

# Chapter Two

## Fundamental Parameters

The Gibbs Dalton law for a mixture of perfect gases states that the mixture pressure is equal to the sum of the partial pressure of the constituents

$$P = p_1 + p_2 + p_3 + \dots \quad (2.5)$$

For moist air

$$P = p_{N_2} + p_{O_2} + p_{CO_2} + p_A + p_v \quad (2.6)$$

Because the various constituents of the dry air may be considered to be one gas,

$$P_B = p_a + p_v \quad (2.7)$$

Where:

$P_B$  : Barometric pressure (Atmospheric pressure)

$P_a$  : dry air pressure

$P_v$  : water vapor pressure

### Example:

Saturated air at 26°C and atmospheric pressure 101325 N/m<sup>2</sup>. Find the partial pressure for each of the dry air and the water vapor?

### Solution:

From table A-2 at temperature 26°C, the saturated pressure of the water vapor is

$P_v = 3363 \text{ Pa (N/m}^2\text{)}$ , and from Eqn. (2.7)

$$P_B = p_a + p_v \quad \text{or} \quad p_a = P_B - p_v$$

$$= 101325 - 3363 = 97962 \text{ Pa} = 97.962 \text{ kPa}$$

### Vapor partial pressure in unsaturated air

These cases are the most common in nature, the vapor pressure can be calculated from empirical formula:

$$P_v = P_{wss} - P_B A (t_d - t_w) \quad (2.8)$$

Where:

$P_v$ : partial pressure of water vapor

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$P_{wss}$ : saturated pressure of vapor at wet bulb temperature ( $t_w$ )

$P_B$ : barometric pressure

$t_d$ : dry bulb temperature

$t_w$ : wet bulb temperature

A: constant

$$A = 6.66 \times 10^{-4} \quad \text{if } t_w \geq 0^\circ\text{C}$$

$$A = 5.94 \times 10^{-4} \quad \text{if } t_w < 0^\circ\text{C}$$

### **Example:**

Calculate the water vapor pressure in wet air under  $20^\circ\text{C}$  dry bulb and  $15^\circ\text{C}$  wet bulb temperature and  $P_B = 950 \text{ mbar}$ ?

### **Solution:**

$$P_v = P_{wss} - P_B A (t_d - t_w)$$

From steam table A-2 at  $t_w = 15^\circ\text{C}$

$$P_{wss} = 1.705 \text{ kPa}$$

$$P_v = 1.705 - 95 \times 6.66 \times 10^{-4} (20 - 15) = 1.388 \text{ kPa}$$

### **Moisture content (W)**

Sometimes called the specific humidity or humidity ratio, it is the ratio of the mass of water vapor ( $m_v$ ) to the mass of the dry air ( $m_a$ ) in the mixture:

$$W = m_v / m_a \dots\dots\dots (2.8)$$

To find the values of ( $m_v$ ) and ( $m_a$ ) using equation (2.3)

$$m_v = P_v V_v / R_v T_v, \quad m_a = P_a V_a / R_a T_a$$

but  $W = m_v / m_a$

and  $V_v = V_a, \quad T_v = T_a$

$$W = (P_v/R_v) / (P_a/R_a) = (P_v/P_a) \cdot (R_a/R_v) = 0.622 P_v / P_a = 0.622 P_v / (P_B - P_v) \dots\dots\dots (2.9)$$

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### Example:

Air at 20°C dry bulb and 15°C wet bulb and the barometric pressure is 95 kPa. Calculate the moisture content of the air and the density of the vapor.

### Solution:

From steam table A-2 at  $t_w = 15^\circ\text{C}$

$$P_{wss} = 1.705 \text{ kPa}$$

$$P_v = P_{wss} - P_B \cdot A \cdot (t_d - t_w) \\ = 1.705 - 95 \times 6.66 \times 10^{-4} (20 - 15) = 1.389 \text{ kPa}$$

$$W = 0.622 (P_v) / (P_B - P_v) = 0.622 (1.389) / (95 - 1.389) = 0.00923 \text{ kg/kg dry air}$$

$$\text{Density of the vapor, } P_v = \rho_v \cdot R_v \cdot T_v \rightarrow \rho_v = (1389) / (461)(20 + 273) = 0.01028 \text{ kg/m}^3$$

### Relative humidity ( $\Phi$ )

Is the ratio of the mole fraction of the water vapor ( $X_v$ ) in a mixture to the mole fraction of the water vapor in a saturated mixture ( $X_s$ ) at the same temperature and pressure.

$$\Phi = [X_v / X_s]_{t, p} \dots\dots\dots (2.10)$$

For a mixture of perfect gases, the mole fraction is equal to the partial pressure ratio of each constituent. The mole fraction of water vapor is:

$$X_v = P_v / P$$

$$\text{Then; } \Phi = (P_v / P) / (P_{ss} / P) = P_v / P_{ss} \dots\dots\dots (2.11)$$

Since the temperature of the dry air and the water vapor are assumed to be the same in the mixture;

$$\Phi = (P_v / R_v \cdot T) / (P_{ss} / R_v \cdot T) = [\rho_v / \rho_{ss}]_{t, p} \dots\dots\dots (2.12)$$

By using the perfect gas law we can derive a relation between the relative humidity ( $\Phi$ ) and the moisture content ( $W$ ). Combining Eqns. (2.9) and (2.11)

$$\Phi = (W \cdot P_B) / (0.622 + W) P_{ss} \dots\dots\dots (2.13)$$

### Example:

Air at 24°C and 40% RH, and  $P_B = 92 \text{ kPa}$ . Find the vapor density ( $\rho_v$ ) and vapor pressure ( $P_v$ ).

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### Solution:

From Table A-2 at  $t_d=24^\circ\text{C}$ ,  $v = 45.883 \text{ m}^3/\text{kg}$

$$\rho_{ss} = 1/v = 1/45.883 = 0.02179 \text{ kg/m}^3$$

$$\Phi = [\rho_v / \rho_{ss}]_{t, p} \rightarrow \rho_v = \Phi \cdot \rho_{ss} = 0.4 \times 0.02179 = 0.008716 \text{ kg/m}^3$$

$$P_v = \rho_v R_v T = 0.008716 \times 461 \times (24+273) = 1193.4 \text{ Pa}$$

Or by other way;

$$\Phi = [\rho_v / \rho_{ss}]_{t, p}, \text{ from Table A-2 at } t=24^\circ\text{C} \rightarrow P_{ss} = 2985 \text{ Pa}$$

$$P_v = 0.4 \times 2985 = 1194 \text{ Pa}$$

### Degree of saturation ( $\mu$ )

Is the ratio of the moisture content (W) to the moisture content of a saturated mixture ( $W_{ss}$ ) at the same temperature and pressure.

$$\mu = [W/W_{ss}]_{t, p} \dots\dots\dots (2.14)$$

$$W = m_s/m_a = (P_s/(P_B - P_s)) \cdot (R_a/R_s)$$

$$W_{ss} = (P_{ss}/(P_B - P_{ss})) \cdot (R_a/R_s)$$

$$\mu = (P_s/(P_B - P_s)) \cdot ((P_B - P_{ss})/P_{ss}) \cdot (R_a/R_s)(R_s/R_a) = (P_s/P_{ss}) \cdot (P_B - P_{ss})/(P_B - P_s)$$

$$= (P_s/P_{ss}) \cdot (1 - P_{ss}/P_B)/(1 - P_s/P_B) = \Phi [ (1 - P_{ss}/P_B)/(1 - \Phi \cdot P_{ss}/P_B) ] \dots\dots\dots (2.15)$$

### Example:

Moist air at  $40^\circ\text{C}$  DBT,  $30^\circ\text{C}$  WBT and 101 kPa barometric pressure, calculate for the air:

- a) Relative humidity ( $\Phi$ )
- b) Moisture content (W)
- c) Degree of saturation ( $\mu$ )

### Solution:

$$P_v = P_{wss} - P_B \cdot A \cdot (t_d - t_w), \text{ from Table A-2}$$

$$P_{wss} = 4.246 \text{ kPa at } 30^\circ\text{C}, P_{dss} = 7.384 \text{ kPa at } 40^\circ\text{C}$$

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$$P_v = 4.246 - 101 \times 6.66 \times 10^{-4} (40 - 30) = 3.57 \text{ kPa}$$

$$a) \Phi = P_v / P_{dss} \times 100\% = (3.57) / (7.384) \times 100\% = 48.4\%$$

$$b) W = 0.622 P_v / (P_B - P_v) = 0.622 (3.57) / (101 - 3.57) = 0.0228 \text{ kg/kg}_{d.a}$$

$$c) \mu = [W / W_{ss}]_{t,p}$$

$$W_{ss} = 0.622 (7.384) / (101 - 7.384) = 0.049 \text{ kg/kg}_{d.a}$$

$$\mu = 0.0228 / 0.049 = 0.465 = 46.5\%$$

Or:

$$\begin{aligned} \mu &= \Phi [ (1 - P_{ss}/P_B) / (1 - \Phi \cdot P_{ss}/P_B) ] = (0.484) [1 - (7.384/101)] / [1 - 0.484(7.384/101)] \\ &= 46.5\% \end{aligned}$$

### Dew point temperature ( $t_{dp}$ )

This is defined as the temperature of saturated air which has the same vapor pressure as the moist air under consideration. It may also be stated as a mixture is cooled at constant pressure, the temperature at which condensation first begins is the dew point.

### Example:

Find the dew point temperature for air at 20°C DBT and 15°C WBT and atmospheric pressure is 95 kPa.

### Solution:

From Table A-2 at  $t_w = 15^\circ\text{C}$ ,  $P_{wss} = 1.705 \text{ kPa}$

$$P_v = P_{wss} - P_B \cdot A \cdot (t_d - t_w)$$

$$= 1.705 - 95 \times 6.66 \times 10^{-4} (20 - 15) = 1.388 \text{ kPa}$$

From Table A-2, we find the saturation temperature that corresponds to  $P_v$

$$t_{sat} = t_{dp} = 11.84^\circ\text{C}$$

### Specific volume ( $v$ )

This is defined as the volume of one kg of moist air, it can be calculated by three ways:

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1. Used of dry air mass and partial pressure
2. Used of vapor mass and partial pressure
3. Used of mass of mixture and its total pressure.

### Example:

Calculate the specific volume ( $v$ ) of air at 35°C DBT and 25°C WBT at a barometric pressure of 95 kPa.

### Solution:

$$P_a V_a = m_a R_a T_a \rightarrow v_a = R_a T_a / P_a$$

$$\text{But; } P_a = P_B - P_v$$

$$P_{wss} = 3.169 \text{ kPa at } t = 25^\circ\text{C}$$

$$P_v = 3.169 - 95 \times 6.66 \times 10^{-4} (35 - 25) = 2.5363 \text{ kPa}$$

$$\text{Then } P_a = 95 - 2.5363 = 92.46 \text{ kPa}$$

$$v_a = (0.287)(35 + 273)/(92.46) = 0.956 \text{ m}^3/\text{kg}$$

Or by other way;

$$W = 0.622 (P_v)/(P_B - P_v) = 0.622(2.5363)/(95 - 2.5363) = 0.01706 \text{ kg/kg}_{d.a}$$

So for 1 kg of dry air there is 0.01706 kg of steam ;  $m_s = 0.01706 \text{ kg}$

$$v_v = (0.01706)(0.461)(35 + 273)/(2.5363) = 0.956 \text{ m}^3/\text{kg}$$

### Enthalpy (h)

If a heat exchange occurs at constant pressure, as well as a change in internal energy taking place, work may be done.

This leads to a definition of enthalpy, H:

$$H = U + pV$$

The equation is strictly true for a pure gas of mass  $m$ , pressure  $p$ , and volume  $V$ . However it may be applied without appreciable error to the mixtures of gases associated with air conditioning.

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The enthalpy,  $h$ , used in psychrometry is the specific enthalpy of moist air, expressed in kJ/kg dry air, defined by the equation:

$$h = h_a + W h_v \dots\dots\dots (2.16)$$

where ;  $h_a$  is the enthalpy of dry air,  $h_v$  is the enthalpy of water vapor, both expressed in kJ kg<sup>-1</sup>, and  $W$  is the moisture content.

An approximate equation for the enthalpy of dry air over the range 0°C to 60°C is, however  $h_a = 1.007 t_d - 0.026 \dots\dots\dots (2.17)$

and for lower temperatures, down to – 10°C, the approximate equation is

$$h_a = 1.005 t_d \dots\dots\dots (2.18)$$

For purposes of approximate calculation, we may assume that, in the range 0°C to 60°C, the vapor is generated from liquid water at 0°C and that the specific heat of superheated steam is a constant. The following equation can then be used for the enthalpy of water vapor:

$$h_v = 2501 + 1.84 t_d \dots\dots\dots (2.19)$$

Equations (2.17) and (2.19) can now be combined, as typified by equation (2.16), to give an approximate expression for the enthalpy of humid air at a barometric pressure of 101.325 kPa:

$$h = (1.007 t_d - 0.026) + W(2501 + 1.84 t_d) \dots\dots\dots (2.20)$$

### **Example:**

Compute the enthalpy of saturated air at 15°C and standard atmospheric pressure.

### **Solution:**

Because the air is saturated,  $W = W_s$

$$W_s = 0.622(P_{vs})/(P_B - P_{vs}) \quad \text{from Table A-2 at } t=15^\circ\text{C, } P_{vs} = 1705 \text{ Pa}$$

$$W_s = 0.622(1705)/(101325 - 1705) = 0.01065 \text{ kg/kg}_{d.a}$$

$$h = (1.007 t_d - 0.026) + W(2501 + 1.84 t_d)$$

$$= (1.007 \times 15 - 0.026) + 0.01065(2501 + 1.84 \times 15) = 42 \text{ kJ/kg}$$

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### Example:

Compute the enthalpy of moist air at 20°C DBT and 15°C WBT and standard atmospheric pressure.

### Solution:

$$P_v = P_{wss} - P_B \cdot A \cdot (t_d - t_w)$$

$$= 1.705 - 101.325 \times 6.66 \times 10^{-4} (20 - 15) = 1.3676 \text{ kPa}$$

$$W_s = 0.622(P_{vs})/(P_B - P_{vs})$$

$$= 0.622(1.3676)/(101.325 - 1.3676) = 0.00843 \text{ kg/kg}_{d.a}$$

$$h = (1.007 \times 20 - 0.026) + 0.00843(2501 + 1.84 \times 20) = 41.508 \text{ kJ/kg}$$

### Example:

Moist air at 42°C DBT, 26°C WBT and 100 kPa barometric pressure. Calculate:

- a) vapor pressure    b) relative humidity    c) moisture content    d) specific enthalpy  
e) specific volume    f) dew point temperature    g) degree of saturation

### Solution:

From Table A-2

$$P_{wss} = 3.363 \text{ kPa} \quad \text{at } t_w = 26^\circ\text{C}$$

$$P_{dss} = 8.268 \text{ kPa} \quad \text{at } t_d = 42^\circ\text{C}$$

$$\text{a) } P_v = 3.363 - 100 \times 6.66 \times 10^{-4} (42 - 26) = 2.2974 \text{ kPa}$$

$$\text{b) } \Phi = P_v/P_{dss} = 2.2974/8.268 = 0.2779 = 27.79\%$$

$$\text{c) } W = 0.622 (2.2974)/(100 - 2.2974) = 0.0146 \text{ kg/kg}_{d.a}$$

$$\begin{aligned} \text{d) } h &= (1.007 t_d - 0.026) + W(2501 + 1.84 t_d) \\ &= (1.007 \times 42 - 0.026) + 0.0146(2501 + 1.84 \times 42) = 79.91 \text{ kJ/kg} \end{aligned}$$

$$\text{e) } v_a = R_a T_a / P_a = (0.287)(42+273)/(100 - 2.2974) = 0.9253 \text{ m}^3/\text{kg}$$

f) the dew point temperature is the temperature corresponds to  $P_v$

$$t_{dp} = 19.7^\circ\text{C}$$



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$$g) \quad \mu = \Phi [ (1 - P_{ss}/P_B) / (1 - \Phi \cdot P_{ss}/P_B) ] = 0.2779 [ (1 - 8.268/100) / (1 - 0.2779 \times 8.268/100) ] \\ = 0.26092$$

### Example:

In a check on an air-conditioning system, a room was maintained at 22°C DBT and 53% RH, by air supply at 1.6 m<sup>3</sup>/s with 10°C DBT and 9°C WBT. The barometric pressure was 920 mbar. Find:

a) sensible heat gain    b) latent heat gain    c) total heat gain

### Solution:

From Table A-2 at  $t_w = 9^\circ\text{C}$ ,  $P_{wss} = 1.15 \text{ kPa}$

$$P_v = 1.15 - 92 \times 6.66 \times 10^{-4} (10 - 9) = 1.0887 \text{ kPa}$$

$$P_a = 92 - 1.0887 = 90.91 \text{ kPa}$$

$$m_a = (P_a V_a) / (R_a T_a) = (90.91 \times 1.6) / (0.287 \times 295) = 1.718 \text{ kg/s}$$

$$h_{a1} = 1.007 \times 22 - 0.026 = 22.128 \text{ kJ/kg}$$

$$h_{a2} = 1.007 \times 10 - 0.026 = 10.044 \text{ kJ/kg}$$

$$Q_s = m_a (h_{a1} - h_{a2}) = 1.718 (22.128 - 10.044) = 20.7 \text{ kW}$$

Also, from Table A-2 at  $t = 22^\circ\text{C}$   $P_{dss} = 2.645 \text{ kPa}$

$$\Phi = P_v / P_{dss} \rightarrow P_v = 0.53 \times 2.645 = 1.4 \text{ kPa}$$

$$W_1 = 0.622 (1.4) / (92 - 1.4) = 0.00961 \text{ kg/kg}_{d.a}$$

$$W_2 = 0.622 (1.0887) / (92 - 1.0887) = 0.00745 \text{ kg/kg}_{d.a}$$

$$h_{v1} = 2501 + 1.84 \times 22 = 2541.48 \text{ kJ/kg}$$

$$h_{v2} = 2501 + 1.84 \times 10 = 2519.4 \text{ kJ/kg}$$

$$Q_L = m_a (W_1 \cdot h_{v1} - W_2 \cdot h_{v2}) = 1.718 (0.00961 \times 2541.48 - 0.00745 \times 2519.4) = 9.714 \text{ kW}$$

$$Q_{\text{Tot}} = Q_s + Q_L = 20.7 + 9.714 = 30.414 \text{ kW}$$

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### Adiabatic saturation and thermodynamic wet bulb temperature

Adiabatic saturation temperature is defined as that temperature at which water, by evaporating into air, can bring the air to saturation at the same temperature adiabatically. An adiabatic saturator is a device using which one can measure theoretically the adiabatic saturation temperature of air.

As shown in Fig.2.1 an adiabatic saturator is a device in which air flows through an infinitely long duct containing water. As the air comes in contact with water in the duct, there will be heat and mass transfer between water and air. If the duct is infinitely long, then at the exit, there would exist perfect equilibrium between air and water at steady state. Air at the exit would be fully saturated and its temperature is equal to that of water temperature. The device is adiabatic as the walls of the chamber are thermally insulated. In order to continue the process, make-up water has to be provided to compensate for the amount of water evaporated into the air. The temperature of the make-up water is controlled so that it is the same as that in the duct.

After the adiabatic saturator has achieved a steady-state condition, the temperature indicated by the thermometer immersed in the water is the thermodynamic wet-bulb temperature. The thermodynamic wet bulb temperature will be less than the entering air DBT but greater than the dew point temperature.

Certain combinations of air conditions will result in a given sump temperature, and this can be defined by writing the energy balance equation for the adiabatic saturator. Based on a unit mass flow rate of dry air, this is given by:

$$h_1 = h_2 - (W_2 - W_1)h_f \quad \dots\dots\dots (2.21)$$

where  $h_f$  is the enthalpy of saturated liquid at the sump or thermodynamic wet-bulb temperature,  $h_1$  and  $h_2$  are the enthalpies of air at the inlet and exit of the adiabatic saturator, and  $W_1$  and  $W_2$  are the humidity ratio of air at the inlet and exit of the adiabatic saturator, respectively.

It is to be observed that the thermodynamic wet-bulb temperature is a thermodynamic property, and is independent of the path taken by air. Assuming the humid specific heat to be constant, from the enthalpy balance, the thermodynamic wet-bulb temperature can be written as:

$$t_2 = t_1 - \frac{h_{fg,2}}{c_{pm}} (w_2 - w_1) \quad \dots\dots\dots (2.22)$$