

A SIMPLE MINIMIZATION PROBLEM

M&D Chemicals produces two products that are sold as raw materials to companies manufacturing bath soaps and laundry detergents. Based on an analysis of current inventory levels and potential demand for the coming month, M&D's management specified that the combined production for products A and B must total at least 350 gallons. Separately, a major customer's order for 125 gallons of product A must also be satisfied. Product A requires 2 hours of processing time per gallon and product B requires 1 hour of processing time per gallon. For the coming month, 600 hours of processing time are available. M&D's objective is to satisfy these requirements at a minimum total production cost. Production costs are \$2 per gallon for product A and \$3 per gallon for product B.

To find the minimum-cost production schedule, we will formulate the M&D Chemicals problem as a linear program. Following a procedure similar to the one used for the Par, Inc., problem, we first define the decision variables and the objective function for the problem.

Let

A = number of gallons of product A

B = number of gallons of product B

With production costs at \$2 per gallon for product A and \$3 per gallon for product B, the objective function that corresponds to the minimization of the total production cost can be written as

$$\text{Min } Z = 2A + 3B$$

Next, consider the constraints placed on the M&D Chemicals problem. To satisfy the major customer's demand for 125 gallons of product A, we know A must be at least 125. Thus, we write the constraint.

$$A \geq 125,$$

For the combined production for both products, which must total at least 350 gallons, we can write the constraint as

$$A + B \geq 350,$$

Finally, for the limitation of 600 hours on available processing time, we add the constraint

$$2A + B \leq 600,$$

After adding the non-negativity constraints ($A, B \geq 0$), we arrive at the following linear program for the M&D Chemicals problem:

$$\text{Min } 2A + 3B$$

s.t.

$$1A \geq 125 \quad \text{Demand for product A}$$

$$1A + 1B \geq 350 \quad \text{Total production}$$

$$2A + 1B \leq 600 \quad \text{Processing time}$$

$$A, B \geq 0$$