Chapter One

What Is Operations Research?

1.1 Introduction

The first formal activities of Operations Research (OR) were initiated in England during World War II, when a team of British scientists set out to assess the best utilization of war materiel based on scientific principles rather than rules. After the war, the ideas advanced in military operations were adapted to improve efficiency and productivity in the civilian sector.

This chapter introduces the basic terminology of OR, including mathematical modelling, feasible solutions, optimization, and iterative algorithmic computations. It stresses that defining the problem correctly is the most important (and most difficult) phase of practicing OR.

1.2 Operations Research Models

Consider the following tickets purchasing problem. A businessperson has a 5-week commitment traveling between Fayetteville (FYV) and Denver (DEN). Weekly departure from Fayetteville occurs on Mondays for return on Wednesdays. A regular roundtrip ticket costs $400, but a 20% discount is granted if the roundtrip dates span a weekend. A one-way ticket in either direction costs 75% of the regular price. How should the tickets be bought for the 5-week period?

We can look at the situation as a decision-making problem whose solution requires answering three questions:
1. What are the decision **alternatives**?
2. Under what **restrictions** is the decision made?
3. What is an appropriate **objective criterion** for evaluating the alternatives?

Three plausible alternatives come to mind:

1. Buy five regular FYV-DEN-FYV for departure on Monday and return on Wednesday of the same week.
2. Buy one FYV-DEN, four DEN-FYV-DEN that span weekends, and one DEN-FYV.
3. Buy one FYV-DEN-FYV to cover Monday of the first week and Wednesday of the last week and four DEN-FYV-DEN to cover the remaining legs. All tickets in this alternative span at least one weekend.

The restriction on these options is that the businessperson should be able to leave FYV on Monday and return on Wednesday of the same week.
An obvious objective criterion for evaluating the proposed alternatives is the price of the tickets. The alternative that yields the smallest cost is the best. Specifically, we have:
Alternative 1 cost = 5 * $400 = $2000
Alternative 2 cost = (.75 * $400) + (4 * 0.8 * $400) + (.75 * $400) = $1880
Alternative 3 cost = 5 * (0.8 * $400) = $**1600**

Alternative 3 is the cheapest.

Consider the following **garden problem**: A home owner is in the process of starting a backyard vegetable garden. The garden must take on a rectangular shape to facilitate row irrigation. To keep critters out, the garden must be fenced. The owner has enough material to build a fence of length L = 100 ft. The goal is to fence the largest possible rectangular area. In contrast with the tickets example, where the number of alternatives is finite, the number of alternatives in the present example is infinite; that is, the *width* and *height* of the rectangle can each assume (theoretically) infinity of values between 0 and L. In this case, the width and the height are **continuous variables**. Because the variables of the problem are continuous, it is impossible to find the solution by exhaustive enumeration. However, we can *sense* the trend toward the best value of the garden area by fielding increasing values of width (and hence decreasing values of height). For example, for L = 100 ft, the combinations (width, height) = (10, 40), (20, 30), (25, 25), (30, 20), and (40, 10), respectively yield (area) = (400, 600, 625, 600, and 400), which demonstrates, but not proves, that the largest area occurs when width = height = $L > 4$ = 25 ft. Clearly, this is no way to compute the optimum, particularly for situations with several decision variables. For this reason, it is important to express the problem mathematically in terms of its unknowns, in which case the best solution is found by applying appropriate solution methods.

To demonstrate how the *garden problem* is expressed mathematically in terms of its two unknowns, width and height, define

w = width of the rectangle in feet

h = height of the rectangle in feet

Based on these definitions, the restrictions of the situation can be expressed verbally as

1. Width of rectangle + Height of rectangle = Half the length of the garden fence
2. Width and height cannot be negative
These restrictions are translated algebraically as
1. $2(w + h) = L$
2. $w \geq 0,\ h \geq 0.$

The only remaining component now is the objective of the problem; namely, maximization of the area of the rectangle. Let z be the area of the rectangle, then the complete model becomes
Maximize $z = wh$
subject to
$2(w + h) = L$
$w \geq 0,\ h \geq 0.$

Based on the preceding two examples, the general OR model can be organized in the following general format:

Maximize or minimize **Objective Function** subject to **Constraints**

1.3 Solving The OR Model

In practice, OR does not offer a single general technique for solving all mathematical models. Instead, the type and complexity of the mathematical model dictate the nature of the solution method. For example, in Section 1.2 the solution of the *tickets purchasing problem* requires simple ranking of alternatives based on the total purchasing price, whereas the solution of the *garden problem* utilizes differential calculus to determine the maximum area.

The most prominent OR technique is **linear programming**. It is designed for models with linear objective and constraint functions. Other techniques include **integer programming** (in which the variables assume integer values), **dynamic programming** (in which the original model can be decomposed into smaller more manageable sub-problems), **network programming** (in which the problem can be modeled as a network), and **nonlinear programming** (in which functions of the model are nonlinear). These are only a few among many available OR tools.

1.4 Phases Of An OR Study

OR studies are rooted in *teamwork*, where the OR analysts and the client work side by side. The OR analysts' expertise in modeling is complemented by the experience and cooperation of the client for whom the study is being carried out.

As a decision-making tool, OR is both a science and an art: It is a science by virtue of the mathematical techniques it embodies, and an art because the success of the phases leading to the solution of the mathematical model depends largely on the creativity and experience of the OR team.

The principal phases for implementing OR in practice include the following:

1. Definition of the problem.
2. Construction of the model.
3. Solution of the model.
4. Validation of the model.
5. Implementation of the solution.