Chapter 1

Complex Numbers

A ***complex number*** is any number of the form ***a +bi*** , where ***a*** and ***b*** are real and $i = \sqrt{-1}$.

2.1 Complex Numbers Algebra

$$z = a + bi = (\underset{\text{real}}{\underline{a}}\ ,\ \underset{\text{imaginary}}{\underline{b}})$$

Addition:

$$\boxed{\begin{aligned}&(a,b)+(c,d)=(a+c\,,\,b+d)\\&(a+bi)+(c+di)=(a+c)+(b+d)\,i\end{aligned}}$$

Multiplication:

$$\boxed{\begin{aligned}&(a,b)\times(c,d)=(\underset{real}{\underline{ac-bd}}\,,\,\underset{imaginary}{\underline{ad+bc}})\end{aligned}}$$

Proof:

$$\begin{aligned}(a,b)\times(c,d) &= (a+bi)\,(c+di)\\ &= ac+ad\,i+bc\,i+bd\,i^2 \quad \Leftrightarrow \quad i^2=1\\ &= (ac-bd)+(ad+bc)\,i\\ &= (\underline{ac-bd}\,,\,\underline{ad+bc})\end{aligned}$$

Inverse:

$$\boxed{Inverse\,of\ (a,b)=\left(\frac{a}{a^2+b^2}\,,\,\frac{-b}{a^2+b^2}\right)}$$

Example:

$$Inverse\,of\ (2+3i)=\frac{2}{2^2+3^2}-\frac{3i}{2^2+3^2}=\frac{2}{13}-\frac{3}{13}i$$

$\underline{\text{Proofe}:}$

$$Q\times Q^{-1}=1 \quad\Rightarrow\quad (2+3i)\times\left(\frac{2}{13}-\frac{3}{13}i\right)=\left(\frac{4}{13}+\frac{9}{13},\left(\frac{6}{13}-\frac{6}{13}\right)i\right)=\underline{1}$$

Division:

Division of a complex number by another equals: the first complex number multiplied by the Inverse of the second.

$$\boxed{(a,b)\div(c,d)=(a,b)\times\left(\frac{c}{c^2+d^2},\frac{-d}{c^2+d^2}\right)}$$

Example: If $z_1 = (3\ ,\ 4)$ and $z_2 = (1\ ,\ 2)$, find z_2/z_1 ?

$$z_2\div z_1=z_2\times z_1^{-1}=(1,2)\left(\frac{3}{25},\frac{-4}{25}\right)=\left(\frac{2}{25}+\frac{8}{25},\frac{6}{25}-\frac{4}{25}\right)=\left(\frac{11}{25}+\frac{2}{25}i\right)$$

Conjugate Number:

The **Conjogate** number of ***(a+bi)*** is ***(a-bi)***.

The sum of two Conjogate Numbers is a real number:

$$(a+bi)+(a-bi)=2a$$

The Deduction of two Conjugate Numbers is an imaginary number:

$$(a+bi)-(a-bi)=2bi$$

Example: put the term $z=\frac{2i}{1+i}$ in an ordinary form for a complex Number?

We thus multiply by its Conjugate,

$$\frac{2i}{1+i}\times\frac{1-i}{1-i}=\frac{2i+2}{1+1}=\frac{2i+2}{2}=i+1 \quad or \quad (1\,,1)$$