● Stress Path

A ‘‘Stress–Path’’ is a curve or a straight line which is the locus of a series of stress points depicting the changes in stress in a test specimen or in a soil element in-situ, during loading or unloading, engineered as in a triaxial test in the former case or caused by forces of nature.

An elementary way to monitor stress changes is by showing the Mohr’s stress circles at different stages of loading/unloading. But this may be cumbersome as well as confusing when a number of circles are to be shown in the same diagram.

Stress-path approach enables the engineer to predict and monitor the shear strength mobilized at any stage of loading/unloading in order to ensure the stability of foundation soil.

Lambe and Whitman (1969) have suggested the locus of points representing the maximum shear stress acting on the soil at different stages be treated as a 'stress path', which can be drawn and studied in place of the corresponding Mohr's circles. This is shown in Fig. below.

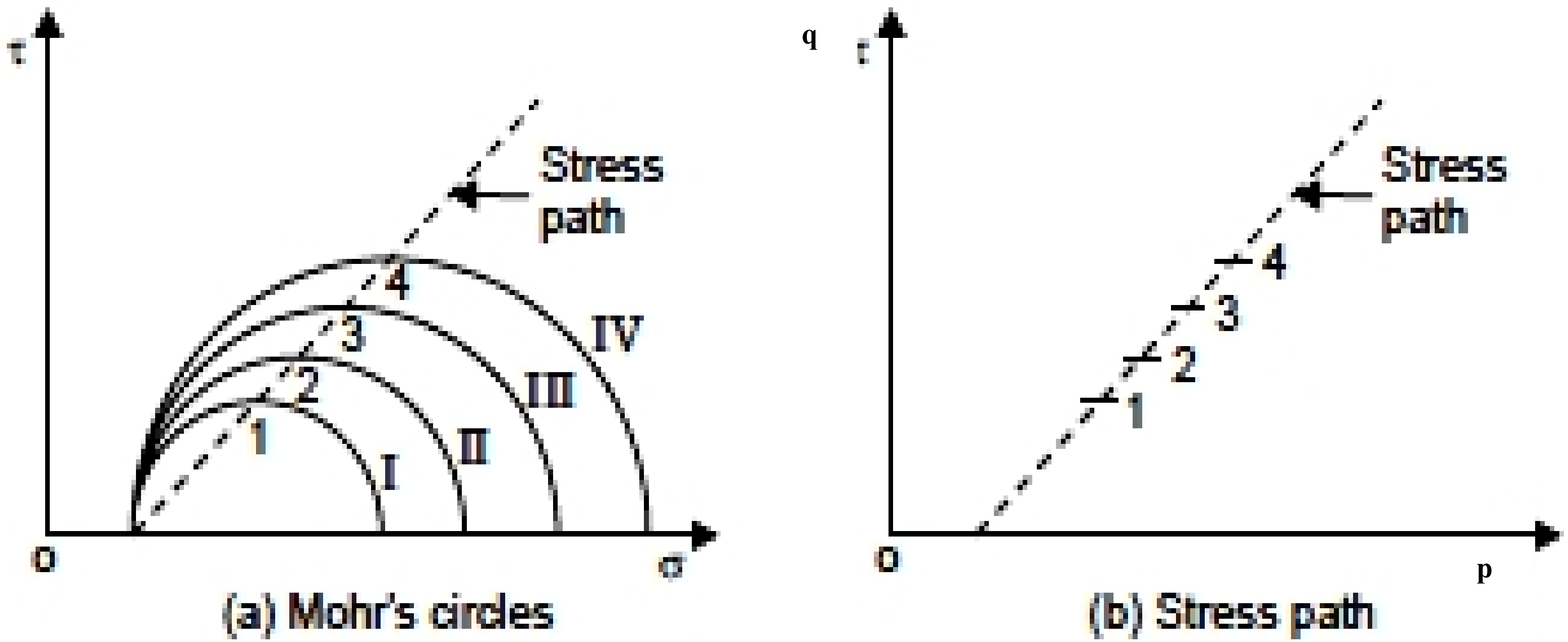


Fig. 8.23 Stress path (Lambe and Whitman, 1969) for the case of σ_1 increasing and σ_3 constant

The co-ordinates of the points on the stress path

$$q = \frac{\sigma_1 - \sigma_3}{2} \qquad p = \frac{\sigma_1 + \sigma_3}{2}$$

If σ_1 and σ_3 are the vertical and horizontal principal stresses, these become

$$q = \frac{\sigma_v - \sigma_h}{2} \qquad p = \frac{\sigma_v + \sigma_h}{2}$$

Either the effective stresses or the total stresses may be used for this purpose. The basic types of stress path and the co-ordinates are:

(a) Effective Stress Path (ESP) $\left[\left(\dfrac{\bar{\sigma}_1+\bar{\sigma}_3}{2}\right),\left(\dfrac{\bar{\sigma}_1-\bar{\sigma}_3}{2}\right)\right]$ p', q'

$p = p' + u$, $q = q'$

(b) Total Stress Path (TSP) $\left[\left(\dfrac{\sigma_1+\sigma_3}{2}\right),\left(\dfrac{\sigma_1-\sigma_3}{2}\right)\right]$

(c) Stress path of total stress less static pore water pressure (TSSP)

$$\left[\left(\frac{\sigma_1+\sigma_3}{2}-u_0\right),\left(\frac{\sigma_1-\sigma_3}{2}\right)\right]$$

u_0 : Static pore water pressure

u_0 = zero in the conventional triaxial test, and (b) and (c) coincide in this case. But if back pressure is used in the test, u_0 = the back pressure.

For an in-situ element, the static pore water pressure depends upon the level of the ground water table.

$$\text{Slope of stress path line} = \frac{\Delta q}{\Delta p} = \frac{q_f - q_o}{p_f - p_o}$$

Where q_f and p_f are coordinates at failure and q_o and p_o are coordinates at initial condition.

$$\tau_f = c + \sigma \tan \phi$$

Modified Failure Envelope

For N.C soil

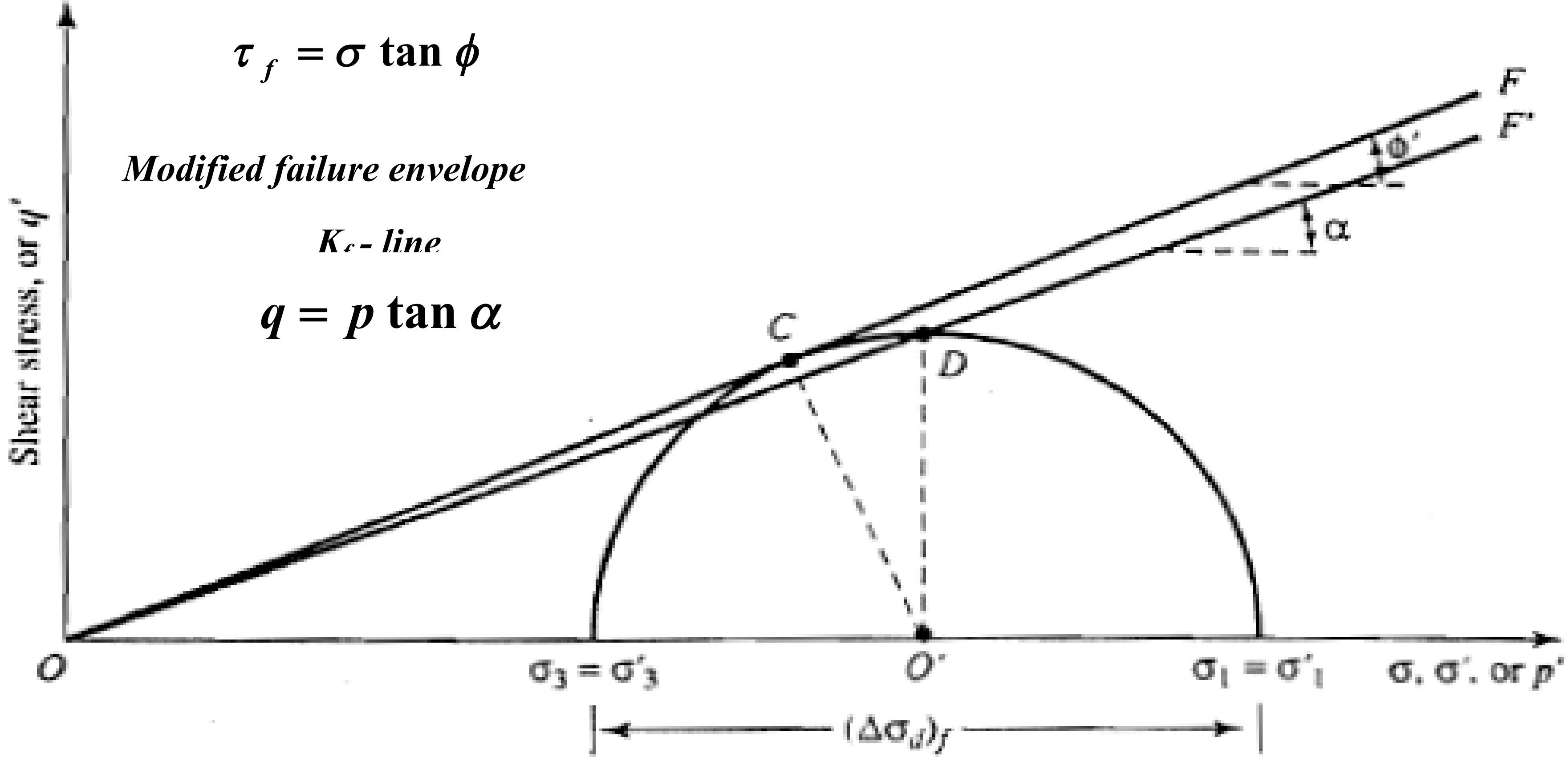
Mohr- coulomb failure envelope

$$\tau_f = \sigma \tan \phi$$

Modified failure envelope

K_f - line

$$q = p \tan \alpha$$



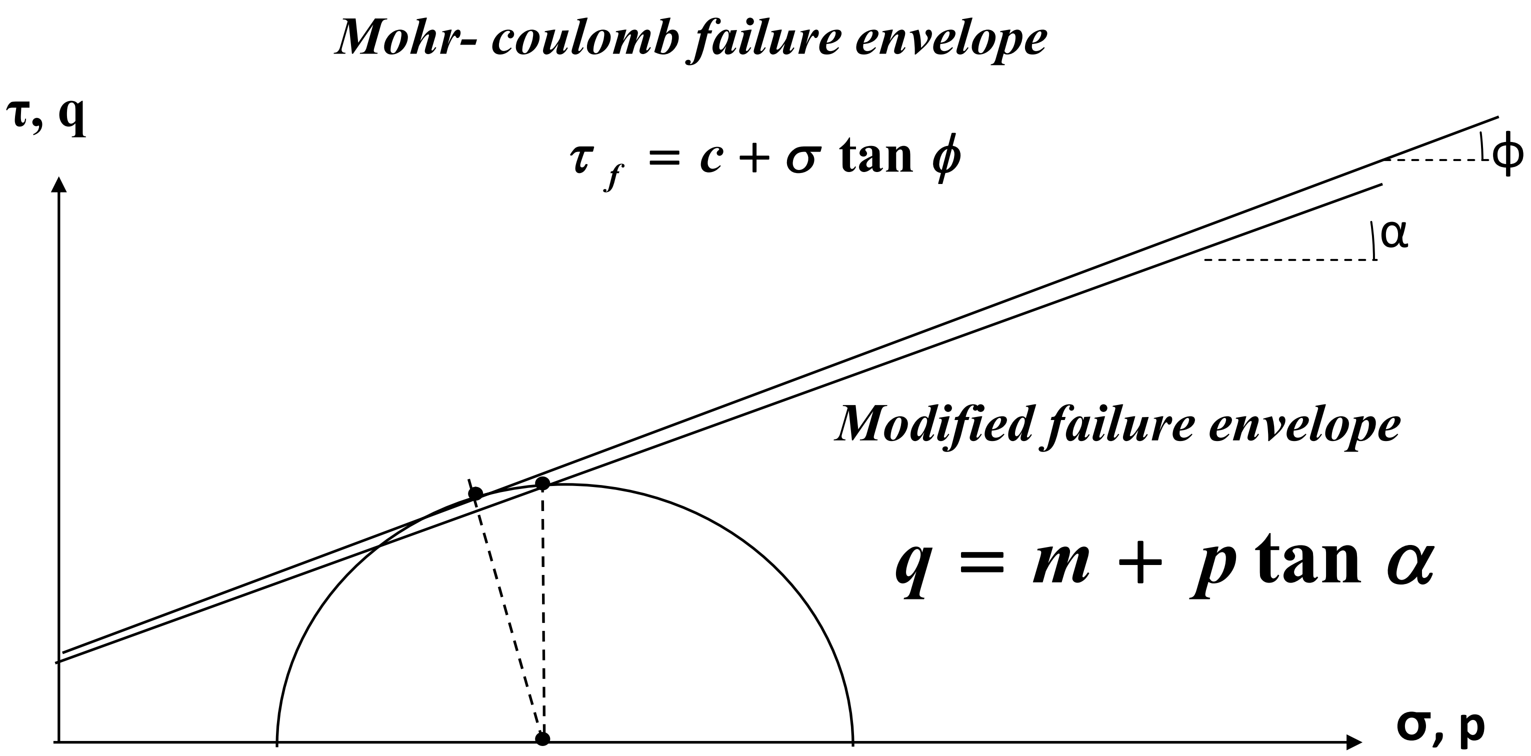
$$\frac{DO'}{OO'} = \tan\alpha \qquad \tan\alpha = \frac{\frac{\sigma_1' - \sigma_3'}{2}}{\frac{\sigma_1' + \sigma_3'}{2}} = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'}$$

Again,

$$\frac{CO'}{OO'} = \sin\phi' \quad \text{or} \quad \sin\phi' = \frac{\frac{\sigma_1' - \sigma_3'}{2}}{\frac{\sigma_1' + \sigma_3'}{2}} = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'}$$

$$\boxed{\sin\phi' = \tan\alpha}$$

For O.C. clay



Where m = c cosφ

Typical stress paths for triaxial compression and extension tests (loading as well as unloading cases) are shown in Fig. below

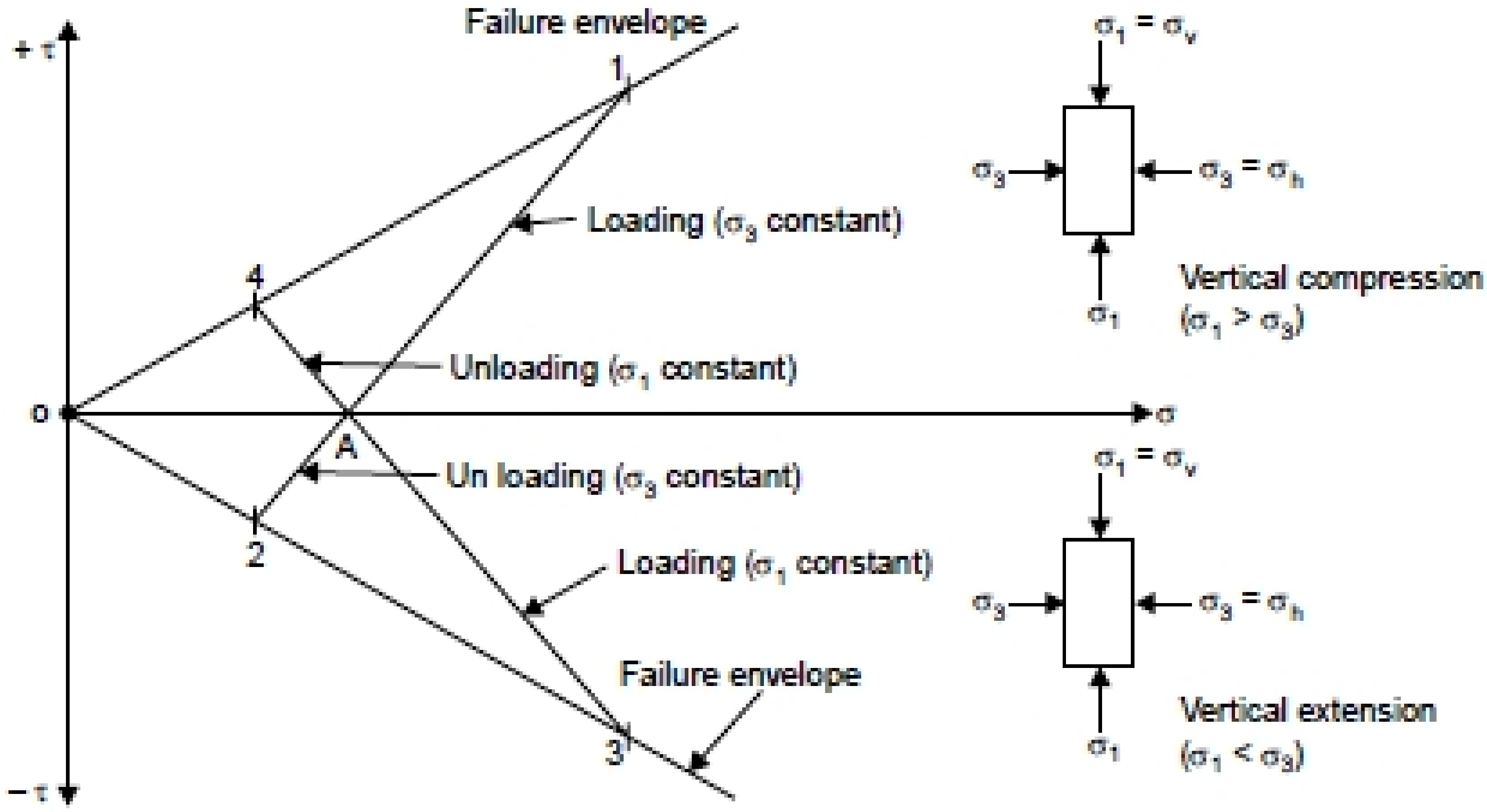


Fig. 8.24 Typical stress paths for triaxial compression and extension tests (loading/unloading)

A-1 is the effective stress path for conventional triaxial compression test during loading. ($\Delta\sigma_v$ = positive and $\Delta\sigma_h = 0$, i.e., σ_h is constant). A typical field case is a footing subjected to vertical loading.

A-2 is the unloading case of the triaxial extension text ($\Delta\sigma_h = 0$ and $\Delta\sigma_v$ = negative). Foundation excavation is a typical field example.

A-3 is the loading case of the triaxial extension test ($\Delta\sigma_v = 0$ and $\Delta\sigma_h$ = positive). Passive earth resistance is represented by this stress path.

A-4 is the unloading case of the triaxial compression test ($\Delta\sigma_v = 0$ and $\Delta\sigma_h$ = negative). Passive earth pressure on retaining walls is the typical field example for this stress path

For a drained test

Figure below shows the typical stress paths. Point **A** corresponds to the stress condition with only the confining pressure acting ($\sigma_1 = \sigma_3$ and $\tau = 0$). Point **F** represents failure. Stress paths for effective stresses, total stresses, and total stresses less static pore water pressure are shown separately in the same figure.

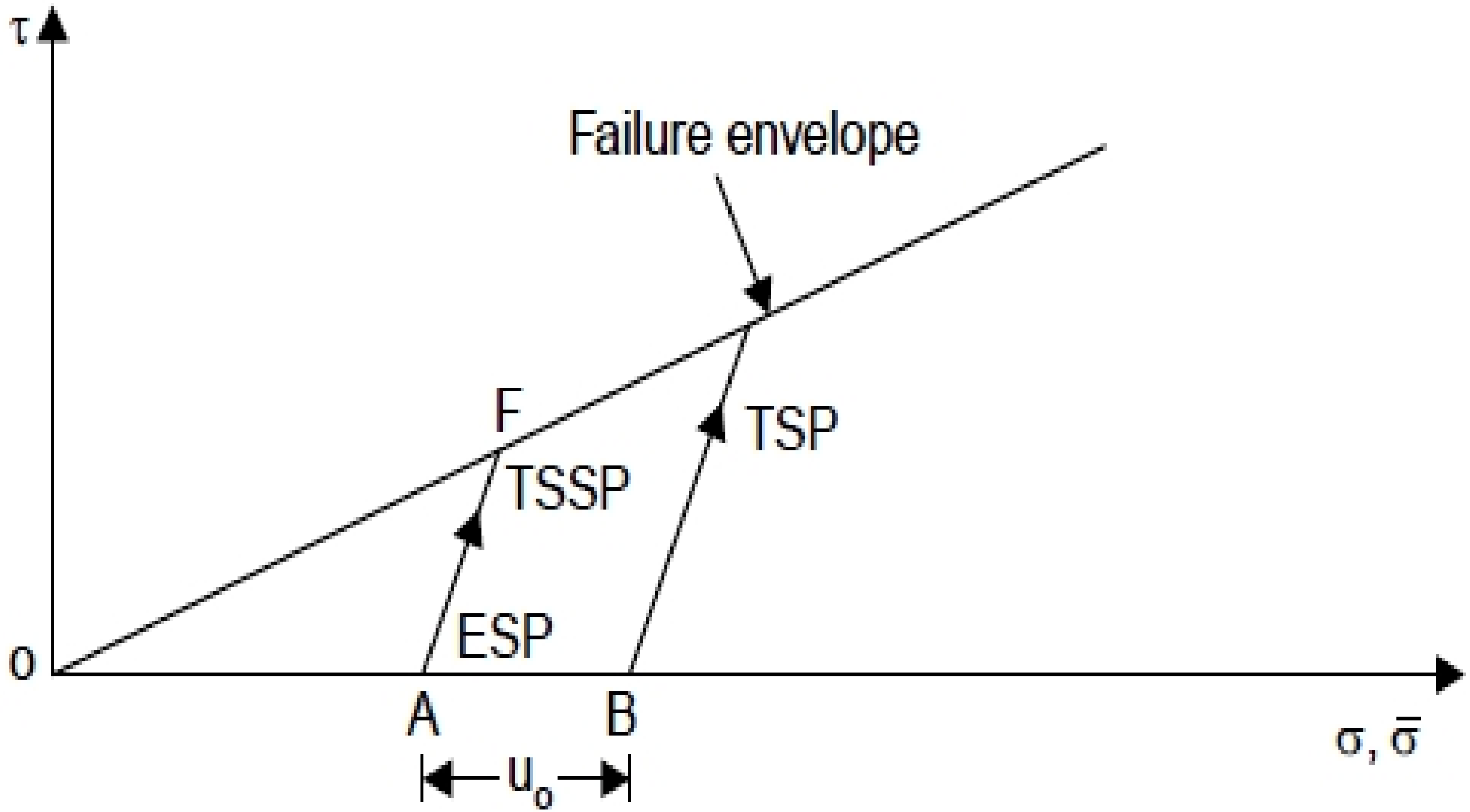


Fig. 8.25 Stress paths for drained test

For a consolidated Undrained test on a normally consolidated clay.

Figure below shows the typical stress paths.

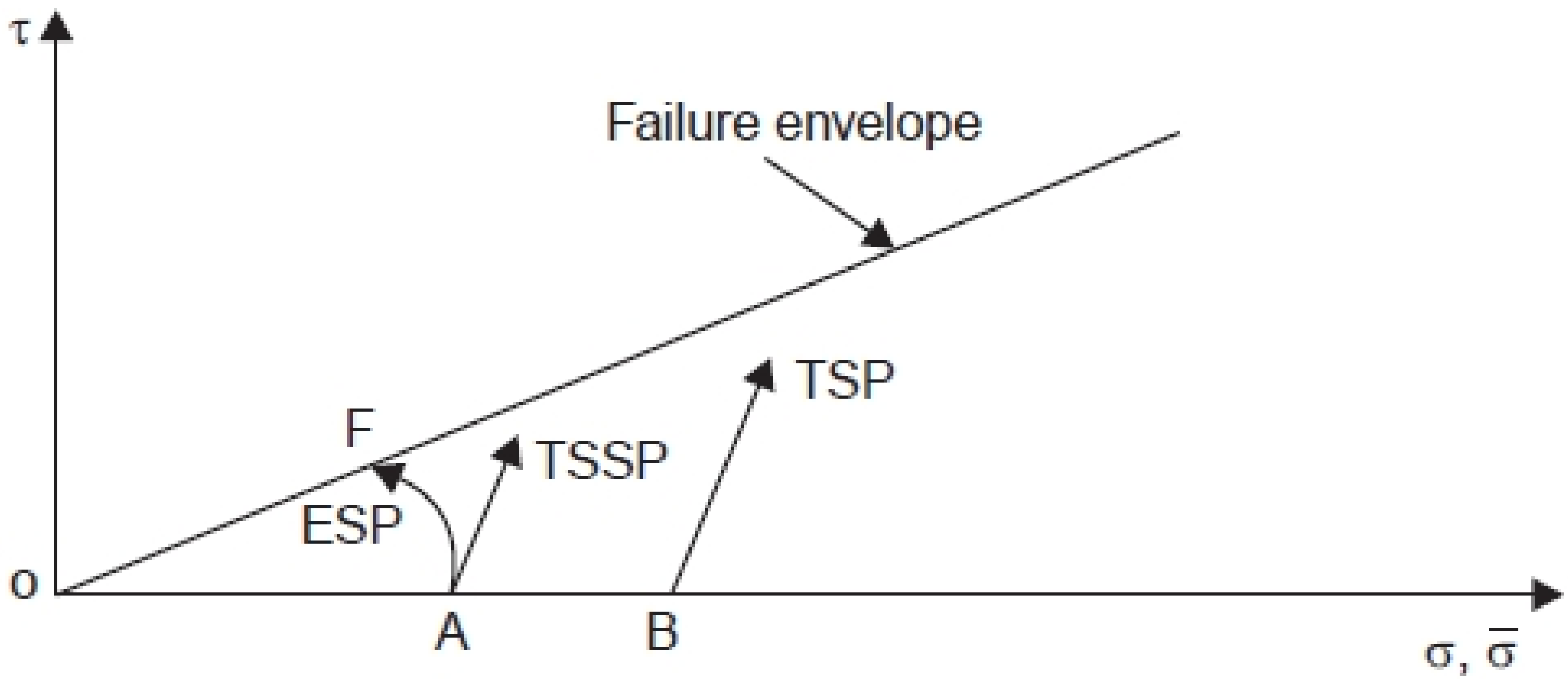


Fig. 8.26 Stress paths for consolidated undrained test on a normally consolidated clay

For a consolidated Undrained test on over consolidated clay

Figure below shows the typical stress paths.

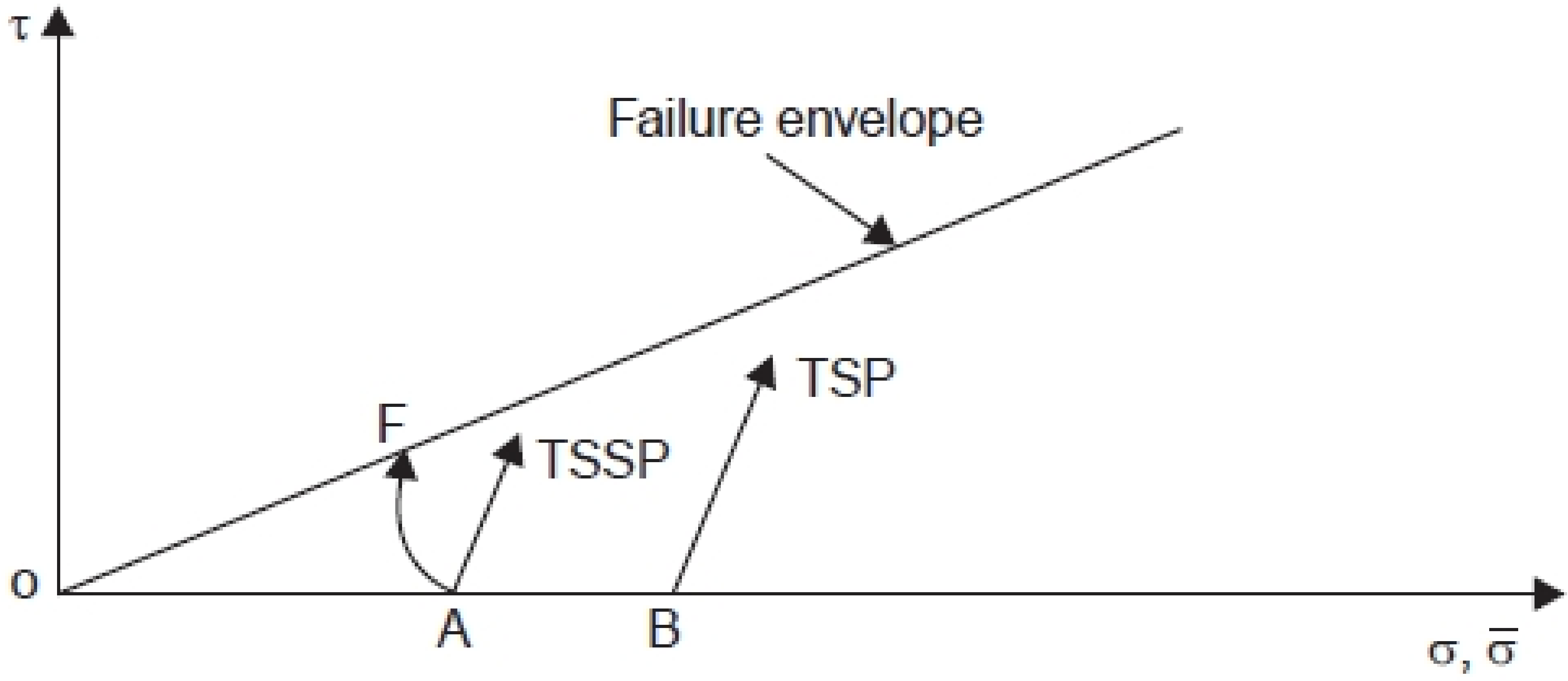


Fig. 8.27 Stress paths for consolidated undrained test on an overconsolidated clay

[Note : TSSP to the right of ESP indicates of positive excess pore pressure; TSSP to the left of ESP indicates negative excess pore pressure. Both coincide for zero excess pore pressure].

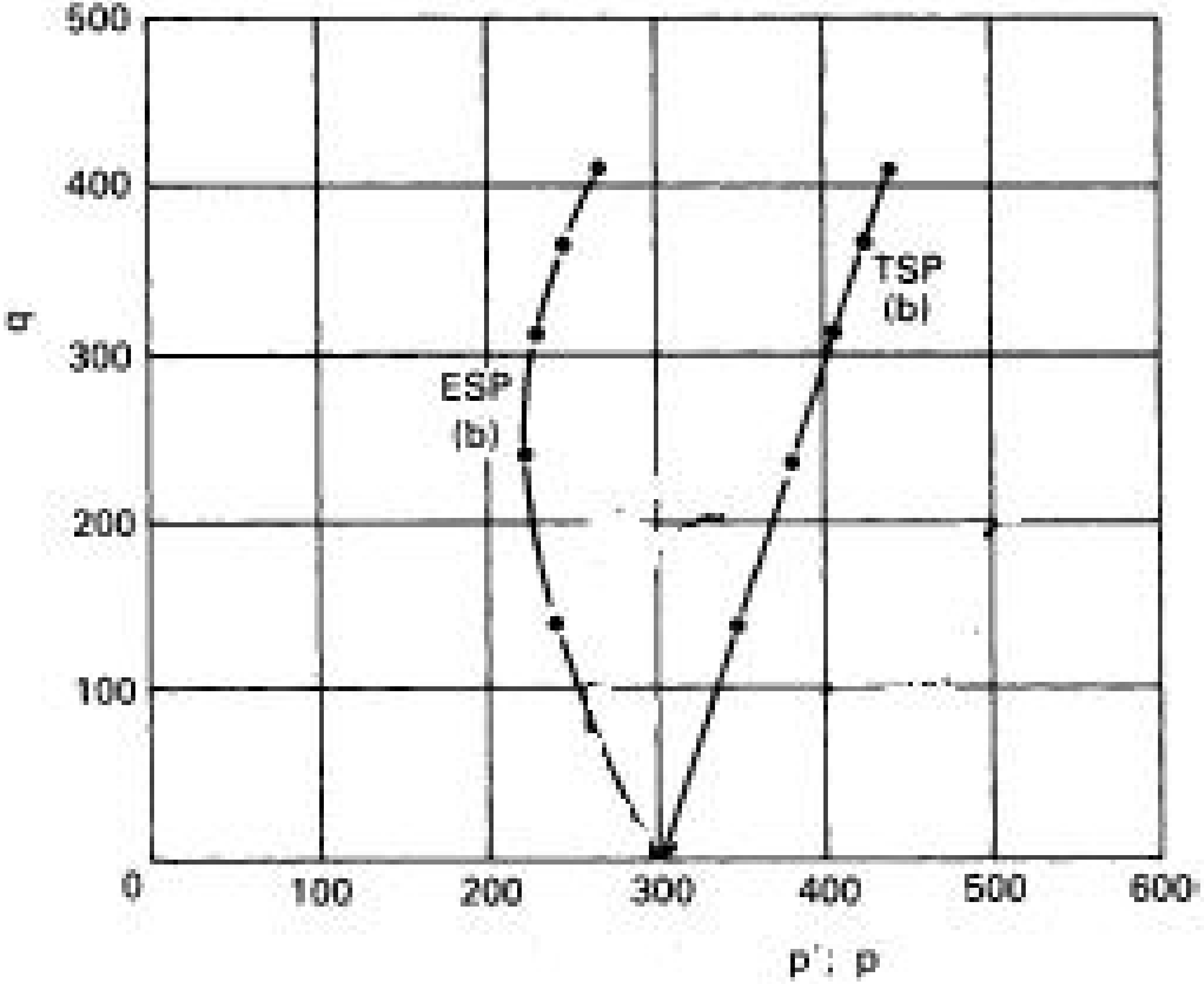
Example 4

The following results refer to a consolidated–undrained triaxial test on a saturated clay specimen under an all-round pressure of 300 kN/m^2:

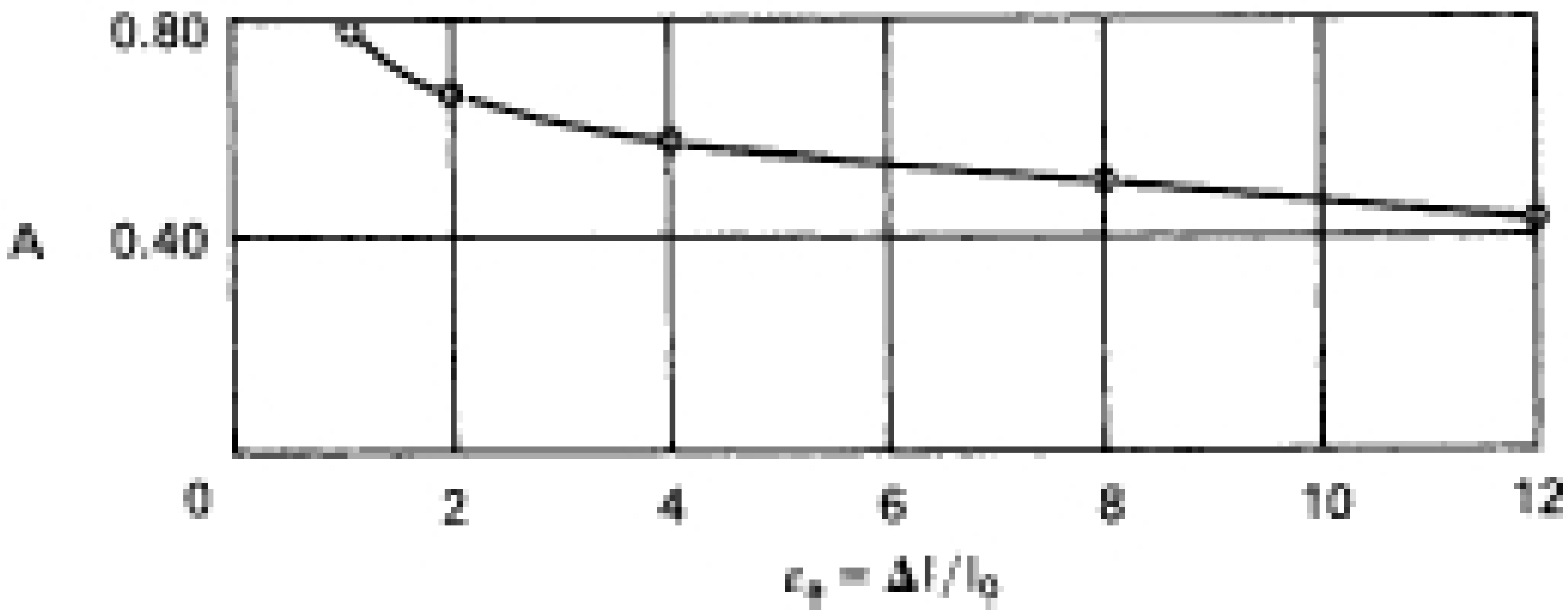
$\Delta l/l_o$	0	0.01	0.02	0.04	0.08	0.12
σ_1 - σ_3 (kN/m^2)	0	138	240	312	368	410
u (kN/m^2)	0	108	158	178	182	172

Draw the total and effective stress paths and plot the variation of the pore pressure coefficient A during the test.

$\Delta l/l_0$	0	0.01	0.02	0.04	0.08	0.12
q	0	138	240	312	368	410
p	300	346	380	404	423	437
p'	300	238	222	226	241	265
A	–	0.78	0.66	0.57	0.50	0.42



500
400
300
200
100
0
100
200
300
400
500
600
q
p' ; p
ESP
(b)
TSP
(b)



From the shape of the effective stress path and the value of A at failure it can be concluded that the clay is overconsolidated.

Examples :-

The results of two CD - triaxial tests on a clay are given below

Test no.	$\bar{\sigma}_3$ (lb/in^2)	$\bar{\sigma}_1$ failure (lb/in^2)
1	26.6	73.4
2	11.96	48.04

use the modified failure envelope (that is $\bar{q} = a + \bar{p} \tan \beta$)

do not plot the graph :

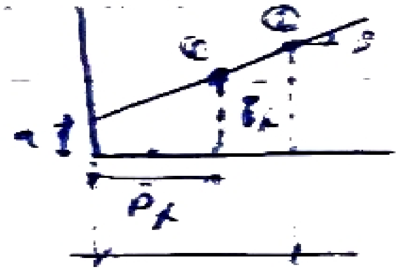
a- find a & β

c- find c & ϕ

Solution -

∴ CD test total stress = effective stress

$$p = \bar{p} \quad ; \quad q = \bar{q}$$



$$\bar{q}_f = \frac{\bar{\sigma}_1 - \bar{\sigma}_3}{2} = \frac{73.4 - 26.6}{2} = 23.4 \ \frac{lb}{in^2} = q$$

$$\bar{p}_f = \frac{\bar{\sigma}_1 + \bar{\sigma}_3}{2} = \frac{73.4 + 26.6}{2} = 50 = p$$

①

$$\bar{q}_f = \frac{48.04 - 11.96}{2} = 18.04 \ \frac{lb}{in^2}$$

$$\bar{p}_f = \frac{48.04 + 11.96}{2} = 30 \ \text{"}$$

②

$$23.4 = a + 50 \tan\beta \quad \text{——— ①}$$

$$\text{Subt.} \ \mp 18.04 = \mp a \mp 30 \tan\beta \quad \text{——— ②}$$

$$5.36 = 20 \tan\beta \longrightarrow \beta = 15^\circ$$

$$\rightarrow 23.4 = a + 50 \tan 15$$

$$\therefore a = 10 \ lb/in^2$$

$$c = \frac{a}{\cos\phi}$$

$$\sin\phi = \tan\beta$$
$$\sin\phi = \tan 15^\circ \longrightarrow \phi = 15.55^\circ$$

$$= \frac{10}{\cos 15.55} = 10.38 \ lb/in^2$$