Chapter Two

Operational Amplifiers (Op Amp)

2.1- Introduction to Op. Amp.

- An op amp is an active circuit element designed to perform mathematical operations of addition, subtraction, multiplication, division, differentiation, and integration.

- A typical Op amp is the eight-pin dual in-line package (or DIP), shown in Fig. 2.1(a). Pin or terminal 8 is unused, and terminals 1 and 5 are of little concern to us. The five important terminals are:
  1- The inverting input, pin 2.
  2- The noninverting input, pin 3.
  3- The output, pin 6.
  4- The positive power supply V^{+}, pin 7.
  5- The negative power supply V^{-}, pin 4.

- The circuit symbol for the op amp is the triangle in Fig. 2.1(b); as shown, the op amp has two inputs and one output. The inputs are marked with minus (-) and plus (+) to specify inverting and noninverting inputs, respectively. An input applied to the noninverting terminal will appear with the same polarity at the output, while an input applied to the inverting terminal will appear inverted at the output.

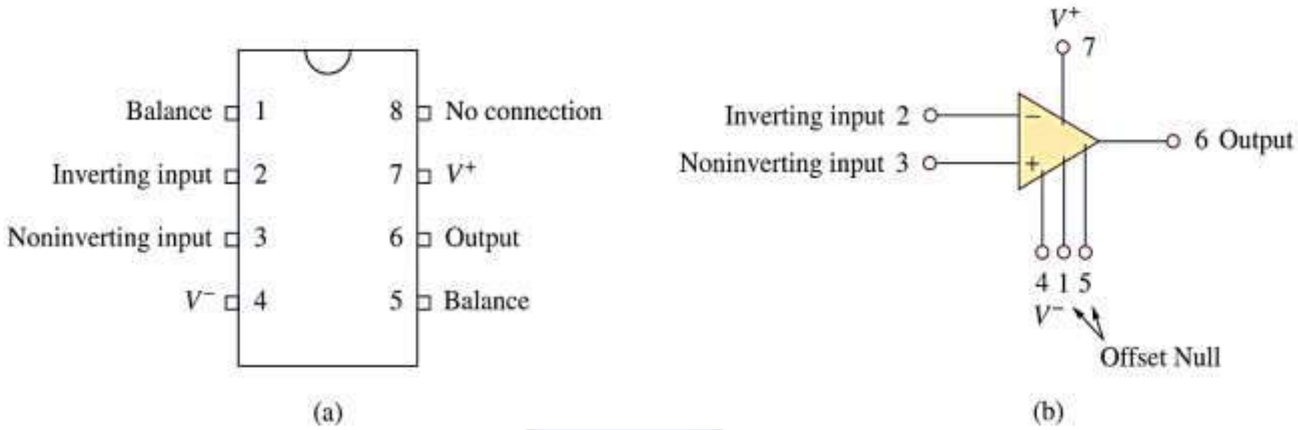


Figure 2.1: A typical op amp: (a) pin configuration, (b) circuit symbol

- As an active element, the op amp must be powered by a voltage supply as typically shown in Fig. 2.2. The power supply currents must not be overlooked. By KCL:

$$i_o = i_1 + i_2 + i_+ + i_- \tag{2.1}$$

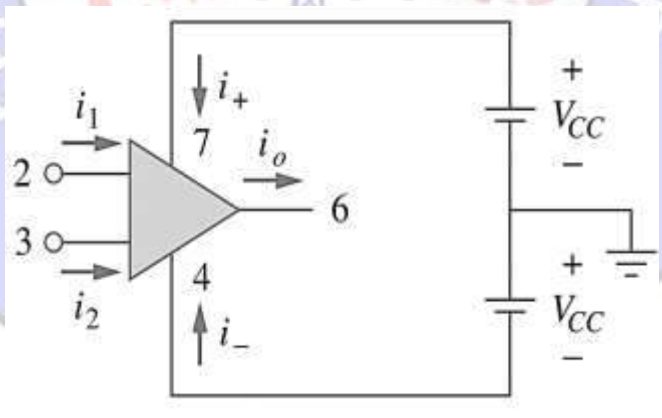


Figure 2.2: Powering the op amp

- The equivalent circuit model of an op amp is shown in Fig. 2.3.
- The output section consists of a voltage-controlled source in series with the output resistance R_o.
- It is evident from Fig. 2.3 that the input resistance R_i is the Thevenin equivalent resistance seen at the input terminals, while the output resistance R_o is the Thevenin equivalent resistance seen at the output.
- The differential input voltage V_d is given by:

$$v_d = v_2 - v_1 \quad (2.2)$$

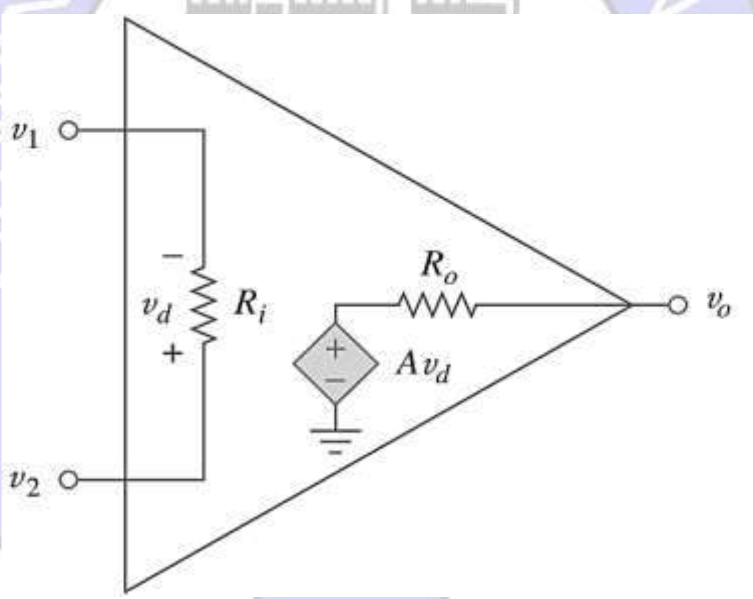


Figure 2.3: The equivalent circuit of the nonideal op amp.

- Where v_1 is the voltage between the inverting terminal and ground, and v_2 is the voltage between the noninverting terminal and ground.

- The op amp senses the difference between the two inputs, multiplies it by the gain A, and causes the resulting voltage to appear at the output. Thus, the output v_o is given by

$$v_o = Av_d = A(v_2 - v_1) \tag{2.3}$$

- A is called the open-loop voltage gain because it is the gain of the op amp without any external feedback from output to input. Table 2.1 shows typical values of voltage gain A, input resistance R_i, output resistance R_o, and supply voltage V_{cc}.

Table 2.1
Typical ranges for op amp parameters.

Parameter	Typical range	Ideal values
Open-loop gain, A	10^5 to 10^8	∞
Input resistance, R_i	10^5 to 10^{13} Ω	∞ Ω
Output resistance, R_o	10 to 100 Ω	0 Ω
Supply voltage, V_{CC}	5 to 24 V	

- A practical limitation of the op amp is that the magnitude of its output voltage cannot exceed $|V_{cc}|$.
- In other words, the output voltage is dependent on and is limited by the power supply voltage.
- Figure 2.4 illustrates that the op amp can operate in three modes, depending on the differential input voltage v_d:

1- Positive saturation, $v_o = V_{cc}$.

2- Linear region, $-V_{cc} \leq v_o = Av_d \leq V_{cc}$.

3- Negative saturation, $v_o = -V_{cc}$

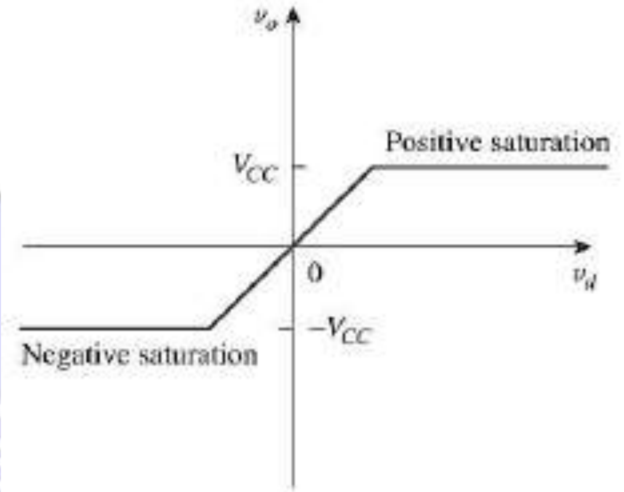
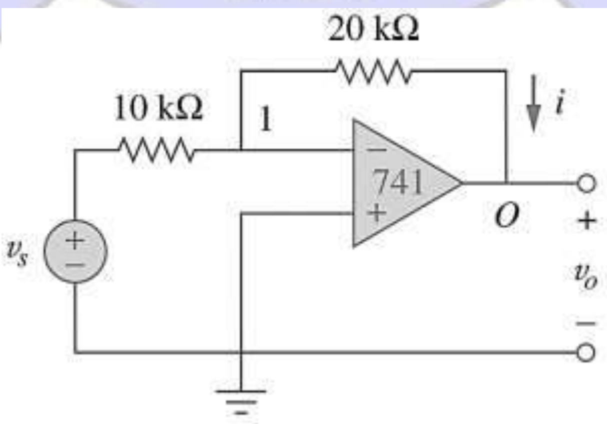


Figure 2.4: Op amp output voltage v_o as a function of the differential input voltage v_d

Example 5.1:

A 741 op amp has an open-loop voltage gain of 2×10^5, input resistance of 2 MΩ, and output resistance of 50 Ω. The op amp is used in the circuit of Fig. 5.6(a). Find the closed-loop gain v_o/v_s. Determine current i when $v_s = 2$ V.



(a)

Solution:

Using the op amp model in Fig. 5.4, we obtain the equivalent circuit of Fig. 5.6(a) as shown in Fig. 5.6(b). We now solve the circuit in Fig. 5.6(b) by using nodal analysis. At node 1, KCL gives

$$\frac{v_s - v_1}{10 \times 10^3} = \frac{v_1}{2000 \times 10^3} + \frac{v_1 - v_o}{20 \times 10^3}$$

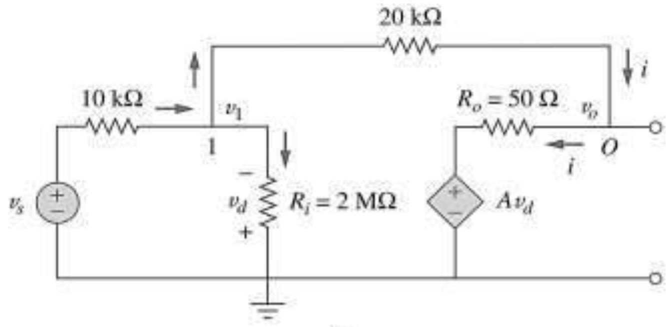
Multiplying through by 2000×10^3, we obtain

$$200v_s = 301v_1 - 100v_o$$

or

$$2v_s \simeq 3v_1 - v_o \quad \Rightarrow \quad v_1 = \frac{2v_s + v_o}{3} \tag{5.1.1}$$

At node O,



(b)

Figure 5.6
For Example 5.1: (a) original circuit, (b) the equivalent circuit

$$\frac{v_1 - v_o}{20 \times 10^3} = \frac{v_o - Av_d}{50}$$

But $v_d = -v_1$ and $A = 200{,}000$. Then

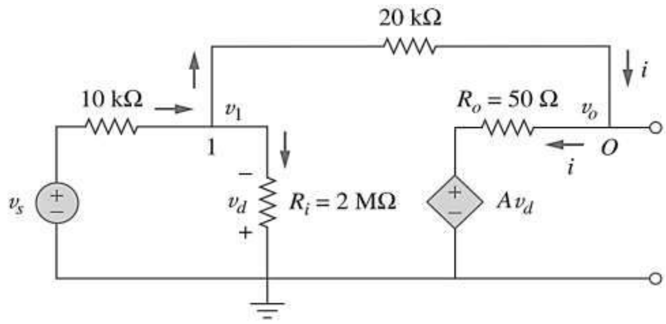
$$v_1 - v_o = 400(v_o + 200{,}000v_1) \tag{5.1.2}$$

Substituting v_1 from Eq. (5.1.1) into Eq. (5.1.2) gives

$$0 \simeq 26{,}667{,}067v_o + 53{,}333{,}333v_s \quad \Rightarrow \quad \frac{v_o}{v_s} = -1.9999699$$

This is closed-loop gain, because the 20-kΩ feedback resistor closes the loop between the output and input terminals. When $v_s = 2$ V, $v_o = -3.9999398$ V. From Eq. (5.1.1), we obtain $v_1 = 20.066667\ \mu$V. Thus,

$$i = \frac{v_1 - v_o}{20 \times 10^3} = 0.19999 \text{ mA}$$



(b)

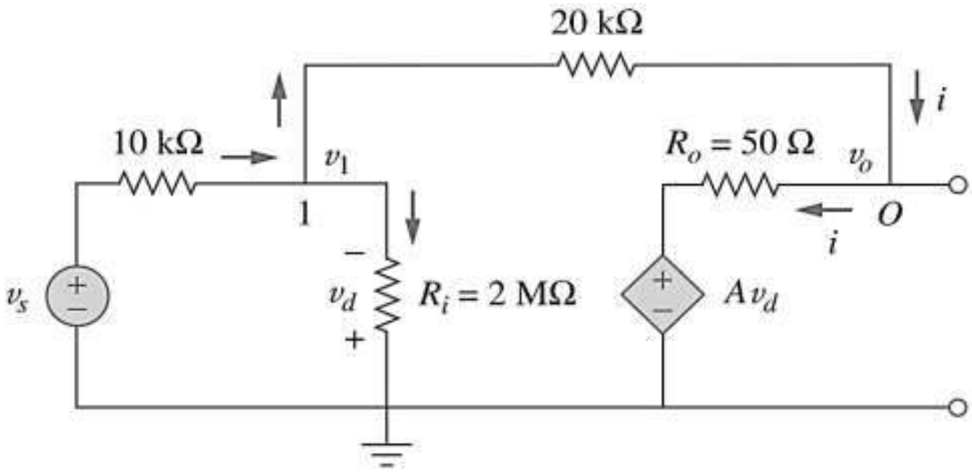


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Figure 5.6
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Multiplying through by 2000×10^3, we obtain

$$200v_s = 301v_1 - 100v_o$$

or

$$2v_s \simeq 3v_1 - v_o \quad \Rightarrow \quad v_1 = \frac{2v_s + v_o}{3} \qquad \textbf{(5.1.1)}$$

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2.2- Ideal Op. Amp.

- An ideal op amp is an amplifier with infinite open-loop gain, infinite input resistance, and zero output resistance.

  1- Infinite open-loop gain, $A = \infty$

  2- Infinite input resistance, $R_i = \infty$

  3- Zero output resistance, $R_o = 0$

- For circuit analysis, the ideal op amp is illustrated in Fig. 5.8.

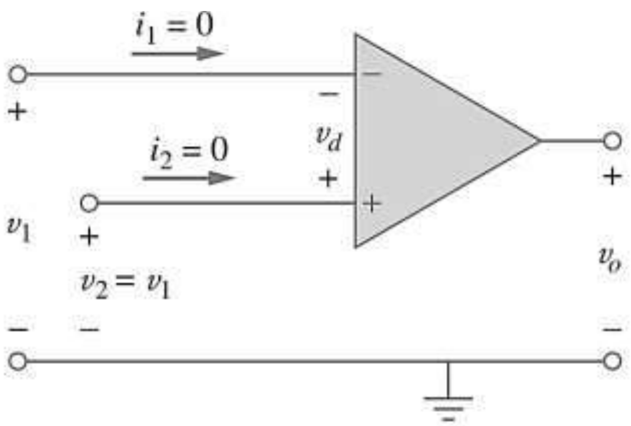


Figure 5.8
Ideal op amp model.

- Two important characteristics of the ideal op amp are:

1- The currents into both input terminals are zero:

$$i_1 = 0, \qquad i_2 = 0 \tag{5.5}$$

2- The voltage across the input terminals is equal to zero; i.e.,

$$v_d = v_2 - v_1 = 0 \tag{5.6}$$

or

$$v_1 = v_2 \tag{5.7}$$

Example 5.2:

Rework Practice Prob. 5.1 using the ideal op amp model.

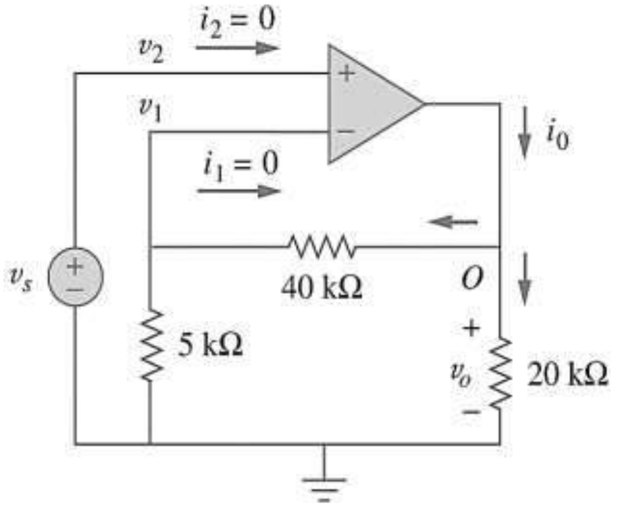
If the same 741 op amp in Example 5.1 is used in the circuit of Fig. 5.7, calculate the closed-loop gain v_o/v_s. Find i_o when $v_s = 1$ V.

Solution:
We may replace the op amp in Fig. 5.7 by its equivalent model in Fig. 5.9 as we did in Example 5.1. But we do not really need to do this. We just need to keep Eqs. (5.5) and (5.7) in mind as we analyze the circuit in Fig. 5.7. Thus, the Fig. 5.7 circuit is presented as in Fig. 5.9. Notice that

$$v_2 = v_s \tag{5.2.1}$$

Since $i_1 = 0$, the 40-kΩ and 5-kΩ resistors are in series; the same current flows through them. v_1 is the voltage across the 5-kΩ resistor. Hence, using the voltage division principle,

$$v_1 = \frac{5}{5+40}v_o = \frac{v_o}{9} \tag{5.2.2}$$



According to Eq. (5.7),

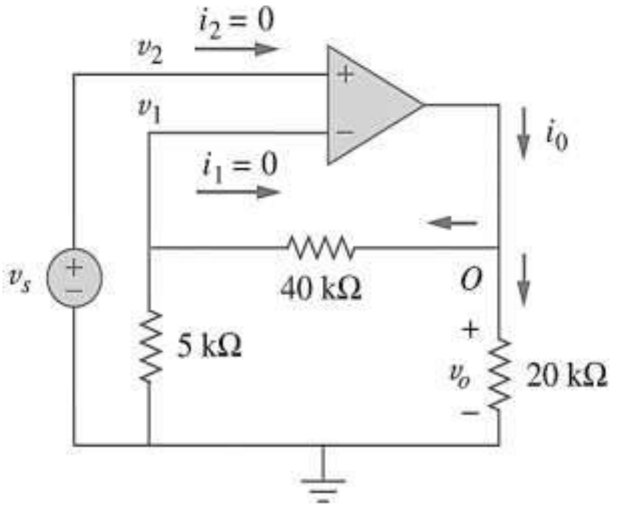
$$v_2 = v_1 \tag{5.2.3}$$

Substituting Eqs. (5.2.1) and (5.2.2) into Eq. (5.2.3) yields the closed-loop gain,

$$v_s = \frac{v_o}{9} \quad \Rightarrow \quad \frac{v_o}{v_s} = 9 \tag{5.2.4}$$

which is very close to the value of 9.00041 obtained with the nonideal model in Practice Prob. 5.1. This shows that negligibly small error results from assuming ideal op amp characteristics.

At node O,

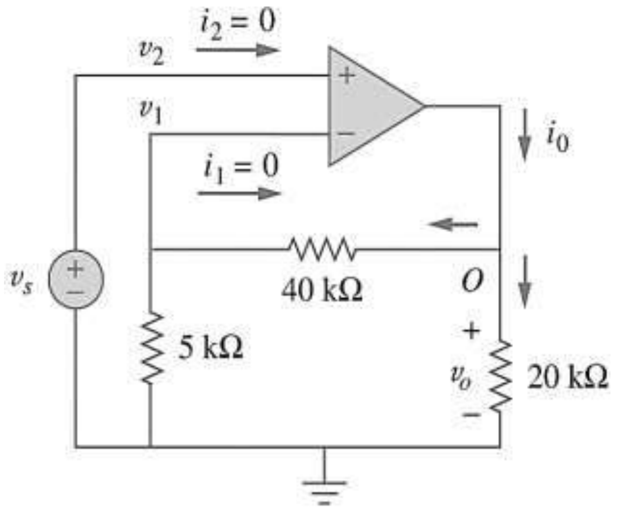


$$i_o = \frac{v_o}{40+5} + \frac{v_o}{20}\text{mA} \tag{5.2.5}$$

From Eq. (5.2.4), when $v_s = 1$ V, $v_o = 9$ V. Substituting for $v_o = 9$ V in Eq. (5.2.5) produces

$$i_o = 0.2 + 0.45 = 0.65 \text{ mA}$$

This, again, is close to the value of 0.657 mA obtained in Practice Prob. 5.1 with the nonideal model.



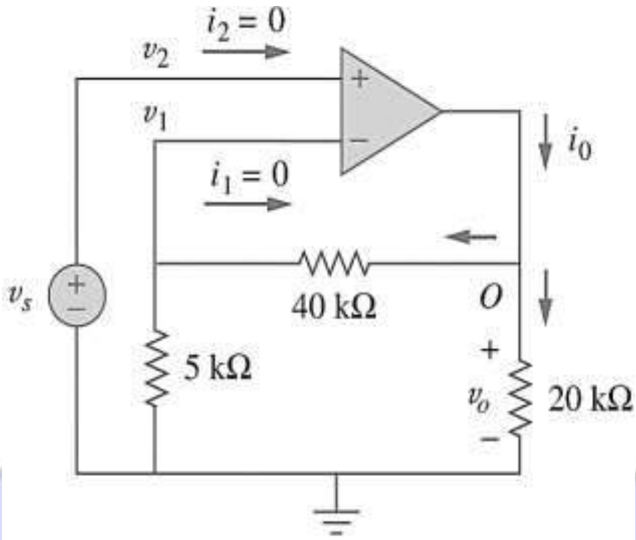


Figure 5.9
For Example 5.2.

Solution:

We may replace the op amp in Fig. 5.7 by its equivalent model in Fig. 5.9 as we did in Example 5.1. But we do not really need to do this. We just need to keep Eqs. (5.5) and (5.7) in mind as we analyze the circuit in Fig. 5.7. Thus, the Fig. 5.7 circuit is presented as in Fig. 5.9. Notice that

$$v_2 = v_s \tag{5.2.1}$$

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$$v_1 = \frac{5}{5 + 40} v_o = \frac{v_o}{9} \tag{5.2.2}$$

According to Eq. (5.7),

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At node O,

$$i_o = \frac{v_o}{40 + 5} + \frac{v_o}{20} \text{mA} \tag{5.2.5}$$

From Eq. (5.2.4), when $v_s = 1$ V, $v_o = 9$ V. Substituting for $v_o = 9$ V in Eq. (5.2.5) produces

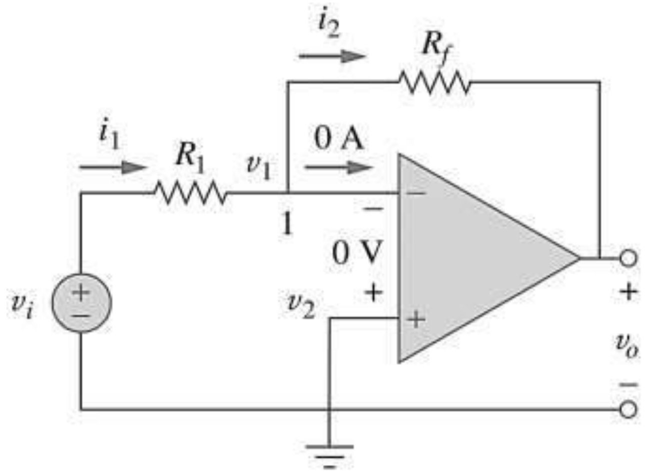
$$i_o = 0.2 + 0.45 = 0.65 \text{ mA}$$

This, again, is close to the value of 0.657 mA obtained in Practice Prob. 5.1 with the nonideal model.

2.3- Inverting Op. Amp.

- The first of op amp circuits is the inverting amplifier shown in Fig. 5.10.

- In this circuit, the noninverting input is grounded, v_i is connected to the inverting input through R_1, and the feedback resistor R_f is connected between the inverting input and output.
- Our goal is to obtain the relationship between the input voltage v_i and the output voltage v_o.



- Applying KCL at node 1,

$$i_1 = i_2 \quad \Rightarrow \quad \frac{v_i - v_1}{R_1} = \frac{v_1 - v_o}{R_f} \tag{5.8}$$

But $v_1 = v_2 = 0$ for an ideal op amp, since the noninverting terminal is grounded. Hence,

$$\frac{v_i}{R_1} = -\frac{v_o}{R_f}$$

or

$$\boxed{v_o = -\frac{R_f}{R_1} v_i} \tag{5.9}$$

The voltage gain is $A_v = v_o/v_i = -R_f/R_1$. The designation of the circuit in Fig. 5.10 as an *inverter* arises from the negative sign.

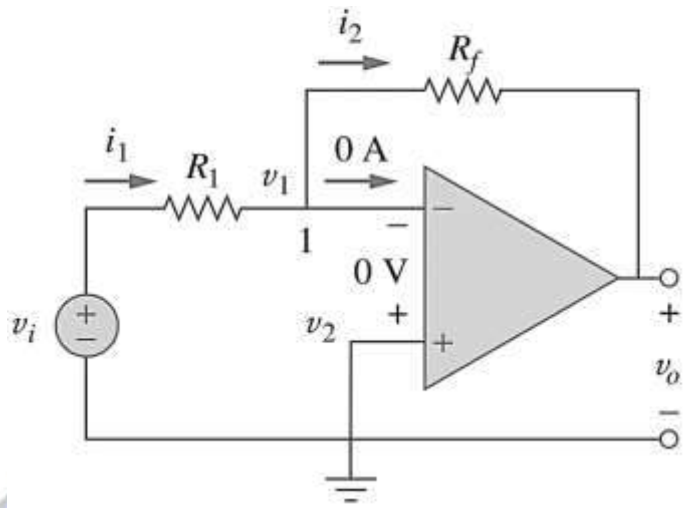


Figure 5.10
The inverting amplifier.

- Applying KCL at node 1,

$$i_1 = i_2 \quad \Rightarrow \quad \frac{v_i - v_1}{R_1} = \frac{v_1 - v_o}{R_f} \tag{5.8}$$

But $v_1 = v_2 = 0$ for an ideal op amp, since the noninverting terminal is grounded. Hence,

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or

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The voltage gain is $A_v = v_o/v_i = -R_f/R_1$. The designation of the circuit in Fig. 5.10 as an *inverter* arises from the negative sign.

Example 5.4:

Determine v_o in the op amp circuit shown in Fig. 5.14.

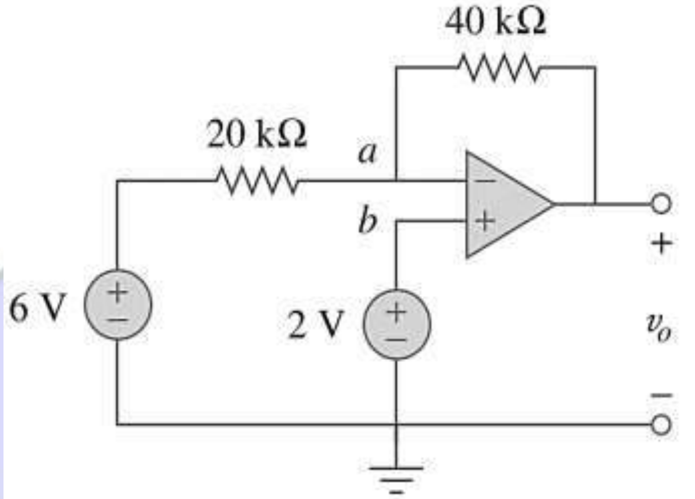

Figure 5.14
For Example 5.4.

Solution:
Applying KCL at node a,

$$\frac{v_a - v_o}{40\text{ k}\Omega} = \frac{6 - v_a}{20\text{ k}\Omega}$$

$$v_a - v_o = 12 - 2v_a \quad \Rightarrow \quad v_o = 3v_a - 12$$

But $v_a = v_b = 2$ V for an ideal op amp, because of the zero voltage drop across the input terminals of the op amp. Hence,

$$v_o = 6 - 12 = -6\text{ V}$$

Notice that if $v_b = 0 = v_a$, then $v_o = -12$, as expected from Eq. (5.9).