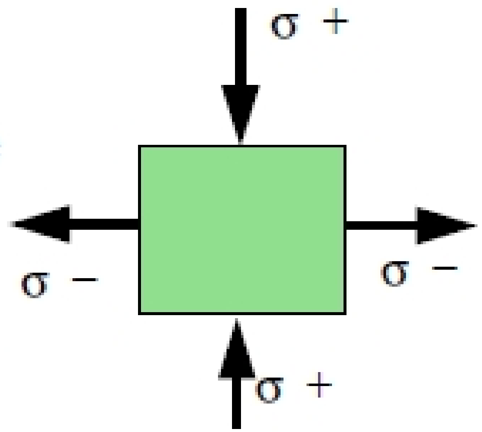
$$\sigma_n = \sigma_3 = \frac{\sigma_y + \sigma_x}{2} - \sqrt{\left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau^2{}_{xy}} \qquad \textbf{min}\textit{or principle stress}$$

These stresses on any plane can be found using *Mohr's circle*

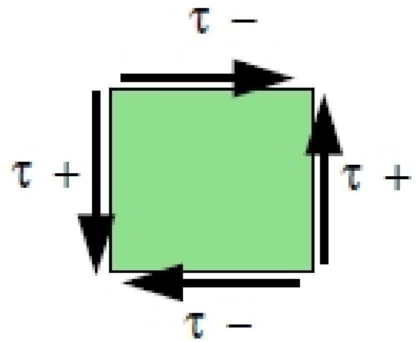
♦ ***Mohr's circle***

Mohr's Circle Sign Conventions:

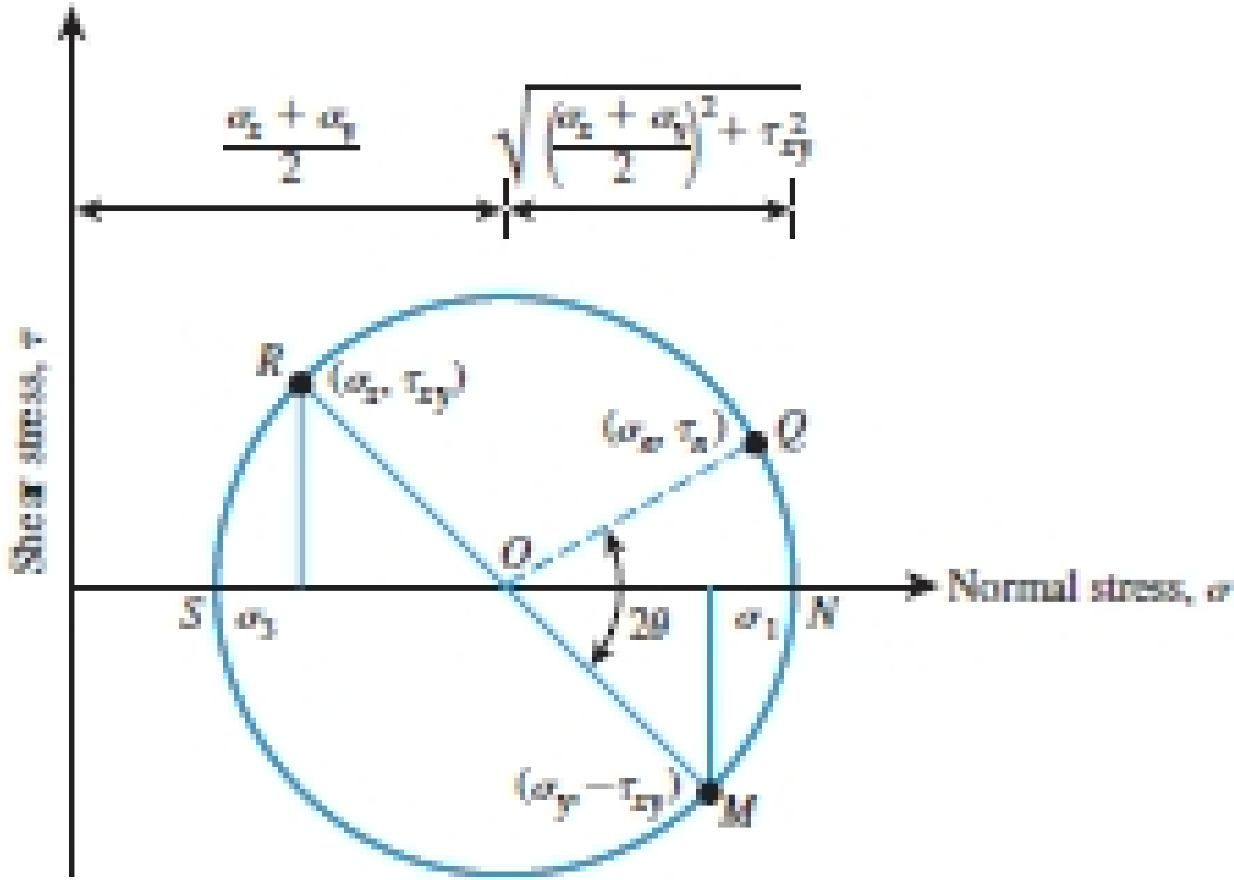
- Compressive normal stresses are positive

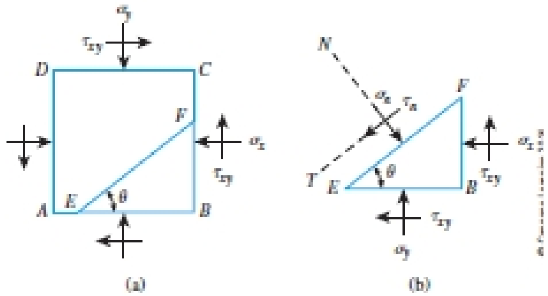


- Shear stresses are positive, if when they act on two opposing faces, they tend to produce a counterclockwise rotation.

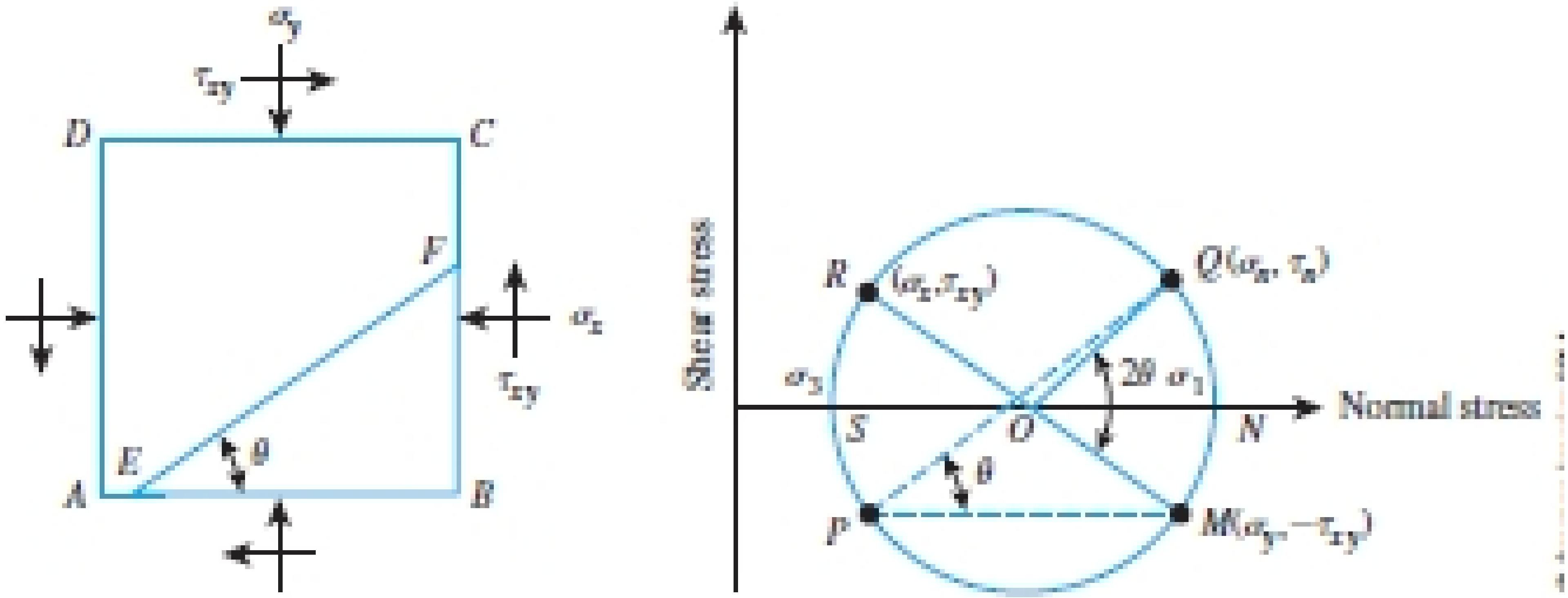


Refer to the element shown in Fig. above

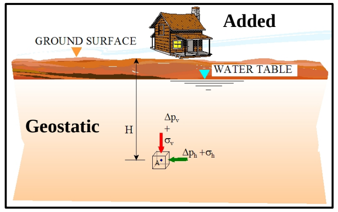




♦ ***Pole Method***



(a) (b)

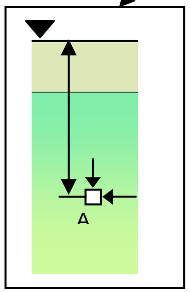


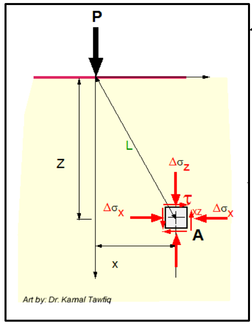
Stress Distribution in Soils

Geostatic stresses

Total Stress
Effective Stress
Pore Water Pressure

Total Stress= Effective Stress+ Pore Water





Added Stresses (Point, line, strip, triangular, circular, rectangular)

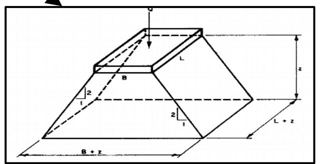
Westergaard’s Method

Bossinisque Equations

- Point Load
- Line Load
- Strip Load
- Triangular Load
- Circular Load
- Rectangular Load

σ_x
σ_y
τ_{xy}

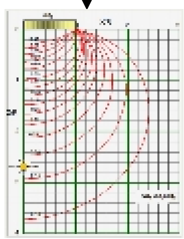
Approximate Method



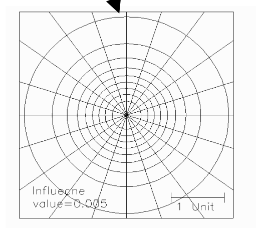
Influence Charts

$$\Delta\sigma_z = I_\sigma \cdot q$$

Stress Bulbs

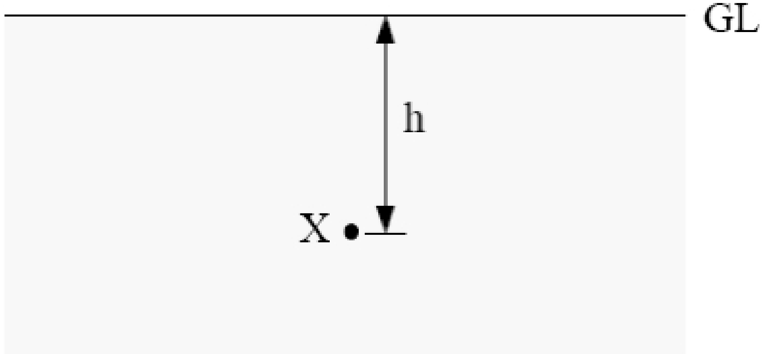


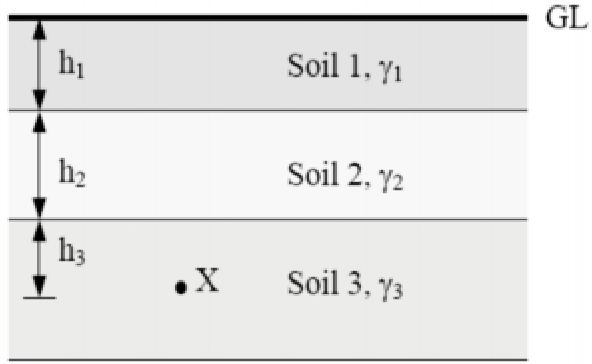
Newmark Charts



Geostatic stresses

The vertical geostatic stress at point X will be computed as following





$\sigma_V = \gamma\, h$ homogenous soils

$\sigma_V = \sum_{1}^{n} \gamma_i h_i$ stratified soils

$\sigma_v = \int_{0}^{h} \gamma\, dz$ density varies continuously with depth

The horizontal geostatic stress can be computed as following

$$\sigma_h = K\sigma_v$$

where K is the coefficient of lateral stress or lateral stress ratio

$$K = \frac{\sigma_h}{\sigma_v} \qquad\qquad 1 < K \leq 1$$

♦ Geostatic stress are principle stresses (σ_1, σ_2 and σ_3 major, intermediate and minor principle stresses) and hence the horizontal and vertical planes through any point are principle planes.

$K < 1 \qquad \sigma_v = \sigma_1 \qquad \sigma_h = \sigma_3 \qquad \sigma_2 = \sigma_3 = \sigma_h$

$K = 1 \qquad \sigma_v = \sigma_h = \sigma_1 = \sigma_2 = \sigma_3 \qquad$ *Isotropic*

$K > 1 \qquad \sigma_h = \sigma_1 \qquad \sigma_v = \sigma_3 \qquad \sigma_2 = \sigma_1 = \sigma_h$

The largest shear stress will found on plane lying at 45^{o} to the horizontal

$K < 1 \qquad \tau_{\max} = \dfrac{\sigma_v}{2}(1 - K)$

$K = 1 \qquad \tau_{\max} = 0$

$K > 1 \qquad \tau_{\max} = \dfrac{\sigma_v}{2}(K - 1)$

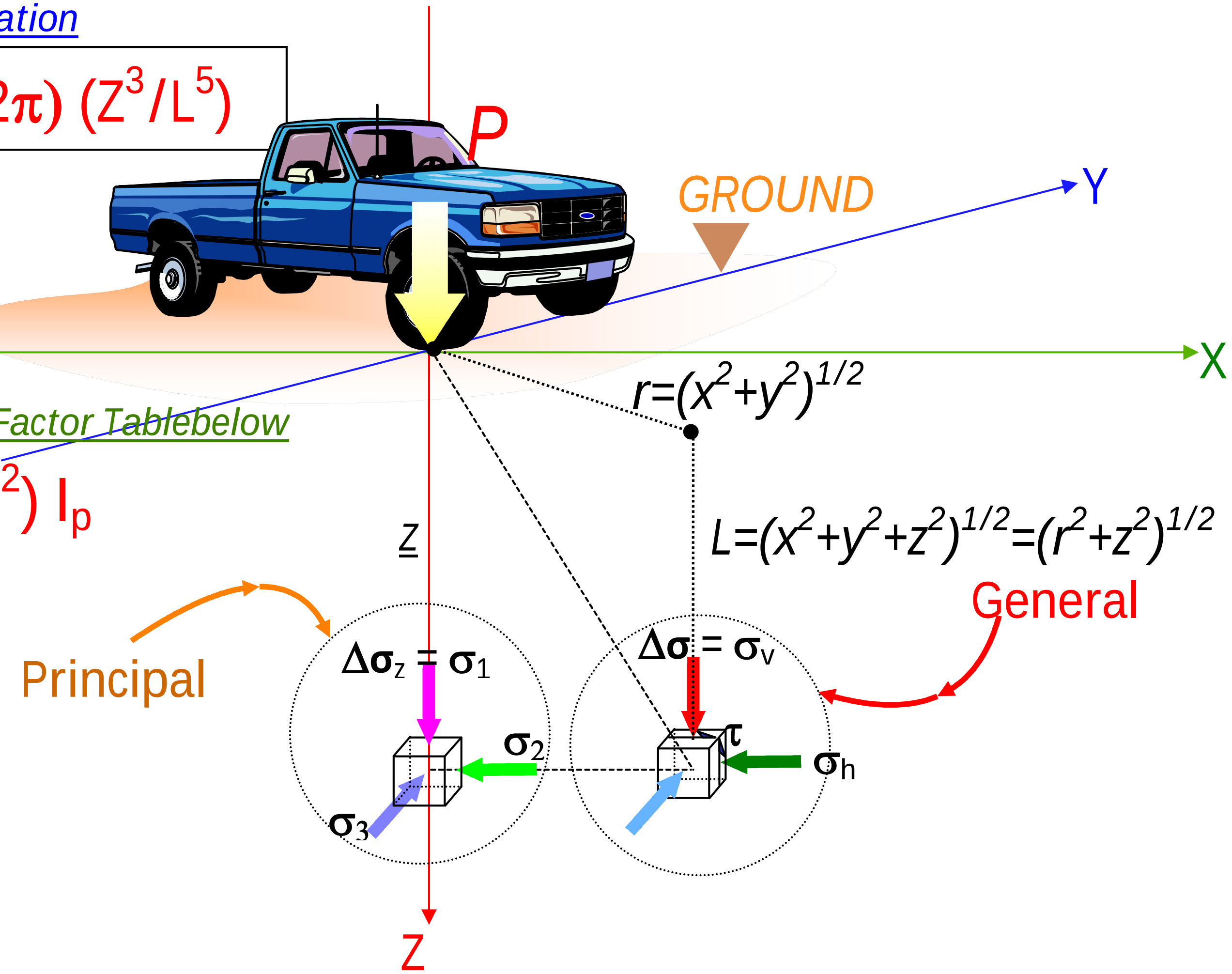
● Stress Caused by a Point Load

Boussinesq's Equation

$$\Delta\sigma_z = (3P/2\pi)\,(Z^3/L^5)$$

Using Influence Factor Tablebelow

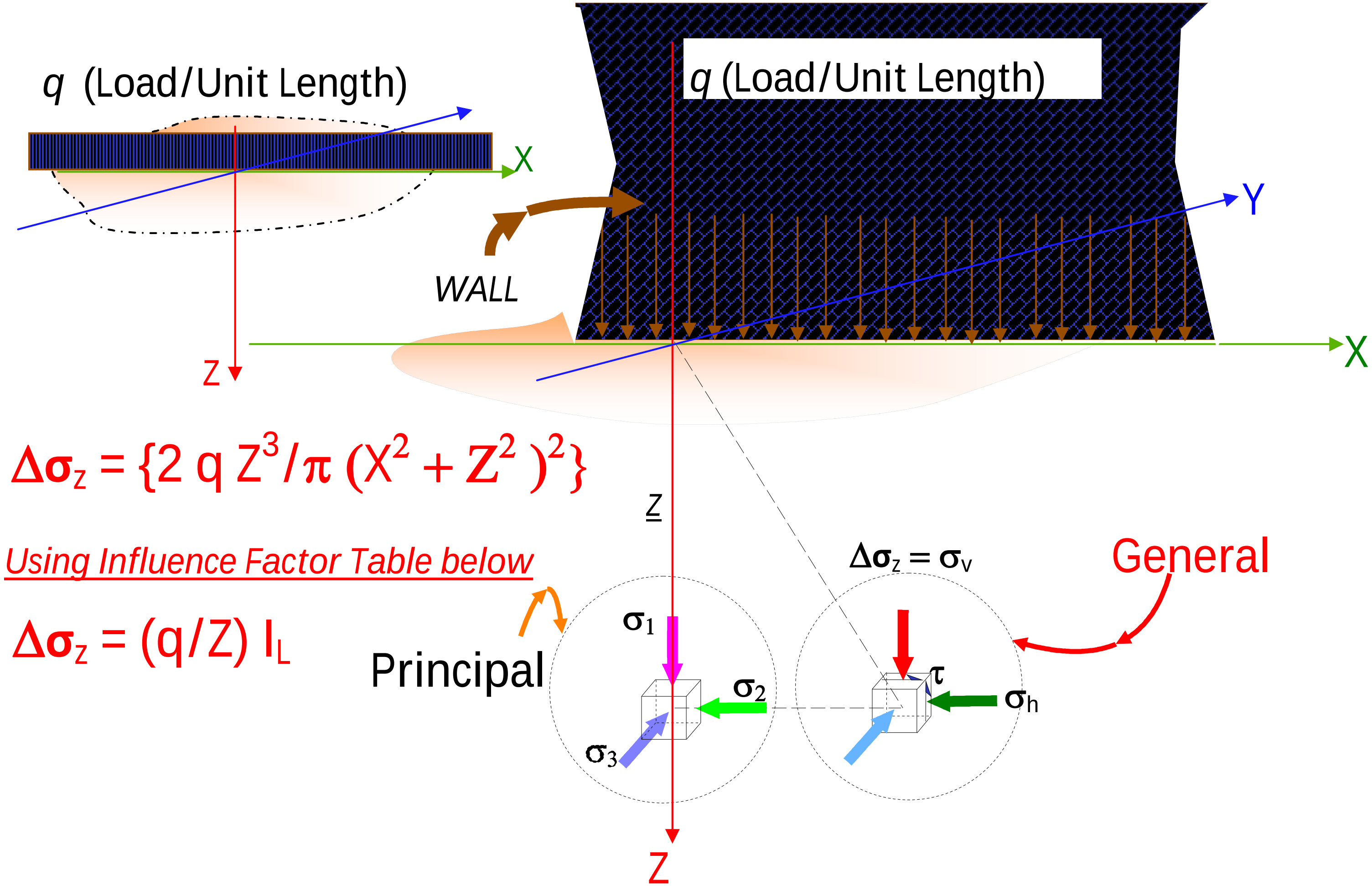
$$\Delta\sigma_z = (P/Z^2)\; I_p$$



Variation of I_P

r/z	I_1	r/z	I_1
0	0.4775	0.9	0.1083
0.1	0.4657	1.0	0.0844
0.2	0.4329	1.5	0.0251
0.3	0.3849	1.75	0.0144
0.4	0.3295	2.0	0.0085
0.5	0.2733	2.5	0.0034
0.6	0.2214	3.0	0.0015
0.7	0.1762	4.0	0.0004
0.8	0.1386	5.0	0.00014

● Vertical Stress Caused by a Line Load



$$\Delta\sigma_z = \{2\, q\, Z^3 / \pi\, (X^2 + Z^2)^2\}$$

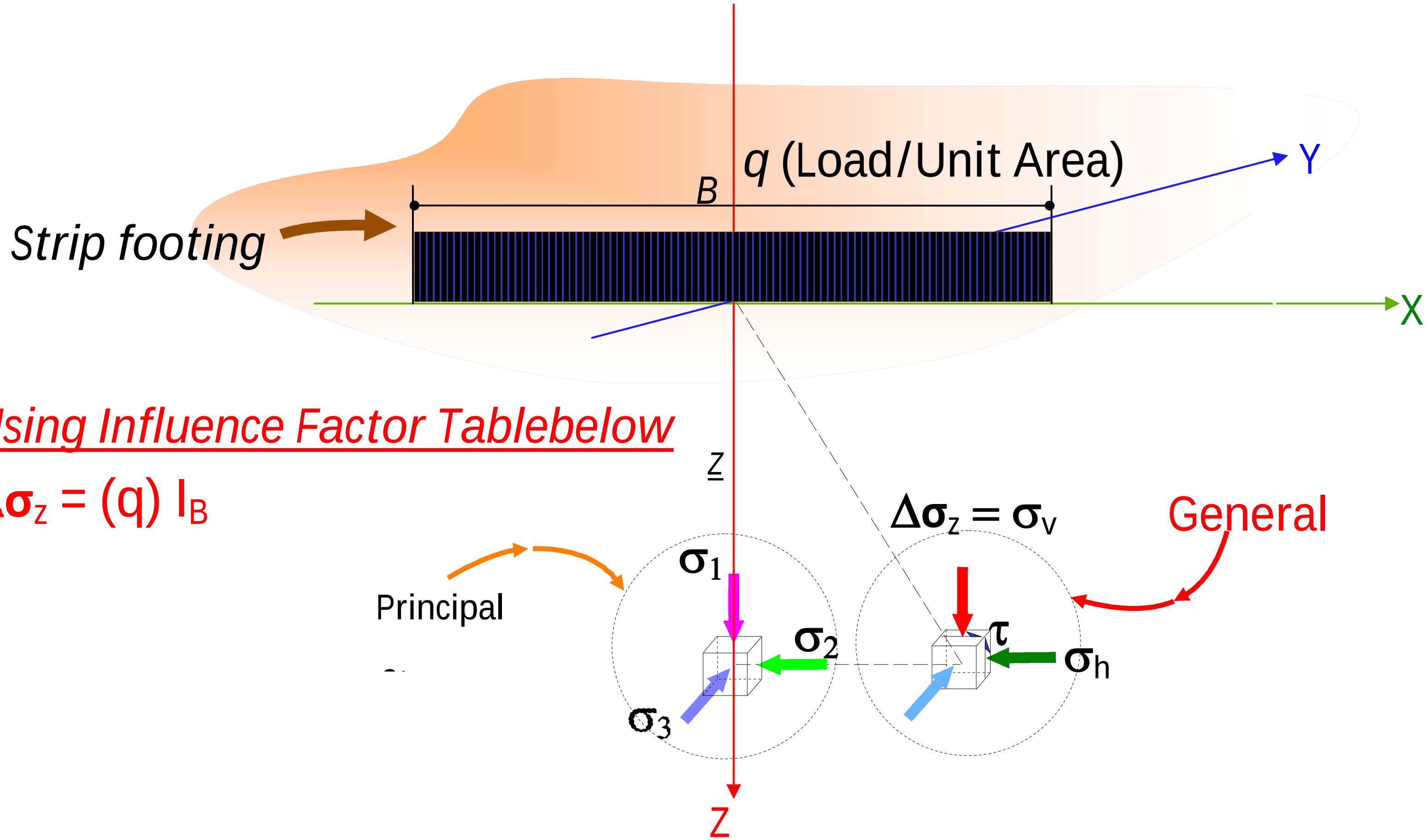
Using Influence Factor Table below

$$\Delta\sigma_z = (q/Z)\, I_L$$

Variation of I_L

x/z	$\frac{\Delta\sigma}{q/z}$	x/z	$\frac{\Delta\sigma}{q/z}$
0	0.637	0.7	0.287
0.1	0.624	0.8	0.237
0.2	0.589	0.9	0.194
0.3	0.536	1.0	0.159
0.4	0.473	1.5	0.060
0.5	0.407	2.0	0.025
0.6	0.344	3.0	0.006

• Vertical Stress Caused by a Strip Load



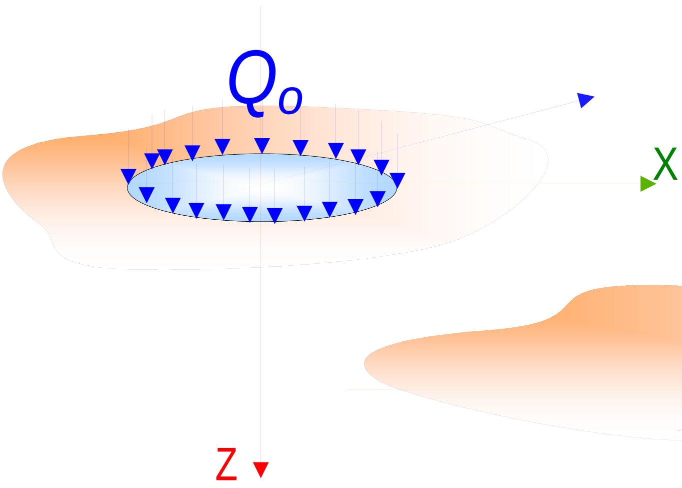
Using Influence Factor Tablebelow

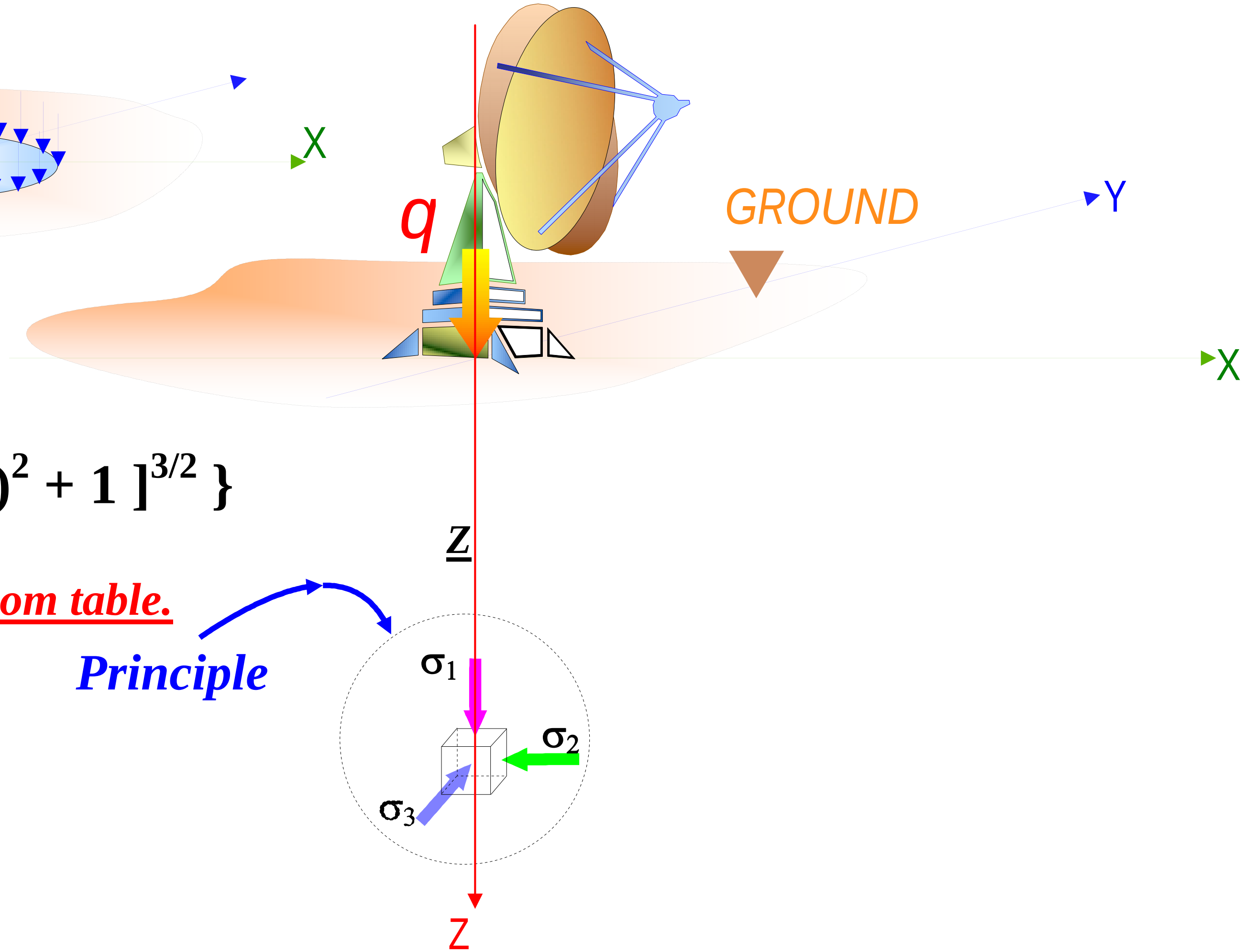
$\Delta\sigma_z = (q)\ I_B$

Variation of $\Delta\sigma/q$ with $2z/B$ and $2x/B$

	$2x/B$				
$2z/B$	0	0.5	1.0	1.5	2.0
0	1.000	1.000	0.500	—	—
0.5	0.959	0.903	0.497	0.089	0.019
1.0	0.818	0.735	0.480	0.249	0.078
1.5	0.668	0.607	0.448	0.270	0.146
2.0	0.550	0.510	0.409	0.288	0.185
2.5	0.462	0.437	0.370	0.285	0.205
3.0	0.396	0.379	0.334	0.273	0.211
3.5	0.345	0.334	0.302	0.258	0.216
4.0	0.306	0.298	0.275	0.242	0.205
4.5	0.274	0.268	0.251	0.226	0.197
5.0	0.248	0.244	0.231	0.212	0.188

• Vertical Stress below the Center of a uniformly Loaded Circular Area





$$\Delta\sigma_z = q \{ 1 - 1/[(R/Z)^2 + 1]^{3/2} \}$$

Using Influence Factorfrom table.

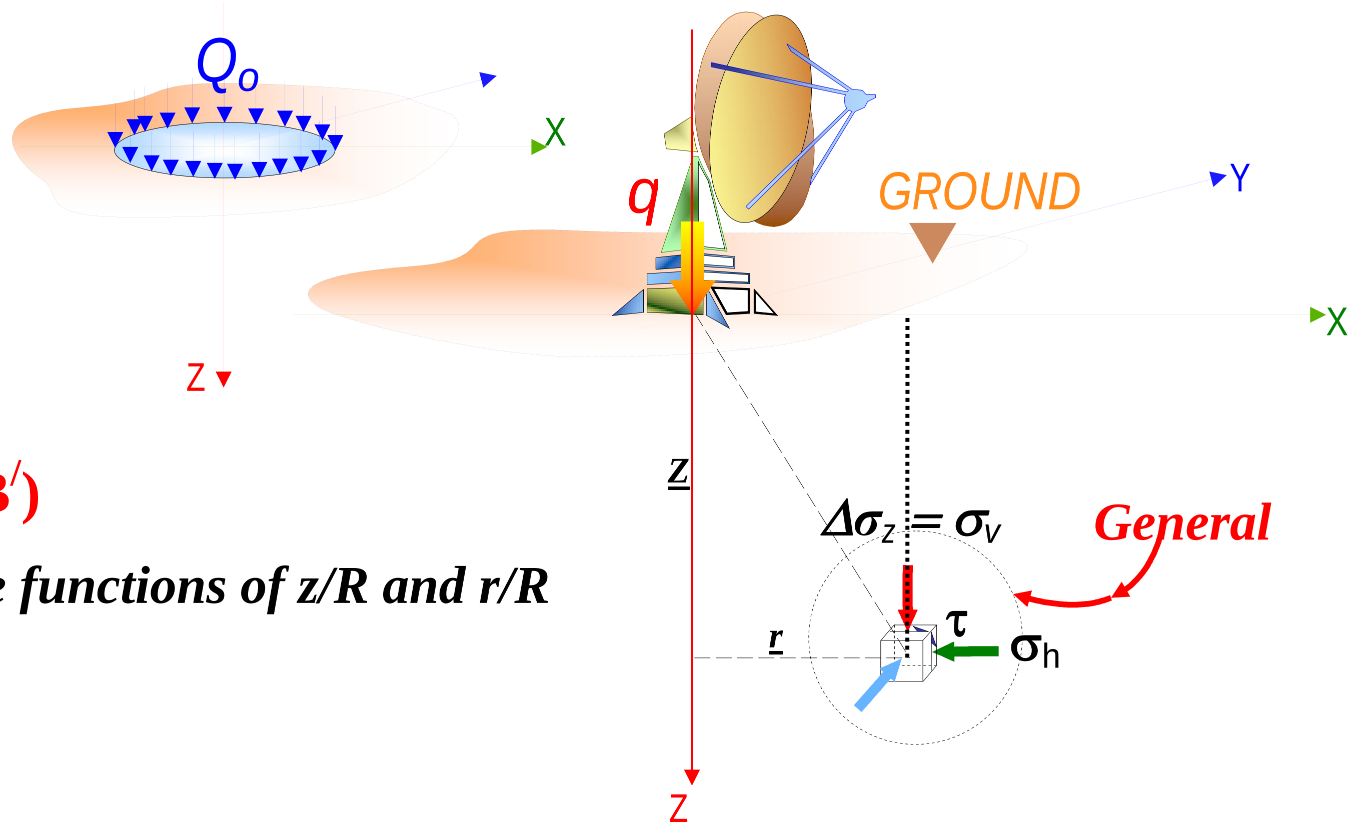
$$\Delta\sigma_z = qI_c$$

Principle

Variation of I_c

z/R	$\Delta\sigma/q$	z/R	$\Delta\sigma/q$
0	1	1.0	0.6465
0.02	0.9999	1.5	0.4240
0.05	0.9998	2.0	0.2845
0.10	0.9990	2.5	0.1996
0.2	0.9925	3.0	0.1436
0.4	0.9488	4.0	0.0869
0.5	0.9106	5.0	0.0571
0.8	0.7562		

• Vertical Stress at any Point below a uniformly Loaded Circular Area



$$\Delta\sigma_z = q\,(A^{/} + B^{/})$$

Where (A/&B/) are functions of z/R and r/R from tables

Table 10.7 Variation of A' with z/R and r/R*

	r/R								
z/R	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2
0	1.0	1.0	1.0	1.0	1.0	0.5	0	0	0
0.1	0.90050	0.89748	0.88679	0.86126	0.78797	0.43015	0.09645	0.02787	0.00856
0.2	0.80388	0.79824	0.77884	0.73483	0.63014	0.38269	0.15433	0.05251	0.01680
0.3	0.71265	0.70518	0.68316	0.62690	0.52081	0.34375	0.17964	0.07199	0.02440
0.4	0.62861	0.62015	0.59241	0.53767	0.44329	0.31048	0.18709	0.08593	0.03118
0.5	0.55279	0.54403	0.51622	0.46448	0.38390	0.28156	0.18556	0.09499	0.03701
0.6	0.48550	0.47691	0.45078	0.40427	0.33676	0.25588	0.17952	0.10010	
0.7	0.42654	0.41874	0.39491	0.35428	0.29833	0.21727	0.17124	0.10228	0.04558
0.8	0.37531	0.36832	0.34729	0.31243	0.26581	0.21297	0.16206	0.10236	
0.9	0.33104	0.32492	0.30669	0.27707	0.23832	0.19488	0.15253	0.10094	
1	0.29289	0.28763	0.27005	0.24697	0.21468	0.17868	0.14329	0.09849	0.05185
1.2	0.23178	0.22795	0.21662	0.19890	0.17626	0.15101	0.12570	0.09192	0.05260
1.5	0.16795	0.16552	0.15877	0.14804	0.13436	0.11892	0.10296	0.08048	0.05116
2	0.10557	0.10453	0.10140	0.09647	0.09011	0.08269	0.07471	0.06275	0.04496
2.5	0.07152	0.07098	0.06947	0.06698	0.06373	0.05974	0.05555	0.04880	0.03787
3	0.05132	0.05101	0.05022	0.04886	0.04707	0.04487	0.04241	0.03839	0.03150
4	0.02986	0.02976	0.02907	0.02802	0.02832	0.02749	0.02651	0.02490	0.02193
5	0.01942	0.01938				0.01835			0.01573
6	0.01361					0.01307			0.01168
7	0.01005					0.00976			0.00894
8	0.00772					0.00755			0.00703
9	0.00612					0.00600			0.00566
10								0.00477	0.00465

Table 10.7 *(continued)*

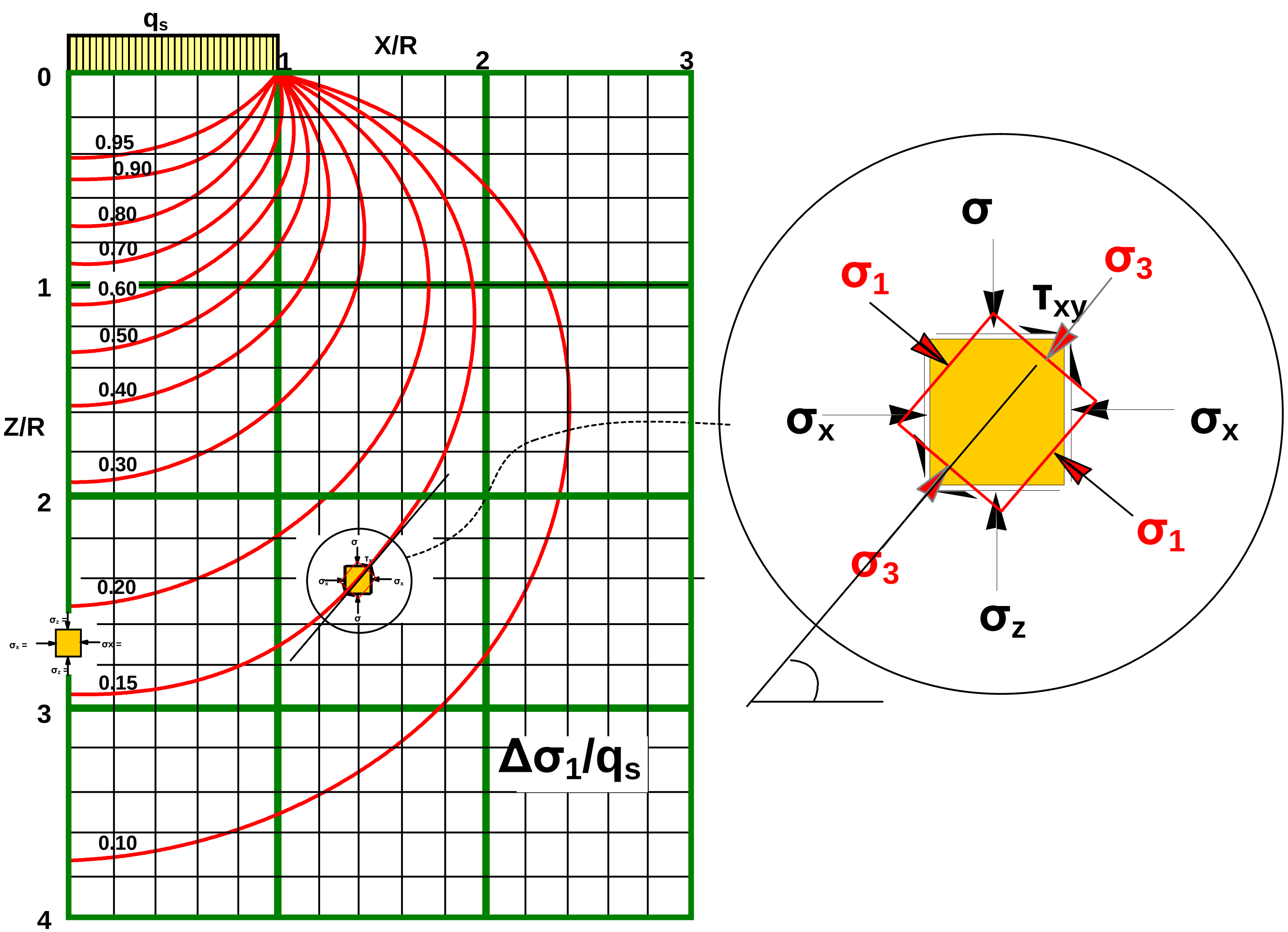
3	4	5	6	7	8	10	12	14
0	0	0	0	0	0	0	0	0
0.00211	0.00084	0.00042						
0.00419	0.00167	0.00083	0.00048	0.00030	0.00020			
0.00622	0.00250							
0.01013	0.00407	0.00209	0.00118	0.00071	0.00053	0.00025	0.00014	0.00009
0.01742	0.00761	0.00393	0.00226	0.00143	0.00097	0.00050	0.00029	0.00018
0.01935	0.00871	0.00459	0.00269	0.00171	0.00115			
0.02142	0.01013	0.00548	0.00325	0.00210	0.00141	0.00073	0.00043	0.00027
0.02221	0.01160	0.00659	0.00399	0.00264	0.00180	0.00094	0.00056	0.00036
0.02143	0.01221	0.00732	0.00463	0.00308	0.00214	0.00115	0.00068	0.00043
0.01980	0.01220	0.00770	0.00505	0.00346	0.00242	0.00132	0.00079	0.00051
0.01592	0.01109	0.00768	0.00536	0.00384	0.00282	0.00160	0.00099	0.00065
0.01249	0.00949	0.00708	0.00527	0.00394	0.00298	0.00179	0.00113	0.00075
0.00983	0.00795	0.00628	0.00492	0.00384	0.00299	0.00188	0.00124	0.00084
0.00784	0.00661	0.00548	0.00445	0.00360	0.00291	0.00193	0.00130	0.00091
0.00635	0.00554	0.00472	0.00398	0.00332	0.00276	0.00189	0.00134	0.00094
0.00520	0.00466	0.00409	0.00353	0.00301	0.00256	0.00184	0.00133	0.00096
0.00438	0.00397	0.00352	0.00326	0.00273	0.00241			

Table 10.8 Variation of B' with z/R and r/R*

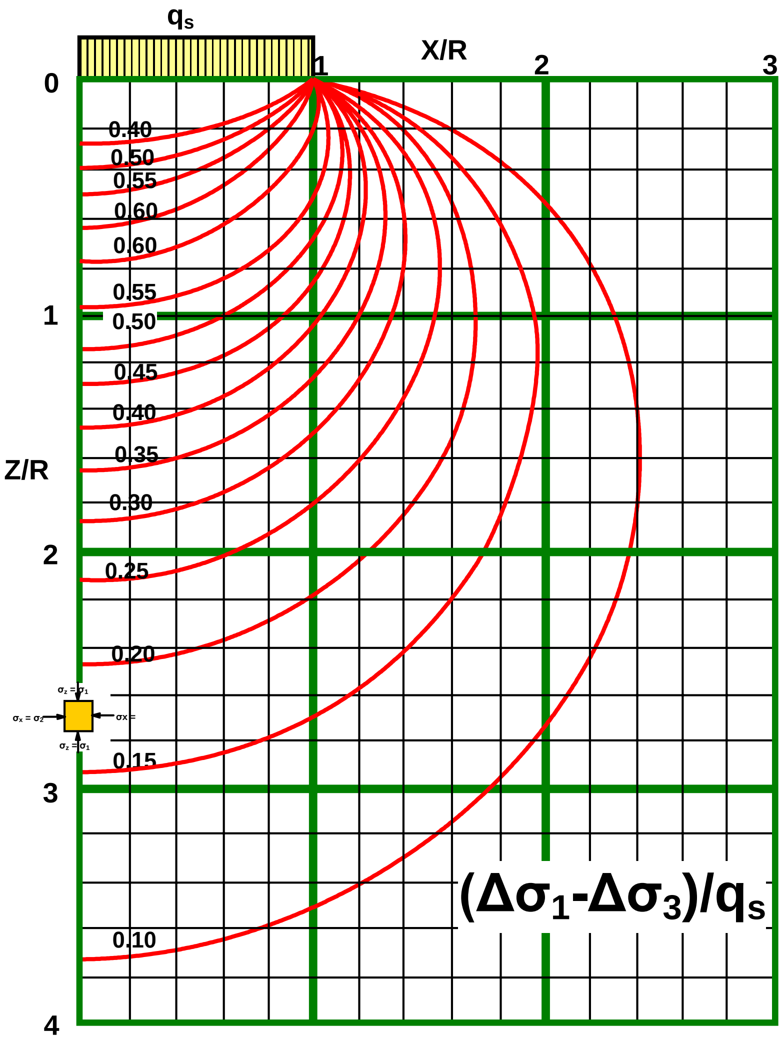
	r/R								
z/R	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2
0	0	0	0	0	0	0	0	0	0
0.1	0.09852	0.10140	0.11138	0.13424	0.18796	0.05388	−0.07899	−0.02672	−0.00845
0.2	0.18857	0.19306	0.20772	0.23524	0.25983	0.08513	−0.07759	−0.04448	−0.01593
0.3	0.26362	0.26787	0.28018	0.29483	0.27257	0.10757	−0.04316	−0.04999	−0.02166
0.4	0.32016	0.32259	0.32748	0.32273	0.26925	0.12404	−0.00766	−0.04535	−0.02522
0.5	0.35777	0.35752	0.35323	0.33106	0.26236	0.13591	0.02165	−0.03455	−0.02651
0.6	0.37831	0.37531	0.36308	0.32822	0.25411	0.14440	0.04457	−0.02101	
0.7	0.38487	0.37962	0.36072	0.31929	0.24638	0.14986	0.06209	−0.00702	−0.02329
0.8	0.38091	0.37408	0.35133	0.30699	0.23779	0.15292	0.07530	0.00614	
0.9	0.36962	0.36275	0.33734	0.29299	0.22891	0.15404	0.08507	0.01795	
1	0.35355	0.34553	0.32075	0.27819	0.21978	0.15355	0.09210	0.02814	−0.01005
1.2	0.31485	0.30730	0.28481	0.24836	0.20113	0.14915	0.10002	0.04378	0.00023
1.5	0.25602	0.25025	0.23338	0.20694	0.17368	0.13732	0.10193	0.05745	0.01385
2	0.17889	0.18144	0.16644	0.15198	0.13375	0.11331	0.09254	0.06371	0.02836
2.5	0.12807	0.12633	0.12126	0.11327	0.10298	0.09130	0.07869	0.06022	0.03429
3	0.09487	0.09394	0.09099	0.08635	0.08033	0.07325	0.06551	0.05354	0.03511
4	0.05707	0.05666	0.05562	0.05383	0.05145	0.04773	0.04532	0.03995	0.03066
5	0.03772	0.03760				0.03384			0.02474
6	0.02666					0.02468			0.01968
7	0.01980					0.01868			0.01577
8	0.01526					0.01459			0.01279
9	0.01212					0.01170			0.01054
10								0.00924	0.00879

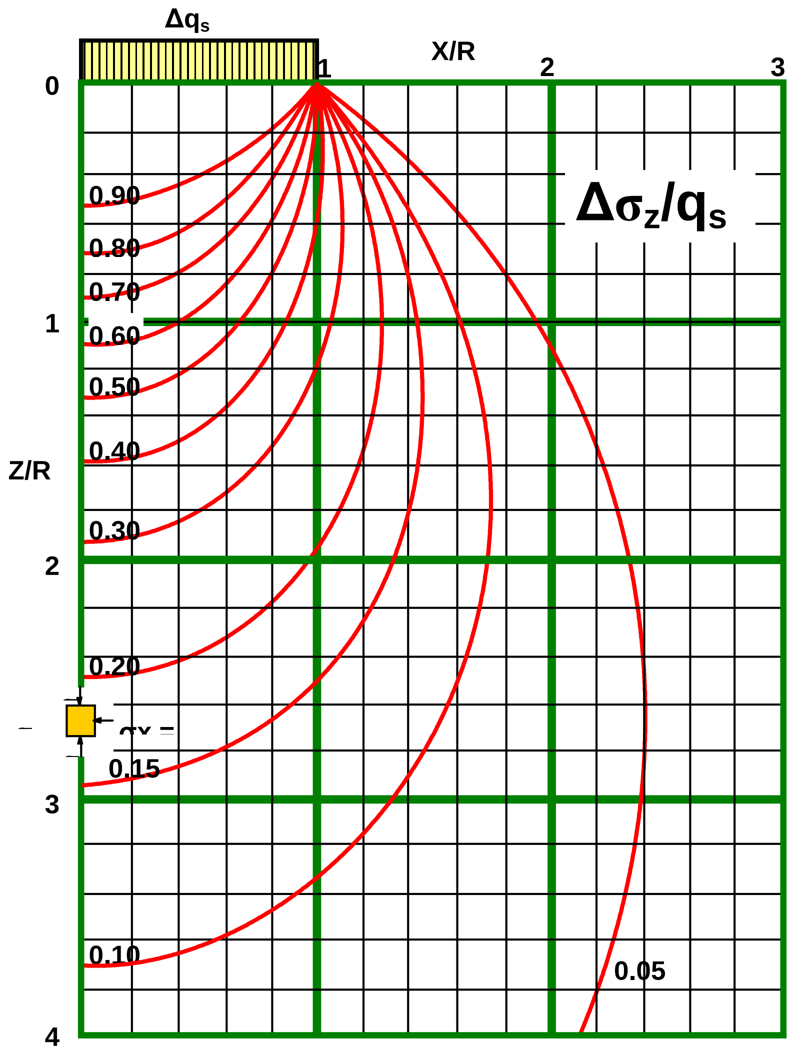
Table 10.8 *(continued)*

3	4	5	6	7	8	10	12	14
0	0	0	0	0	0	0	0	0
−0.00210	−0.00084	−0.00042						
−0.00412	−0.00166	−0.00083	−0.00024	−0.00015	−0.00010			
−0.00599	−0.00245							
−0.00991	−0.00388	−0.00199	−0.00116	−0.00073	−0.00049	−0.00025	−0.00014	−0.00009
−0.01115	−0.00608	−0.00344	−0.00210	−0.00135	−0.00092	−0.00048	−0.00028	−0.00018
−0.00995	−0.00632	−0.00378	−0.00236	−0.00156	−0.00107			
−0.00669	−0.00600	−0.00401	−0.00265	−0.00181	−0.00126	−0.00068	−0.00040	−0.00026
0.00028	−0.00410	−0.00371	−0.00278	−0.00202	−0.00148	−0.00084	−0.00050	−0.00033
0.00661	−0.00130	−0.00271	−0.00250	−0.00201	−0.00156	−0.00094	−0.00059	−0.00039
0.01112	0.00157	−0.00134	−0.00192	−0.00179	−0.00151	−0.00099	−0.00065	−0.00046
0.01515	0.00595	0.00155	−0.00029	−0.00094	−0.00109	−0.00094	−0.00068	−0.00050
0.01522	0.00810	0.00371	0.00132	0.00013	−0.00043	−0.00070	−0.00061	−0.00049
0.01380	0.00867	0.00496	0.00254	0.00110	0.00028	−0.00037	−0.00047	−0.00045
0.01204	0.00842	0.00547	0.00332	0.00185	0.00093	−0.00002	−0.00029	−0.00037
0.01034	0.00779	0.00554	0.00372	0.00236	0.00141	0.00035	−0.00008	−0.00025
0.00888	0.00705	0.00533	0.00386	0.00265	0.00178	0.00066	0.00012	−0.00012
0.00764	0.00631	0.00501	0.00382	0.00281	0.00199			



Circular Load: (Major Principal Stress)/(Surface Stress)


qs
X/R
0
1
2
3
1
2
3
4
Z/R
0.40
0.50
0.55
0.60
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
(Δσ1-Δσ3)/qs

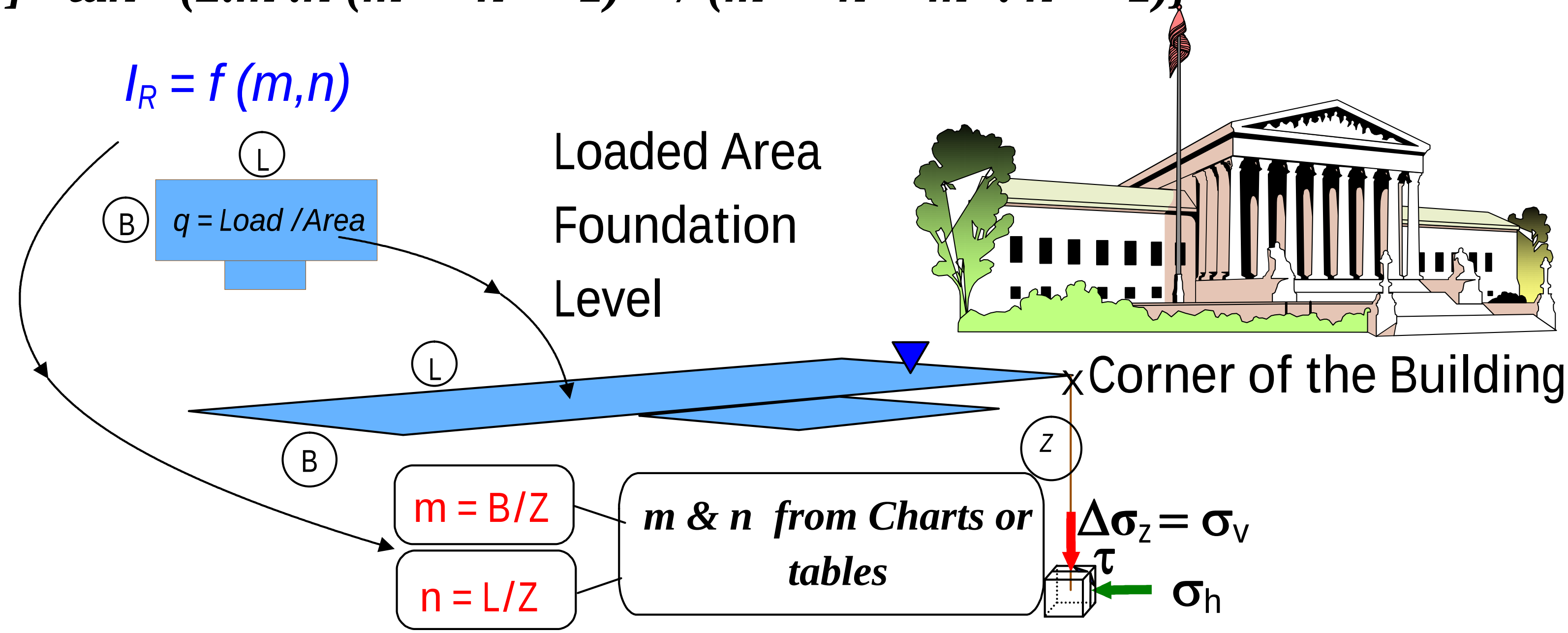


Circular Load: (Vertical Stress)/(Surface

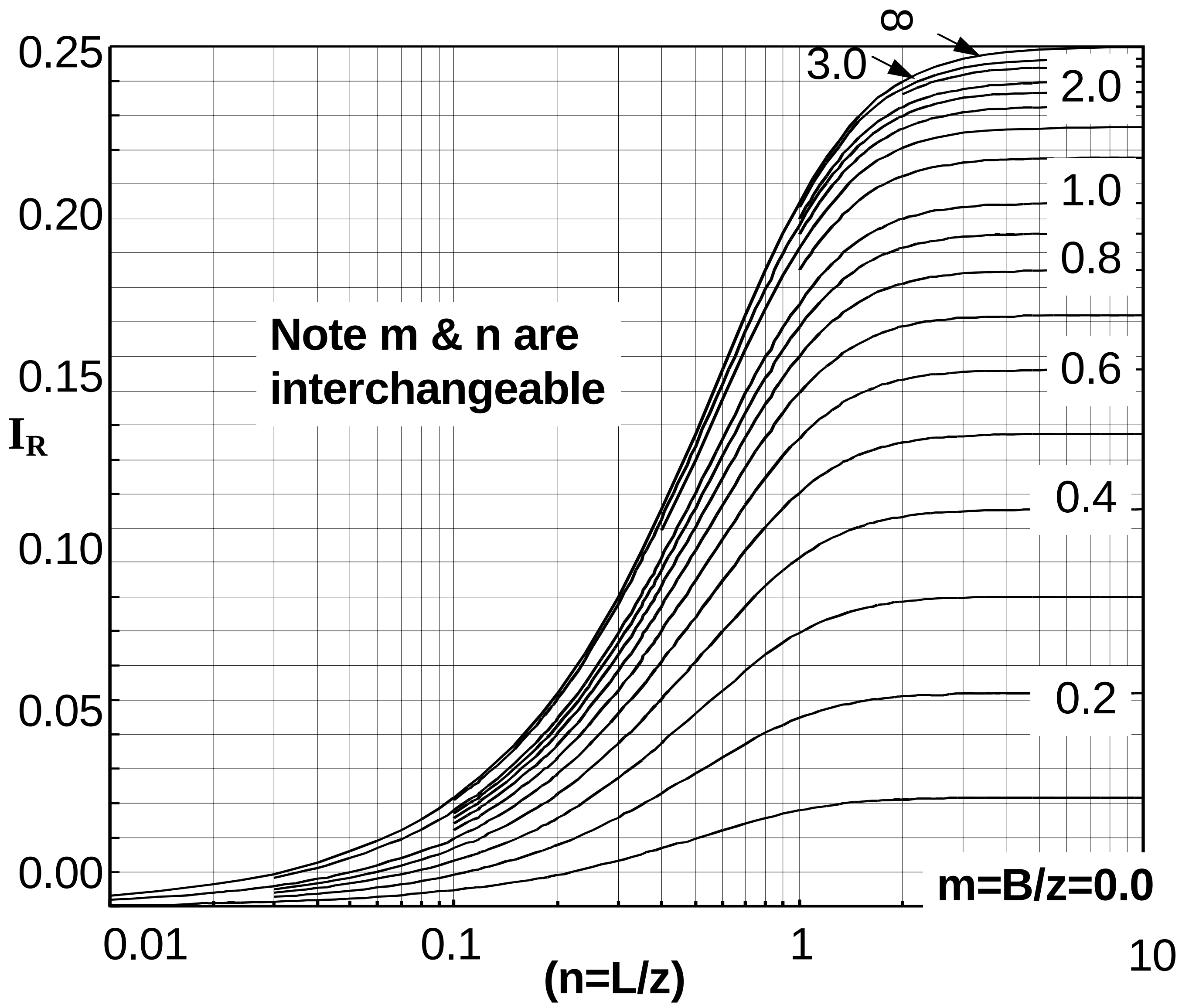
• Vertical Stress Caused by a Rectangularly Loaded Area

$$\Delta P = q\, I_R$$

$$I_R = 1/4\pi \{[\, (2.\, m.n\, (m^2 + n^2 + 1)^{1/2}\,) / (m^2 + n^2 + m^2.\, n^2 + 1)\,]\, [(m^2 + n^2 + 2)/(m^2 + n^2 + 1)] + \tan^{-1}\, (2.m\,.n\, (m^2 + n^2 + 1)^{1/2} / (m^2 + n^2 - m^2\,.\, n^2 + 1))\}$$

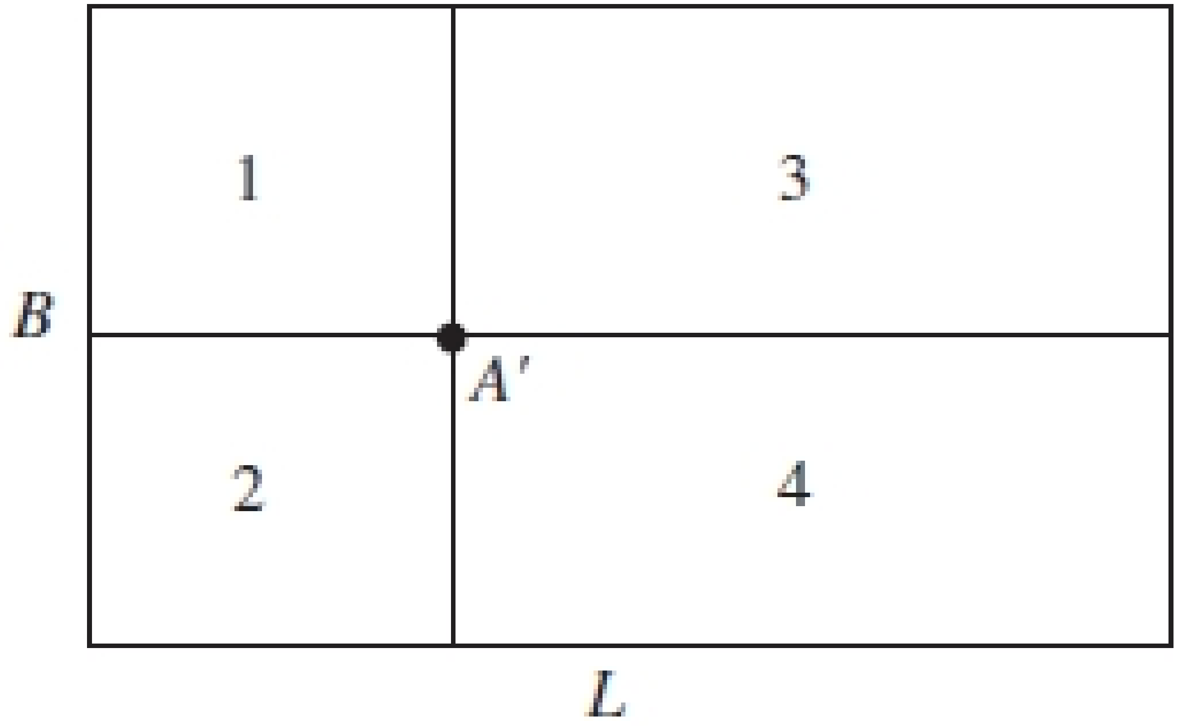


from tables or one can use the charts below


∞
3.0
2.0
1.0
0.8
0.6
0.4
0.2
m=B/z=0.0
Note m & n are interchangeable
0.25
0.20
0.15
0.10
0.05
0.00
I_R
0.01
0.1
1
10
(n=L/z)

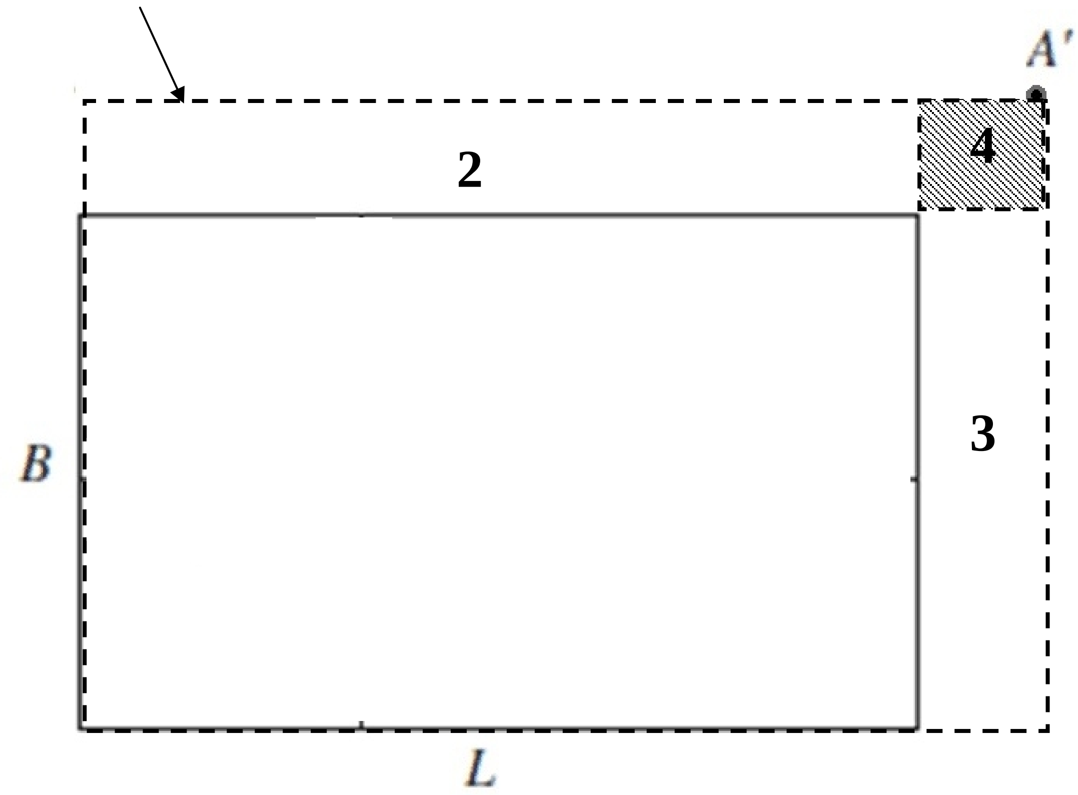
♦ ***Calculation of Stress below an interior point of the loaded area***

$$\Delta\sigma_z = q[I_1 + I_2 + I_3 + I_3 + I_4]$$



♦ ***Calculation of Stress below a point outside of the loaded area***

$$\Delta\sigma_z = q[I_1 - I_2 - I_3 + I_4]$$



• Approximate Methods

2:1 Method

Q = q x Area of foundation

Rectangular area

$$\Delta\sigma_z = \frac{Q}{(B+Z)(L+Z)}$$

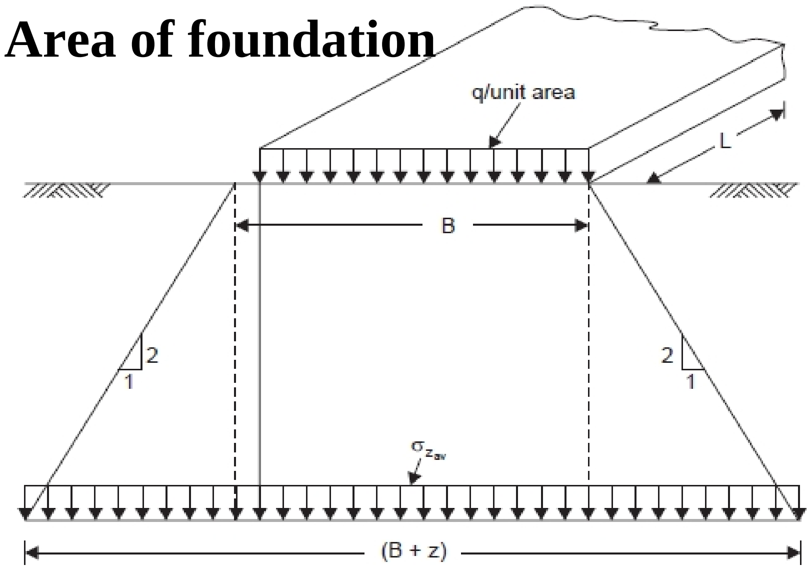
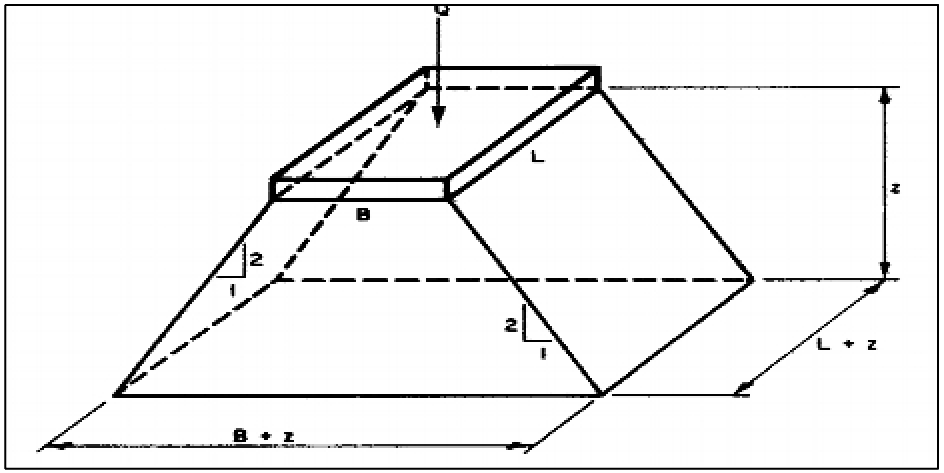


Fig. 10.21 Two is to one method

Square area

$$\Delta\sigma_z = \frac{Q}{(B+Z)^2}$$



Circular area

$$\Delta\sigma_z = \frac{Q}{\frac{\pi}{4}(D+Z)^2}$$

Examples (1-3)

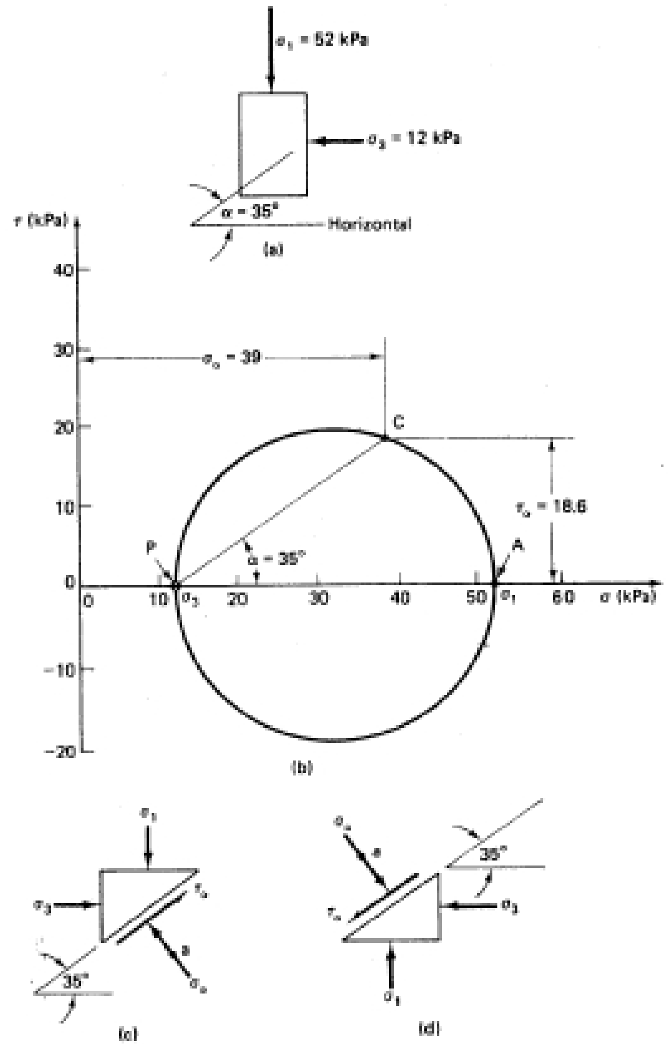
Given:

Stresses on an element as shown in Fig.

Plot the Mohr circle to some convenient sc

$$\text{center of circle} = \frac{\sigma_1 + \sigma_3}{2} = \frac{52 + 12}{2} = 32 \text{ kP}$$

$$\text{radius of circle} = \frac{\sigma_1 - \sigma_3}{2} = \frac{52 - 12}{2} = 20 \text{ kP}$$

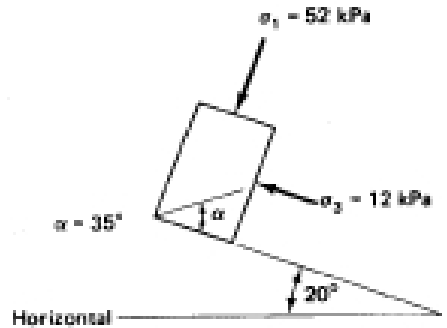


Given:

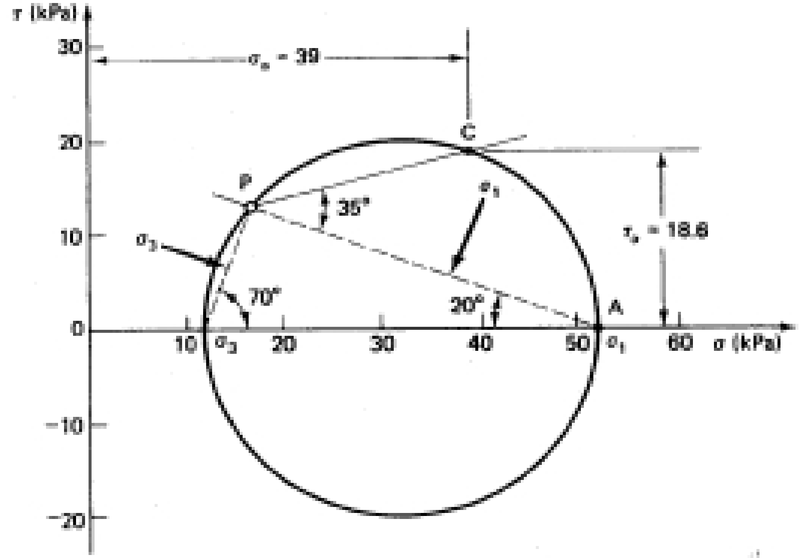
The same element and stresses as in Fig. Ex. 1 , except that the
is rotated 20° from the horizontal, as shown in Fig.

Required:

As in Example 10.1, find the normal stress σ_α and the shear stress
plane inclined at $\alpha = 35°$ from the base of the element.



(a)



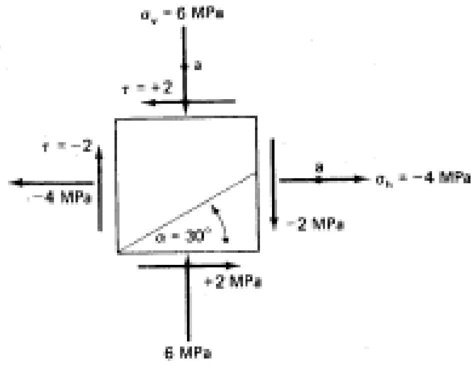
(b)

Given:

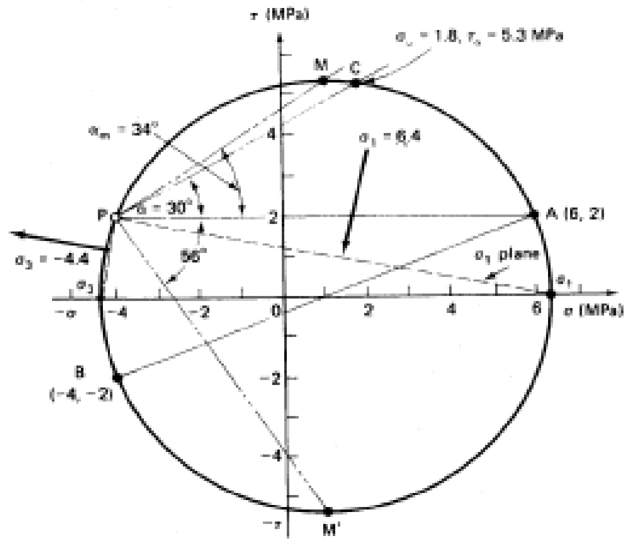
The stress shown on the element in Fig.

Required:

a. Evaluate σ_α and τ_α when $\alpha = 30°$.
b. Evaluate σ_1 and σ_3 when $\alpha = 30°$.



(a)



(b)

Example 4

The plan of a uniformly loaded rectangular area is shown in Figure 6.23a. Determine the vertical stress increase, $\Delta\sigma$, below point A' at a depth $z = 4$ m.

Solution

The stress increase, $\Delta\sigma$ can be written as

$$\Delta\sigma = \Delta\sigma_1 - \Delta\sigma_2$$

where $\Delta\sigma_1$ = stress increase due to the loaded area shown in Figure 6.23b
$\Delta\sigma_2$ = stress increase due to the loaded area shown in Figure 6.23c

For the loaded area shown in Figure 6.23b:

$$m' = \frac{B}{z} = \frac{2}{4} = 0.5$$

$$n' = \frac{L}{z} = \frac{4}{4} = 1$$

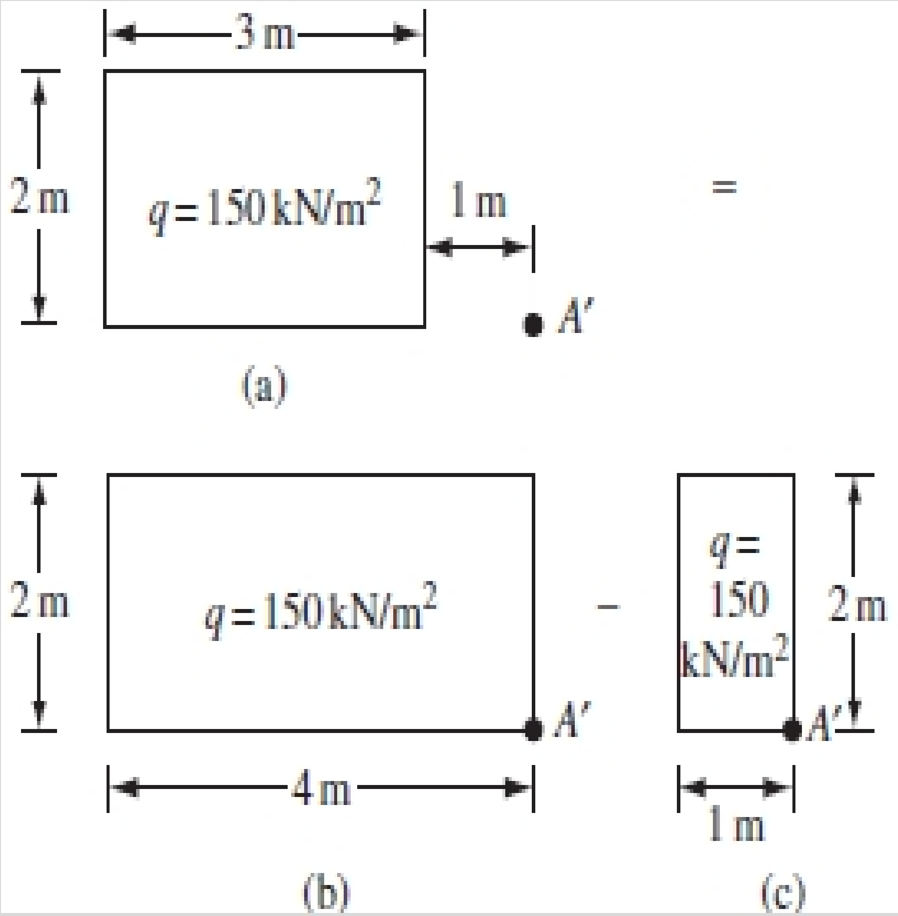


Figure 6.23

From Figure 6.21 for $m' = 0.5$ and $n' = 1$, the value of $I_5 = 0.1225$. So $\Delta\sigma_1 = qI_5 = (150)(0.1225) = 18.38 \text{ kN/m}^2$

Similarly, for the loaded area shown in Figure 6.23c:

$$m' = \frac{B}{z} = \frac{1}{4} = 0.25$$

$$n' = \frac{L}{z} = \frac{2}{4} = 0.5$$

Thus, $I_5 = 0.0473$., Hence

$$\Delta\sigma_2 = (150)(0.0473) = 7.1 \text{ kN/m}^2$$

So

$$\Delta\sigma = \Delta\sigma_1 - \Delta\sigma_2 = 18.38 - 7.1 = \mathbf{11.28\ kN/m^2}$$

≣