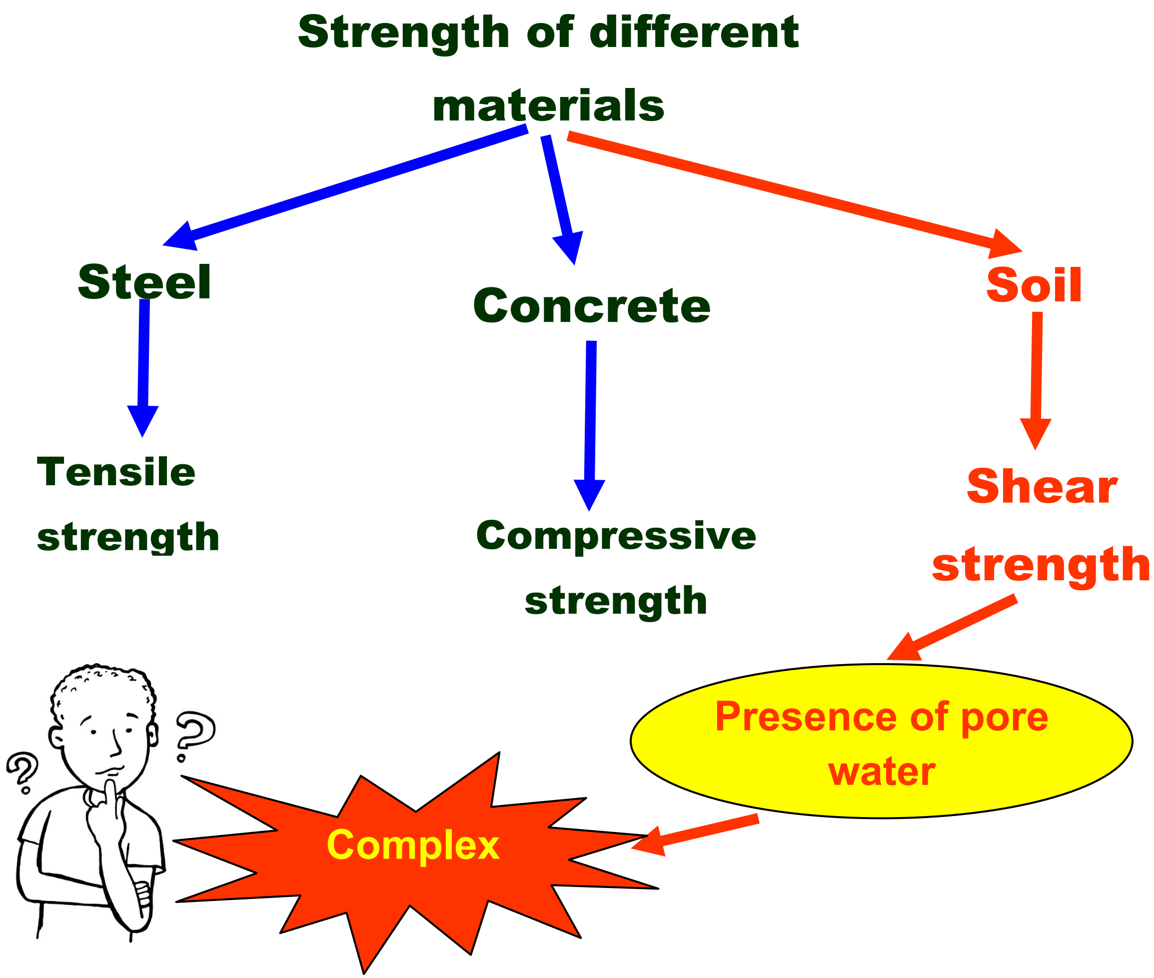
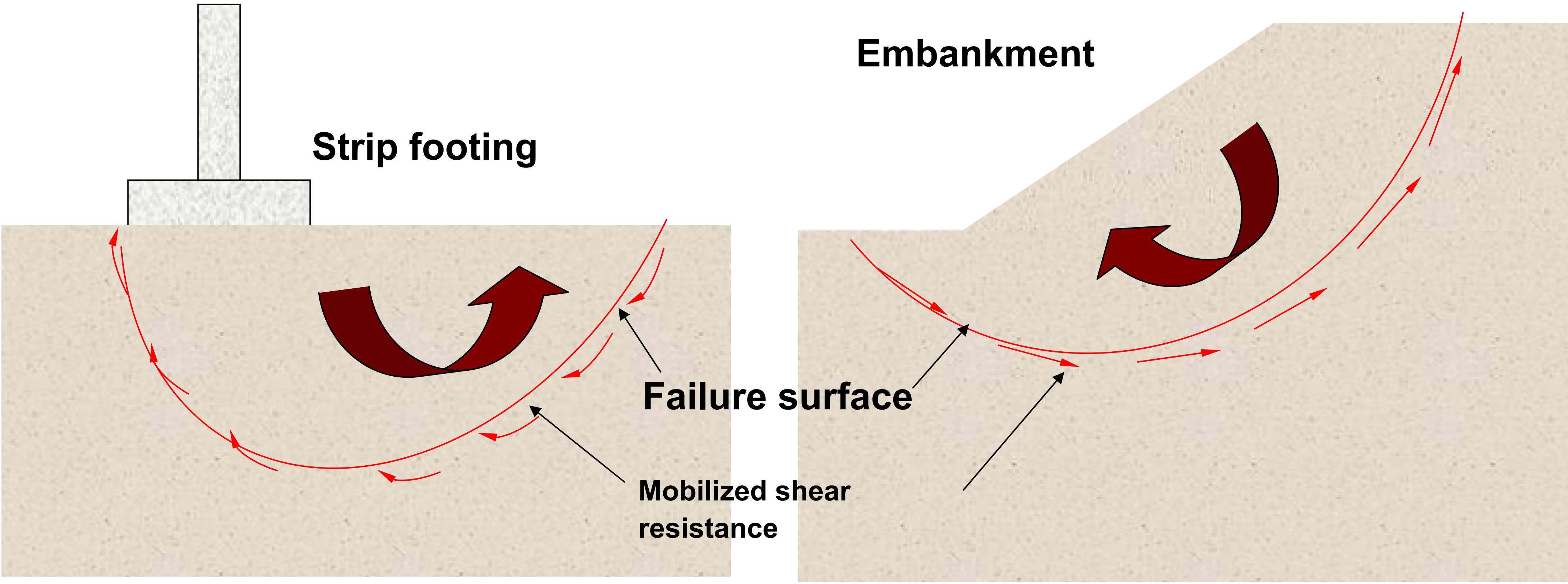

Strength of different materials
Steel
Concrete
Soil
Tensile strength
Compressive strength
Shear strength
Presence of pore water
Complex

Shear Strength of Soils

Topics

- **Introduction**
- **Mohr-Coulomb Failure Criterion**
- **Inclination of the plane of failure due to shear**
- **Determination of Shear Strength Parameters**
  1. **Direct Shear test**
  2. **Triaxial Shear tests**
     - a. **Consolidated-Drained (CD) Test**
     - b. **Consolidated-Undrained (CU) Test**
     - c. **Unconsolidated-Undrained (UU) Test**


Strip footing
Embankment
Failure surface
Mobilized shear resistance

● Introduction

Soils are--

- ♦ essentially frictional materials.
- ♦ are comprised of individual particles that can *slide* and *roll* relative to one another, and
- ♦ it is generally assumed that the particles are not cemented.

Thus, the ***Shear strength of a soil*** *mass* is the internal resistance per unit area that the soil mass can offer to resist failure and sliding along any plane inside it.

One consequence of the frictional nature is that the strength **depends on the effective stresses in the soil**. As the effective stresses increase with depth, so in general will the strength.

The strength will also **depend on whether the soil deformation occurs under fully drained conditions, constant volume (undrained) conditions**, or with some intermediate state of drainage. In each case, different excess pore pressures will occur resulting in different effective stresses, and hence different strengths.

In assessing the stability of soil constructions analyses are usually performed to check the **short term (undrained)** and long term **(fully drained)** conditions.

Shear strength components

The shear strength components are-

- Friction resistance-

  It occurs between the particles of the soil due to the external load consists of-

  - Friction due to sliding
  - Friction due to rolling
  - Friction due to interlocking

- Cohesion

True Cohesion	Apparent Cohesion
• Cementation Due to the presence of cementing agents such as calcium carbonate or iron oxide • Electrostatic and electromagnetic attractions • Primary valence bonding (adhesion) • Occurs primarily during overconsolidation	• Negative pore water pressure • Negative excess pore water pressures due to dilation (expansion) • Apparent mechanical forces • Can not be relied on for soil strength

● Mohr-Coulomb Failure Criterion

Mohr (1900) presented a theory for rupture in materials that held "*a material fails through a critical combination of normal stress (σ) and shear resistance (τ_f), and not through either maximum normal or shear stress alone.*

The functional relationship on a failure plane can be expressed in the form

$$\tau_f = f(\sigma)$$

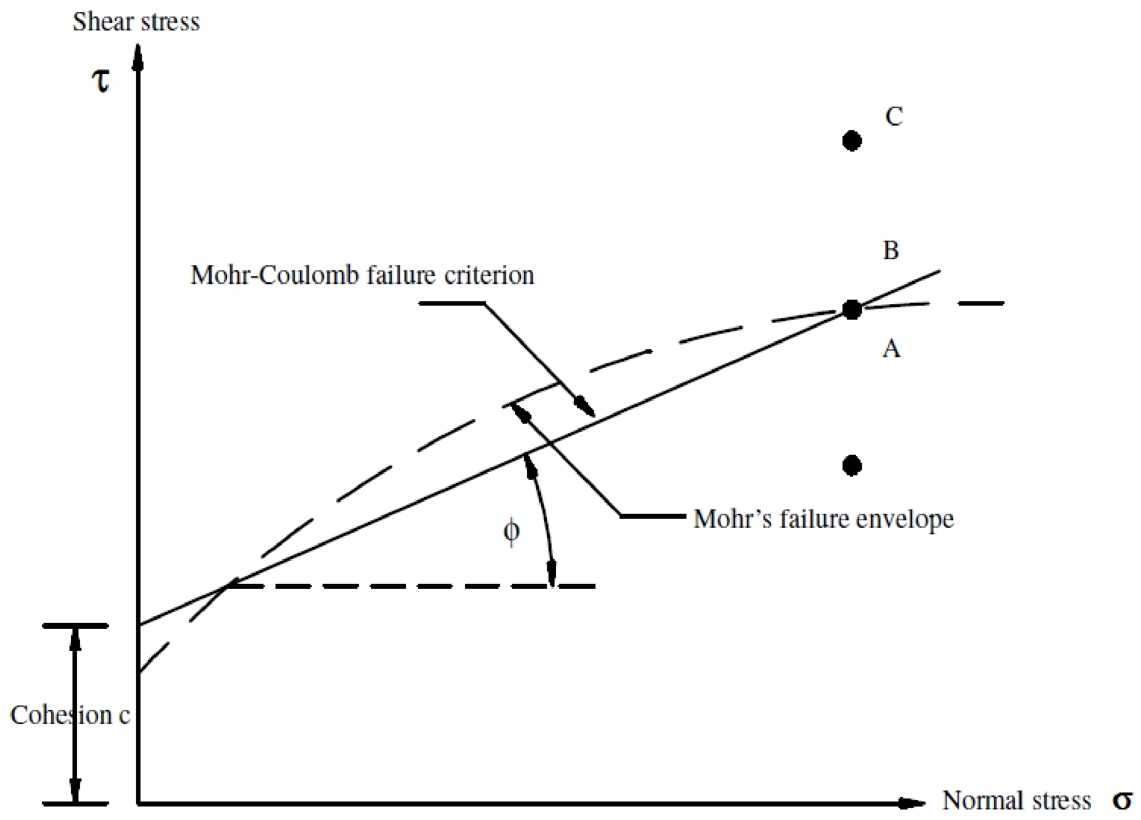
In soils the relationship is approximated as a linear relationship as following

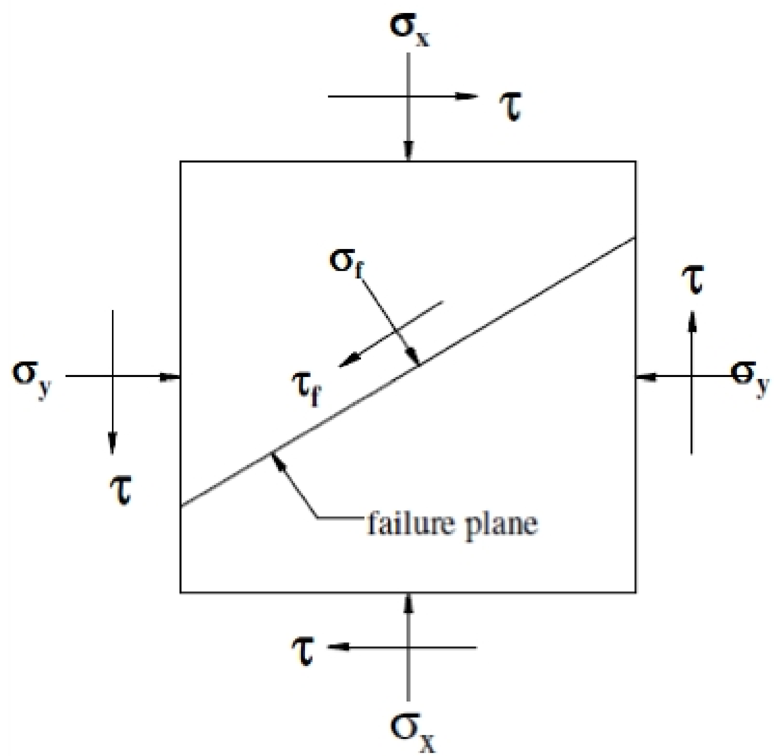
$$\tau_f = c + \sigma \tan\phi$$

This equation is known as the ***Mohr-Coulomb Failure Criterion*.**

where c = cohesion, and
 ϕ = angle of internal friction

What does the failure envelope mean?





Comparison between the Mohr's failure envelope and the Mohr-Coulomb failure criterion.

● Inclination of the plane of failure due to shear

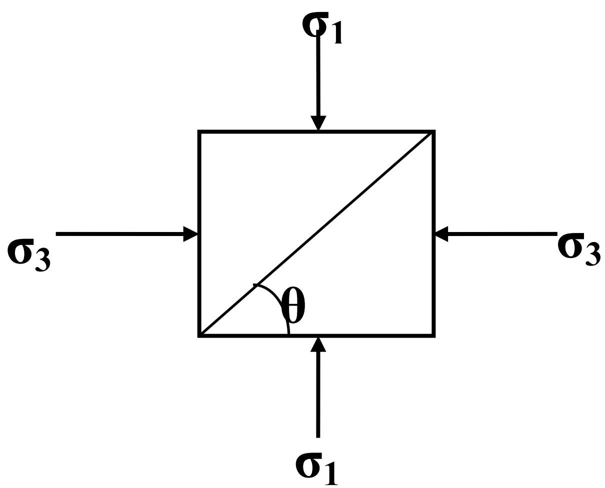
As stated by the Mohr-Coloumb failure criteria, failure by shear will take place then the shear stresses on a plane reaches the value given by the equation

$$\tau_f = c + \sigma \tan\phi \quad \text{.......................1}$$

To determine the inclination of failure plane with major principle plane

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\theta$$

$$\tau_f = \frac{\sigma_1 - \sigma_3}{2}\sin 2\theta \quad \text{..............2}$$



Sub. Eq.2 in Eq.1

$$\frac{\sigma_1 - \sigma_3}{2}\sin 2\theta = c + \left[\frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\theta\right]\tan\phi$$

or

$$\sigma_1 = \sigma_3 + \frac{\sigma_3 \tan\phi + c}{\frac{1}{2}\sin 2\theta - \cos^2\theta\tan\phi} \text{......................} 3$$

For a given value of σ_3 and c, the failure condition will be determined by the minimum value of the major principle stress σ_1, for a minimum value of σ_1 , the term [$\frac{1}{2}\sin 2\theta - \cos^2\theta\tan\phi$] in eq.3 has to be maximum. Thus,

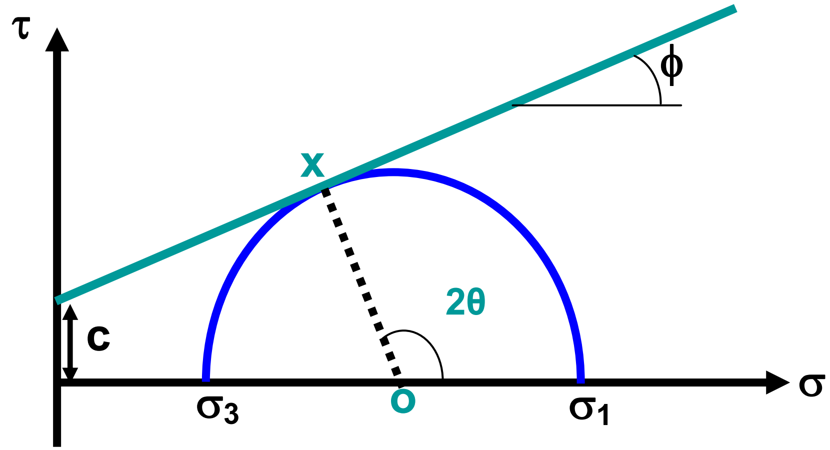
$$\frac{d}{d\theta}\left(\frac{1}{2}\sin 2\theta - \cos^2\theta\tan\phi\right) = 0 \text{..........................} 4$$

or

$$\cos^2\theta - \sin^2\theta + 2\sin\theta\cos\theta\tan\phi = 0 \text{..................} 5$$

Eq.5 gives the relation

$$\theta = 45 + \frac{\phi}{2} \text{..} 6$$



Sub. Eq.6 in Eq.3 we get

$$\sigma_1 = \sigma_3 \tan^2(45 + \frac{\phi}{2}) + 2c \tan(45 + \frac{\phi}{2}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 7$$

$$let \quad N_\phi = \tan^2(45 + \frac{\phi}{2}) = \frac{1 + \sin\phi}{1 - \sin\phi} \quad prove\ it\ from\ geometry$$

$$\therefore \quad \sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 8$$

Shear Failure Law in Saturated Soil

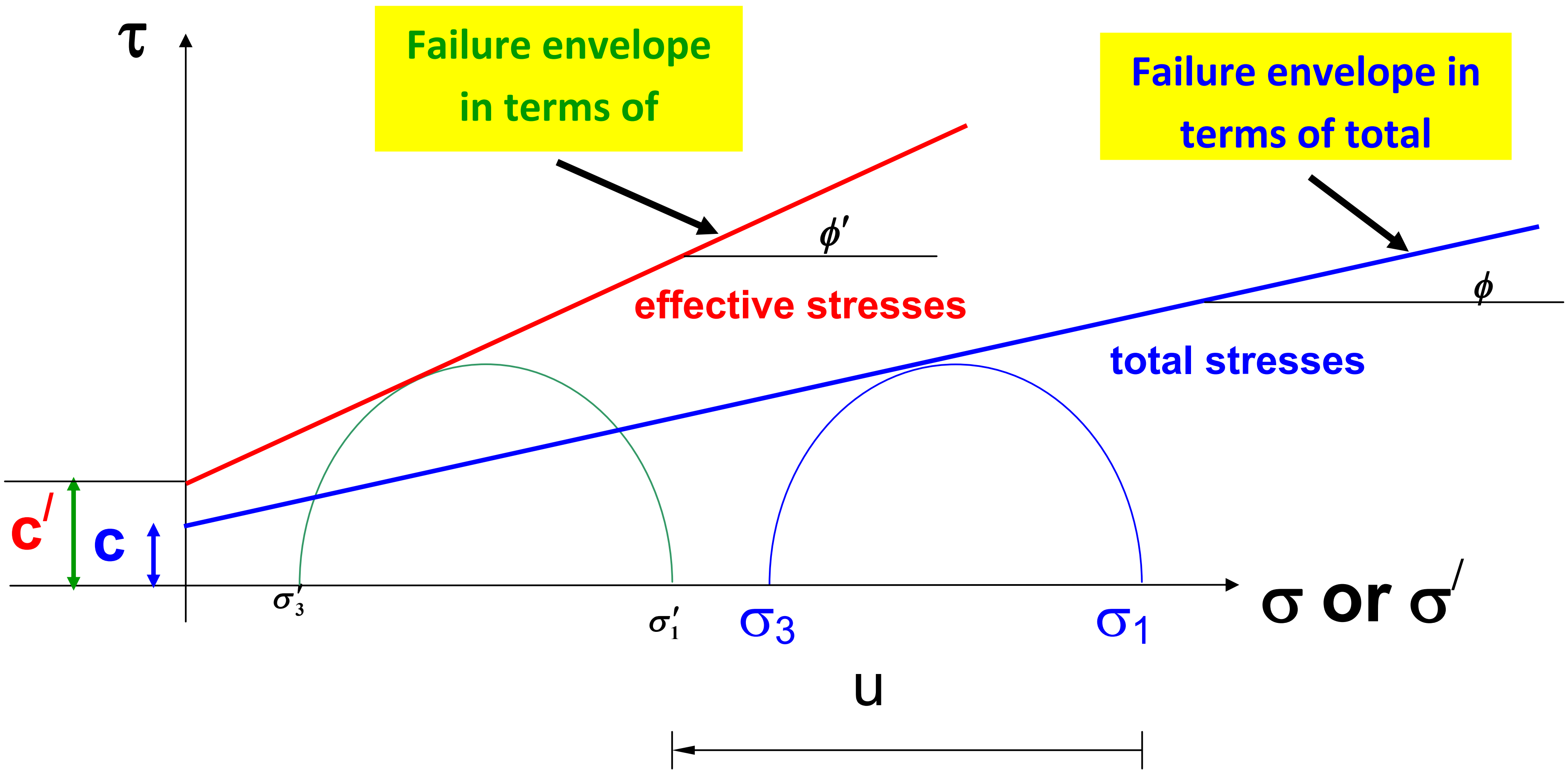
In saturated soil $\sigma = \sigma' + u$ and as stated before shear strength of the soil is a function of effective stress, the shear strength will be in terms of effective stress and eq.1 will be

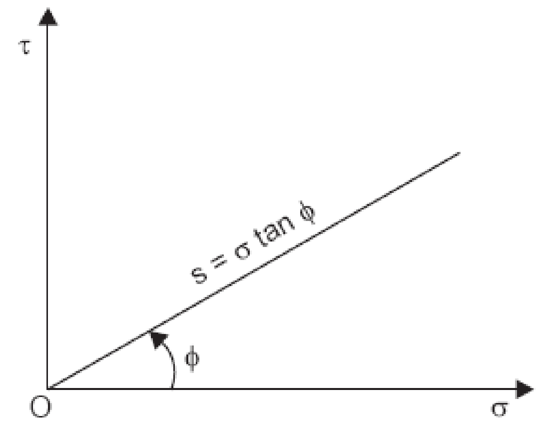
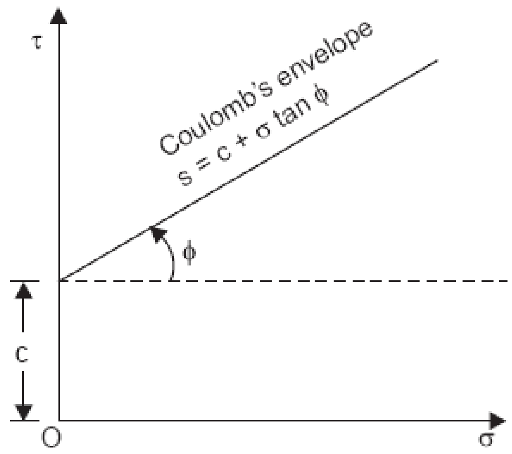
$$\tau_f = c' + (\sigma - u)\tan\phi'$$

$$\tau_f = c' + \sigma' \tan\phi'$$

c and ϕ or c' and ϕ' are measures of shear strength, Higher the values, higher the shear strength.

Failure envelopes in terms of total & effective stresses



Type of soil	Cohesion	
Sand and Inorganic silt	Zero	
Normally consolidated clays	Very small ≈ Zero	
Over consolidated clays	> Zero	

● Determination of Shear Strength Parameters(c, ϕ or $c^{/}$, $\phi^{/}$)

Shear tests

Laboratory tests on specimens taken from representative undisturbed samples

Field tests

Most common laboratory tests to determine the shear strength parameters are,

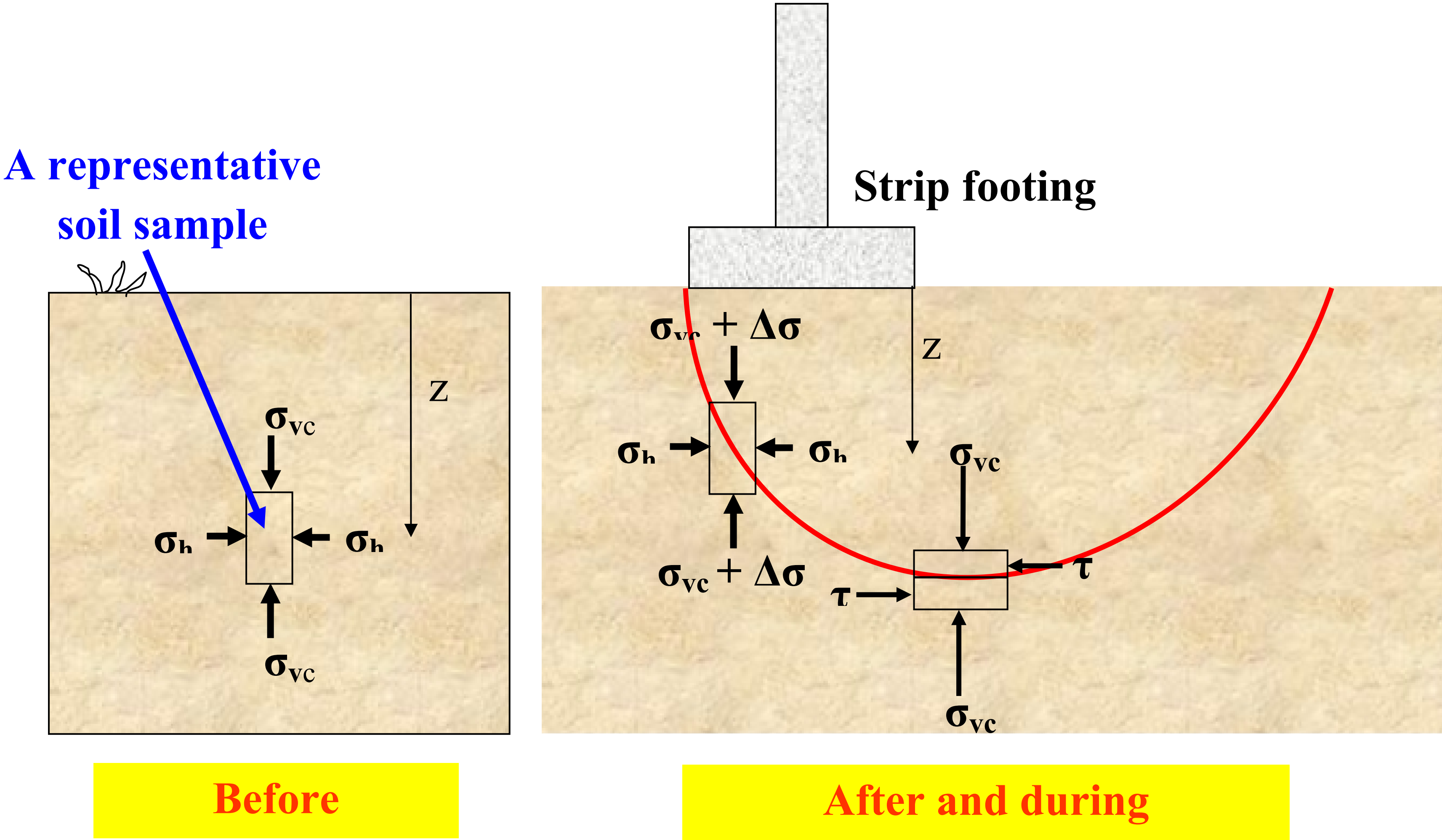
- **Direct shear test**
- **Triaxial shear test**

Other laboratory tests include,

- **Direct simple shear test,**
- **torsional ring shear test,**
- **plane strain triaxial test,**
- **laboratory vane shear test,**
- **laboratory fall cone test**

- **Vane shear test**
- **Torvane**
- **Pocket penetrometer**
- **Fall cone**
- **Pressuremeter**
- **Static cone penetrometer**
- **Standard penetration test**

Simulating field conditions in the laboratory



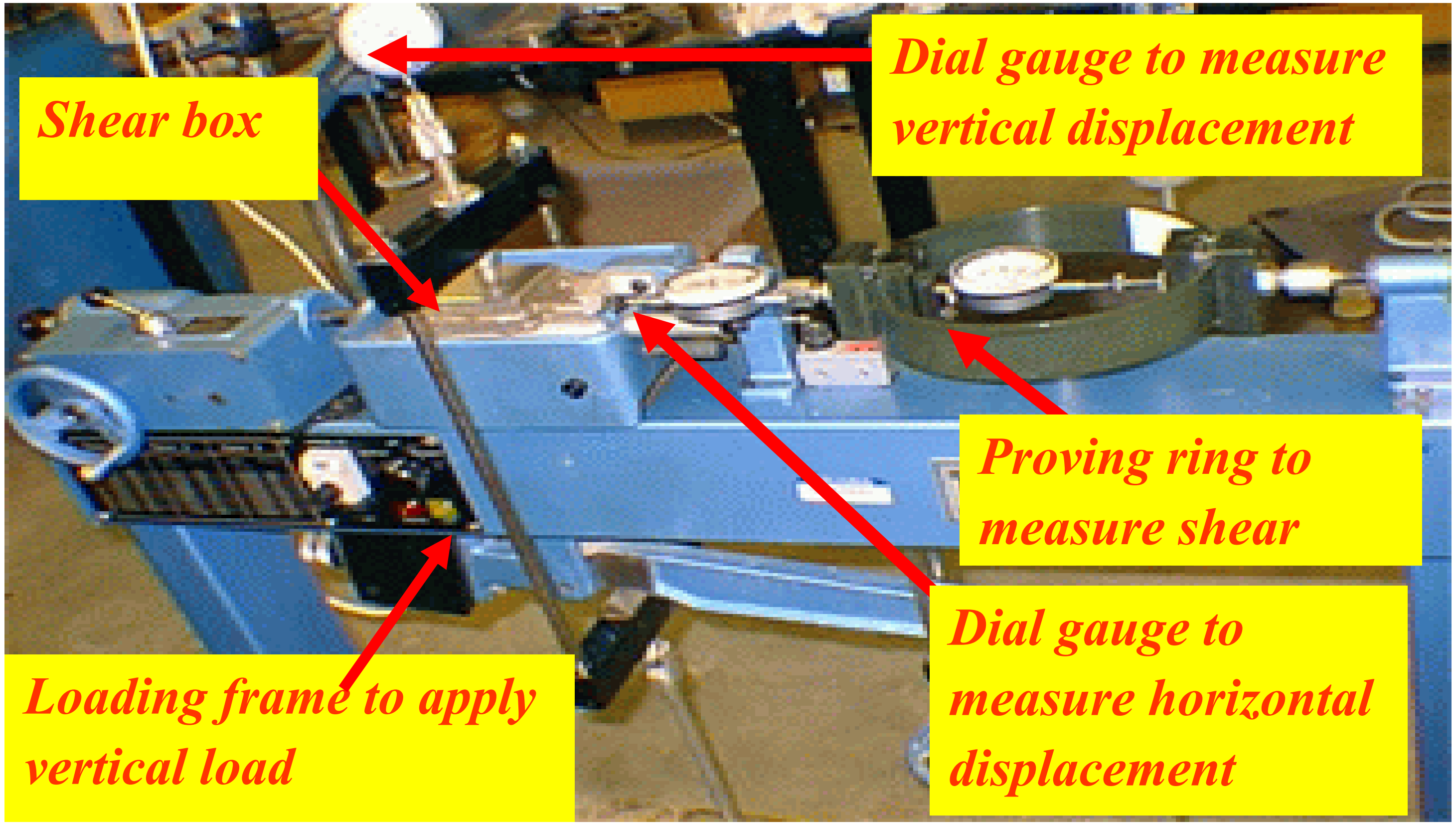
Step 1 Representative soil sample taken from the site

Step 2 Set the specimen in the apparatus and apply the initial stress condition

Step 3 Apply the corresponding field stress conditions

Direct shear test

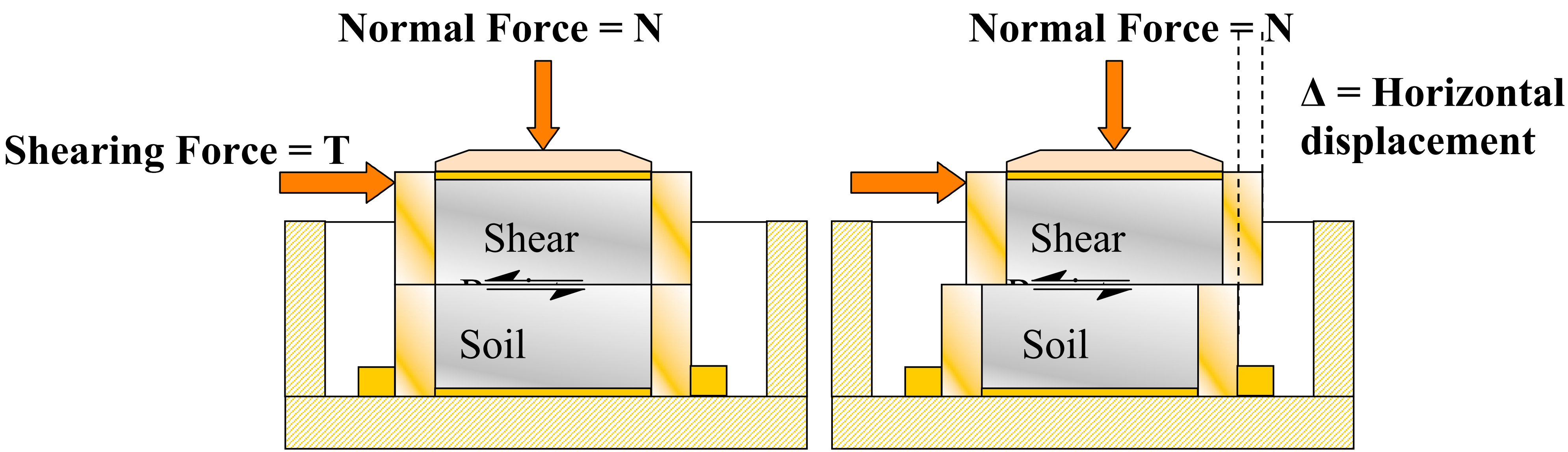
Direct shear test is most suitable for consolidated drained tests specially on granular soils (e.g.: sand) or stiff clays



Schematic diagram of the direct shear apparatus

Step 1: Apply a vertical load to the specimen and wait for consolidation

Step 2: Lower box is subjected to a horizontal displacement at a constant rate.



Step 3: Repeat this test three times. Each time increase "N"

Analysis of test results

$$\sigma = \frac{Normal\ \ force}{Cross\ \sec tional\ \ area} = \frac{N}{A}$$

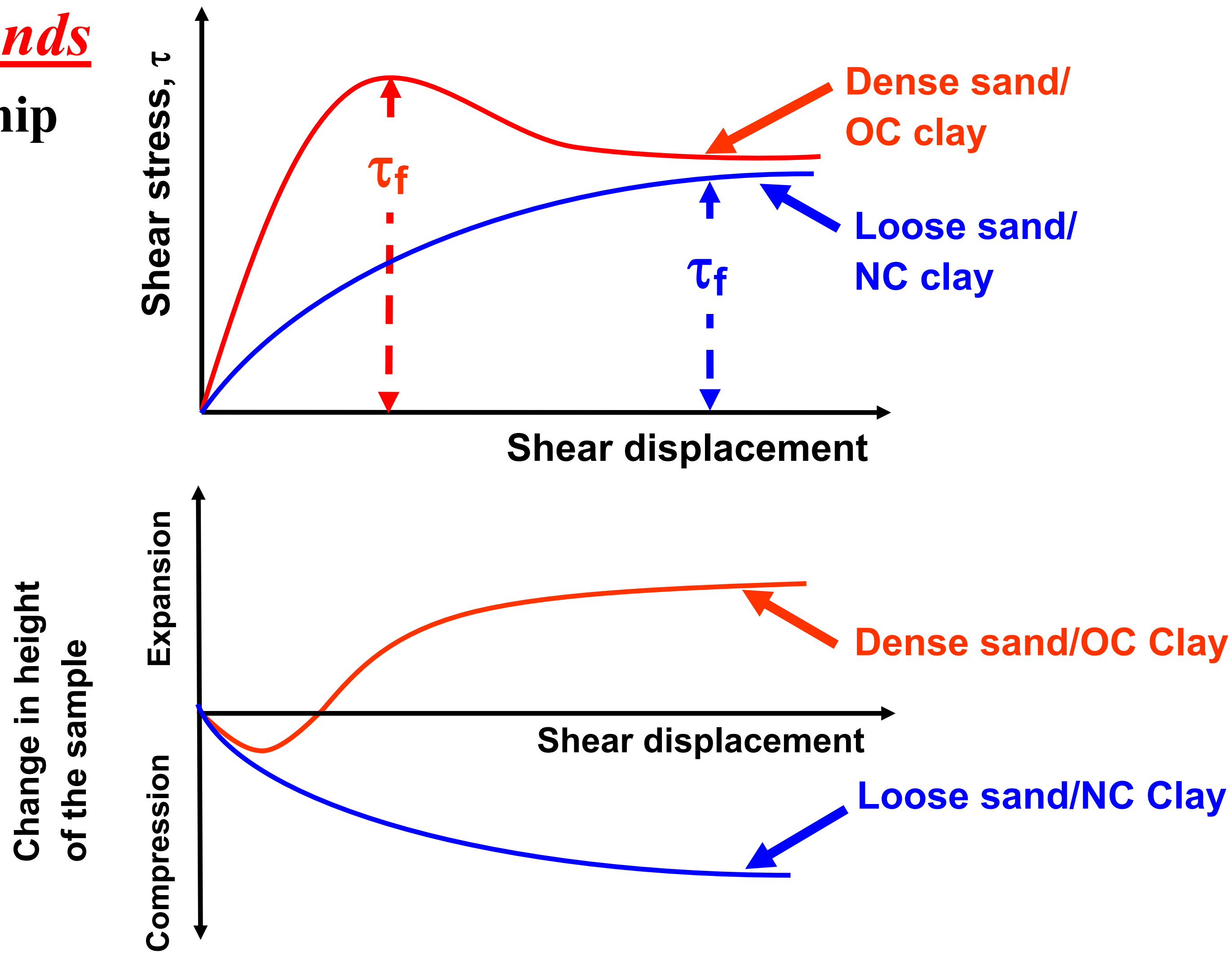
$$\tau = \frac{Shearing\ \ force}{Cross\ \sec tional\ \ area} = \frac{T}{A}$$

Note: Cross-sectional area of the sample changes with the horizontal displacement A= (L - ΔL)L

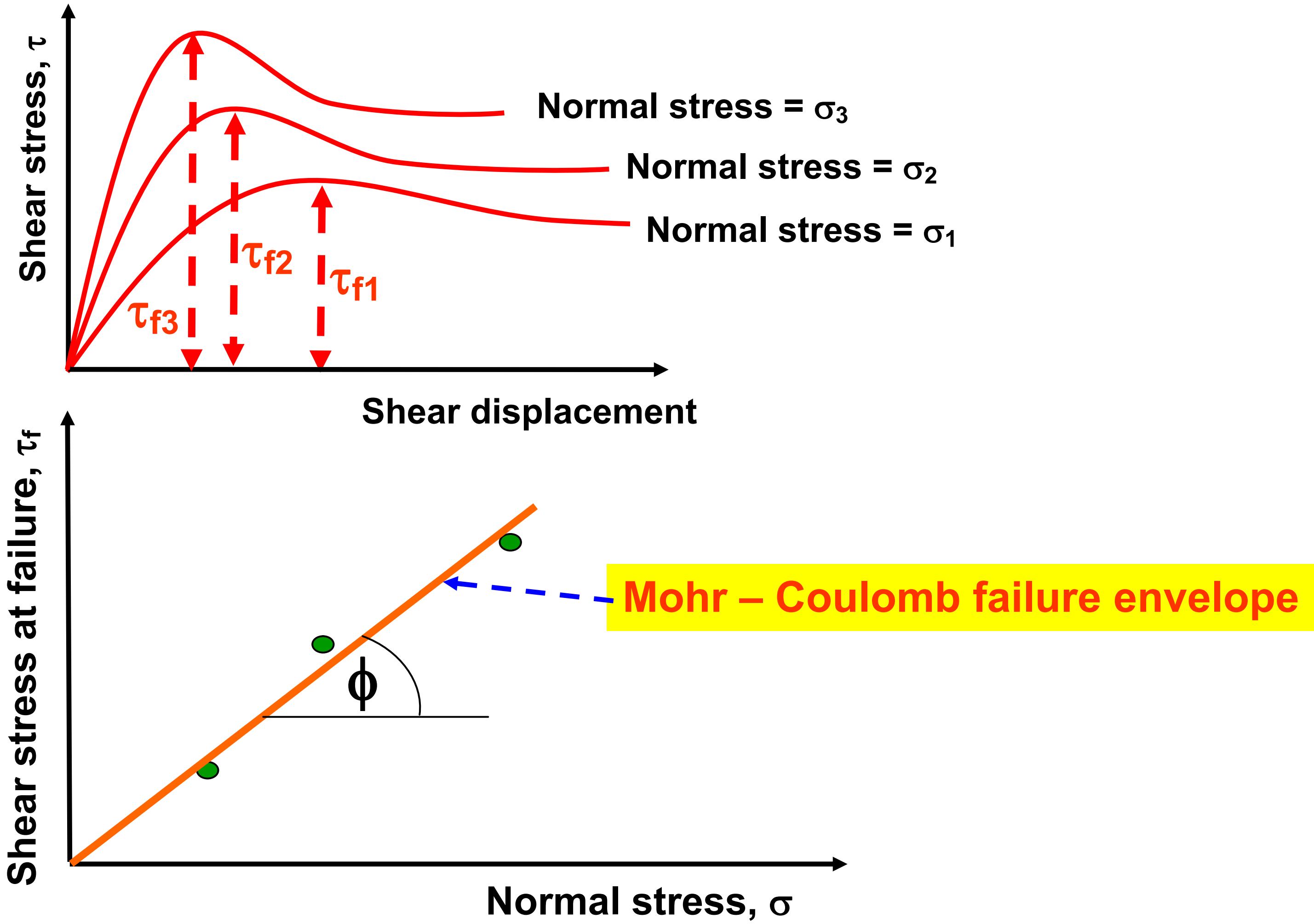
Sometimes A = L^2 as in your textbook

Direct shear tests on sands

Stress-strain relationship



How to determine strength parameters c and φ

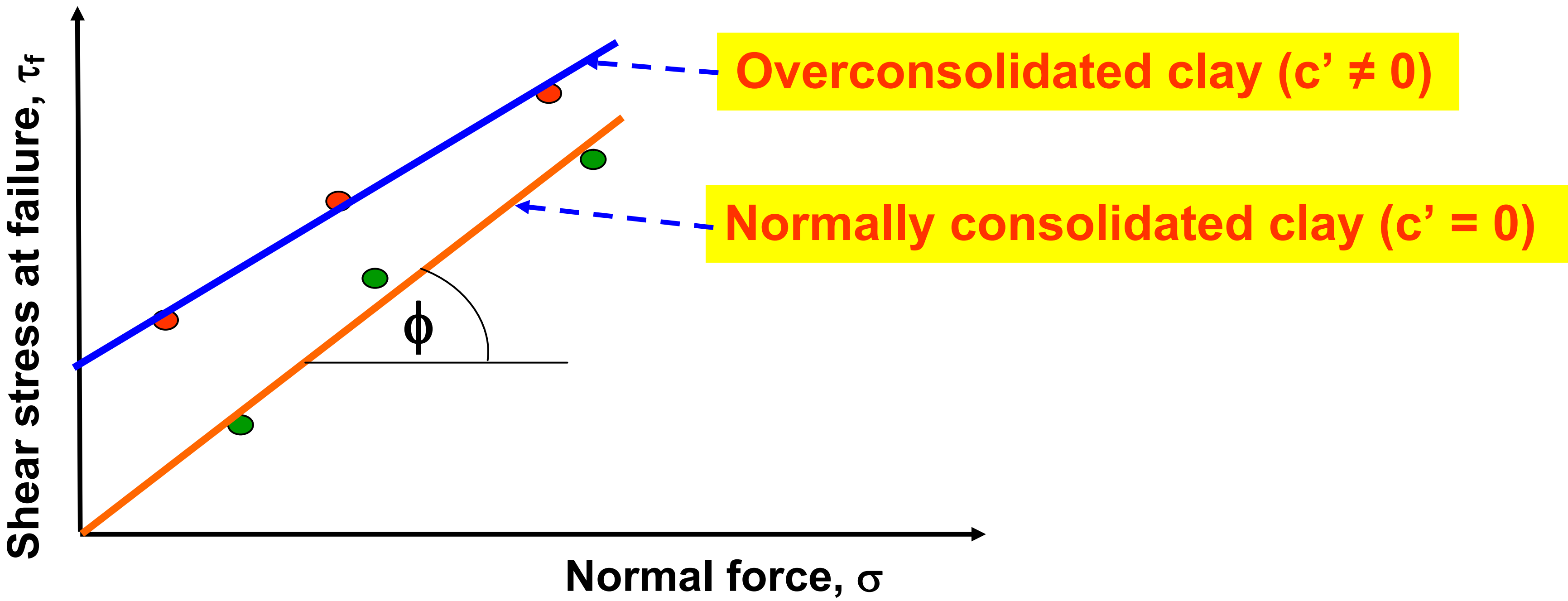


Some important facts on strength parameters c and f of sand

- **Sand is cohesionless hence c = 0**
- **Direct shear tests are drained and pore water pressures are dissipated, hence u = 0, Therefore,**
    - $\phi = \phi'$ **and** $c = c'$

Direct shear tests on clays

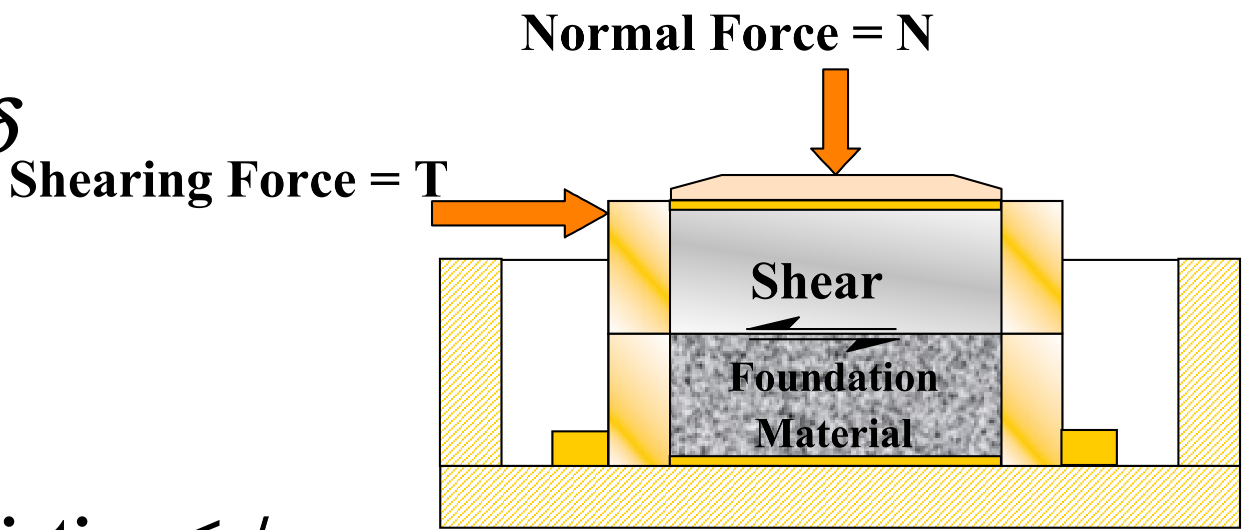
In case of clay, horizontal displacement should be applied at a very slow rate to allow dissipation of pore water pressure (therefore, one test would take several days to finish)



Interface tests on direct shear apparatus

- In many foundation design problems and retaining wall problems, it is required to determine the angle of internal friction between soil and the structural material (concrete, steel or wood)

$$\tau_f = c_a + \sigma' \tan \delta$$



where

c_a = adhesion

δ = angle of wall friction $\leq \phi$

Advantages of direct shear apparatus	**Disadvantages of direct shear apparatus**
• Due to the smaller thickness of the sample, rapid drainage can be achieved • Can be used to determine interface strength parameters • Clay samples can be oriented along the plane of weakness or an identified failure plane	• Failure occurs along a predetermined failure plane • Area of the sliding surface changes as the test progresses • Non-uniform distribution of shear stress along the failure surface

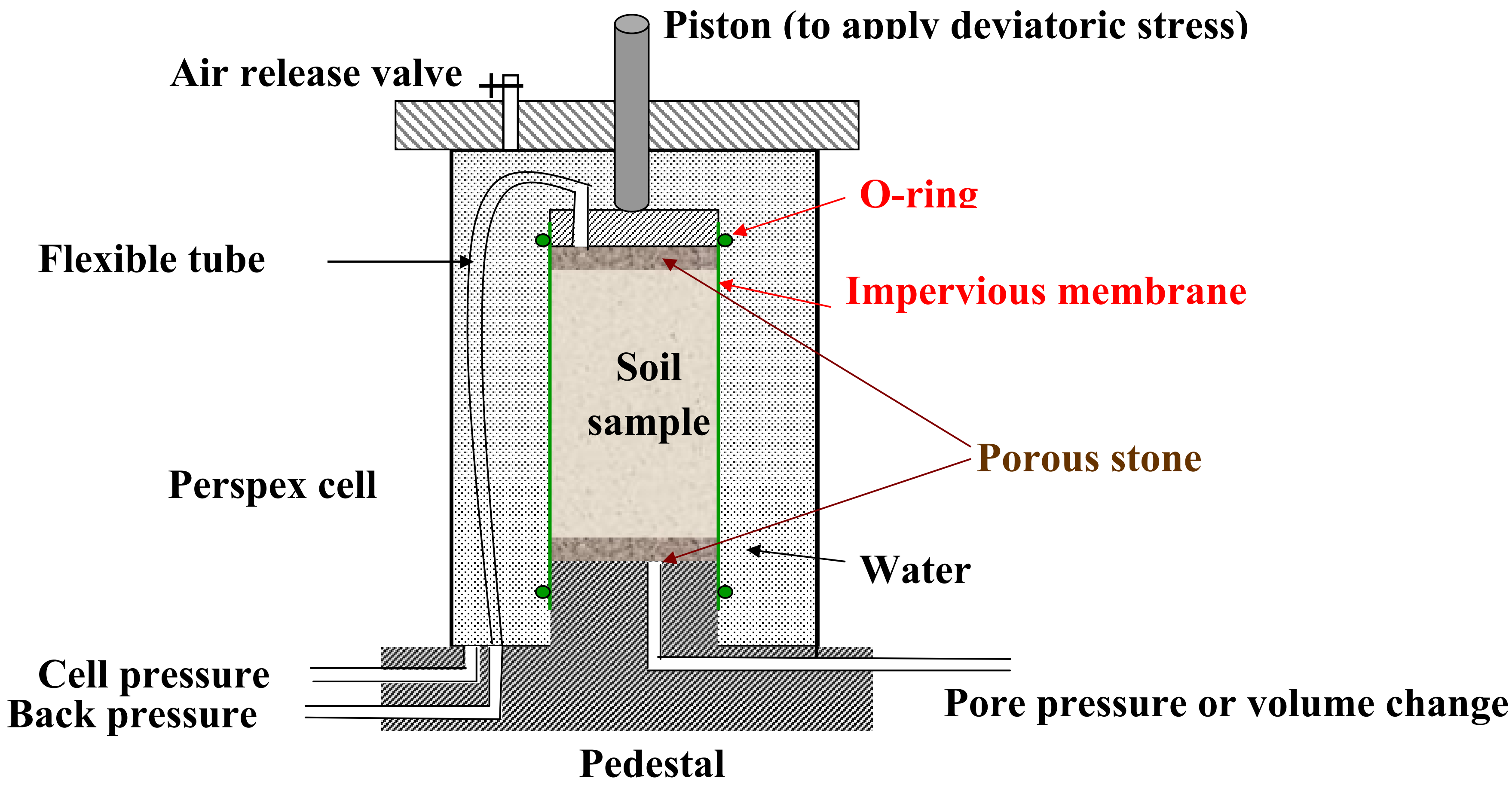
Triaxial Shear Test

Failure plane

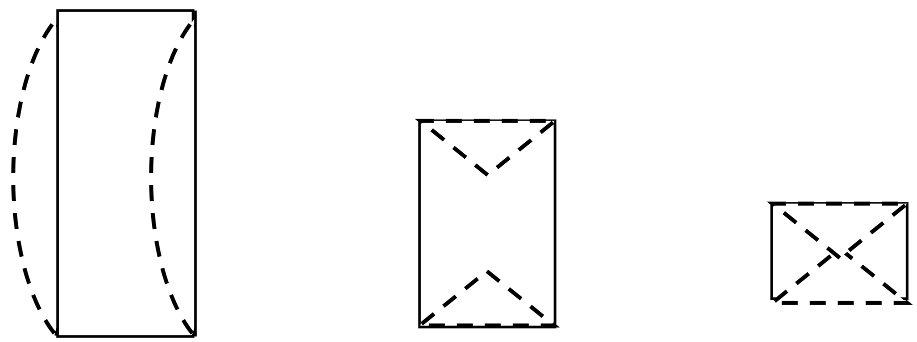
Soil sample at failure

Proving ring to measure the deviator load

Dial gauge to measure vertical displacement


Piston (to apply deviatoric stress)
Air release valve
O-ring
Flexible tube
Impervious membrane
Soil sample
Porous stone
Perspex cell
Water
Cell pressure
Back pressure
Pore pressure or volume change
Pedestal

- In this test, a soil sample about 38 mm (1.5//) in diameter and 76 mm (3//) is generally used (L = 2D – 3D)

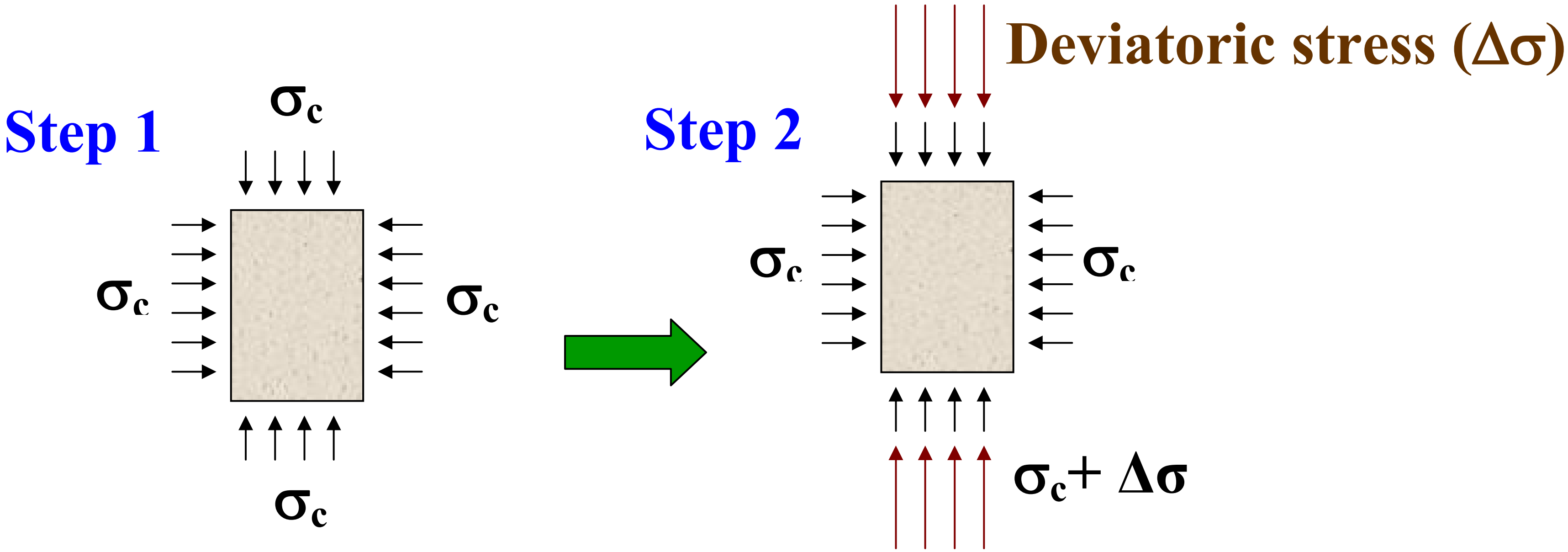


> 3D 3D-2D < D

- Sample is encased by thin rubber membrane and placed inside a plastic cylindrical chamber that is usually filled with water or glycerine.

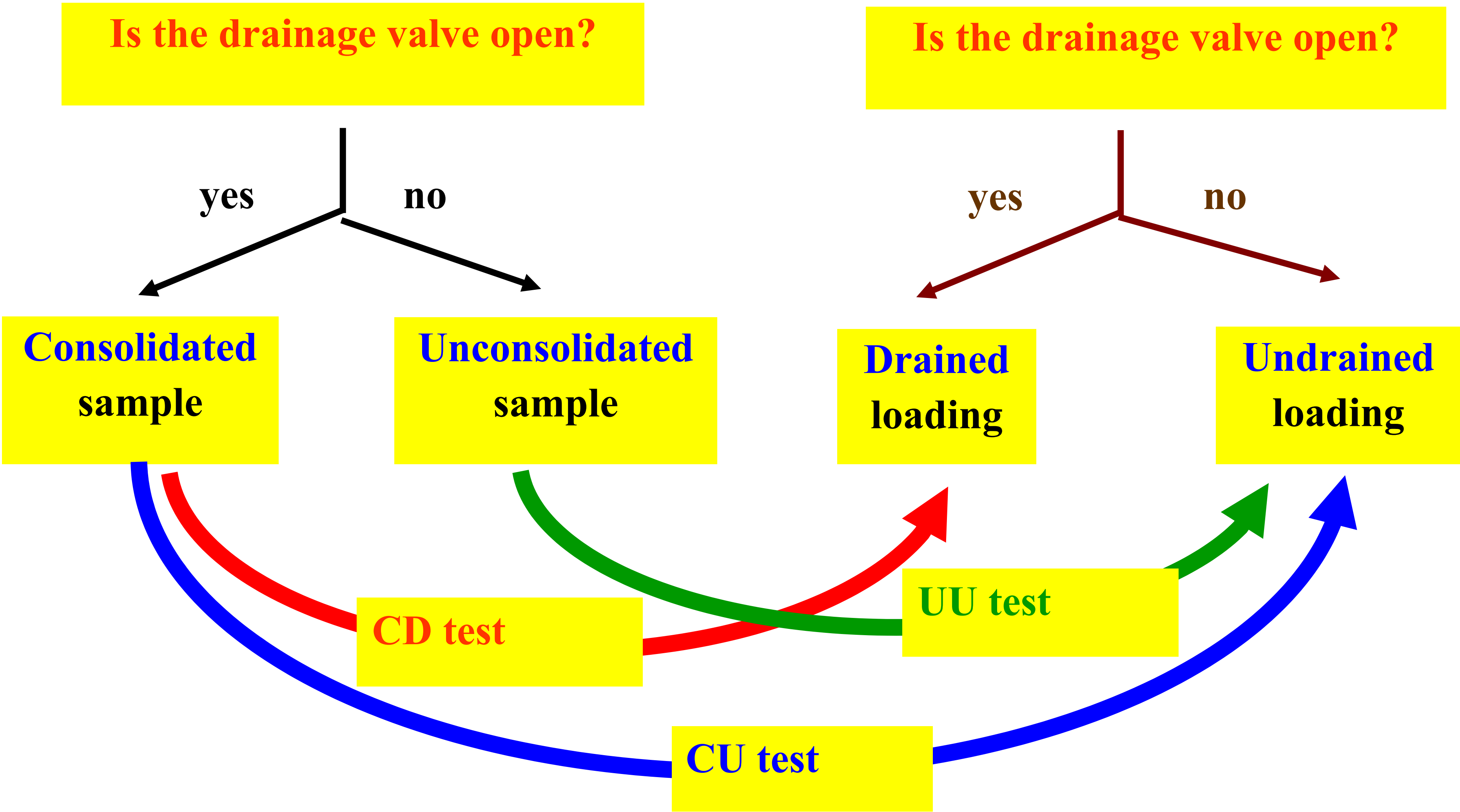
- Confining pressure is applied by compression of fluid in the chamber (air sometimes used as a compression medium)
- To cause shear failure in the sample, axial stress is applied through a vertical loading ram (called deviator stress). This can be done in one of two ways
- Stress-controlled – load is applied in increments and the deformation is measured
- Strain-controlled – load is applied at a constant rate of deformation

Types of Triaxial Tests



Under all-around cell pressure σ_c

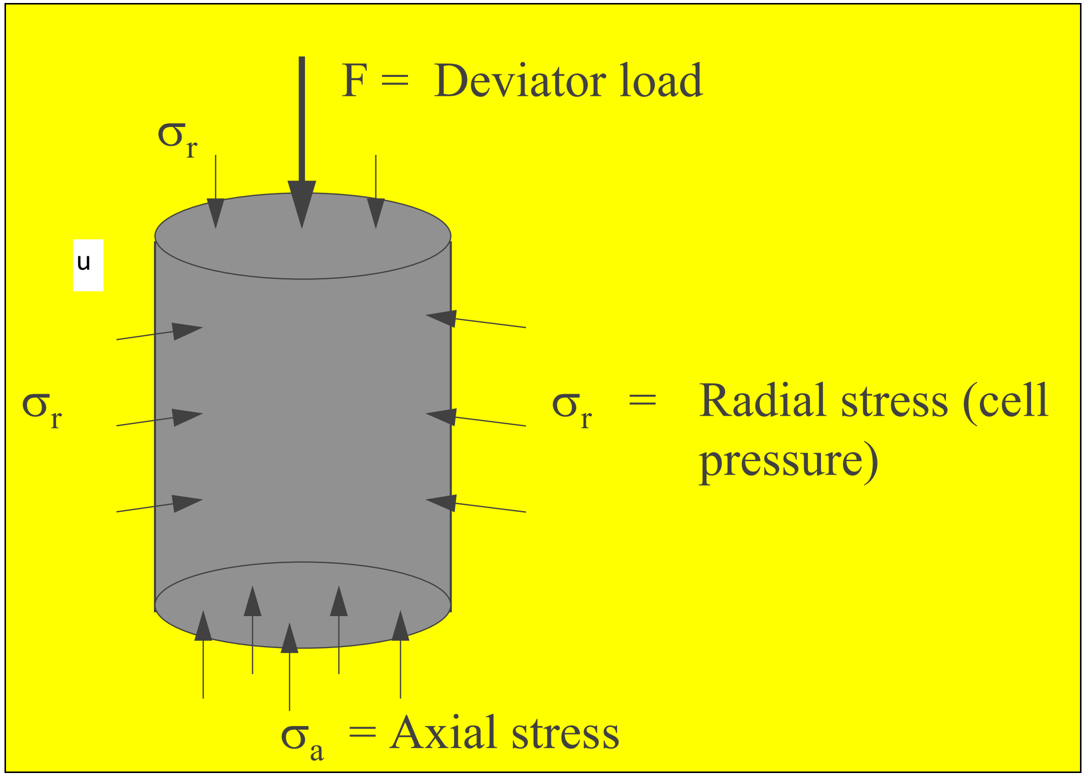
Shearing (loading)


Is the drainage valve open?
yes
no
Consolidated sample
Unconsolidated sample
Is the drainage valve open?
yes
no
Drained loading
Undrained loading
CD test
UU test
CU test

σ_C = confining pressure or all around pressure or cell pressure = σ_3

$\Delta\sigma$ = deviatoric stress = σ_1 - σ_3

Stresses



From vertical equilibrium we have $\sigma_a = \sigma_r + \frac{F}{A}$

The term F/A is known as the deviator stress, and is usually given the symbol $\Delta\sigma$.

Hence we can write $\Delta\sigma = \sigma_a - \sigma_r = \sigma_1 - \sigma_3$ (The axial and radial stresses are principal stresses)

If $\Delta\sigma = 0$ increasing cell pressure will result in:

- Volumetric compression if the soil is free to drain. The effective stresses will increase and so will the strength

- Increasing pore water pressure if soil volume is constant (that is, undrained). As the effective stresses cannot change it follows that $\Delta u = \Delta\sigma_3$

Strains

From the measurements of change in height, dh, and change in volume dV we can determine

Axial strain $\boldsymbol{\varepsilon_a = -dh/h_0}$

Volume strain $\boldsymbol{\varepsilon_v = -dV/V_0}$

Where h_0 is the initial height, and V_0 the initial volume. The conventional small strain assumption is generally used.

It is assumed that the sample deforms as a right circular cylinder. The cross-sectional area, A, can then be determined from

$$A(h_o + \Delta h) = V = V_0 + \Delta V$$

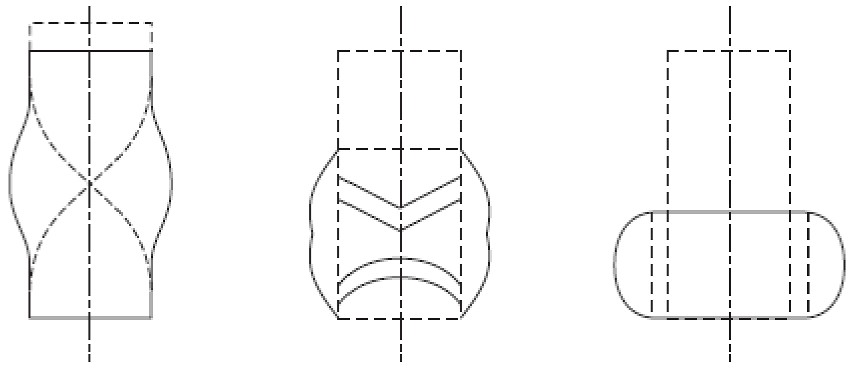
$$A = A_o \left(\frac{1 + \frac{dV}{V_0}}{1 + \frac{dh}{h_0}} \right) = A_o \left(\frac{1 - \varepsilon_v}{1 - \varepsilon_a} \right)$$

For an undrained test $\Delta V = 0$, then

$$A = A_o \left(\frac{1}{1 - \varepsilon_a} \right)$$

It is important to make allowance for the changing area when calculating the deviator stress, $\Delta\sigma = \sigma_1 - \sigma_3 = F/A$

A triaxial compression test specimen may exhibit a particular pattern or shape as failure is reached, depending upon the nature of the soil and its condition, as illustrated in Fig. below



(a) Brittle failure (b) Semi-plastic failure (c) Plastic failure

Failure patterns in triaxial compression tests

- brittle failure with well-defined shear plane,
- semi-plastic failure showing shear cones and some lateral bulging,
- Plastic failure with well-expressed lateral bulging.

In the case of plastic failure, the strain goes on increasing slowly at a reduced rate with increasing stress, with no specific stage to pin-point failure. In such a case, failure is assumed to have taken place when the strain reaches an arbitrary value such as 20%.

Merits of Triaxial Compression Test

The following are the significant points of merit of triaxial compression test:

(1) Failure occurs along the weakest plane unlike along the predetermined plane in the case of direct shear test.

(2) The stress distribution on the failure plane is much more uniform than it is in the direct shear test: the failure is not also progressive, but the shear strength is mobilised all at once. Of course, the effect of end restraint for the sample is considered to be a disadvantage; however, this may not have pronounced effect on the results since

the conditions are more uniform to the desired degree near the middle of the height of the sample where failure usually occurs.

(3) Complete control of the drainage conditions is possible with the triaxial compression test; this would enable one to simulate the field conditions better.

(4) The possibility to vary the cell pressure or confining pressure also affords another means to simulate the field conditions for the sample, so that the results are more meaningfully interpreted.

(5) Precise measurements of pore water pressure and volume changes during the test are possible.

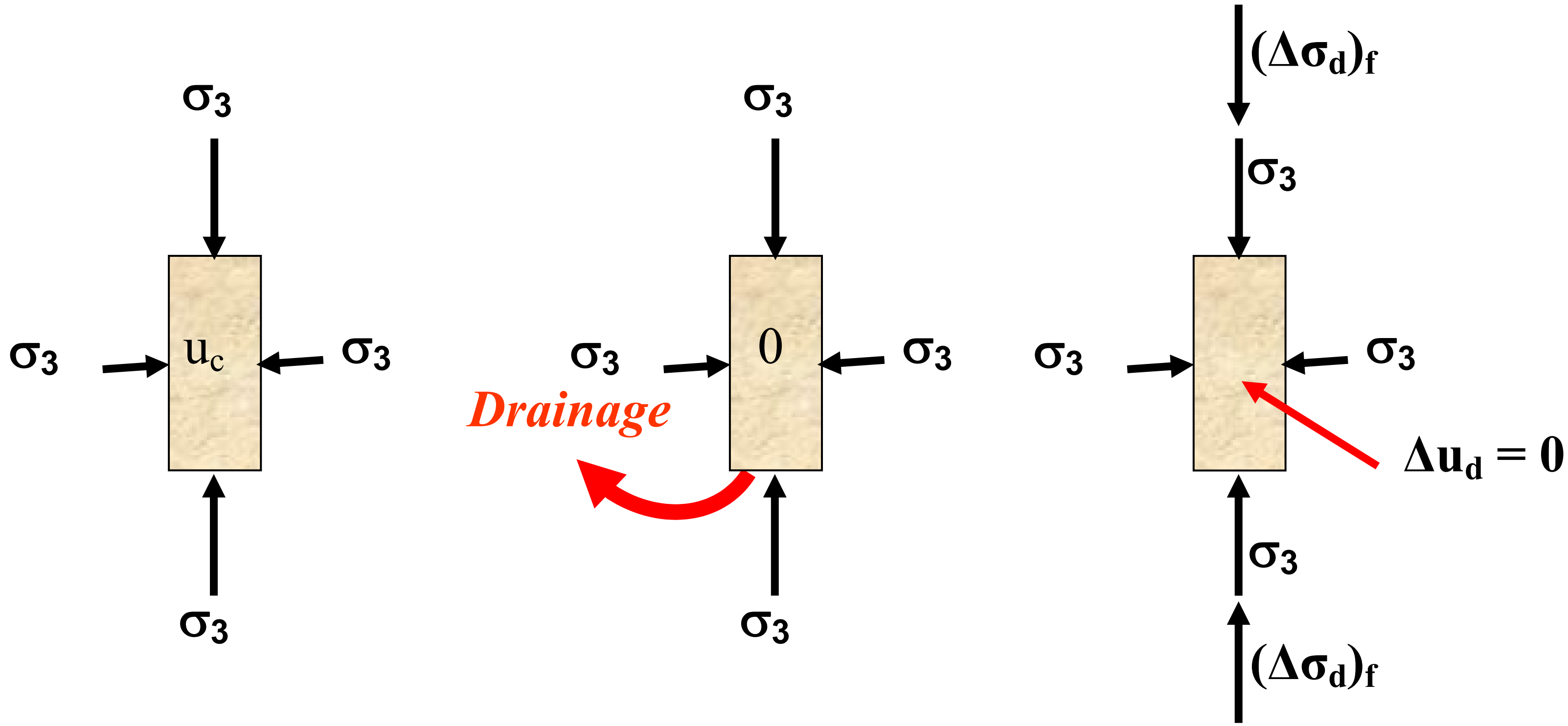
(6) The state of stress within the specimen is known on all planes and not only on a predetermined failure plane as it is with direct shear tests.

(7) The state of stress on any plane is capable of being determined not only at failure but also at any earlier stage.

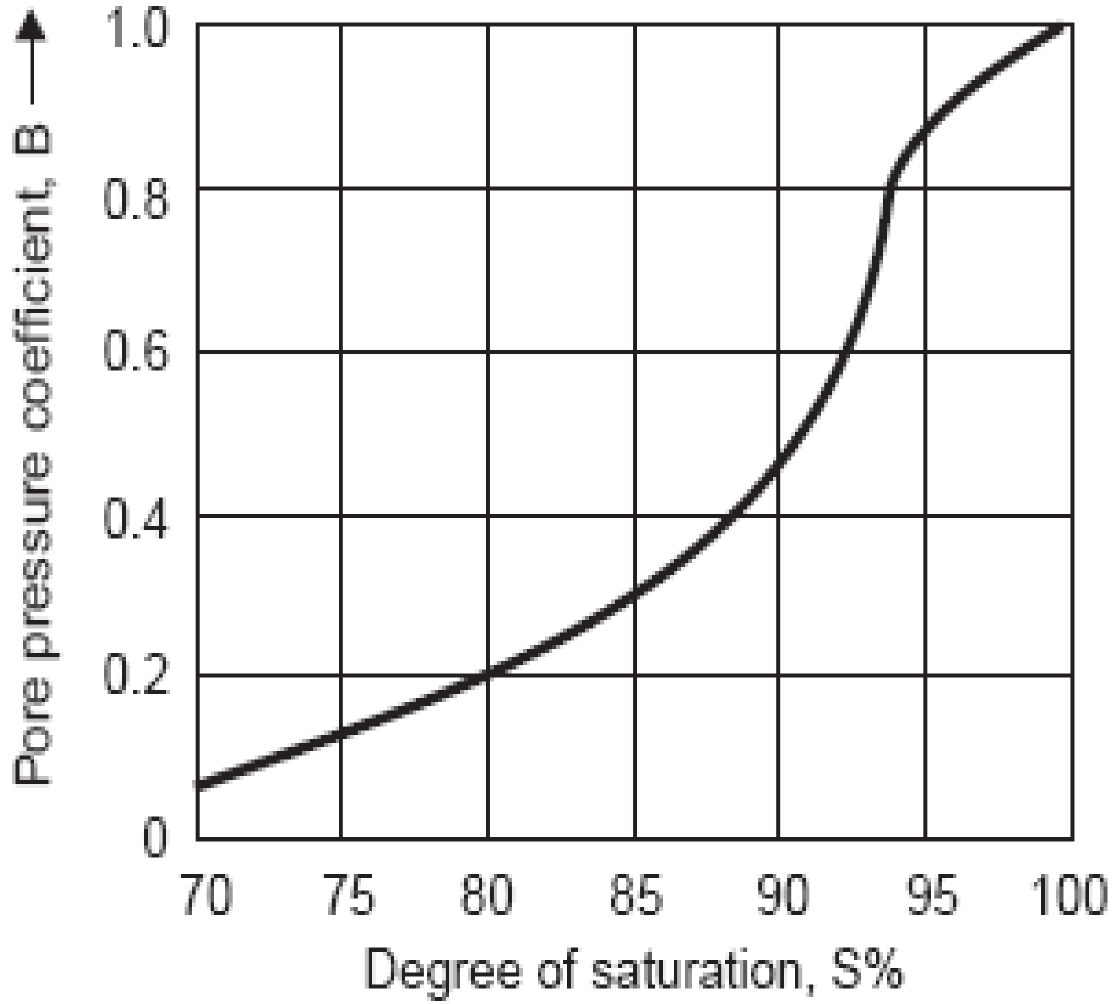
(8) Special tests such as extension tests are also possible to be conducted with the triaxial testing apparatus.

(9) It provides an ingenious and a symmetrical three-dimensional stress system better suited to simulate field conditions.

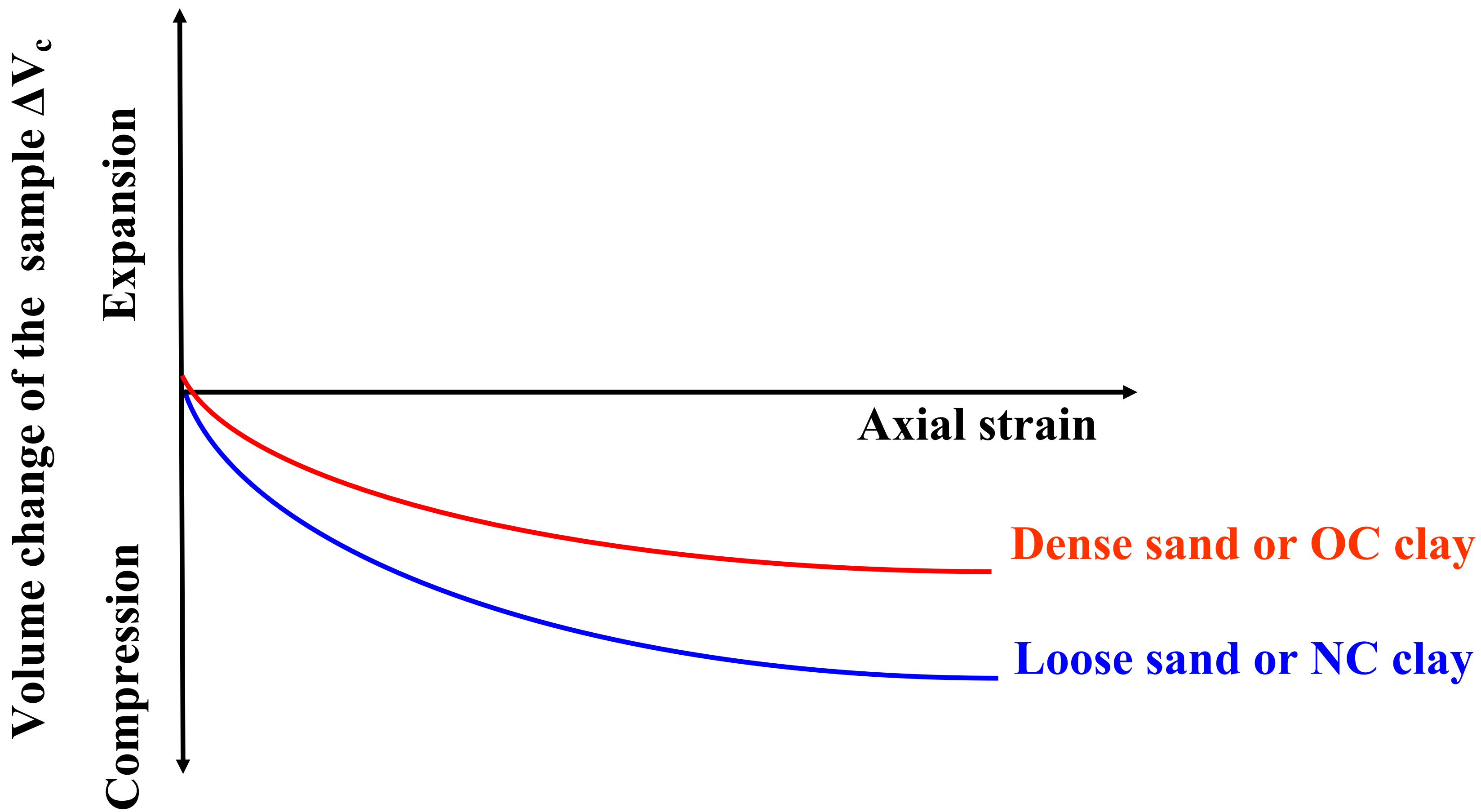
Consolidated - Drained test (CD Test)



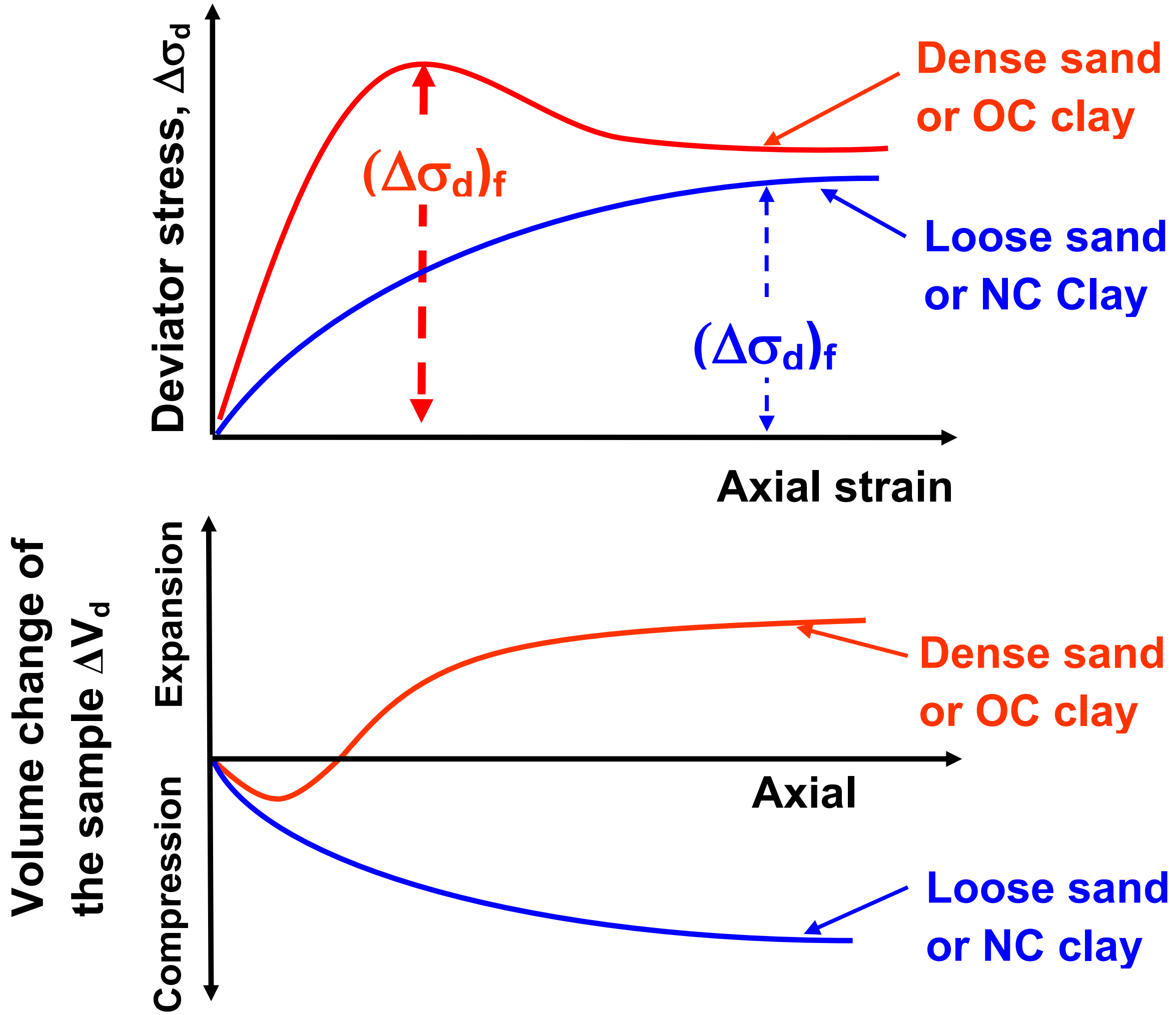
$$B = \frac{u_c}{\sigma_3}$$



$$B = \frac{u_c}{\sigma_3}$$


Volume change of the sample ΔV_c
Expansion
Compression
Axial strain
Dense sand or OC clay
Loose sand or NC clay

Stress-strain relationship during shearing

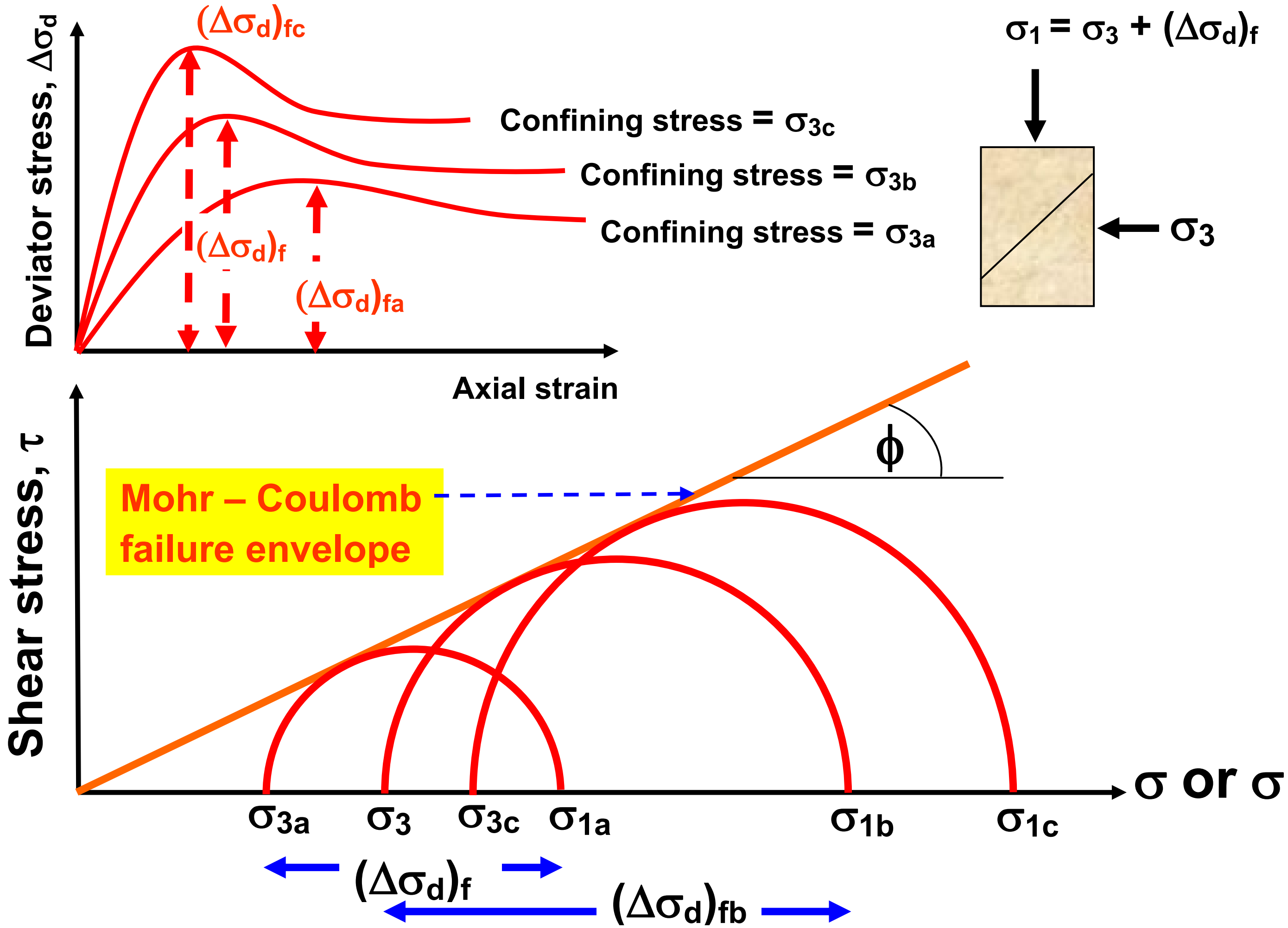


Since u = 0 in CD tests, $\sigma = \sigma'$ Therefore, $c = c'$ and $\phi = \phi'$ and c_d and ϕ_d are used to denote them

$$\sigma_3 = \sigma_3'$$

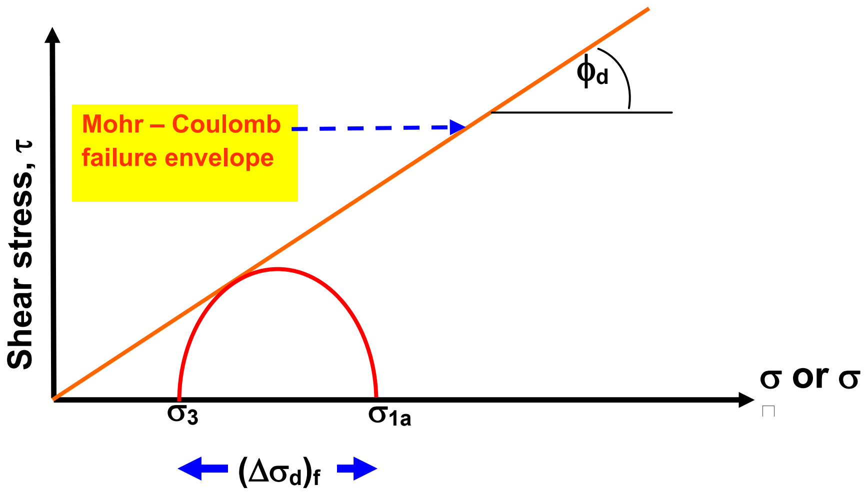
$$\sigma_1 = \sigma_1' = \sigma_3 + (\Delta\sigma_d)_f$$

CD tests How to determine strength parameters c and ϕ



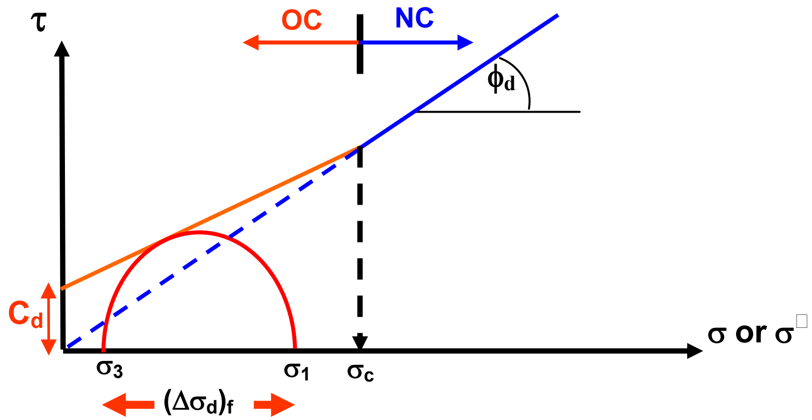
CD tests Failure envelopes

For sand and NC Clay, $c_d = 0$



CD tests Failure envelopes

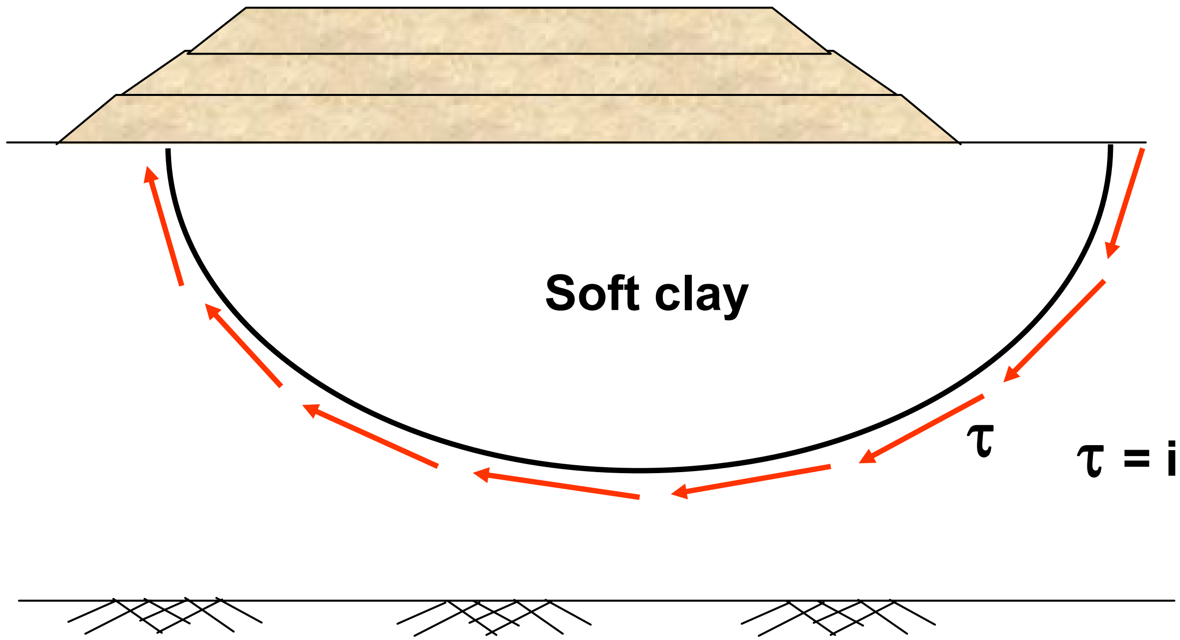
For OC Clay, $c_d \neq 0$



Therefore, one CD test would be sufficient to determine ϕ_d of sand or NC clay

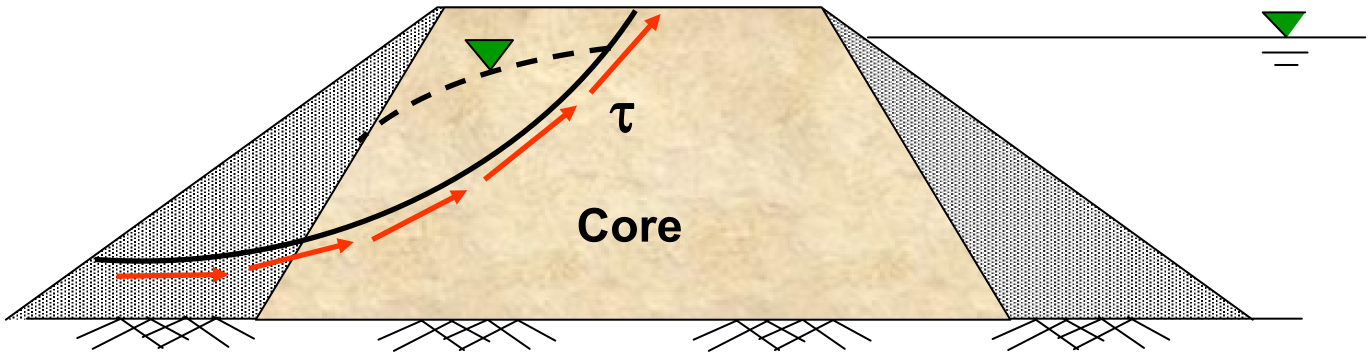
Some practical applications of CD analysis for clays

1. Embankment constructed very slowly, in layers over a soft clay deposit



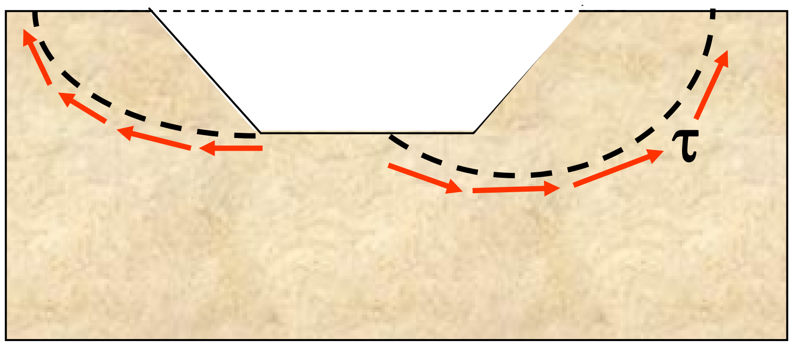
τ = in situ drained shear strength

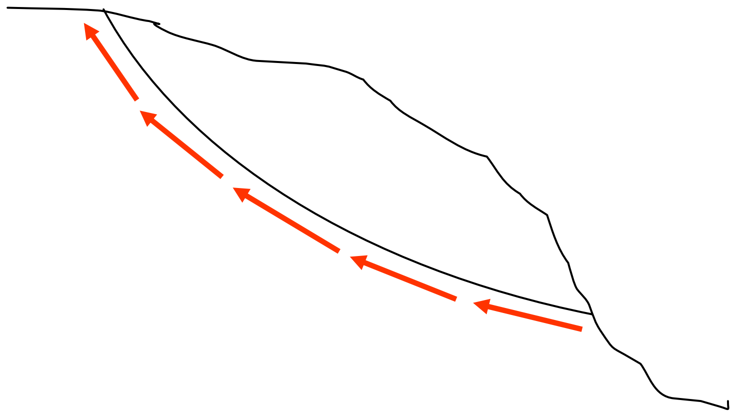
2. Earth dam with steady state seepage



τ = drained shear strength of clay core

3. Excavation or natural slope in clay

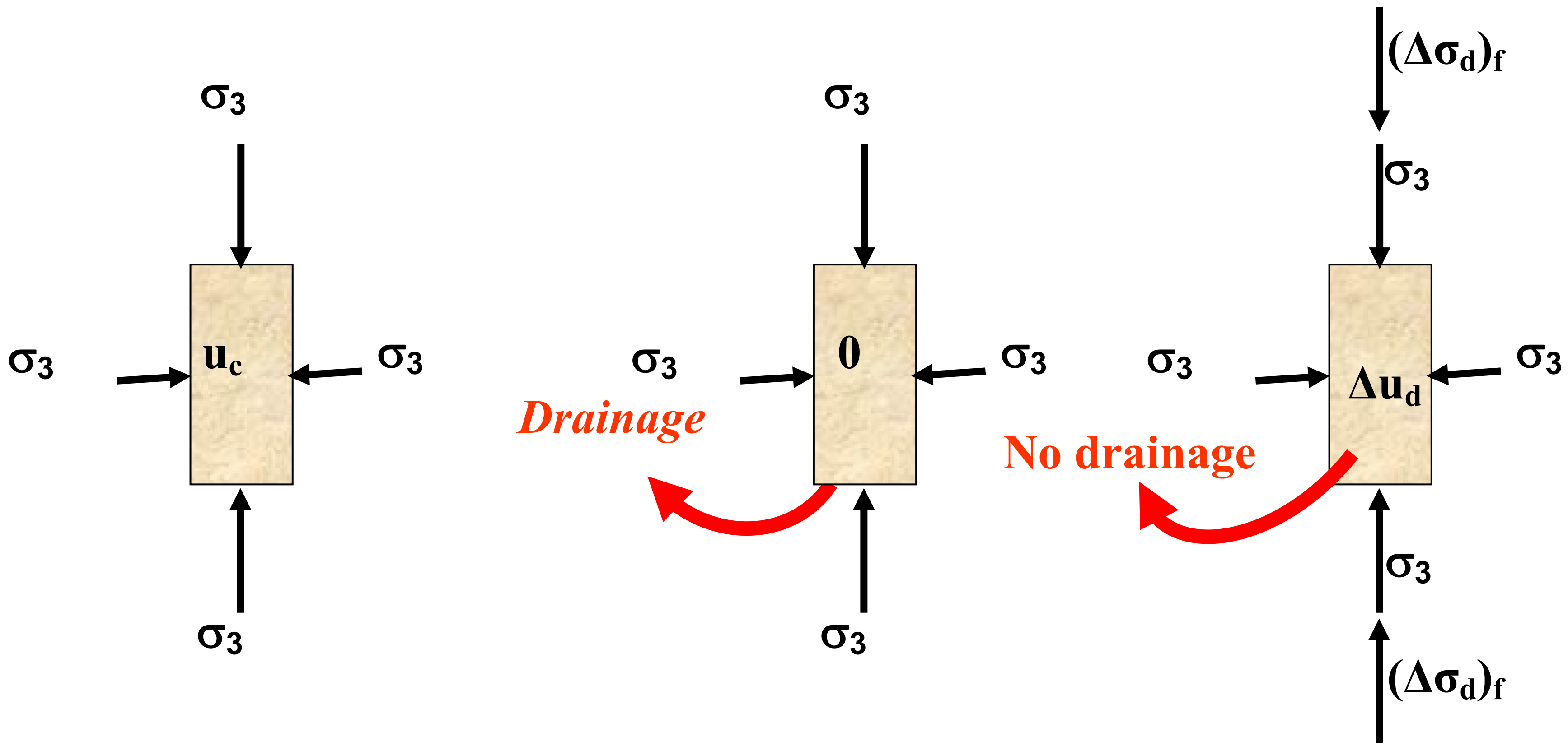




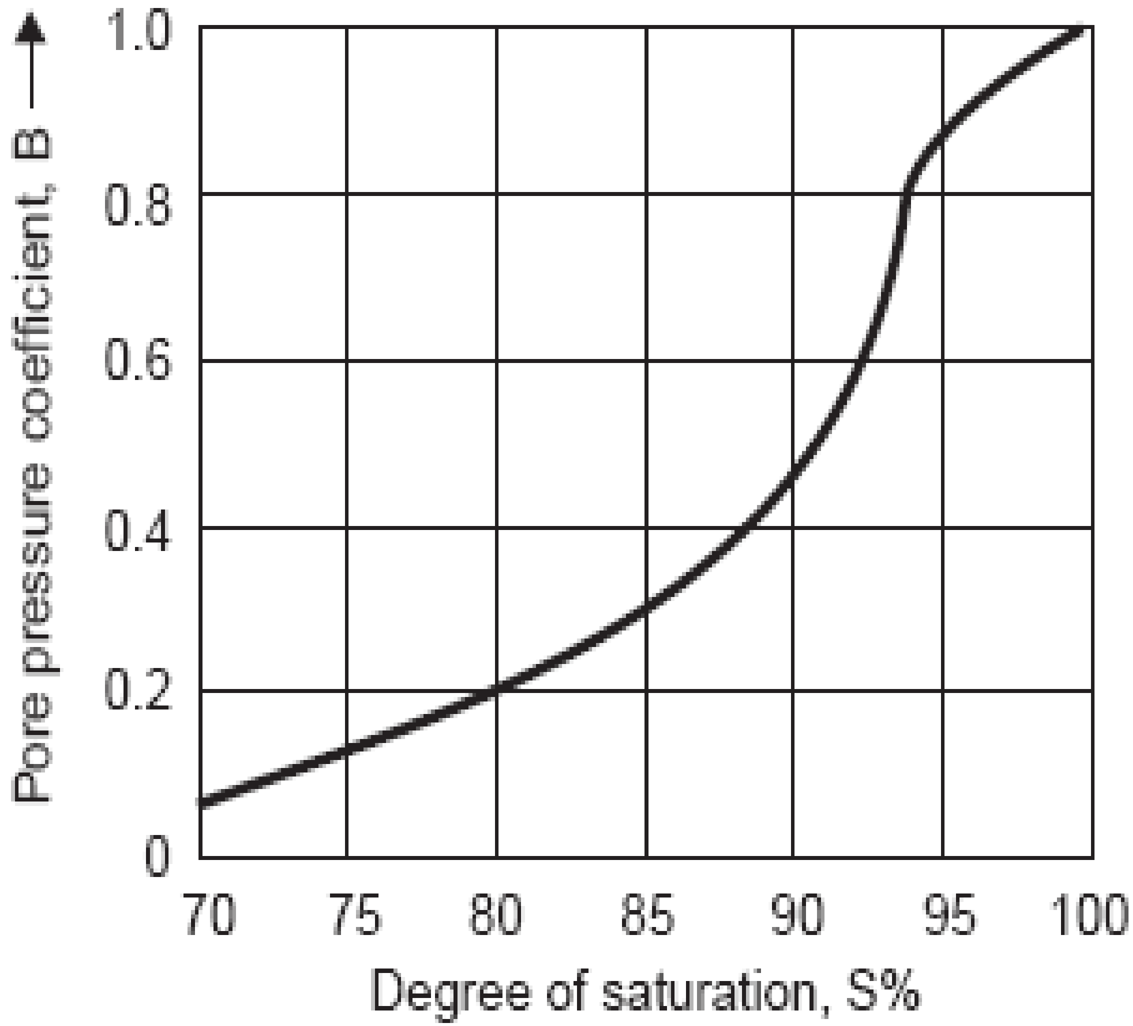
τ = In situ drained shear strength

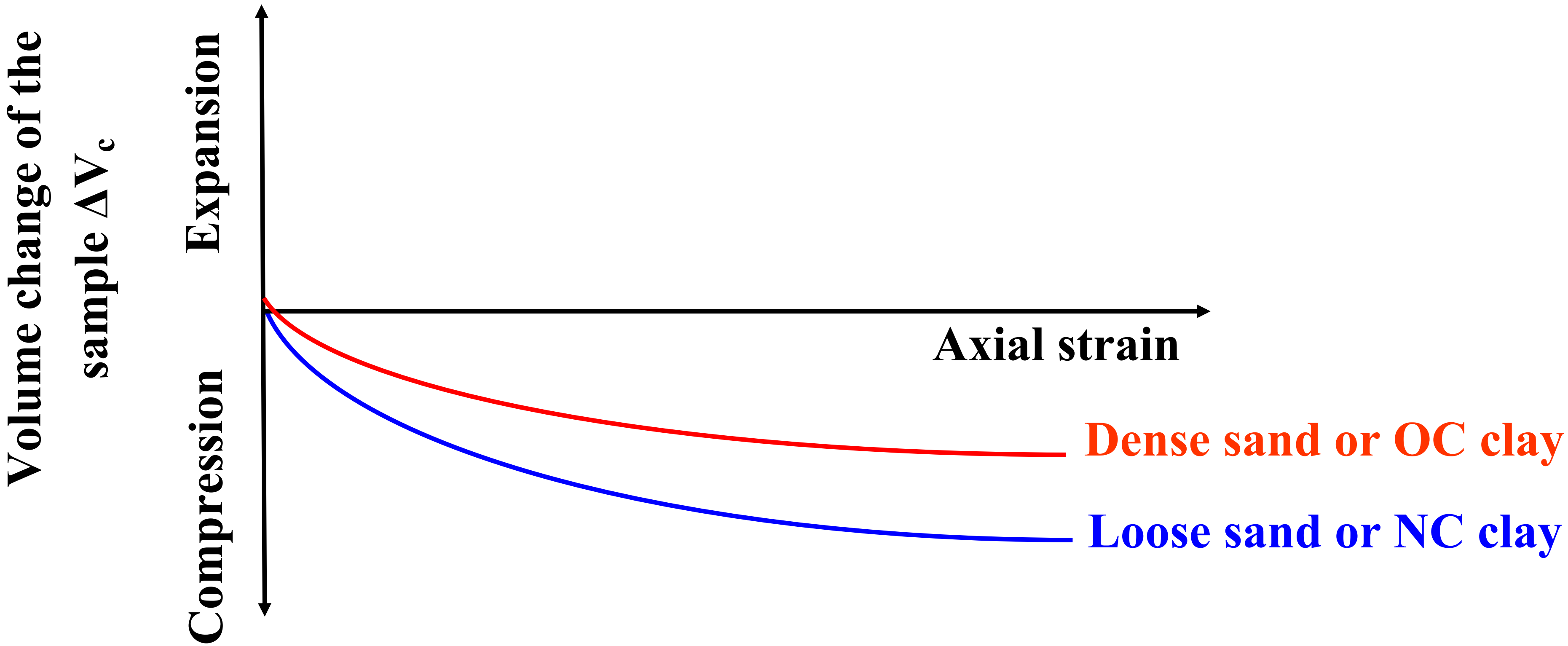
Note: CD test simulates the <u>long term condition</u> in the field. Thus, c_d and ϕ_d should be used to evaluate the long term behavior of soils

Consolidated- Undrained test (CU Test)

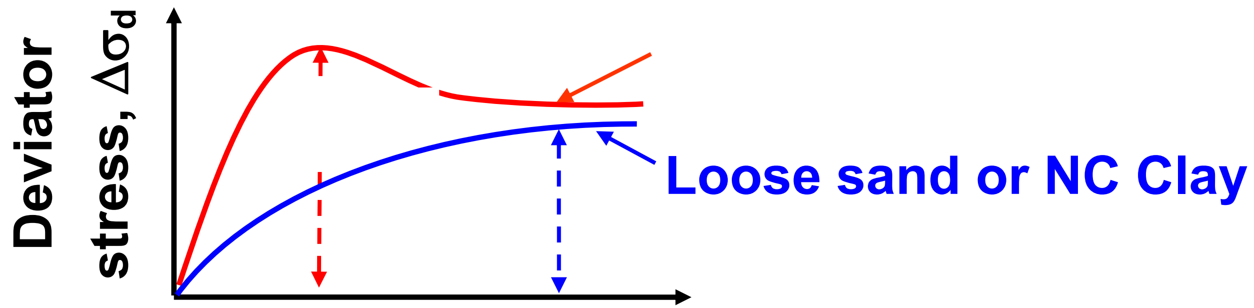


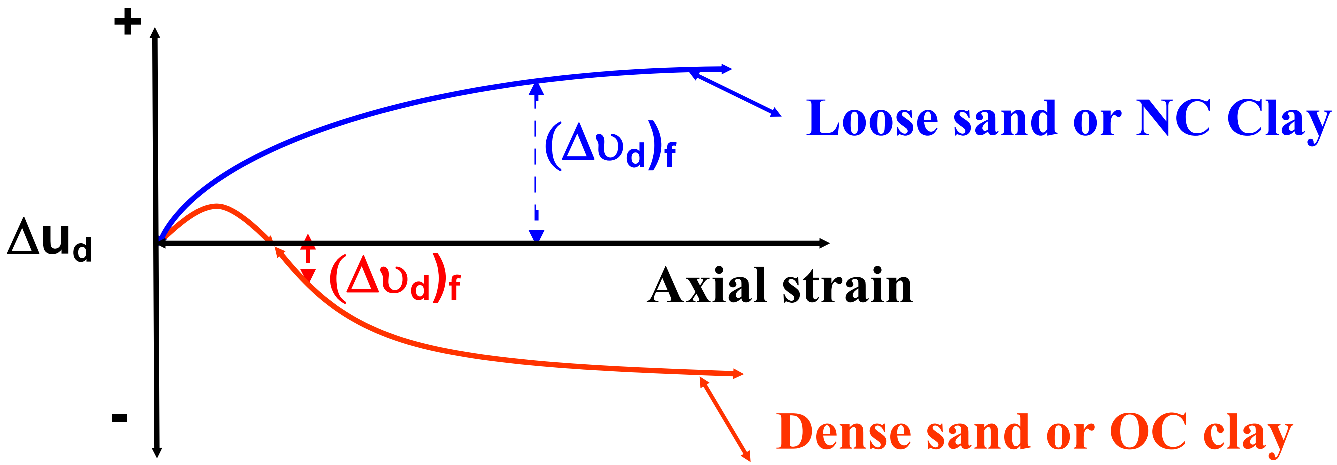
$$B = \frac{u_c}{\sigma_3}$$


Pore pressure coefficient, B
1.0
0.8
0.6
0.4
0.2
0
70
75
80
85
90
95
100
Degree of saturation, S%


Volume change of the sample ΔV_c
Expansion
Compression
Axial strain
Dense sand or OC clay
Loose sand or NC clay

Stress-strain relationship





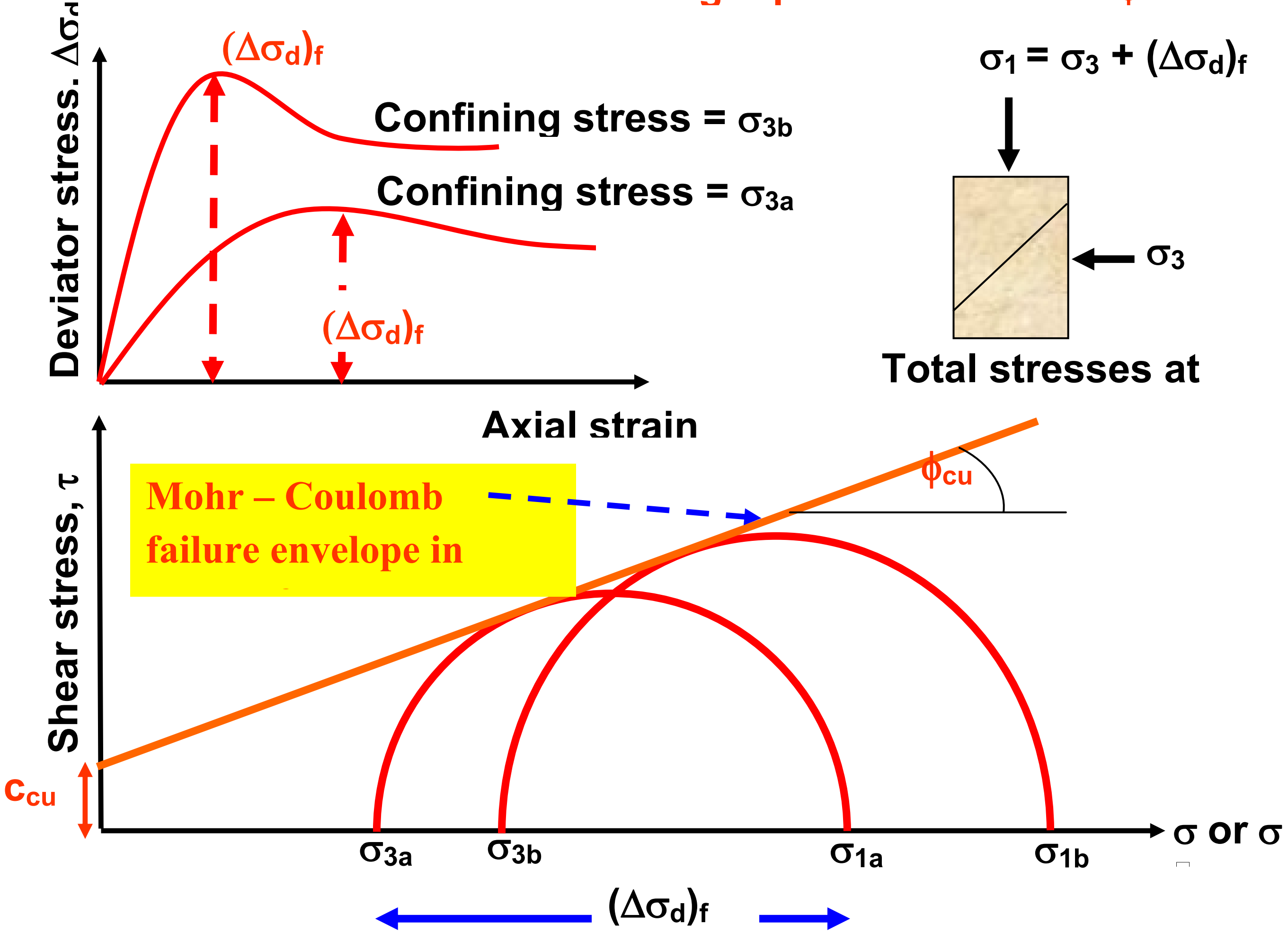
$$A = \frac{\Delta u_d}{\Delta \sigma_d}$$

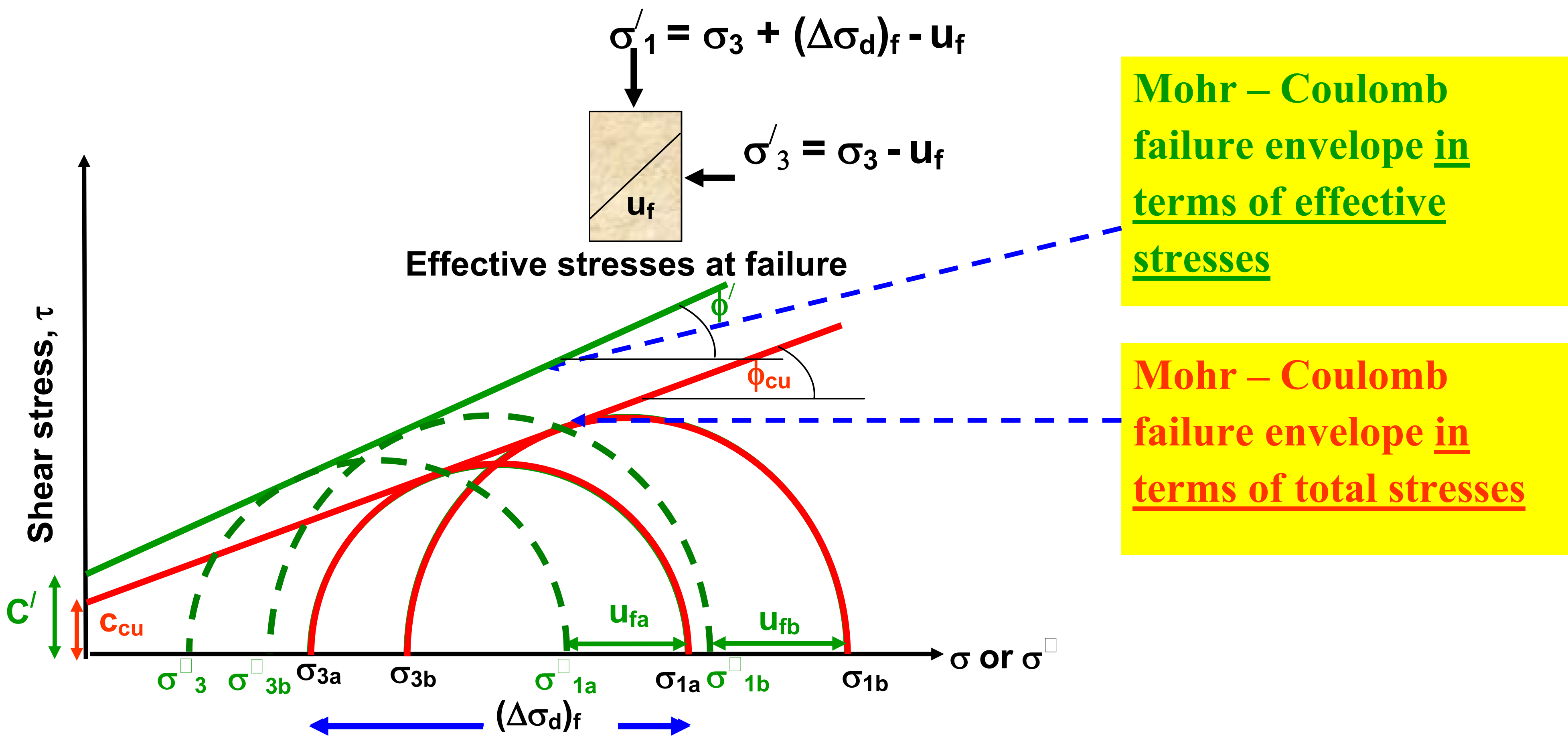
$$A_f = \frac{(\Delta u_d)_f}{(\Delta \sigma_d)_f}$$

Soil	A_f
N.C	0.5-1
O.C	0-0.5

CU tests

How to determine strength parameters c and ϕ




σ/1 = σ3 + (Δσd)f - uf
uf
σ/3 = σ3 - uf
Effective stresses at failure
Mohr – Coulomb failure envelope in terms of effective stresses
Mohr – Coulomb failure envelope in terms of total stresses
Shear stress, τ
φ/
φcu
C/
ccu
ufa
ufb
σ or σ/
σ/3
σ/3b
σ3a
σ3b
σ/1a
σ1a
σ/1b
σ1b
(Δσd)f

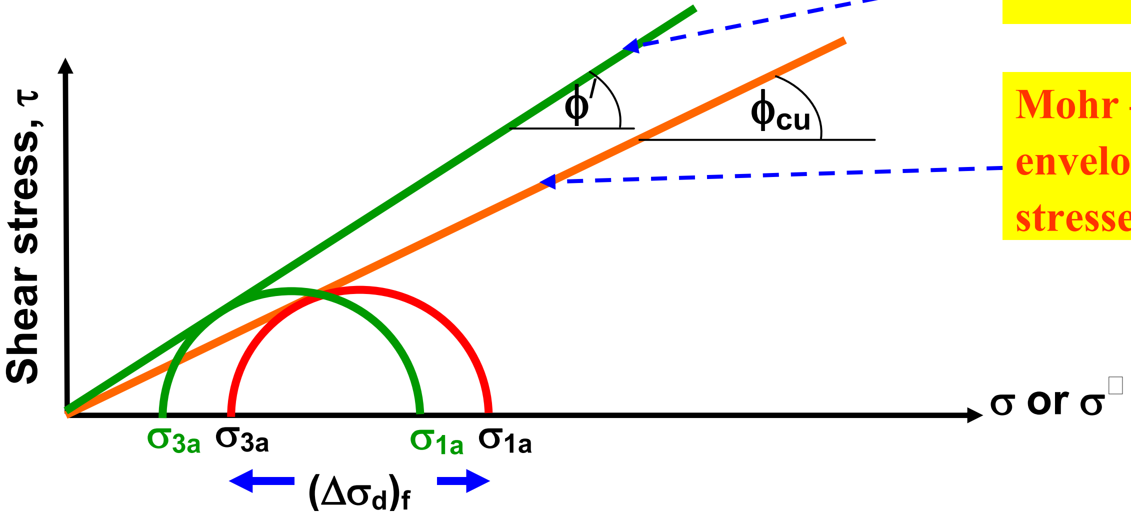
- **Shear strength parameters in terms of total stresses are C_{cu} and Φ_{cu}**
- **Shear strength parameters in terms of effective stresses are $C^{/}$ and $\Phi^{/}$**
- **$C^{/} = C_{drained}$ and $\Phi^{/} = \Phi_{drained}$**

CU tests Failure envelopes

For sand and NC Clay, c_{cu} and $c^{/} = 0$

Mohr – Coulomb failure envelope in terms of effective stresses

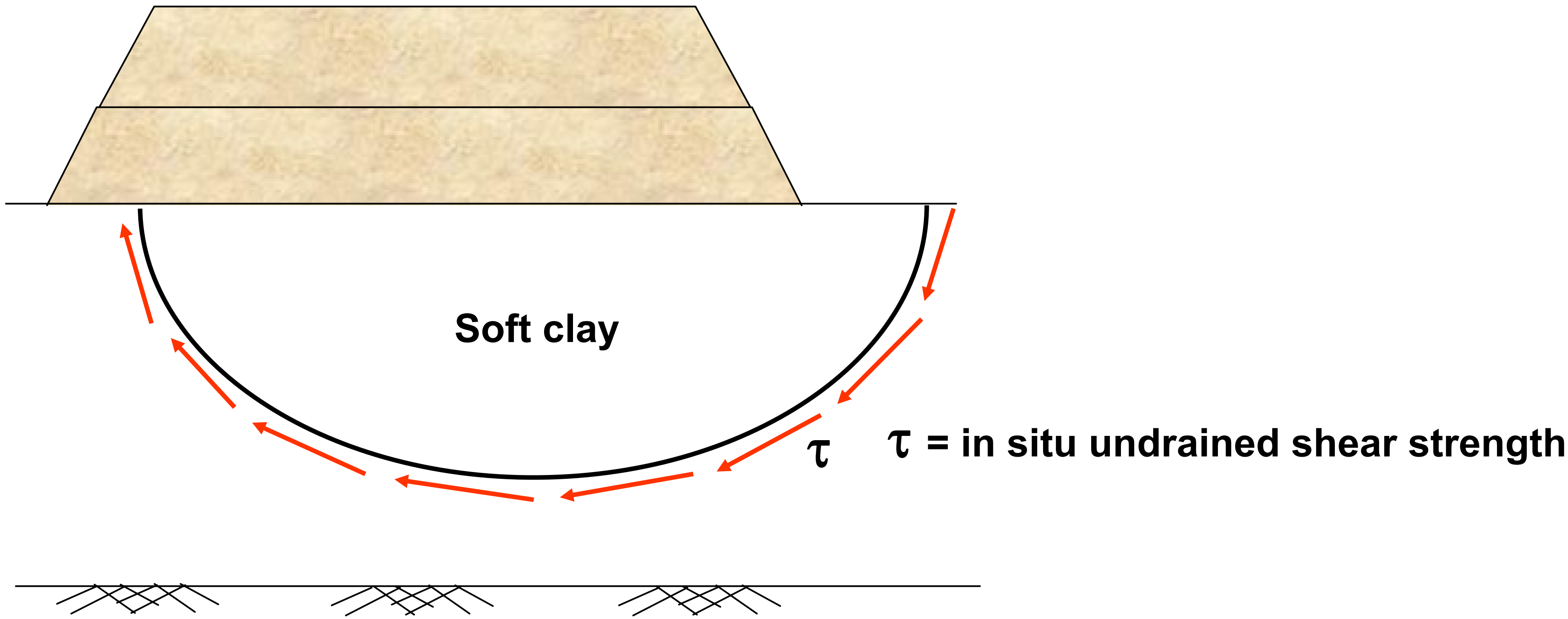
Mohr – Coulomb failure envelope in terms of total stresses



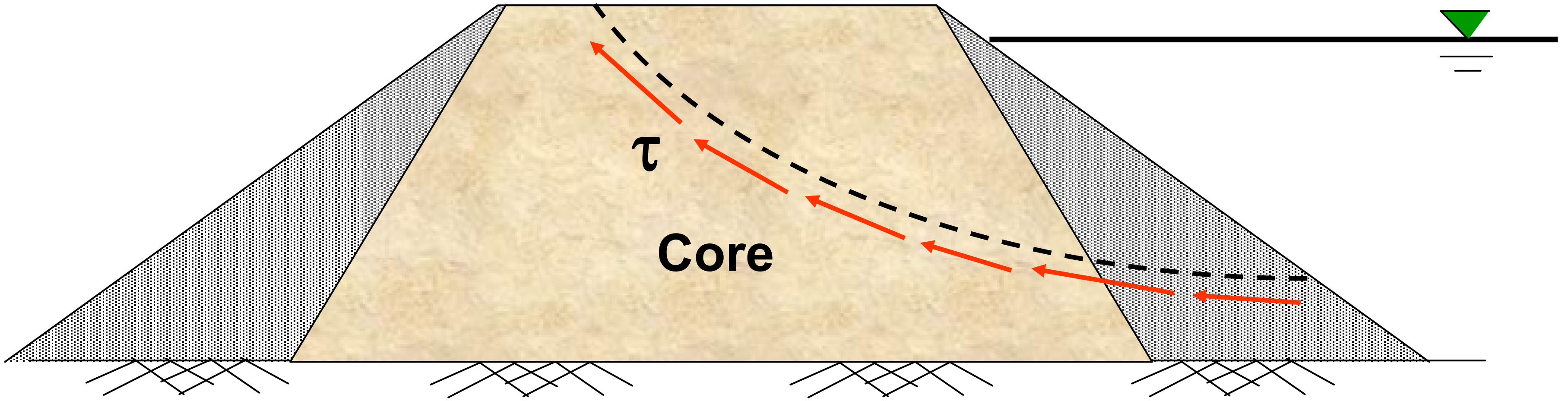
Therefore, one CU test would be sufficient to determine ϕ_{cu} and $\phi^{/} = \phi_d$ of sand or NC clay

Some practical applications of CU analysis for clays

1. Embankment constructed rapidly over a soft clay deposit

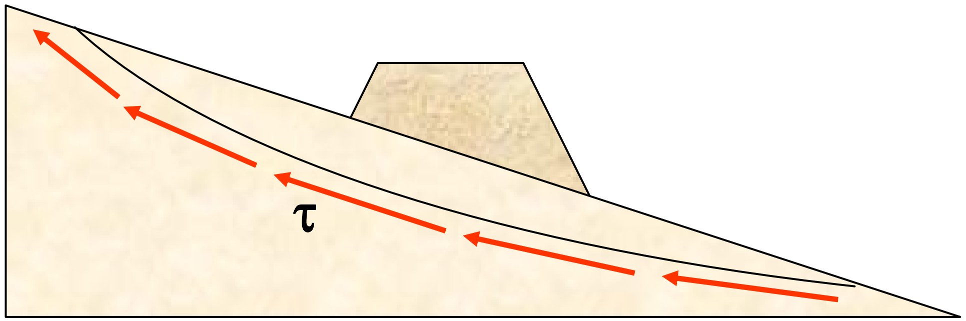


2. Rapid drawdown behind an earth dam



τ = Undrained shear strength of clay core

3. Rapid construction of an embankment on a natural slope



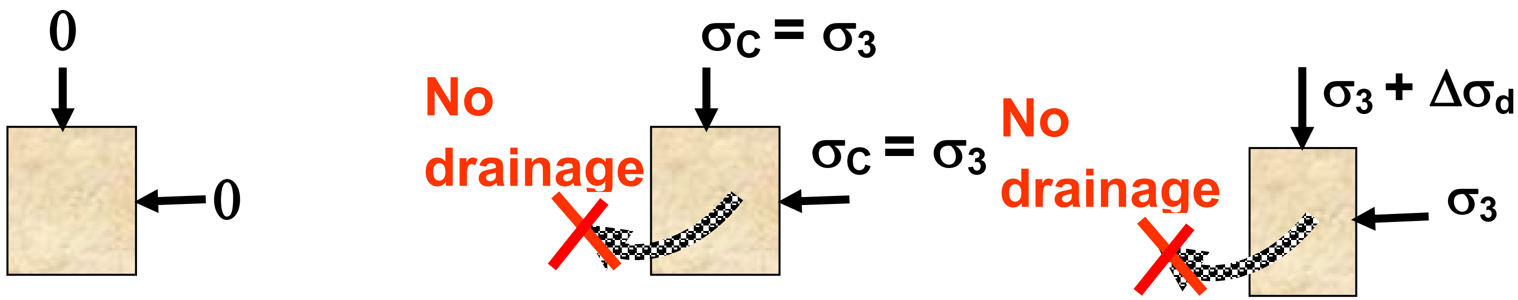
τ = In situ undrained shear strength

Note: Total stress parameters from CU test (C_{cu} and Φ_{cu}) can be used for stability problems where, Soil have become fully consolidated and are at equilibrium with the existing stress state; Then for some reason additional stresses are applied quickly with no drainage occurring

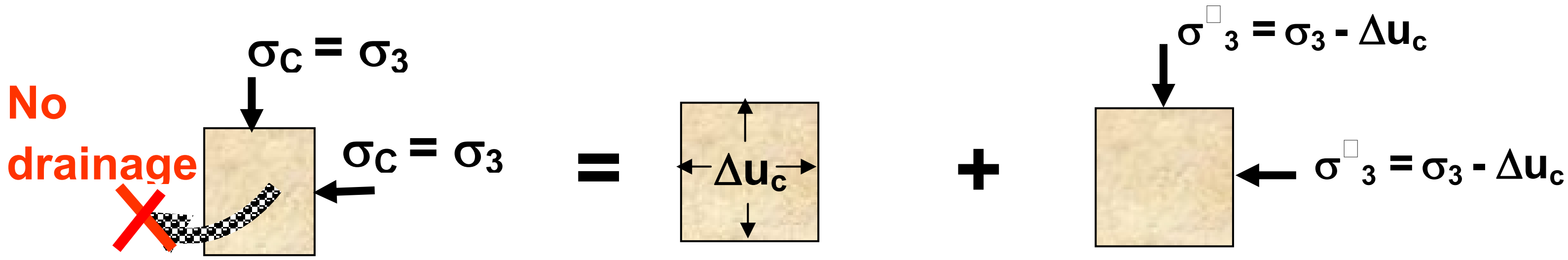
Unconsolidated- Undrained test (UU Test)

Data analysis

Step 1 Immediately after sampling	Step 2 Initial specimen condition	Step 3 Specimen condition during shearing



Step 2: After application of hydrostatic cell pressure



$$\Delta u_c = B \, \Delta\sigma_3$$

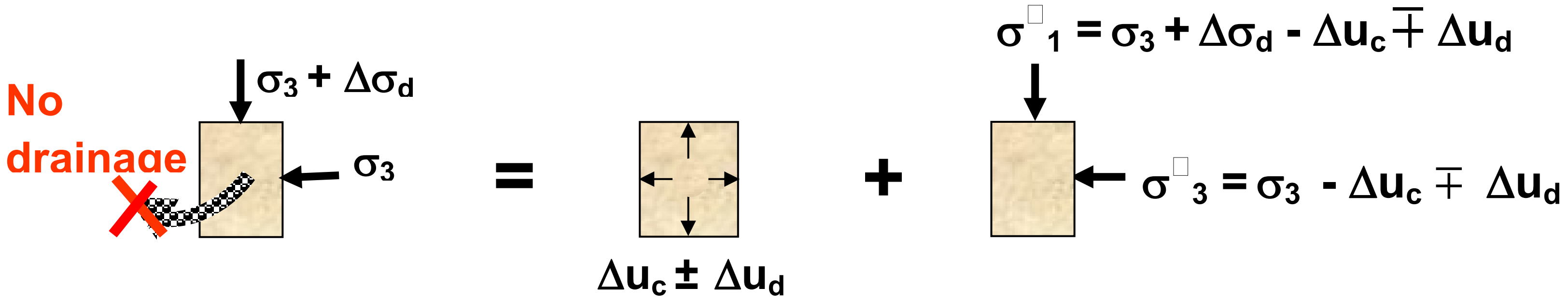
Increase of pwp due to increase of cell

Skempton's pore water pressure

Increase of cell

Note: If soil is fully saturated, then B = 1 (hence, $\Delta u_c = \Delta\sigma_3$)

Step 3: During application of axial load



$$\Delta u_d = A B \Delta\sigma_d$$

Increase of pwp due to increase of deviator

Increase of deviator stress

Skempton's pore water pressure parameter, A

Combining steps 2 and 3,

$$\Delta u_c = B\ \Delta\sigma_3$$

$$\Delta u_d = AB\Delta\sigma_d$$

Total pore water pressure increment at any stage, Δu

$$\Delta u = \Delta u_c + \Delta u_d$$

$$\Delta u = B\ [\Delta\sigma_3 + A\Delta\sigma_d]$$

$$\Delta u = B\ [\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3]$$

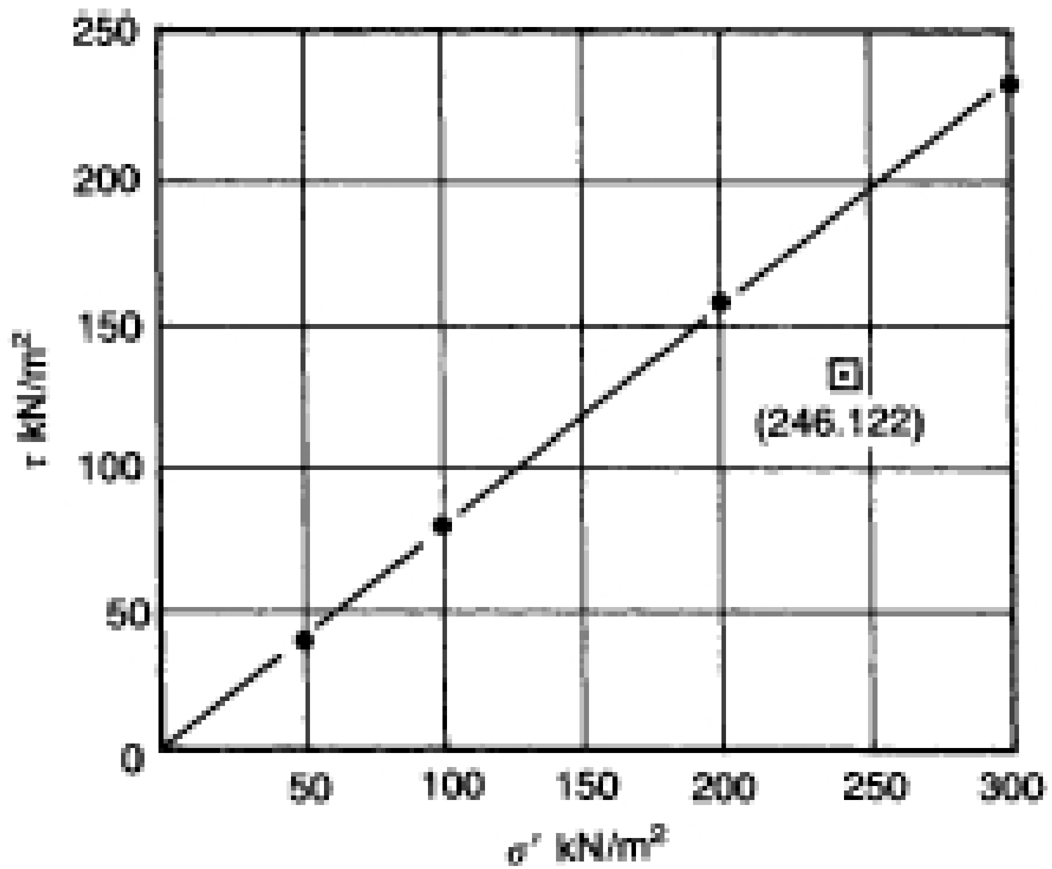
Skempton's pore water pressure equation

Example 1

The following results were obtained from direct shear tests on specimens of a sand compacted to the in-situ density. Determine the value of the shear strength parameter $\varphi^{/}$.

Normal stress (kN/m^2)	**50**	**100**	**200**	**300**
Shear stress at failure (kN/m^2)	**36**	**80**	**154**	**235**

Would failure occur on a plane within a mass of this sand at a point where the shear stress is 122 kN/m^2 and the effective normal stress 246 kN/m^2?


250
200
150
100
50
0
τ kN/m²
(246.122)
50
100
150
200
250
300
σ' kN/m²

The values of shear stress at failure are plotted against the corresponding values of normal stress, as shown in Figure above. The failure envelope is the line having the best fit to the plotted points; in this case a straight line through the origin. If the stress scales are the same, the value of $\varphi^{/}$ can be measured directly and is 38^{o}.

The stress state τ=122kN/m^2, $\varphi^{/}$ = 246 kN/m^2 plots below the failure envelope, and therefore would not produce failure.

Example 2

The results shown in Table below were obtained at failure in a series of triaxial tests on specimens of a saturated clay initially 38mm in diameter by 76mm long. Determine the values of the shear strength parameters with respect to (a) total stress and (b) effective stress.

Solution

The initial values of length, area and volume for each specimen are:

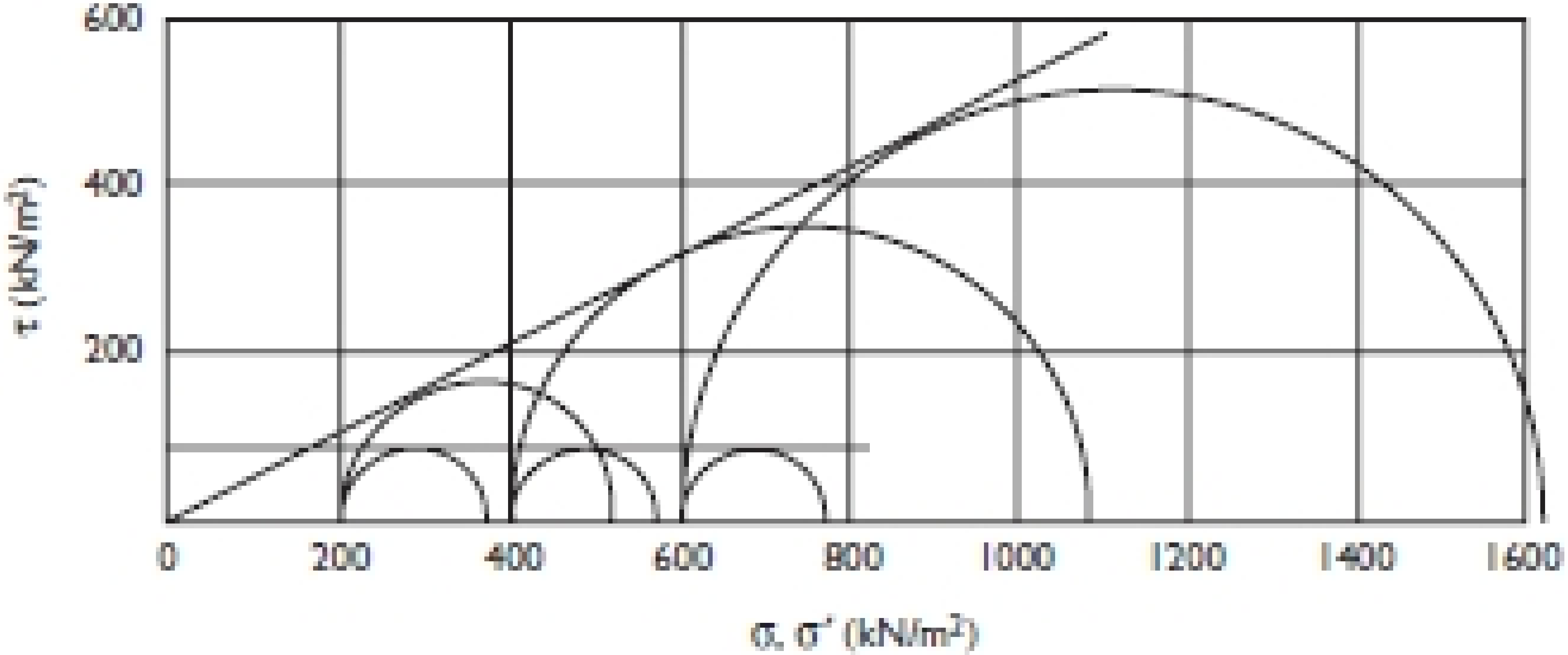
l_o = 76mm; A_0 = 1135mm^2; V_0 = 86 x 103 mm^3

Table 4.2

	Type of test	All-round pressure (kN/m^2)	Axial load (N)	Axial deformation (mm)	Volume change (ml)
(a)	Undrained	200	222	9.83	–
		400	215	10.06	–
		600	226	10.28	–
(b)	Drained	200	403	10.81	6.6
		400	848	12.26	8.2
		600	1265	14.17	9.5

Table 4.3

	σ_3 (kN/m^2)	$\Delta l/l_0$	$\Delta V/V_0$	Area (mm^2)	$\sigma_1 - \sigma_3$ (kN/m^2)	σ_1 (kN/m^2)
(a)	200	0.129	–	1304	170	370
	400	0.132	–	1309	164	564
	600	0.135	–	1312	172	772
(b)	200	0.142	0.077	1222	330	530
	400	0.161	0.095	1225	691	1091
	600	0.186	0.110	1240	1020	1620



Cu = 85 kN/m^2; φ_u = 0 **c$^/$ = 0; $\varphi^/$ = 27^0**

Example 3

The results shown in Table below were obtained for peak failure in a series of consolidated–undrained triaxial tests, with pore water pressure measurement, on specimens of saturated clay. Determine the values of the effective stress parameters.

All-round pressure (kN/m^2)	Principal stress difference (kN/m^2)	Pore water pressure (kN/m^2)
150	192	80
300	341	154

σ_3	σ_1	σ'_3	σ'_1
150	342	70	262
300	641	146	487

$$\sigma_1' = \sigma_3' \tan^2(45 + \frac{\phi'}{2}) + 2c' \tan(45 + \frac{\phi'}{2})$$

$$262 = 70\tan^2(45 + \frac{\phi'}{2}) + 2c' \tan(45 + \frac{\phi'}{2}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$487 = 146\tan^2(45 + \frac{\phi'}{2}) + 2c' \tan(45 + \frac{\phi'}{2}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Solve eqs. 1 and 2 simultaneously we get,

$\varphi' = 29.67^{o} \approx 30^{o}$ $\quad$ $c' \approx 16\ kN/m^2$