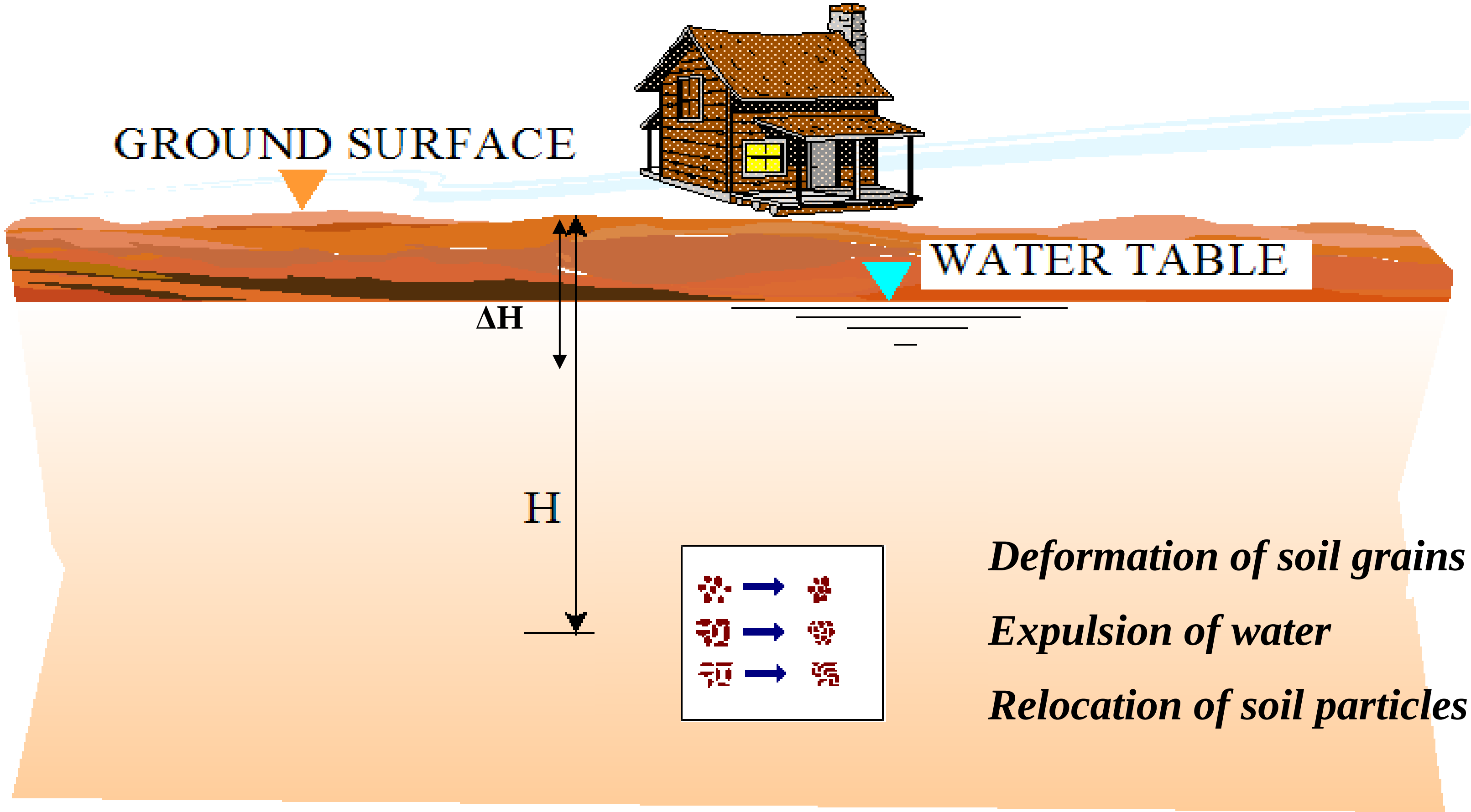
Compressibilty of Soils



Deformation of soil grains

Expulsion of water

Relocation of soil particles

Topics

- **Introduction**
- **Consolidation Settlement (Primary Consolidation)**
- **Secondary Compression (Secondary consolidation) Settlement**
- **Time Rate of Consolidation**

• Introduction

- Soil deformation may occur by change in:
  a) Stress
  b) Water content
  c) Soil mass
  d) Temperature
- The compression is caused by
  a) Elastic Deformation of soil particles
  b) Expulsion of water from the voids
  c) plastic adjustment of soil fabrics

- Types of settlement:
  a) Immediate (Elastic) Settlement $\boldsymbol{\delta_e}$ (4^{th} year)
  b) Consolidation Settlement (primary consolidation) $\boldsymbol{\delta_c}$
  c) Secondary Compression (Consolidation) Settlement $\boldsymbol{\delta_s}$

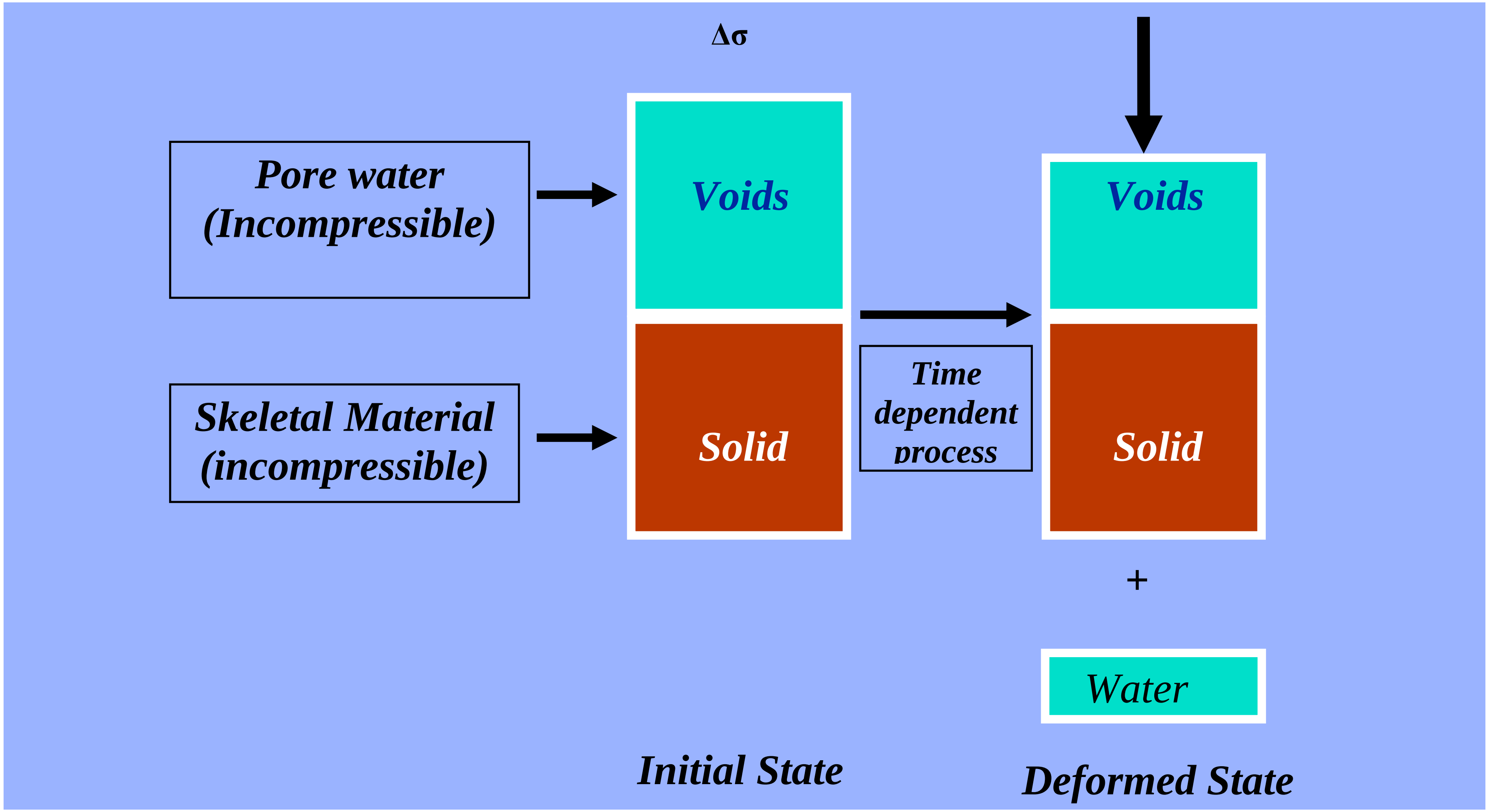
Thus, the total settlement will be $\boldsymbol{\delta_T = \delta_e + \delta_c + \delta_s}$

• Consolidation Settlement (Primary Consolidation)

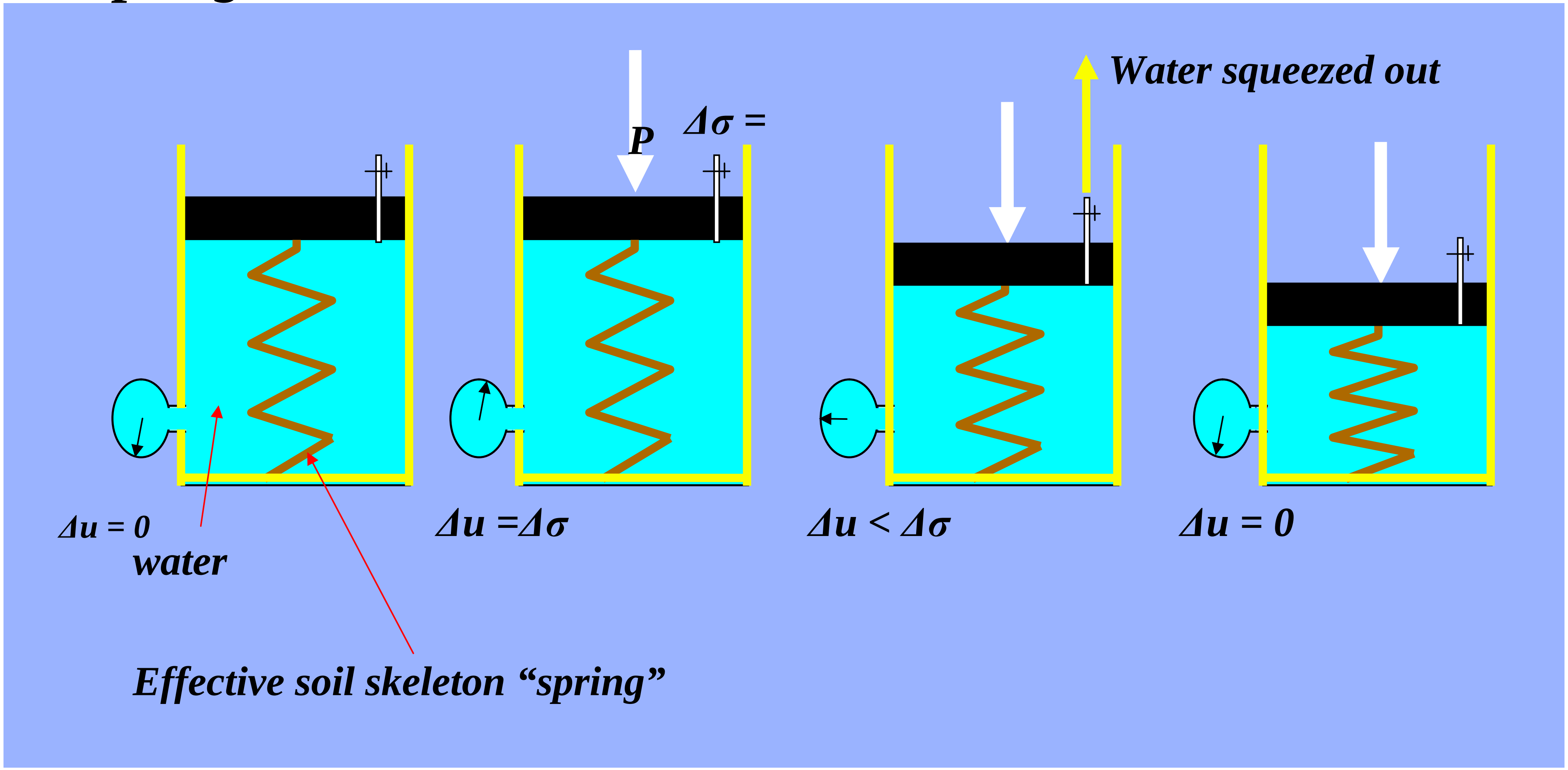
• *Fundamentals of Consolidation*

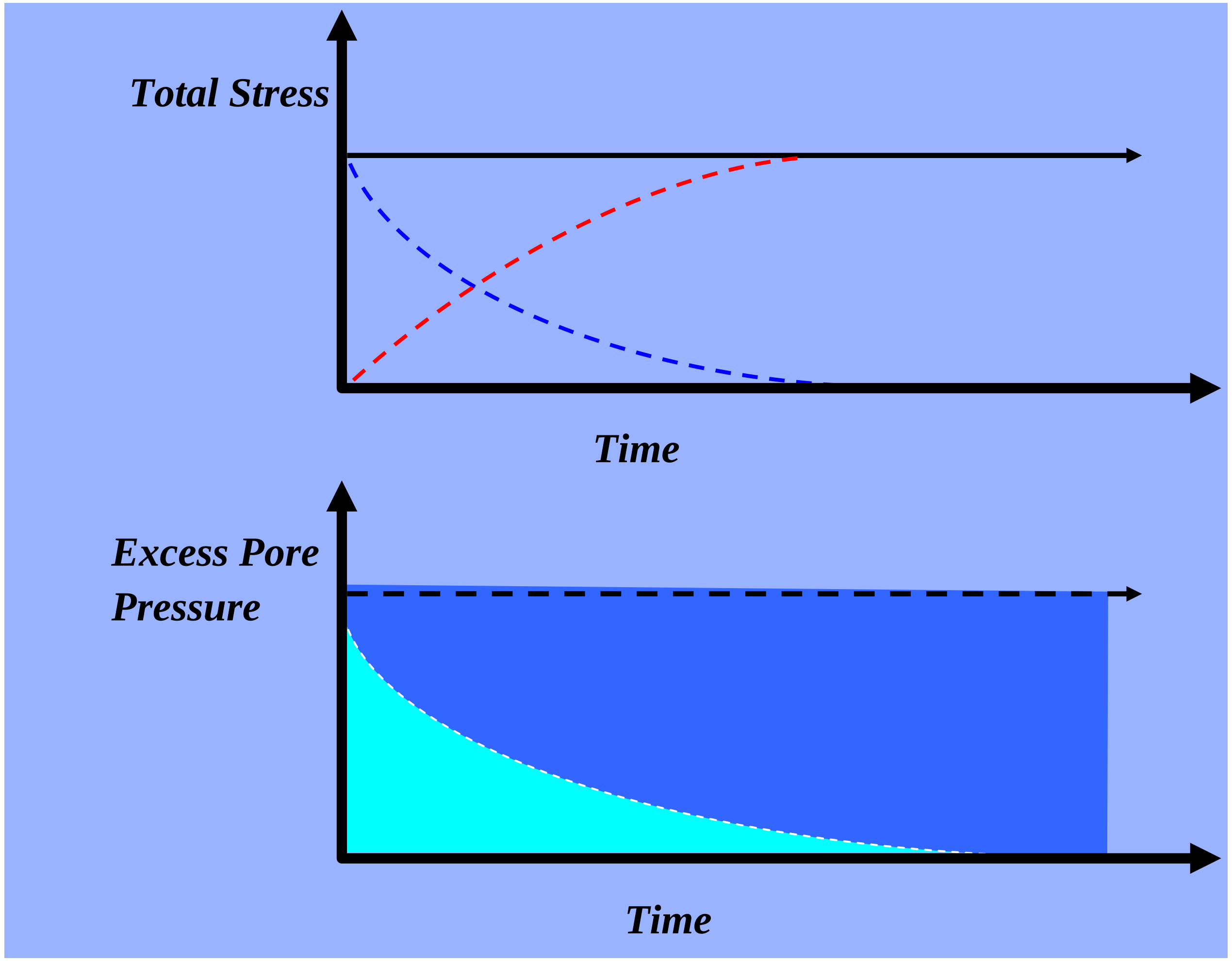
Deformation of saturated soil occurs by reduction of pore space & the squeezing out of pore water.

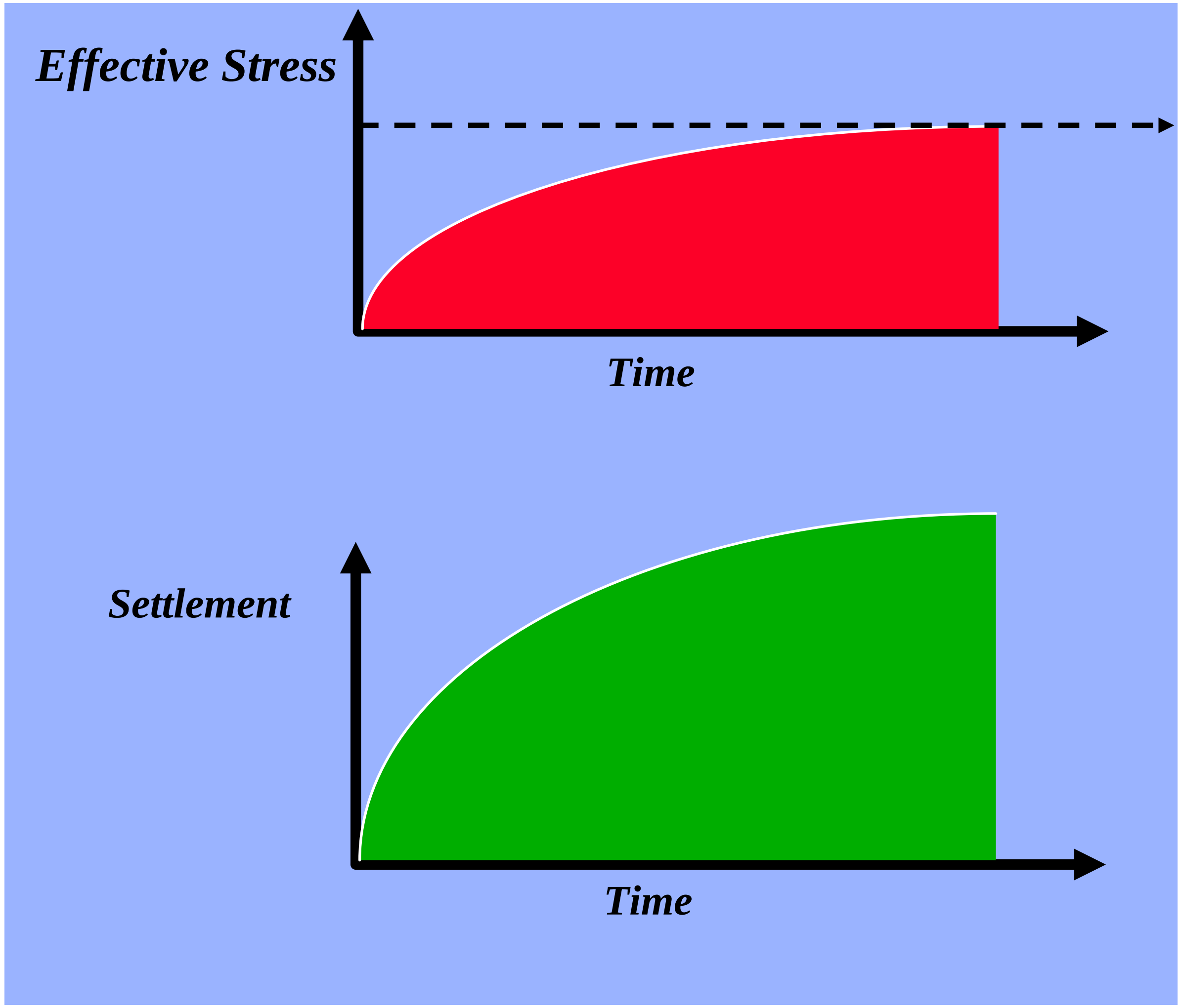
The water can only escape through the pores which for fine-grained soils are very small, while for coarse-grained soils are large enough for the process to occur immediately after the application of load.


Δσ
Pore water
(Incompressible)
Skeletal Material
(incompressible)
Voids
Solid
Time
dependent
process
Voids
Solid
+
Water
Initial State
Deformed State

• *Spring model*



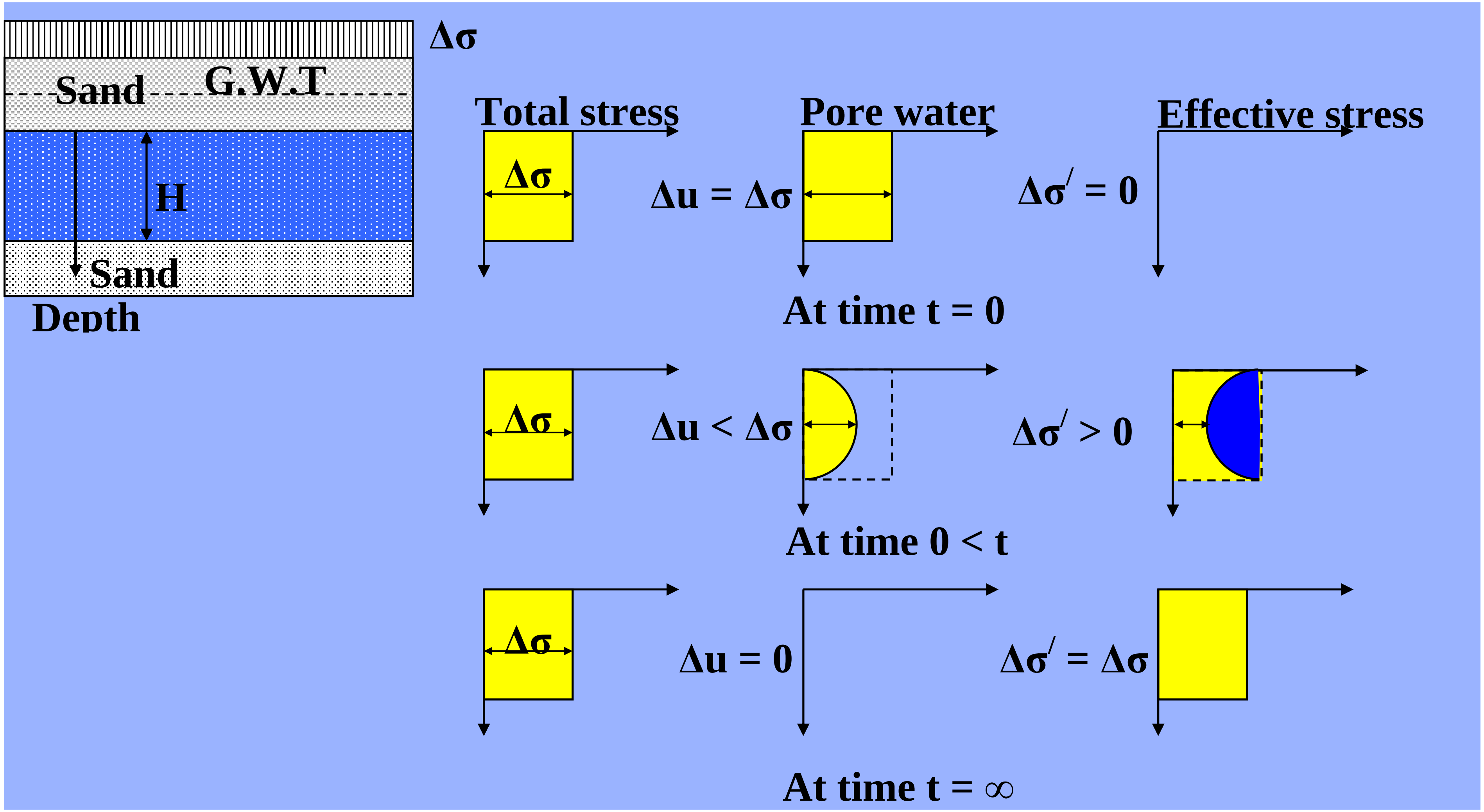

Total Stress
Time
Excess Pore
Pressure
Time


Effective Stress
Time
Settlement
Time

Conclusions

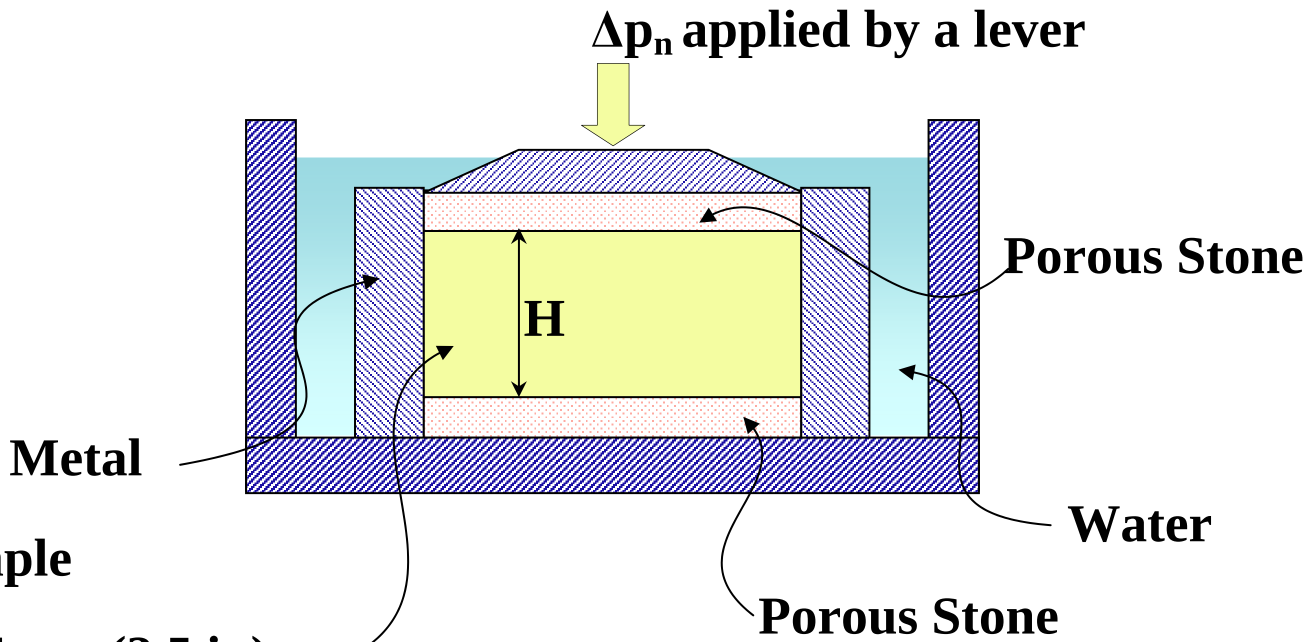
- Especially in low permeability soils (silts and clays) settlement is delayed by the need to squeeze the water out of the soil
- Consolidation is the process of gradual transfer of an applied load from the pore water to the soil structure as pore water is squeezed out of the voids.
- The amount of water that escapes depends on the size of the load and compressibility of the soil.
- The rate at which it escapes depends on the coefficient of permeability, thickness, and compressibility of the soil.

Consider a clay layer with thickness H subjected to an instantaneous increase of total stress $\Delta\sigma$.


Δσ
Sand
G.W.T
H
Sand
Depth
Total stress
Pore water
Effective stress
Δσ
Δu = Δσ
Δσ/ = 0
At time t = 0
Δσ
Δu < Δσ
Δσ/ > 0
At time 0 < t
Δσ
Δu = 0
Δσ/ = Δσ
At time t = ∞

- ***One-Dimensional Laboratory Consolidation Test***

  It was suggested by Terzaghi.

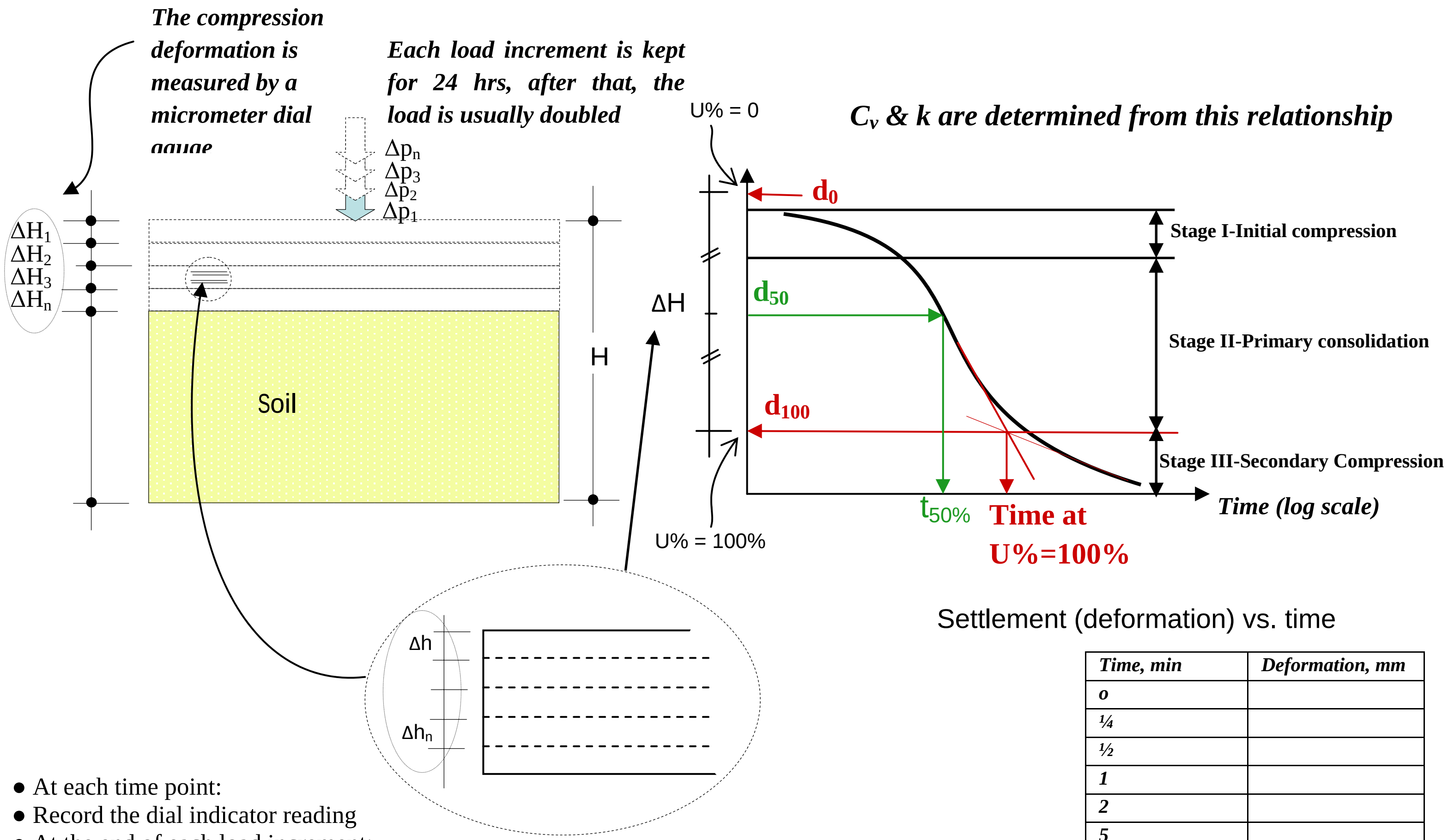


Soil Sample

Dia. ≈ 64 mm(2.5 in)

H ≈ 25 mm (1 in)

Consolidometer (Oedometer)



Settlement (deformation) vs. time

- At each time point:
- Record the dial indicator reading
- At the end of each load increment:
- Record the dial reading at completion of primary consolidation for each load (usually 24 hours)
- At the unloading of the specimen:
- Remove load in decrements, recording dial indicator readings
- Conduct a water content test on specimen after unloading is complete

Time, min	*Deformation, mm*
o	
¼	
½	
1	
2	
5	
10	
20	
30	
1 hr	
2 hr	
……..	
24 hr	

- ***Void Ratio-Pressure (e-log***

1. calculate the height of solids, H_s

$$H_s = \frac{W_s}{AG_s\gamma_w}$$ Prove it.

2. calculate initial height of voids, H_v

$$H = H - H_s$$

3. calculate initial void ratio, e_o

$$e_o = \frac{V_v}{V_v} = \frac{H_v A}{H_s A} = \frac{H_v}{H_s}$$

4. for the 1st incremental loading, σ_1 ($\frac{\Delta P_1}{A}$) which causes a deformation ΔH_1 $\Delta e_1 = \frac{\Delta H_1}{H_s}$

5. calculate new void ratio after consolidation caused by σ_1

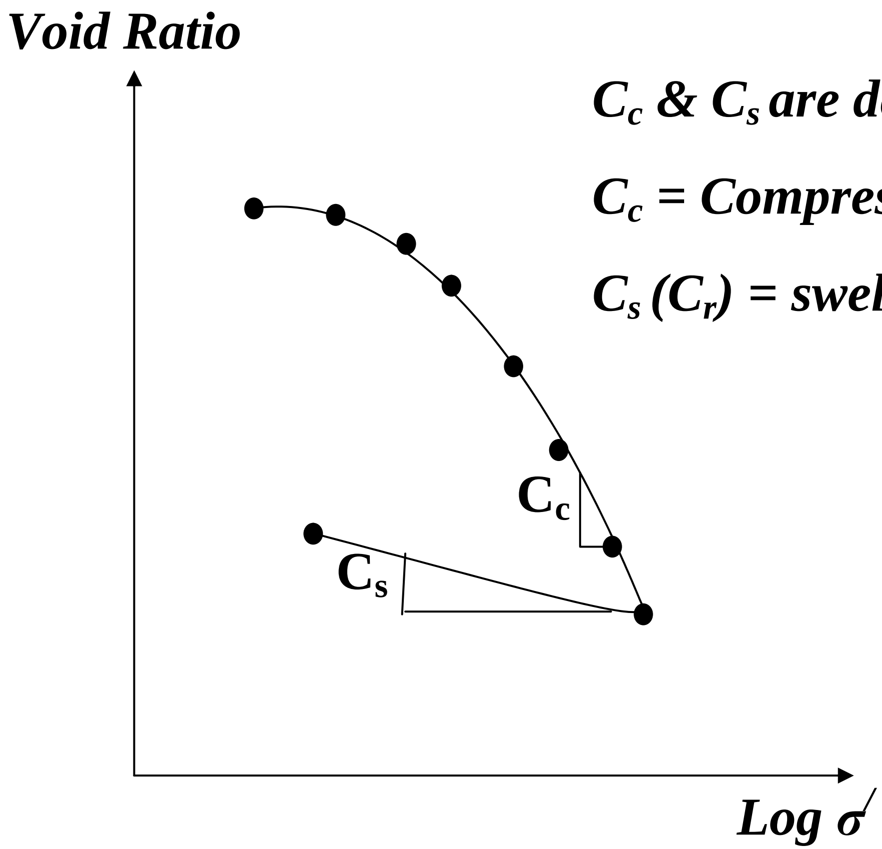
$$e_1 = e_o - \Delta e_1$$

6. for the next loading, $\sigma_2 \left(\frac{\Delta P_1 + \Delta P_2}{A}\right)$, which causes additional deformation ΔH_2, the void ratio at the end of consolidation is

$$e_2 = e_1 - \frac{\Delta H_2}{H_s}$$

<u>Note:-</u> at the end of consolidation $\sigma = \sigma^{/}$

Plot the corresponding e with $\sigma^{/}$ on semi-logarithmic paper. The typical shape of e-log $\sigma^{/}$ will be as shown in the figure.



C_c & C_s are determined from this relationship

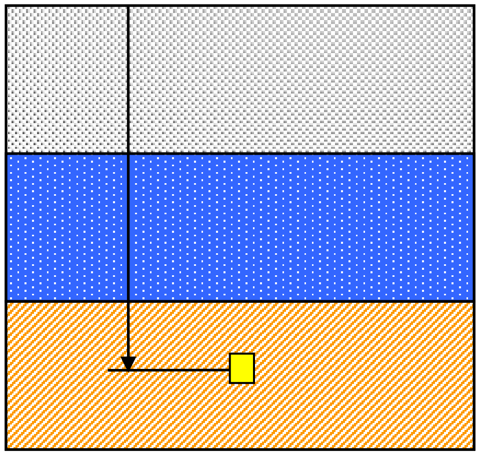
C_c = Compression Index

C_s (C_r) = swelling (rebound) Index

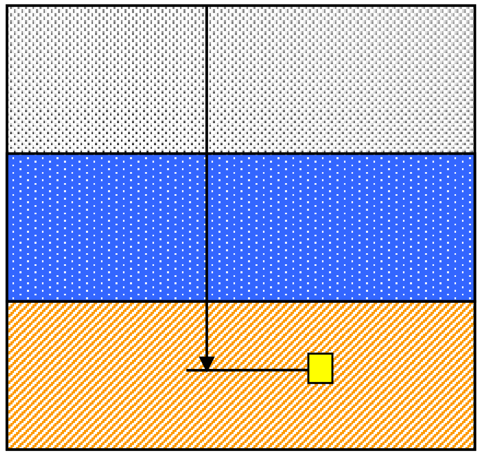
Change in void ratio vs. vertical effective stress

• *Normally Consolidated and Overconsolidated Clays*

Normally Consolidated



$\sigma'_o = \sigma'_P$



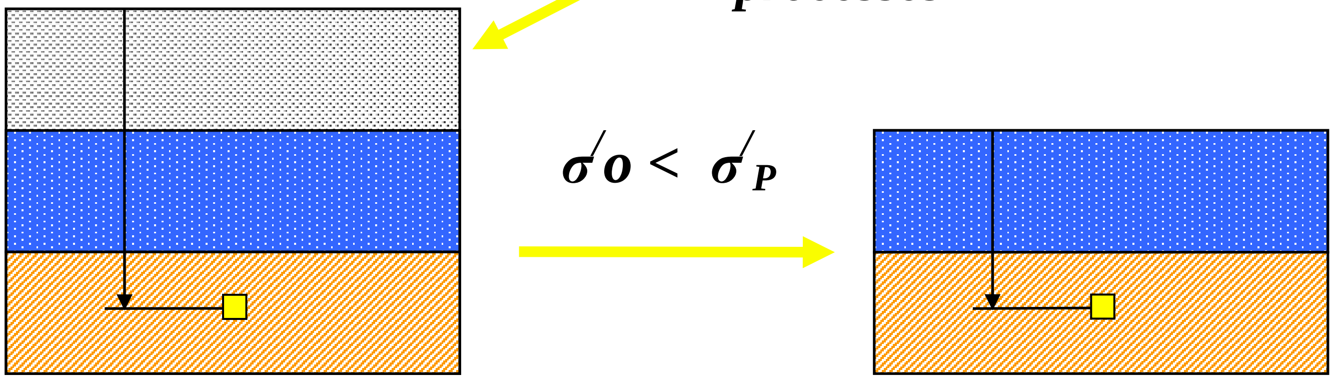
At past ***At present***

σ'_P ***= max. preconsolidation pressure*** σ'_o***= Effective overburden pressure***

Overconsolidated

May be removed due geologic processes or human processes

$\sigma'_o < \sigma'_P$

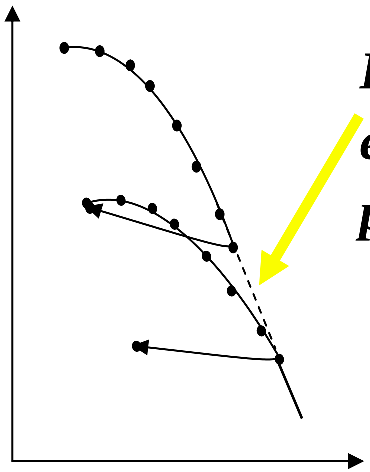


At past

At present

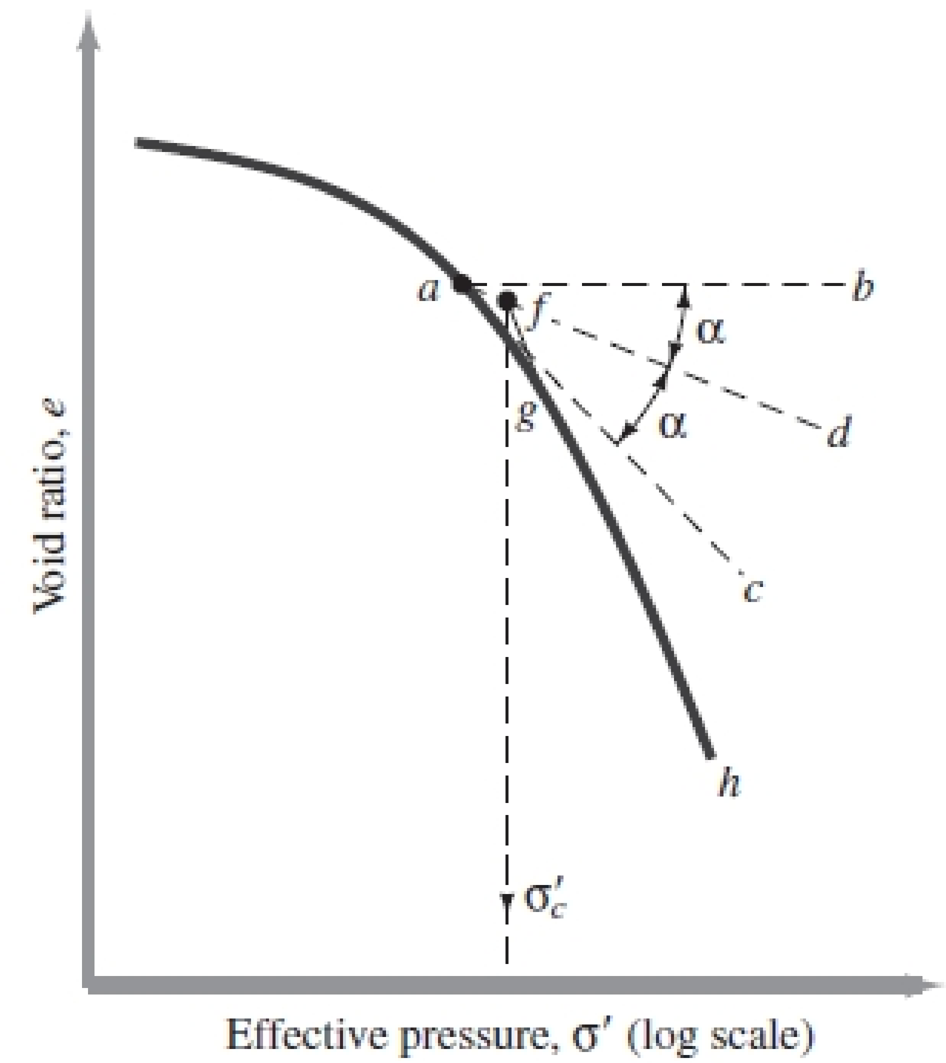
σ'_P ***= max. preconsolidation pressure***

σ'_o ***= Effective overburden pressure***



Relationship is linear and stepper when the effective stresses is exceeding maximum past preconsolidation pressure

σ'_P

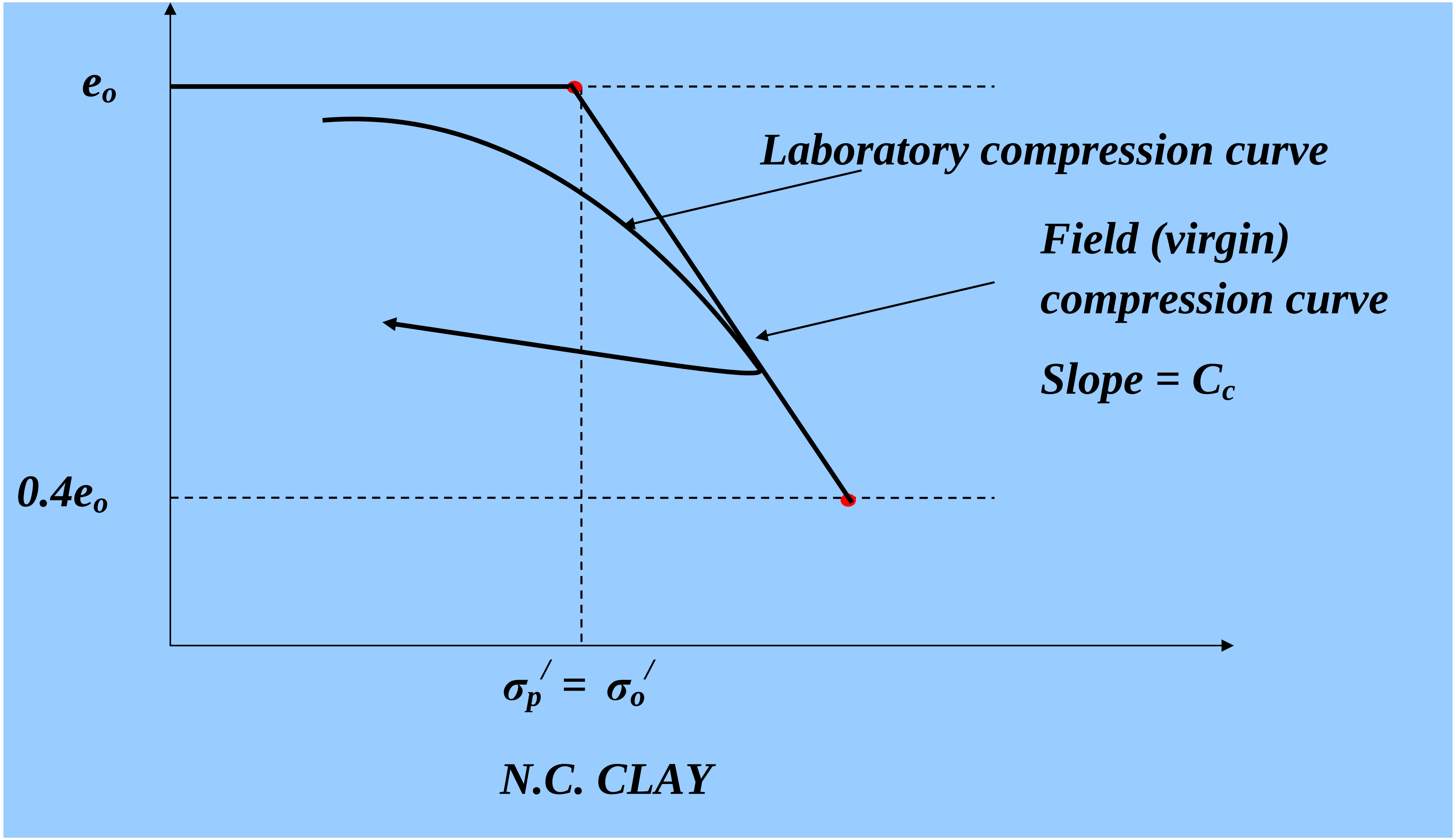


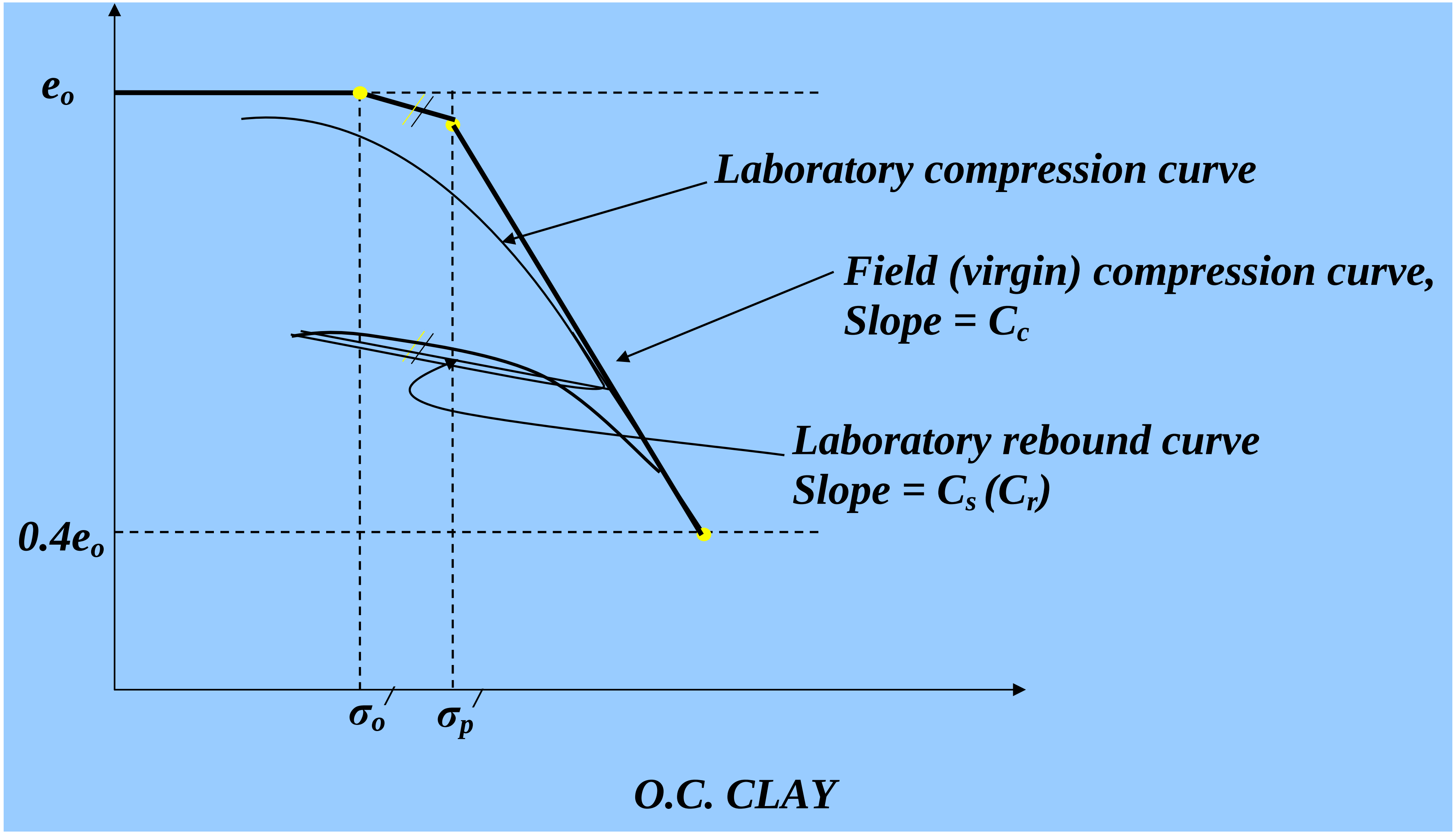
$$O.C.R = \frac{\sigma'_P}{\sigma'_O}$$

• *Effect of Disturbance on Void Ratio-Pressure Relationship*

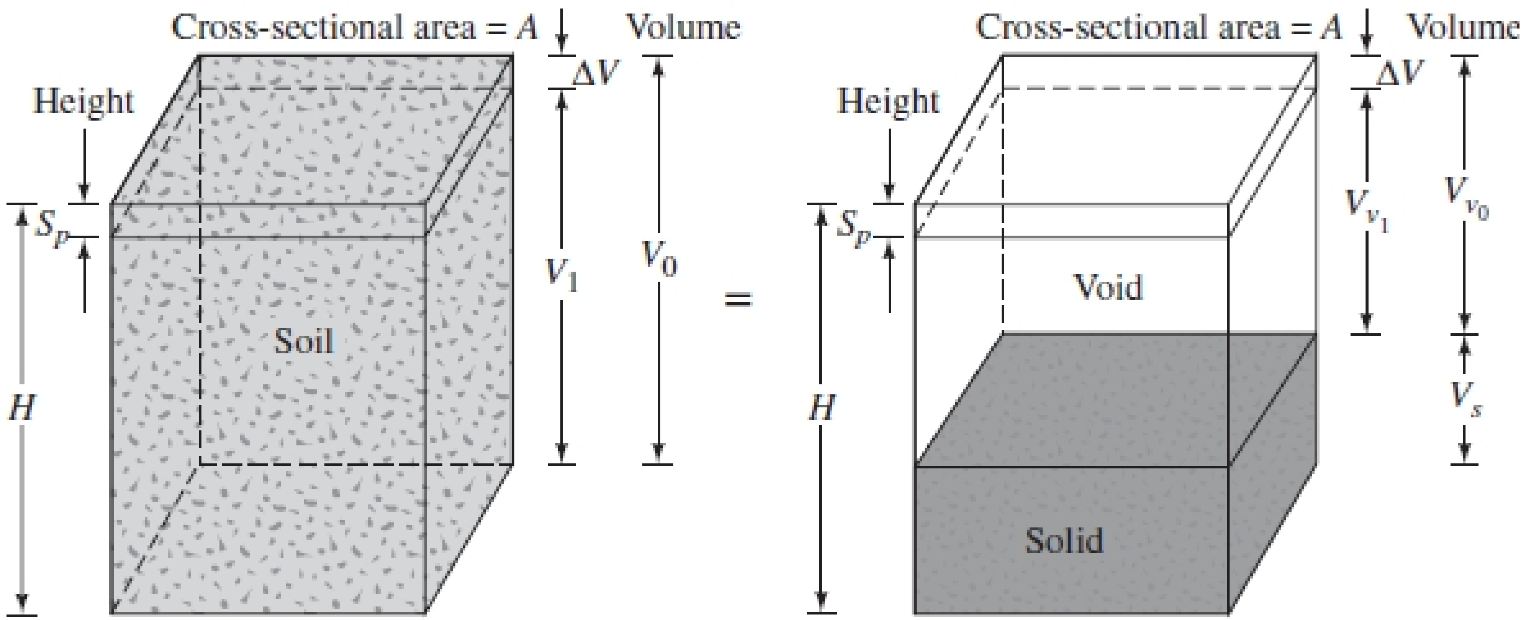
- Usually e-log $\sigma^{/}$ founded by performing consolidation test on undisturbed sample or remolded sample, does not reflects the field (virgin) compression curve.

- This difference is attribute to disturbance due to-
  - Handling and transferring samples into consolidation cells.
  - Sampling and stress relief


e_o
Laboratory compression curve
Field (virgin) compression curve
Slope = C_c
$0.4e_o$
$\sigma_p' = \sigma_o'$
N.C. CLAY


e_o
Laboratory compression curve
Field (virgin) compression curve,
Slope = C_c
Laboratory rebound curve
Slope = C_s (C_r)
$0.4e_o$
σ_o'
σ_p'
O.C. CLAY

• *Calculation of Settlement from One-Dimensional Primary Consolidation*



Settlement caused by one-dimensional consolidation

$$\Delta V = V_o - V_1 = HA - (H - \delta_c)A = \delta_c A$$

$$\Delta V = V_{vo} - V_{v1} = \Delta V_v = \Delta e V_s$$

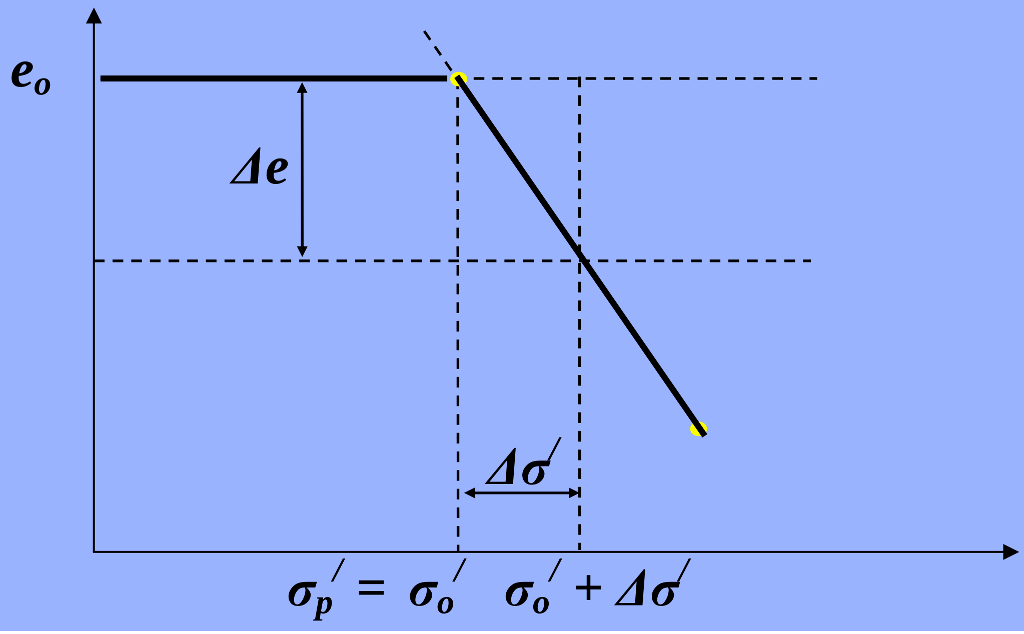
$$V_s = \frac{V_o}{1+e_o} = \frac{AH}{1+e_o}$$

$$\Delta V = \delta_c A = \Delta e V_s = \Delta e \frac{AH}{1+e_o}$$

$$\delta_c = H \frac{\Delta e}{1+e_o}$$

$$C_c = \frac{\Delta e}{\Delta \log \sigma'} \Rightarrow \Delta e = C_c[\log(\sigma'_o + \Delta\sigma') - \log \sigma'_o] = C_c \log(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o})$$

$$\delta_c = H\frac{\Delta e}{1+e_o} = \frac{C_c}{1+e_o} H \log(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o})$$



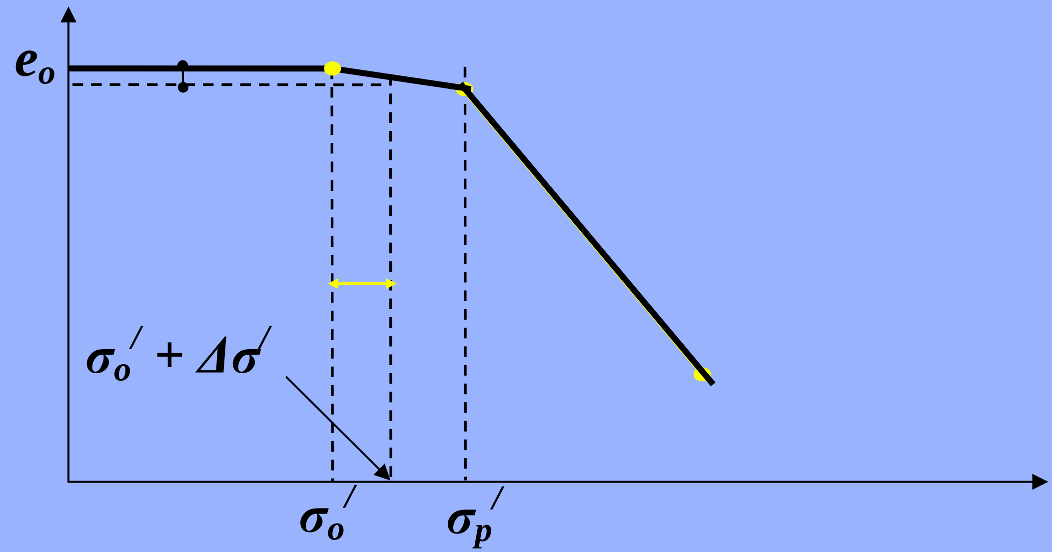
N.C. CLAY

O.C. CLAY

If $\sigma_o' + \Delta\sigma' \leq \sigma_p'$

$$C_s = \frac{\Delta e}{\Delta \log \sigma'} \Rightarrow \Delta e = C_s[\log(\sigma_o' + \Delta\sigma') - \log \sigma_o'] = C_s \log(\frac{\sigma_o' + \Delta\sigma'}{\sigma_o'})$$

$$\delta_c = H \frac{\Delta e}{1+e_o} = \frac{C_s}{1+e_o} H \log(\frac{\sigma_o' + \Delta\sigma'}{\sigma_o'})$$

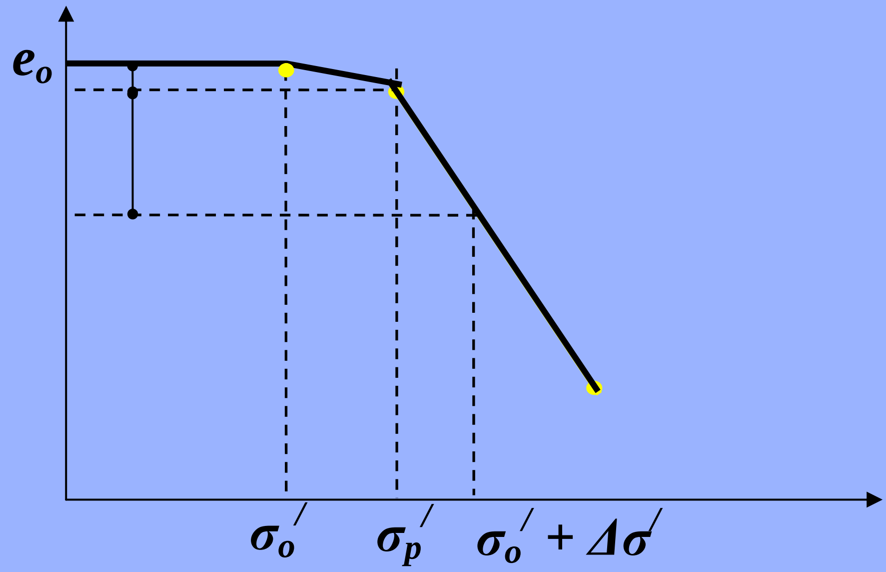


If $\sigma_o' + \Delta\sigma' > \sigma_p'$ **O.C. CLAY**

$$\Delta e = \Delta e_1 + \Delta e_2 = C_s[\log \sigma'_p - \log \sigma'_o] + C_c[\log(\sigma'_o + \Delta\sigma') - \log \sigma'_p]$$

$$= C_s \log(\frac{\sigma'_p}{\sigma'_o}) + C_c \log(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_p})$$

$$\delta_c = H \frac{\Delta e}{1 + e_o} = \frac{H}{1 + e_o}\left[C_s \log(\frac{\sigma'_p}{\sigma'_o}) + C_c \log(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_p})\right]$$



Compression Index (C_c) and Swell Index (C_s)

Several correlations were suggested for C_c besides other eq. and in most cases $C_s \cong \frac{1}{5} \text{ to } \frac{1}{10} C_c$

$C_c = 0.009(LL - 10)$ undisturbed clays LL= liquid limit

$C_c = 0.007(LL - 7)$ remolded clays LL= liquid limit

Table 10.4 Correlations for Compression Index, C_c*

Equation	Reference	Region of applicability
$C_c = 0.007(LL - 7)$	Skempton (1944)	Remolded clays
$C_c = 0.01w_N$		Chicago clays
$C_c = 1.15(e_O - 0.27)$	Nishida (1956)	All clays
$C_c = 0.30(e_O - 0.27)$	Hough (1957)	Inorganic cohesive soil: silt, silty clay, clay
$C_c = 0.0115w_N$		Organic soils, peats, organic silt, and clay
$C_c = 0.0046(LL - 9)$		Brazilian clays
$C_c = 0.75(e_O - 0.5)$		Soils with low plasticity
$C_c = 0.208e_O + 0.0083$		Chicago clays
$C_c = 0.156e_O + 0.0107$		All clays

* After Rendon-Herrero (1980)

Note: e_O = *in situ* void ratio; w_N = *in situ* water content.

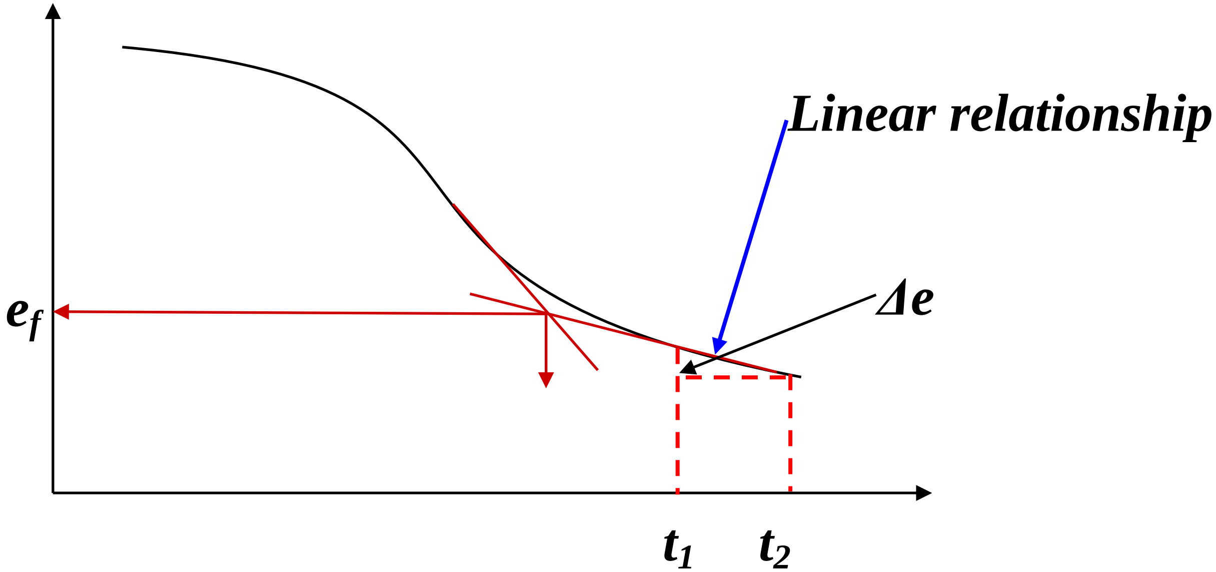
● Secondary Compression (Consolidation) Settlement

- Secondary compression settlement is a form of soil creep that is largely controlled by the rate at which the skeleton of compressible soils, particularly clays, silts, and peats, can yield and compress.
- Secondary compression is often conveniently identified to follow primary consolidation when excess pore fluid pressure can no longer be measured; however, both processes may occur simultaneously.
- Also referred to as the “secular effect”

$$C_\alpha = \frac{\Delta e}{\log t_2 - \log t_1} = \frac{\Delta e}{\log \frac{t_2}{t_1}}$$

$$C'_\alpha = \frac{C_\alpha}{1+e_p}$$

$$\delta_s = C'_\alpha H \log\left(\frac{t_2}{t_1}\right)$$

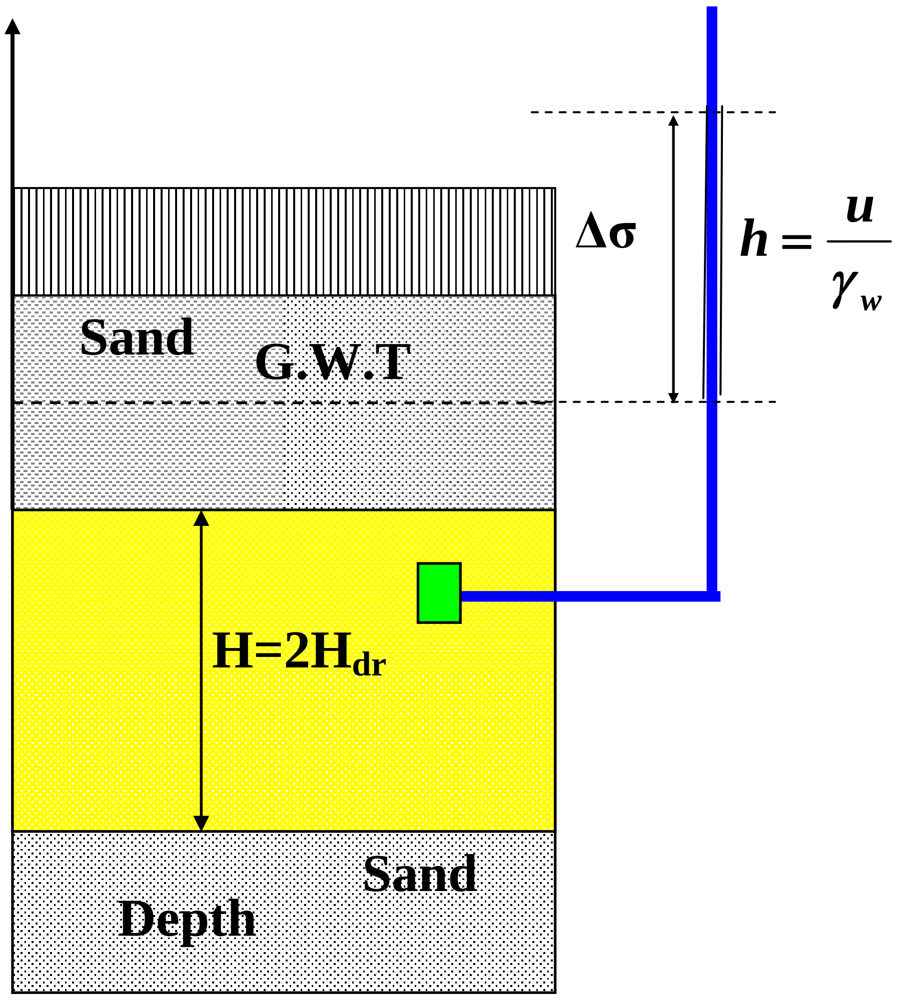


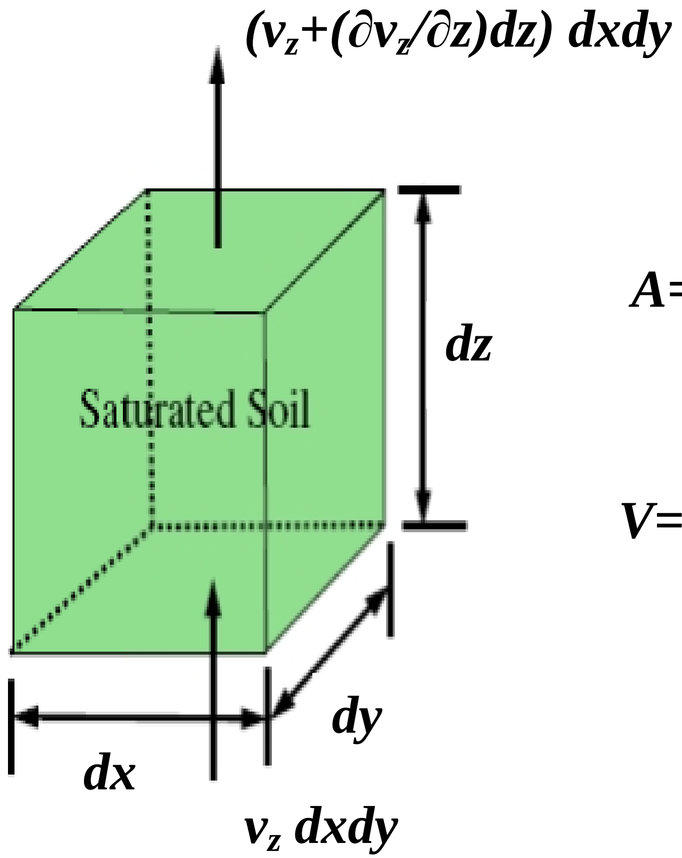
Type of soil	C'_α
O.C clays	0.001 or less
N.C clays	0.005 to 0.03
Organic soil	0.04 or more

• Time Rate of Consolidation

- Terzaghi (1925) proposed the first theory to consider the rate of one-dimensional consolidation for saturated clay soils.
- Assumptions:-
  1. The clay-water system is homogenous.
  2. Saturation is complete.
  3. Compressibility of water is negligible
  4. The flow of water is in one direction only (direction of compression)
  5. Darcy's law is valid

To begin, consider a very small element of soil being subjected to one-dimensional consolidation in the z-direction.





$A=dxdy$

$V=dxdydz$

Volume of pore fluid which flows out = Volume decrease of the soil

and thus

Rate at which pore fluid flows out = Rate of volume decrease of soil

$$[(v_z + \frac{\partial v_z}{\partial z} dz) \; - v_z]dxdy = \frac{\partial V}{\partial t}$$

$$\frac{\partial v_z}{\partial z} dxdydz = \frac{\partial V}{\partial t}$$

It will also be assumed that Darcy's law holds and thus that

$$v_z = ki = -k\frac{\partial h}{\partial z} = -\frac{k}{\gamma_w}\frac{\partial u}{\partial z} \qquad h = \frac{u}{\gamma_w}$$

u = excess pore water pressure caused by the increase of stress

$$-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = \frac{1}{dxdydz}\frac{\partial V}{\partial t}$$

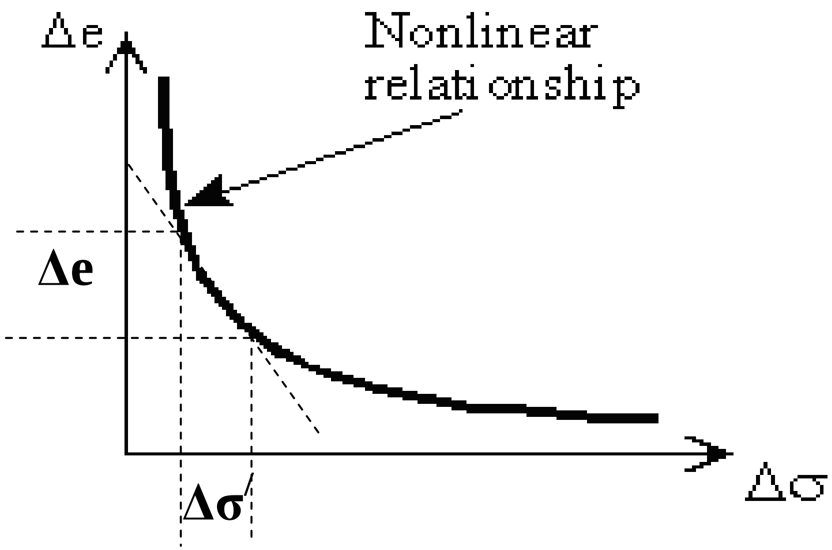
During consolidation

$$\frac{\partial V}{\partial t} = \frac{\partial V_v}{\partial t} = \frac{\partial (V_s + eV_s)}{\partial t} = \frac{\partial V_s}{\partial t} + V_s\frac{\partial e}{\partial t} + e\frac{\partial V_s}{\partial t}$$

but $\dfrac{\partial V_s}{\partial t} = 0$

$$\frac{\partial V}{\partial t} = V_s \frac{\partial e}{\partial t}$$

$$V_s = \frac{V}{1 + e_o} = \frac{dxdydz}{1 + e_o}$$



$$\frac{\partial V}{\partial t} = \frac{dxdydz}{1 + e_o} \frac{\partial e}{\partial t}$$

$$-\frac{k}{\gamma_w} \frac{\partial^2 u}{\partial z^2} = \frac{1}{1 + e_o} \frac{\partial e}{\partial t}$$

But $$\partial e = a_v \partial(\Delta\sigma') = -a_v \partial u$$

a_v = *coefficient of compressibility*

$$\therefore \quad -\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = -\frac{a_v}{1+e_o}\frac{\partial u}{\partial t} = -m_v \frac{\partial u}{\partial t}$$

Where

$$m_v = \frac{a_v}{1+e_o}$$

m_v = *coefficient of volume compressibility* or $$\boxed{\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}}$$ where

$$c_v = \frac{k}{\gamma_w m_v} = \frac{k}{\gamma_w \left(\dfrac{a_v}{1+e_o} \right)}$$

c_v = *coefficient of consolidation has units* L^2/T

k = *coefficient of permeability*

This equation is the basic differential equation of <u>*Terzaghi's consolidation theory*</u> and can be solved with the following boundary conditions

$z = 0$ $u = 0$ at a permeable boundary

$z = 2H_{dr}$ $u = 0$ at a permeable boundary

$t = 0$ $u = u_o = \Delta\sigma$ initial excess pore water pressure

The solutions yields

$$u = \sum_{m=0}^{\infty}\left[\frac{2u_o}{M}\sin\left(\frac{MZ}{H_{dr}}\right)\right]e^{-M^2T_v}$$

Where

$$M = \frac{\pi}{2}(2m+1)$$

a depth factor dimensionless number

$$Z = \frac{z}{H_{dr}}$$

a time factor is a nondimensoinal number

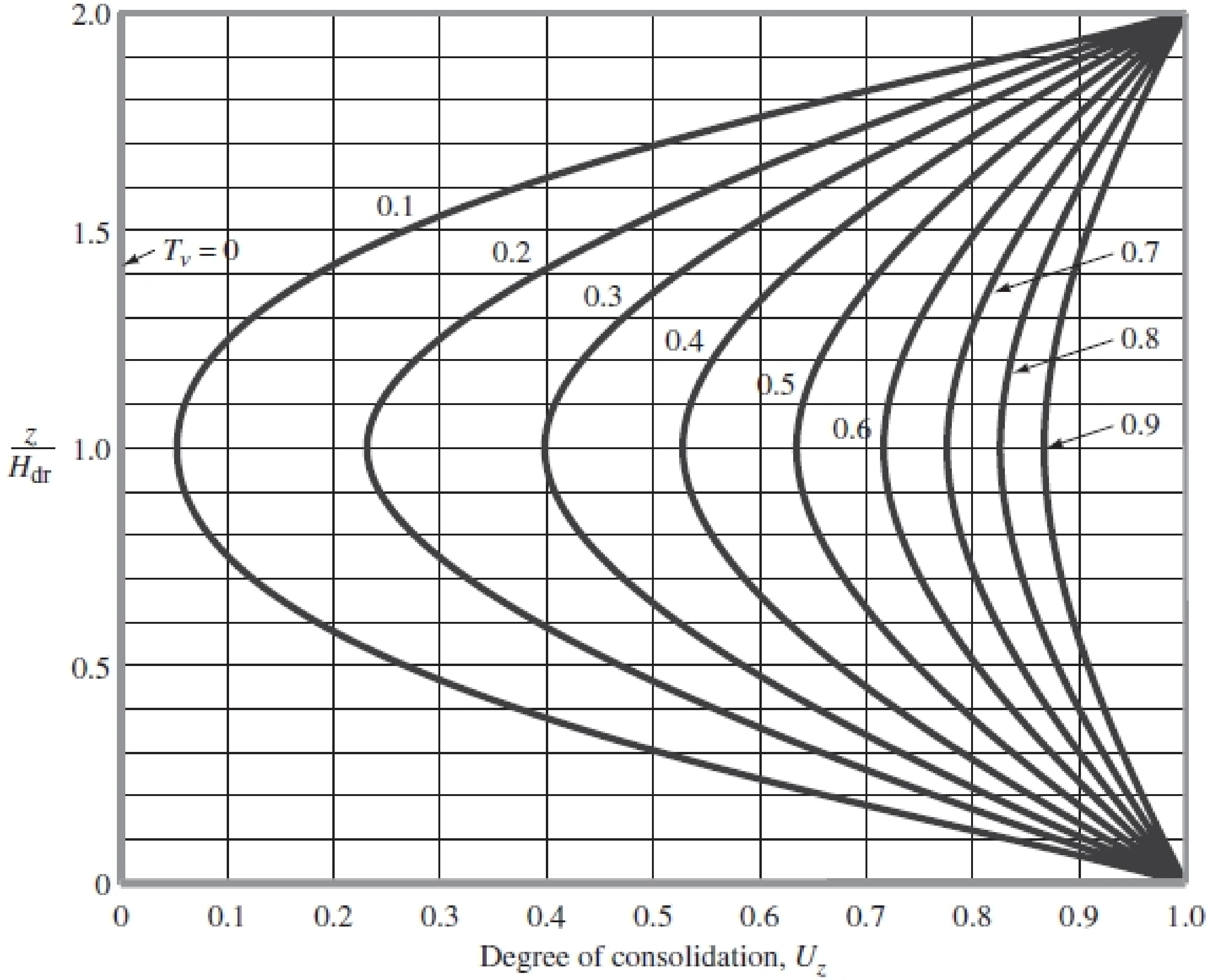
$$T_v = \frac{c_v t}{H_{dr}^{\;2}},$$

Because consolidation progresses by the dissipation of excess pore water pressure, the degree of consolidation at distance ***z*** at any time ***t*** is

$$U_z = \frac{u_o - u_z}{u_o} = 1 - \frac{u_z}{u_o}$$

u_z = *excess pore pressure at time* t

The variation of excess pore pressure within the layer is shown in Figure below


2.0
1.5
1.0
0.5
0
$\frac{z}{H_{dr}}$
$T_v = 0$
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Degree of consolidation, U_z

The average degree of consolidation for the entire depth of the clay layer at any time t can be written as

$$U = \frac{\delta_{c(t)}}{\delta_c} = 1 - \frac{\left(\frac{1}{2H_{dr}}\right)\int_0^{2H_{dr}} u_z\,dz}{u_o} = 1 - \sum_{m=0}^{m=\infty} \frac{2}{M^2} e^{-M^2 T_v}$$

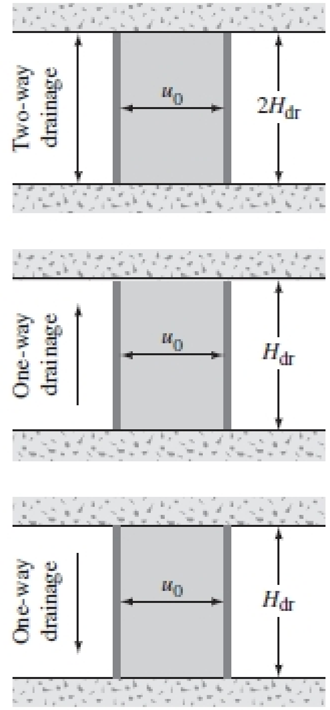
U = average degree of consolidation

$\delta_{c(t)}$ = settlement of the layer at time t

δ_c = ultimate (final) primary consolidation settlement

The U – T_v relationship is represented in the figure below for the case where u_o is uniform for the entire depth of the consolidating layer.

U (%)	T_v	U (%)	T_v	U (%)	T_v
0	0	34	0.0907	68	0.377
1	0.00008	35	0.0962	69	0.390
2	0.0003	36	0.102	70	0.403
3	0.00071	37	0.107	71	0.417
4	0.00126	38	0.113	72	0.431
5	0.00196	39	0.119	73	0.446
6	0.00283	40	0.126	74	0.461
7	0.00385	41	0.132	75	0.477
8	0.00502	42	0.138	76	0.493
9	0.00636	43	0.145	77	0.511
10	0.00785	44	0.152	78	0.529
11	0.0095	45	0.159	79	0.547
12	0.0113	46	0.166	80	0.567
13	0.0133	47	0.173	81	0.588
14	0.0154	48	0.181	82	0.610
15	0.0177	49	0.188	83	0.633
16	0.0201	50	0.197	84	0.658
17	0.0227	51	0.204	85	0.684
18	0.0254	52	0.212	86	0.712
19	0.0283	53	0.221	87	0.742
20	0.0314	54	0.230	88	0.774
21	0.0346	55	0.239	89	0.809
22	0.0380	56	0.248	90	0.848
23	0.0415	57	0.257	91	0.891
24	0.0452	58	0.267	92	0.938
25	0.0491	59	0.276	93	0.993
26	0.0531	60	0.286	94	1.055
27	0.0572	61	0.297	95	1.129
28	0.0615	62	0.307	96	1.219
29	0.0660	63	0.318	97	1.336
30	0.0707	64	0.329	98	1.500
31	0.0754	65	0.304	99	1.781
32	0.0803	66	0.352	100	∞
33	0.0855	67	0.364		



Different types of drainage with u_0 constant

*u_0 constant with depth.

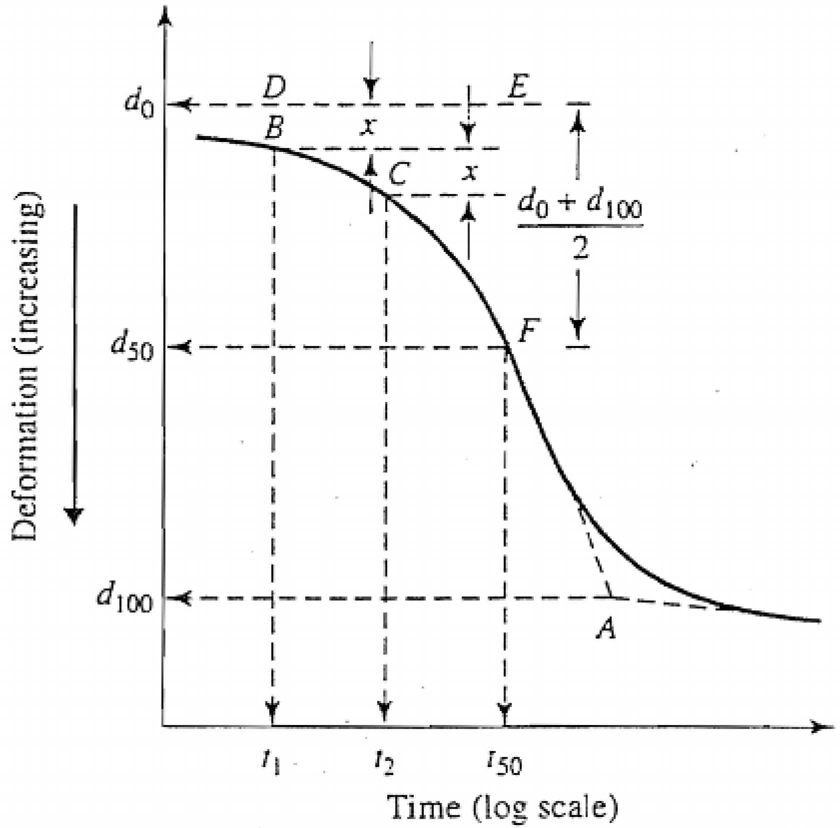
The values of T_v and their corresponding average U for the case presented above may also be approximately by the following relationship:

For U = 0 to 60% $$T_v = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2$$

For U > 60% $$T_v = 1.781 - 0.933\log(100 - U\%)$$

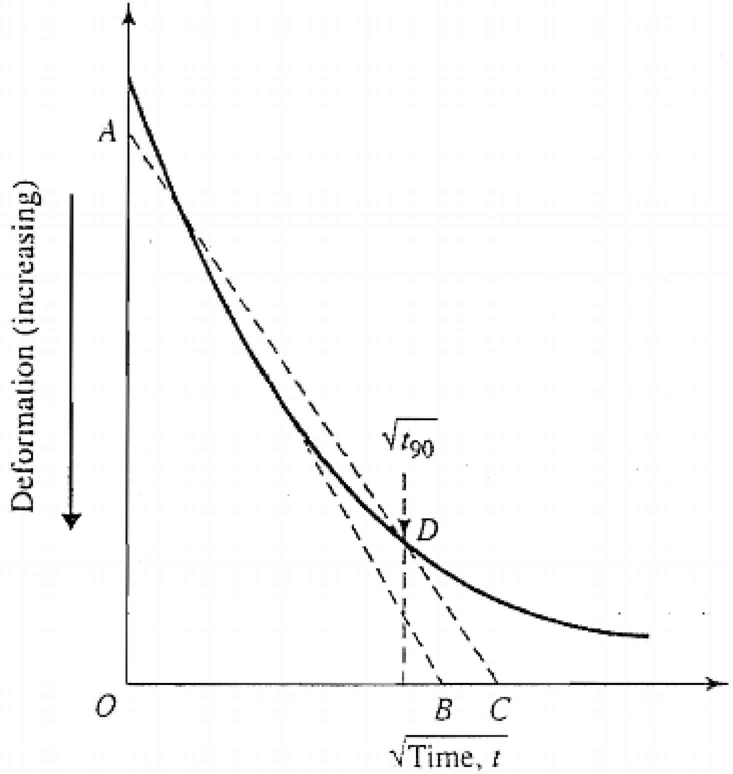
- ***Coefficient of Consolidation***
  - ***Logarithm of Time method***
  - ***Square Root of Time method***
  - ***Hperbola method***
  - ***Early Stage log-t method***

➢ *Logarithm of Time method*



$$T_{50} = \frac{c_v t_{50}}{H_{dr}^2} \quad \Rightarrow \quad c_v = \frac{0.197 H_{dr}^2}{t_{50}}$$

➢ *Square Root of Time method*

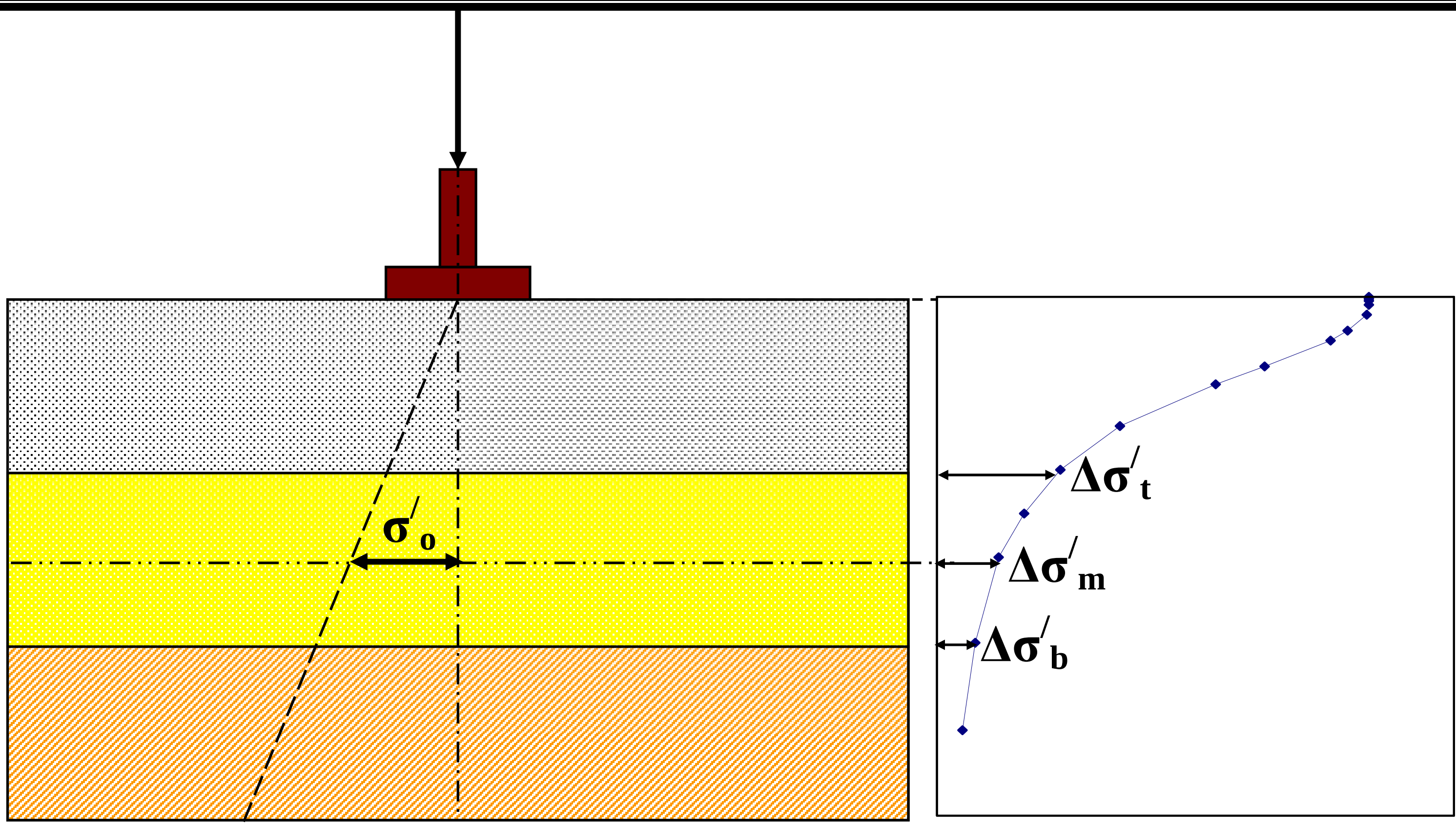


$$T_{90} = \frac{c_v t_{90}}{H_{dr}^2} \quad \Rightarrow \quad c_v = \frac{0.848 H_{dr}^2}{t_{90}}$$

• Calculation of Consolidation Settlement under a Foundation

For limited area foundations (circular, square and rectangular), the increase of effective stress ($\boldsymbol{\Delta\sigma'}$) decrease with depth as shown in figure below which can be estimated as described before in previous chapter.

Estimate $\boldsymbol{\sigma'_o}$ **and** $\boldsymbol{\Delta\sigma'_{av}}$ at the middle of the clay layer, then use the previous equations in to determine final consolidation settlement.



Using Simpson's rule

$$\Delta\sigma'_{av} = \frac{\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b}{6}$$

Alternative approach

Simply divide the clay layer to a number of sub layers, and then estimate $\boldsymbol{\delta_c}$ for each sub layer taking into account effective overburden pressure and an increase in effective stress at the middle of each sub layer, then get the summation of settlements of the sub layers to get the final consolidation of the clay layer.

Rate of consolidation

It is important to determine δ_c – ***time*** relationship, which can be helpful in estimating the differential settlement between adjacent footings if the drainage condition at one footing differs from the other.

$$U = \frac{\delta_{c(t)}}{\delta_c}$$

Examples

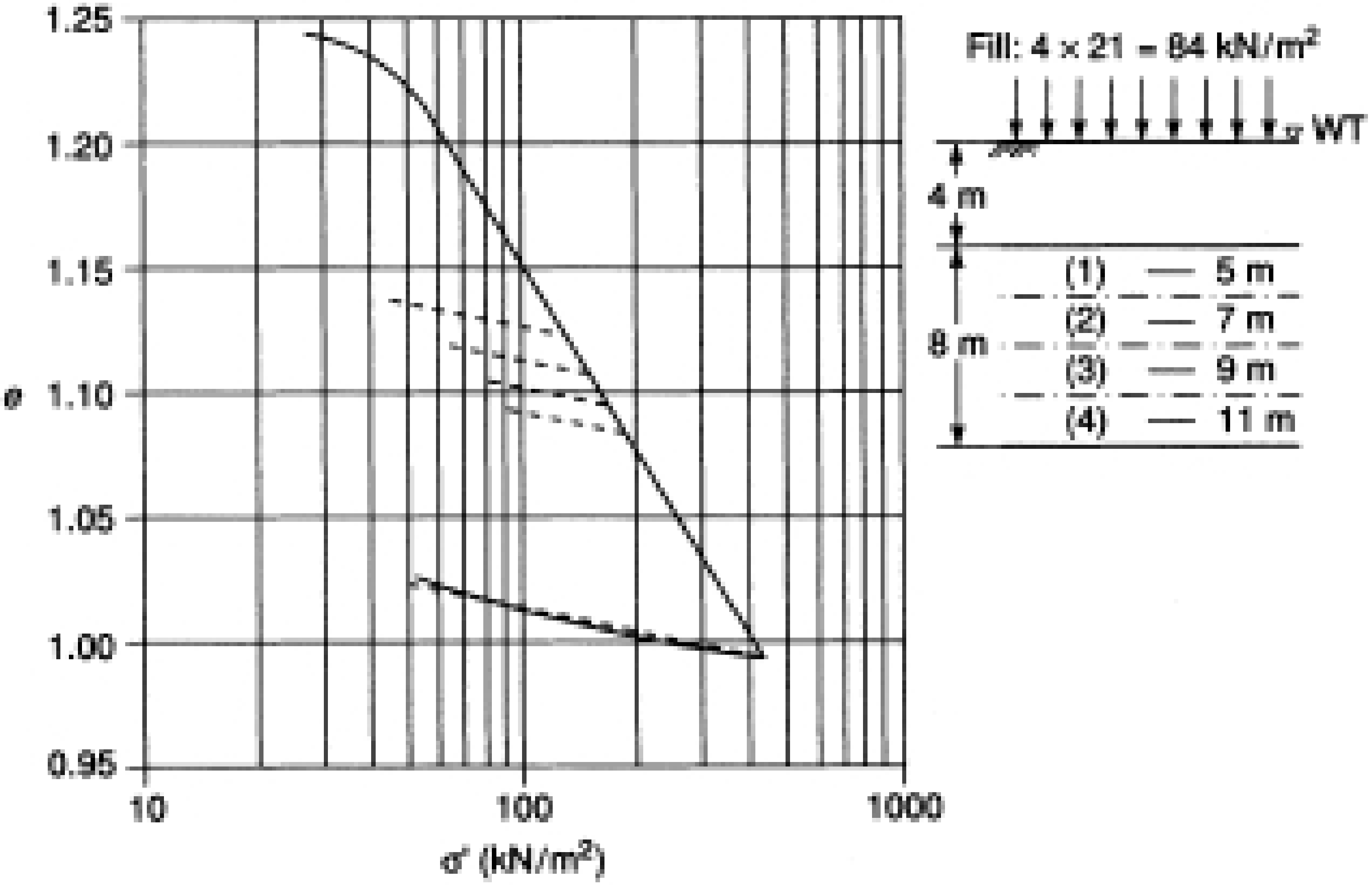
The following results were obtained from an oedometer test on a specimen of saturated clay:

Pressure (kN/m^2)	***27***	***54***	***107***	***214***	***429***	***214***	***107***	***54***
Void ratio	***1.243***	***1.217***	***1.144***	***1.068***	***0.994***	***1.001***	***1.012***	***1.024***

A layer of this clay 8m thick lies below a 4m depth of sand, the water table being at the surface. The saturated unit weight for both soils is 19kN/m^3. A 4m depth of fill of unit weight 21 kN/m^3 is placed on the sand over an extensive area. Determine the final settlement due to consolidation of the clay. If the fill were to be removed some time after the completion of consolidation, what heave would eventually take place due to swelling of the clay?

$$\delta_c = \frac{e_o - e_1}{1 + e_o} H$$

Appropriate values of e are obtained from e-log$\sigma^{/}$ drawn from the result. The clay will be divided into four sub-layers, hence H =2000 mm.


1.25
1.20
1.15
1.10
1.05
1.00
0.95
e
10
100
1000
σ' (kN/m²)
Fill: 4 × 21 = 84 kN/m²
WT
4 m
8 m
(1) — 5 m
(2) — 7 m
(3) — 9 m
(4) — 11 m

Settlement

Layer	σ'_0 (kN/m^2)	σ'_1 (kN/m^2)	e_0	e_1	$e_0 - e_1$	s_c (mm)
1	46.0*	130.0†	1.236	1.123	0.113	101
2	64.4	148.4	1.200	1.108	0.092	84
3	82.8	166.8	1.172	1.095	0.077	71
4	101.2	185.2	1.150	1.083	0.067	62
						318

Notes
* 5×9.2.
† $46.0 + 84$.

Heave

Layer	σ'_0 (kN/m^2)	σ'_1 (kN/m^2)	e_0	e_1	$e_0 - e_1$	s_c (mm)
1	130.0	46.0	1.123	1.136	−0.013	−12
2	148.4	64.4	1.108	1.119	−0.011	−10
3	166.8	82.8	1.095	1.104	−0.009	−9
4	185.2	101.2	1.083	1.091	−0.008	−7
						−38

Assuming the fill in pervious example is dumped very rapidly, what would be the value of excess pore water pressure at the centre of the clay layer after a period of 3 years? Assume Two-way drainage condition and the value of c_v is $2.4 m^2/year$.

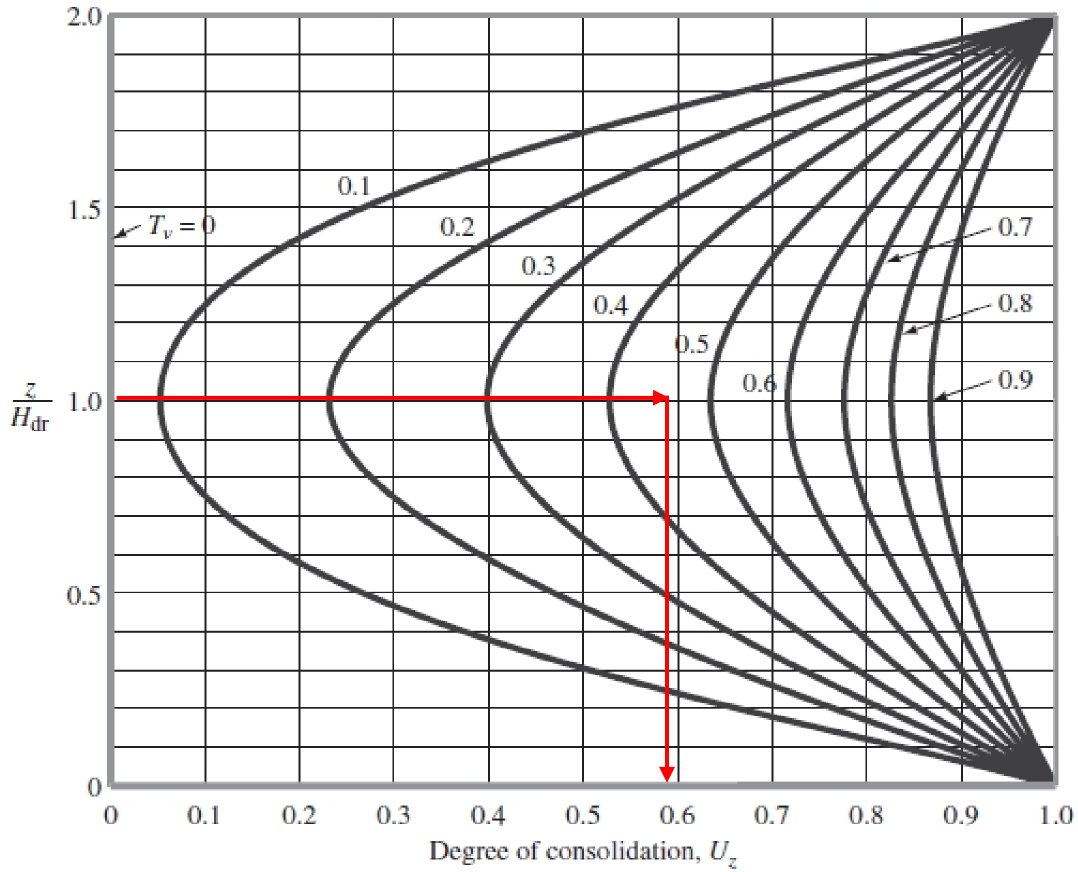
$$d = \frac{8}{2} = 4\,\text{m}$$

$$T_v = \frac{c_v t}{d^2} = \frac{2.4 \times 3}{4^2} = 0.450$$

$$u_i = \Delta\sigma = 84\,\text{kN/m}^2$$

$$U_z = \frac{u_o - u_z}{u_o} = 1 - \frac{u_z}{84} = 0.58$$

$$u_z = 35.2 \ldots kN\ /m^2$$



In an oedometer test a specimen of saturated clay 19mm thick reaches 50% consolidation in 20 min. How long would it take a layer of this clay 5m thick to reach the same degree of consolidation under the same stress and drainage conditions? How long would it take the layer to reach 30% consolidation?

$$U = f(T_v) = f\left(\frac{c_v t}{d^2}\right)$$

Hence if c_v is constant,

$$\frac{t_1}{t_2} = \frac{d_1^2}{d_2^2}$$

where ‘1’ refers to the oedometer specimen and ‘2’ the clay layer.

For double Drainage (Two-way Drainage)

$$d_1 = 9.5\,\text{mm} \quad \text{and} \quad d_2 = 2500\,\text{mm}$$

$$\therefore \text{ for } U = 0.50,\ t_2 = t_1 \times \frac{d_2^2}{d_1^2}$$

$$= \frac{20}{60 \times 24 \times 365} \times \frac{2500^2}{9.5^2} = 2.63\,\text{years}$$

$$\text{for } U < 0.60,\ T_v = \frac{\pi}{4} U^2$$

$$\therefore\ t_{0.30} = t_{0.50} \times \frac{0.30^2}{0.50^2}$$

$$= 2.63 \times 0.36 = 0.95\,\text{years}$$