An Introduction to Linear Programming

Linear programming is a problem-solving approach developed to help managers make decisions. Numerous applications of linear programming can be found in today's competitive business environment. For instance, Eastman Kodak uses linear programming to determine where to manufacture products throughout their worldwide facilities, and GE Capital uses linear programming to help determine optimal lease structuring. Marathon Oil Company uses linear programming for gasoline blending and to evaluate the economics of a new terminal or pipeline

These examples are only a few of the situations in which linear programming has been used successfully, but they illustrate the diversity of linear programming applications

In all linear programming problems, the maximization or minimization of some quantity is the objective.

All linear programming problems also have a second property: restrictions, or **constraints,** that limit the degree to which the objective can be pursued. For example , a manufacturer is restricted by constraints requiring product demand to be satisfied and by the constraints limiting production capacity. Thus, constraints are another general feature of every linear programming problem.

A SIMPLE MAXIMIZATION PROBLEM

Par, Inc., is a small manufacturer of golf equipment and supplies whose management has decided to move into the market for medium- and high-priced golf bags. Par's distributor is enthusiastic about the new product line and has agreed to buy all the golf bags Par produces over the next three months. After a thorough investigation of the steps involved in manufacturing a golf bag, management determined that each golf bag produced will require the following operations:

1. Cutting and dyeing the material
2. Sewing
3. Finishing (inserting umbrella holder, club separators, etc.)
4. Inspection and packaging

The director of manufacturing analyzed each of the operations and concluded that if the company produces a medium-priced standard model, each bag will require 7/10 hour in the cutting and dyeing department, 1/2 hour in the sewing department, 1 hour in the finishing department, and 1/10 hour in the inspection and packaging department. The more expensive deluxe model will require 1 hour for cutting and dyeing, 5/6 hour for sewing, 2/3 hour for finishing, and 1/4 hour for inspection and packaging. This production information is summarized in Table 2.1.

TABLE 2.1 PRODUCTION REQUIREMENTS PER GOLF BAG

	Production Time (hours)	
Department	**Standard Bag**	**Deluxe Bag**
Cutting and Dyeing	$\frac{7}{10}$	1
Sewing	$\frac{1}{2}$	$\frac{5}{6}$
Finishing	1	$\frac{2}{3}$
Inspection and Packaging	$\frac{1}{10}$	$\frac{1}{4}$

After studying departmental workload projections, the director of manufacturing estimates that 630 hours for cutting and dyeing, 600 hours for sewing, 708 hours for finishing, and 135 hours for inspection and packaging will be available for the production of golf bags during the next three months.

The accounting department analyzed the production data, assigned all relevant variable costs, and arrived at prices for both bags that will result in a profit contribution1 of $10 for every standard bag and $9 for every deluxe bag produced. Let us now develop a mathematical model of the Par, Inc., problem that can be used to determine the number of standard bags and the number of deluxe bags to produce in order to maximize total profit contribution.

Problem Formulation

Problem formulation, or **modeling,** is the process of translating the verbal statement of a problem into a mathematical statement.

Describe the Objective The objective is to maximize the total contribution to profit.

Describe Each Constraint Four constraints relate to the number of hours of manufacturing time available; they restrict the number of standard bags and the number of deluxe bags that can be produced.

Constraint 1: Number of hours of cutting and dyeing time used must be less than or equal to the number of hours of cutting and dyeing time available.

Constraint 2: Number of hours of sewing time used must be less than or equal to the number of hours of sewing time available.

Constraint 3: Number of hours of finishing time used must be less than or equal to the number of hours of finishing time available.

Constraint 4: Number of hours of inspection and packaging time used must be less than or equal to the number of hours of inspection and packaging time available.

Define the Decision Variables The controllable inputs for Par, Inc., are (1) the number of standard bags produced, and (2) the number of deluxe bags produced. Let

S = number of standard bags
D= number of deluxe bags

In linear programming terminology, *S* and *D* are referred to as the **decision variables.**

Write the Objective in Terms of the Decision Variables

Total Profit Contribution = 10S + 9D

Because the objective—maximize total profit contribution—is a function of the decision variables *S* and *D*, we refer to 10*S* + 9*D* as the *objective function*. Using "Max" as an abbreviation for maximize, we write Par's objective as follows:

Max 10S + 9D

Write the Constraints in Terms of the Decision Variables

$$\begin{pmatrix} \text{Hours of cutting and} \\ \text{dyeing time used} \end{pmatrix} \leq \begin{pmatrix} \text{Hours of cutting and} \\ \text{dyeing time available} \end{pmatrix}$$

Total hours of cutting and dyeing time used

7/10 *S* + 1 *D* <= 630

Constraint 2:

$$\begin{pmatrix}\text{Hours of sewing}\\ \text{time used}\end{pmatrix} \leq \begin{pmatrix}\text{Hours of sewing}\\ \text{time available}\end{pmatrix}$$

1/2 *S* + 5/6 *D* <= 600

Constraint 3:

$$\begin{pmatrix}\text{Hours of finishing}\\ \text{time used}\end{pmatrix} \leq \begin{pmatrix}\text{Hours of finishing}\\ \text{time available}\end{pmatrix}$$

1*S* + 2/3 *D* <=708

Constraint 4:

$$\begin{pmatrix}\text{Hours of inspection and} \\ \text{packaging time used}\end{pmatrix} \leq \begin{pmatrix}\text{Hours of inspection and} \\ \text{packaging time available}\end{pmatrix}$$

1/10 *S* + 1/4 *D* <= 135

Can Par produce a negative number of standard or deluxe bags? Clearly, the answer is no. Thus, to prevent the decision variables *S* and *D* from having negative values, two constraints.

***S* >=0 and *D* >= 0**

We succeeded in translating the objective and constraints of the problem into a set of mathematical relationships referred to as a **mathematical model.** The complete mathematical model for the Par problem is as follows:

$$\text{Max} \quad 10S + 9D$$

subject to (s.t.)

$$\begin{aligned}
\tfrac{7}{10}S + 1D &\leq 630 && \text{Cutting and dyeing} \\
\tfrac{1}{2}S + \tfrac{5}{6}D &\leq 600 && \text{Sewing} \\
1S + \tfrac{2}{3}D &\leq 708 && \text{Finishing} \\
\tfrac{1}{10}S + \tfrac{1}{4}D &\leq 135 && \text{Inspection and packaging}
\end{aligned}$$

$$S, D \geq 0$$