1.4.2- Thevenin's Theorem:

- Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th} , where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.
- Find the Thevenin equivalent circuit of the circuit shown in Fig. 4.27, to the left of the terminals a-b. Then find the current through $R_L = 6, 16, and\ 36\ \Omega$.

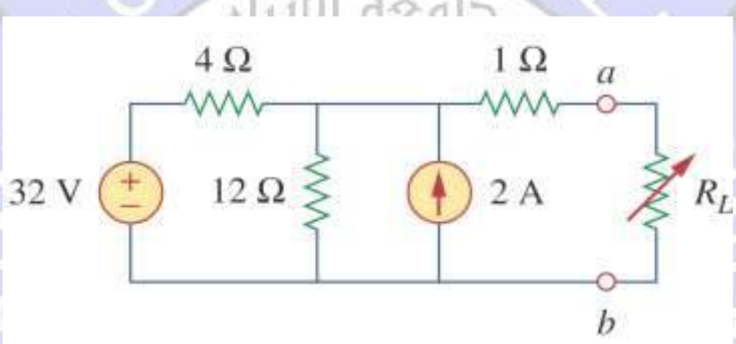


Fig. 4.27: Example 9

Solution:

We find R_{Th} by turning off the 32-V voltage source (replacing it with a short circuit) and the 2-A current source (replacing it with an open circuit). The circuit becomes what is shown in Fig. 4.28(a). Thus,

$$R_{Th} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\ \Omega$$

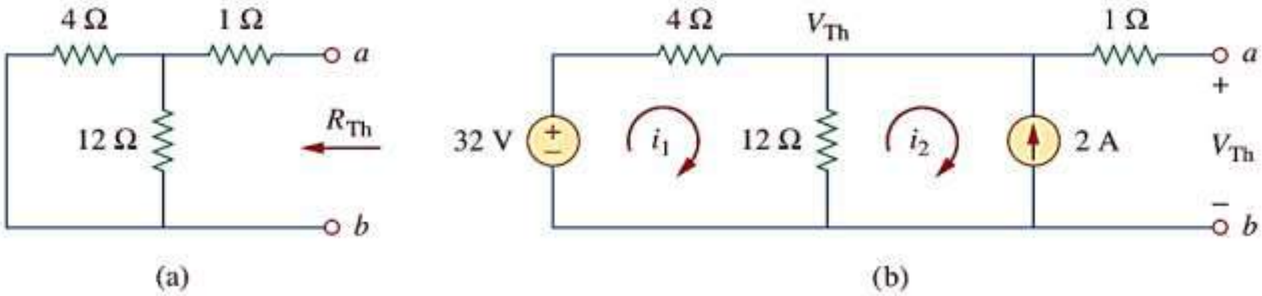


Figure 4.28
For Example 4.8: (a) finding R_{Th}, (b) finding V_{Th}.

To find V_{Th}, consider the circuit in Fig. 4.28(b). Applying mesh analysis to the two loops, we obtain

$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \qquad i_2 = -2 \text{ A}$$

Solving for i_1, we get $i_1 = 0.5$ A. Thus,

$$V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30 \text{ V}$$

Alternatively, it is even easier to use nodal analysis. We ignore the 1-Ω resistor since no current flows through it. At the top node, KCL gives

$$\frac{32 - V_{Th}}{4} + 2 = \frac{V_{Th}}{12}$$

or

$$96 - 3V_{Th} + 24 = V_{Th} \quad \Rightarrow \quad V_{Th} = 30 \text{ V}$$

as obtained before. We could also use source transformation to find V_{Th}.

The Thevenin equivalent circuit is shown in Fig. 4.29. The current through R_L is

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L}$$

When $R_L = 6$,

$$I_L = \frac{30}{10} = 3 \text{ A}$$

When $R_L = 16$,

$$I_L = \frac{30}{20} = 1.5 \text{ A}$$

When $R_L = 36$,

$$I_L = \frac{30}{40} = 0.75 \text{ A}$$

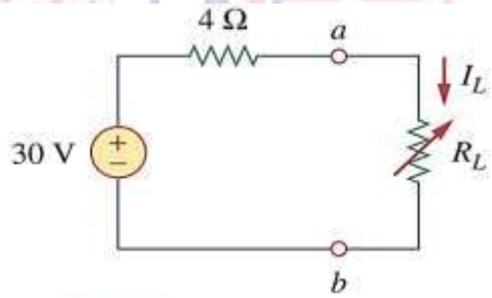


Figure 4.29
The Thevenin equivalent circuit for Example 4.8.

1.4.3- Norton's Theorem:

- Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

$$R_N = R_{Th} \quad (4.9)$$

$$I_N = i_{sc} \quad (4.10)$$

$$I_N = \frac{V_{Th}}{R_{Th}} \quad (4.11)$$

- Find the Norton equivalent circuit of the circuit in Fig. 4.39 at terminals a-b.

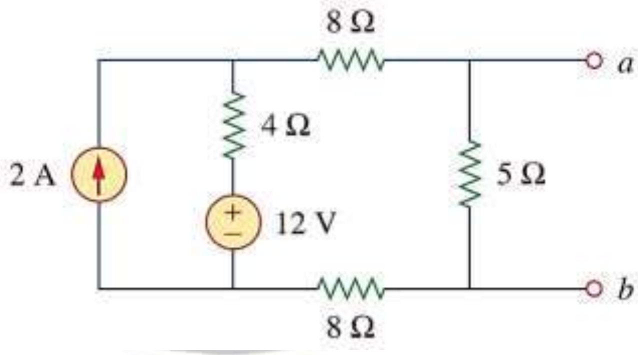


Fig. 4.39: Example 10

Solution:

We find R_N in the same way we find R_{Th} in the Thevenin equivalent circuit. Set the independent sources equal to zero. This leads to the circuit in Fig. 4.40(a), from which we find R_N. Thus,

$$R_N = 5 \parallel (8 + 4 + 8) = 5 \parallel 20 = \frac{20 \times 5}{25} = 4\ \Omega$$

To find I_N, we short-circuit terminals a and b, as shown in Fig. 4.40(b). We ignore the 5-Ω resistor because it has been short-circuited. Applying mesh analysis, we obtain

$$i_1 = 2\text{ A}, \qquad 20i_2 - 4i_1 - 12 = 0$$

From these equations, we obtain

$$i_2 = 1\text{ A} = i_{sc} = I_N$$

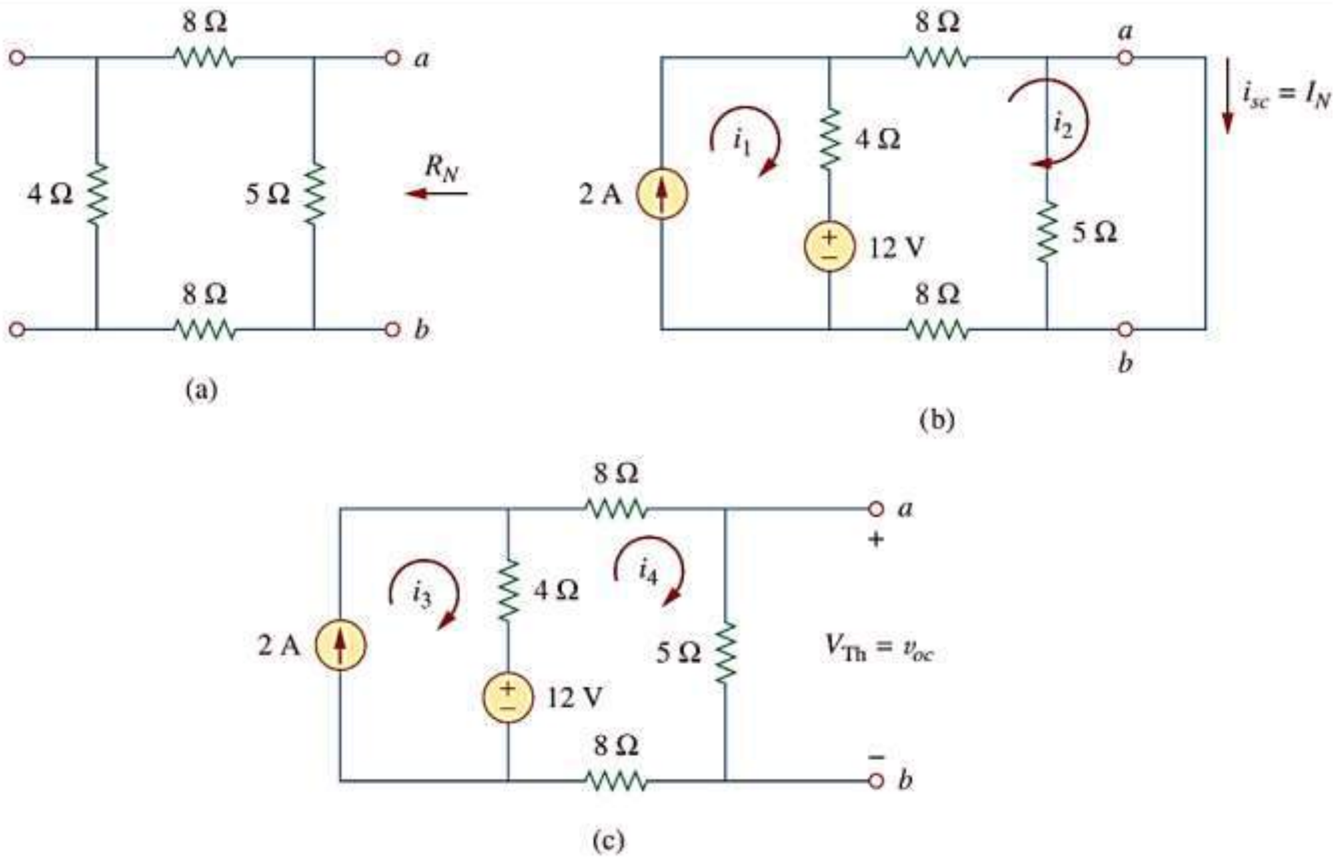


Figure 4.40
For Example 4.11; finding: (a) R_N, (b) $I_N = i_{sc}$, (c) $V_{Th} = v_{oc}$.

Alternatively, we may determine I_N from V_{Th}/R_{Th}. We obtain V_{Th} as the open-circuit voltage across terminals a and b in Fig. 4.40(c). Using mesh analysis, we obtain

$$i_3 = 2 \text{ A}$$

$$25i_4 - 4i_3 - 12 = 0 \quad \Rightarrow \quad i_4 = 0.8 \text{ A}$$

and

$$v_{oc} = V_{Th} = 5i_4 = 4 \text{ V}$$

Hence,

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{4}{4} = 1 \text{ A}$$

as obtained previously. This also serves to confirm Eq. (4.12c) that $R_{Th} = v_{oc}/i_{sc} = 4/1 = 4\ \Omega$. Thus, the Norton equivalent circuit is as shown in Fig. 4.41.

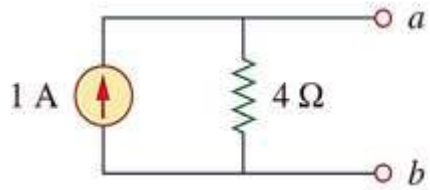
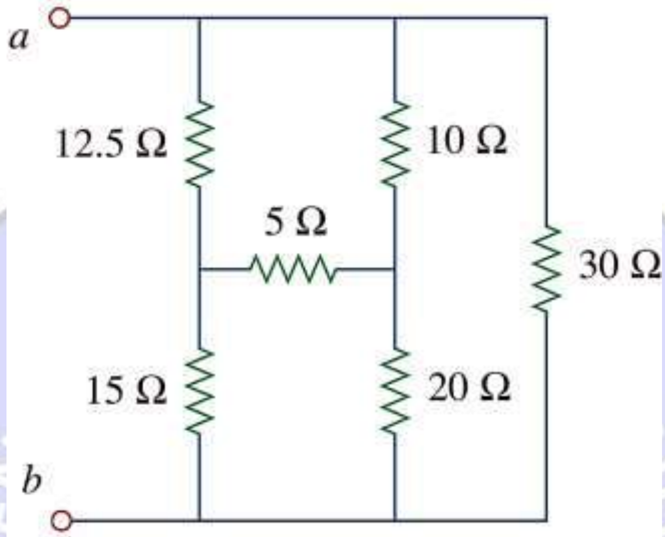


Figure 4.41
Norton equivalent of the circuit in Fig. 4.39.

1.5- Examples:

Example 2.15:

Obtain the equivalent resistance R_{ab} for the circuit in Fig. 2.52



$$R_1 = 10\,\Omega, \qquad R_2 = 20\,\Omega, \qquad R_3 = 5\,\Omega$$

Thus from Eqs. (2.53) to (2.55) we have

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10}$$

$$= \frac{350}{10} = 35\,\Omega$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \frac{350}{20} = 17.5\,\Omega$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{350}{5} = 70\,\Omega$$

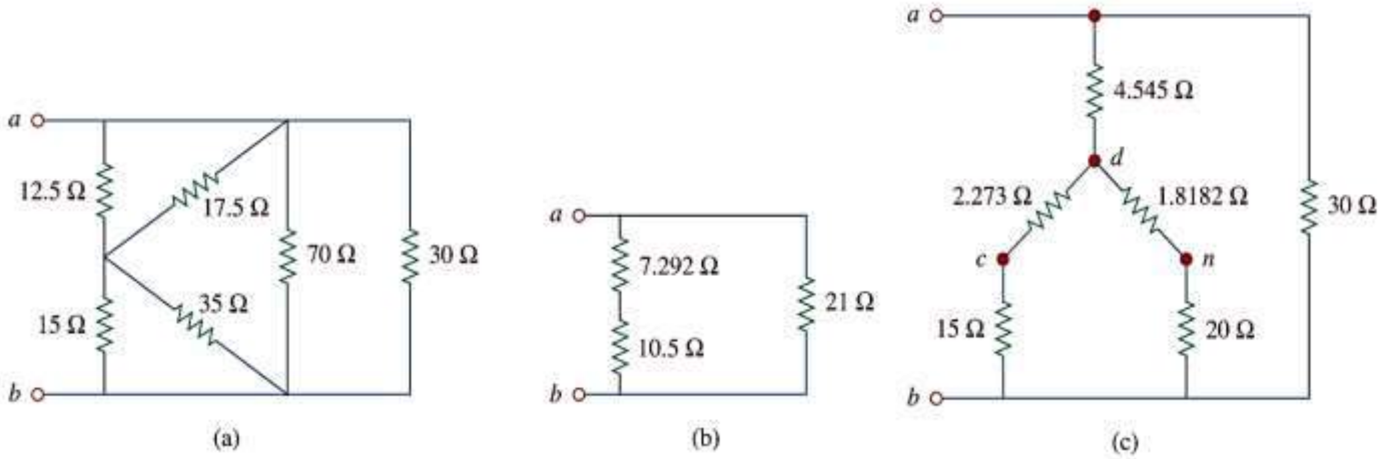


Figure 2.53
Equivalent circuits to Fig. 2.52, with the voltage source removed.

With the Y converted to Δ, the equivalent circuit (with the voltage source removed for now) is shown in Fig. 2.53(a). Combining the three pairs of resistors in parallel, we obtain

$$70 \parallel 30 = \frac{70 \times 30}{70 + 30} = 21\ \Omega$$

$$12.5 \parallel 17.5 = \frac{12.5 \times 17.5}{12.5 + 17.5} = 7.292\ \Omega$$

$$15 \parallel 35 = \frac{15 \times 35}{15 + 35} = 10.5\ \Omega$$

so that the equivalent circuit is shown in Fig. 2.53(b). Hence, we find

$$R_{ab} = (7.292 + 10.5) \parallel 21 = \frac{17.792 \times 21}{17.792 + 21} = \mathbf{9.632\ \Omega}$$

Example 3.4:

Find the node voltages in the circuit of Fig. 3.12.

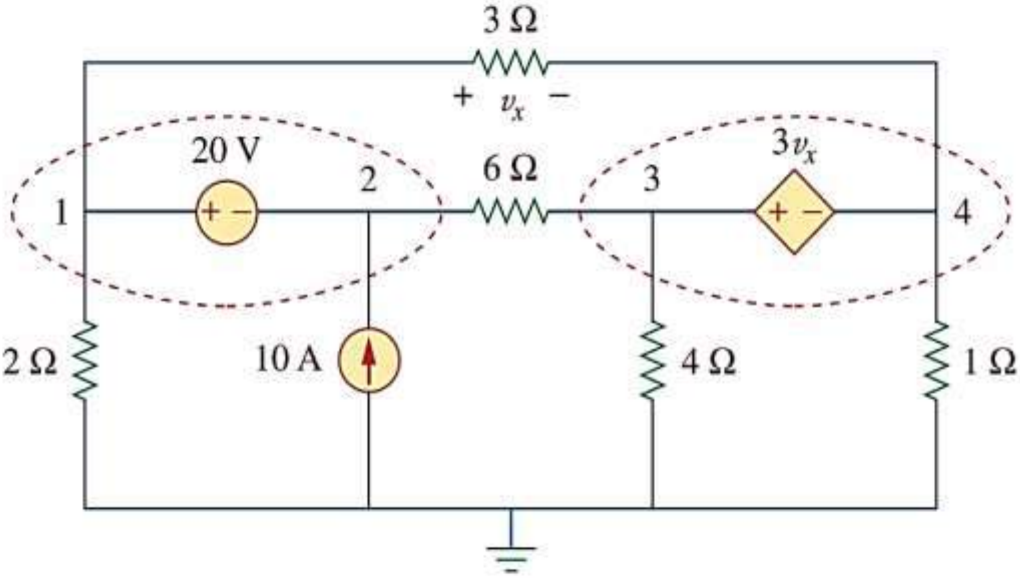


Figure 3.12
For Example 3.4.

Solution:

Nodes 1 and 2 form a supernode; so do nodes 3 and 4. We apply KCL to the two supernodes as in Fig. 3.13(a). At supernode 1-2,

$$i_3 + 10 = i_1 + i_2$$

Expressing this in terms of the node voltages,

$$\frac{v_3 - v_2}{6} + 10 = \frac{v_1 - v_4}{3} + \frac{v_1}{2}$$

or

$$5v_1 + v_2 - v_3 - 2v_4 = 60 \qquad \textbf{(3.4.1)}$$

At supernode 3-4,

$$i_1 = i_3 + i_4 + i_5 \quad \Rightarrow \quad \frac{v_1 - v_4}{3} = \frac{v_3 - v_2}{6} + \frac{v_4}{1} + \frac{v_3}{4}$$

or

$$4v_1 + 2v_2 - 5v_3 - 16v_4 = 0 \qquad \textbf{(3.4.2)}$$

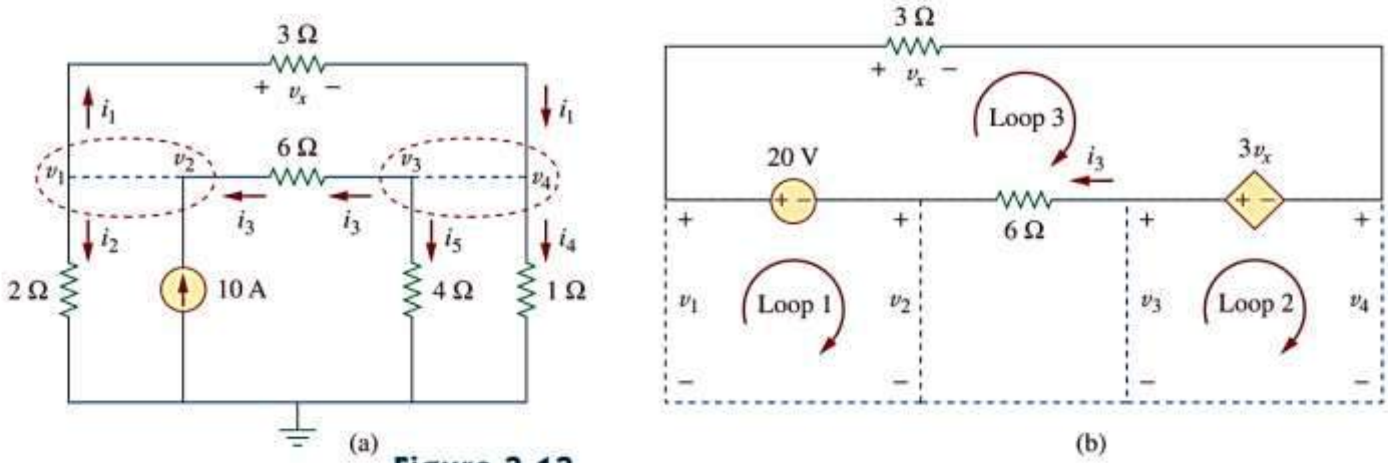


Figure 3.13
Applying: (a) KCL to the two supernodes, (b) KVL to the loops

We now apply KVL to the branches involving the voltage sources as shown in Fig. 3.13(b). For loop 1,

$$-v_1 + 20 + v_2 = 0 \quad \Rightarrow \quad v_1 - v_2 = 20 \qquad \textbf{(3.4.3)}$$

For loop 2,

$$-v_3 + 3v_x + v_4 = 0$$

But $v_x = v_1 - v_4$ so that

$$3v_1 - v_3 - 2v_4 = 0 \qquad \textbf{(3.4.4)}$$

For loop 3,

$$v_x - 3v_x + 6i_3 - 20 = 0$$

But $6i_3 = v_3 - v_2$ and $v_x = v_1 - v_4$. Hence,

$$-2v_1 - v_2 + v_3 + 2v_4 = 20 \qquad \textbf{(3.4.5)}$$

We need four node voltages, v_1, v_2, v_3, and v_4, and it requires only four out of the five Eqs. (3.4.1) to (3.4.5) to find them. Although the fifth equation is redundant, it can be used to check results. We can solve Eqs. (3.4.1) to (3.4.4) directly using *MATLAB*. We can eliminate one node voltage so that we solve three simultaneous equations instead of four. From Eq. (3.4.3), $v_2 = v_1 - 20$. Substituting this into Eqs. (3.4.1) and (3.4.2), respectively, gives

$$6v_1 - v_3 - 2v_4 = 80 \tag{3.4.6}$$

and

$$6v_1 - 5v_3 - 16v_4 = 40 \tag{3.4.7}$$

Equations (3.4.4), (3.4.6), and (3.4.7) can be cast in matrix form as

$$\begin{bmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{bmatrix} \begin{bmatrix} v_1 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 80 \\ 40 \end{bmatrix}$$

Using Cramer's rule gives

$$\Delta = \begin{vmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{vmatrix} = -18, \qquad \Delta_1 = \begin{vmatrix} 0 & -1 & -2 \\ 80 & -1 & -2 \\ 40 & -5 & -16 \end{vmatrix} = -480,$$

$$\Delta_3 = \begin{vmatrix} 3 & 0 & -2 \\ 6 & 80 & -2 \\ 6 & 40 & -16 \end{vmatrix} = -3120, \qquad \Delta_4 = \begin{vmatrix} 3 & -1 & 0 \\ 6 & -1 & 80 \\ 6 & -5 & 40 \end{vmatrix} = 840$$

Thus, we arrive at the node voltages as

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{-480}{-18} = 26.67 \text{ V}, \qquad v_3 = \frac{\Delta_3}{\Delta} = \frac{-3120}{-18} = 173.33 \text{ V},$$

$$v_4 = \frac{\Delta_4}{\Delta} = \frac{840}{-18} = -46.67 \text{ V}$$

and $v_2 = v_1 - 20 = 6.667$ V. We have not used Eq. (3.4.5); it can be used to cross check results.

Example 3.7:

For the circuit in Fig. 3.24, find i_1 to i_4 using mesh analysis

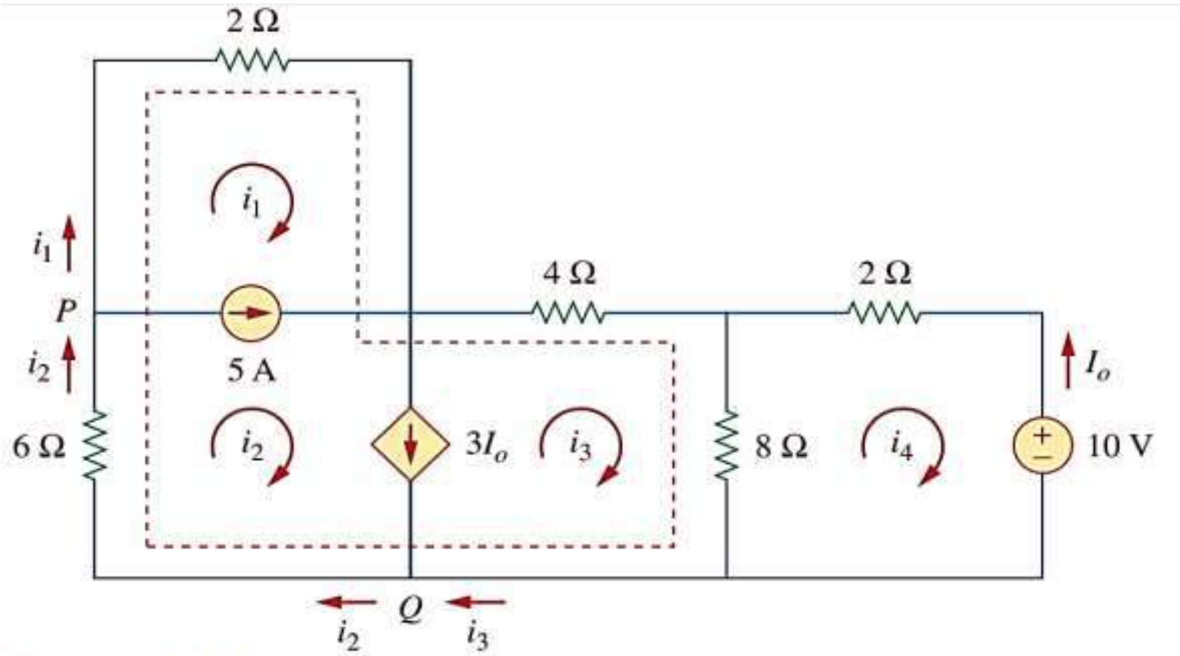


Figure 3.24
For Example 3.7.

Solution:

Note that meshes 1 and 2 form a supermesh since they have an independent current source in common. Also, meshes 2 and 3 form another supermesh because they have a dependent current source in common. The two supermeshes intersect and form a larger supermesh as shown. Applying KVL to the larger supermesh,

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

or

$$i_1 + 3i_2 + 6i_3 - 4i_4 = 0 \qquad \textbf{(3.7.1)}$$

For the independent current source, we apply KCL to node P:

$$i_2 = i_1 + 5 \qquad \textbf{(3.7.2)}$$

For the dependent current source, we apply KCL to node Q:

$$i_2 = i_3 + 3I_o$$

But $I_o = -i_4$, hence,

$$i_2 = i_3 - 3i_4 \quad \textbf{(3.7.3)}$$

Applying KVL in mesh 4,

$$2i_4 + 8(i_4 - i_3) + 10 = 0$$

or

$$5i_4 - 4i_3 = -5 \quad \textbf{(3.7.4)}$$

From Eqs. (3.7.1) to (3.7.4),

$$i_1 = -7.5 \text{ A}, \qquad i_2 = -2.5 \text{ A}, \qquad i_3 = 3.93 \text{ A}, \qquad i_4 = 2.143 \text{ A}$$

Example 4.12:

Using Norton's theorem, find R_N and I_N of the circuit in Fig. 4.43 at terminals *a-b*.

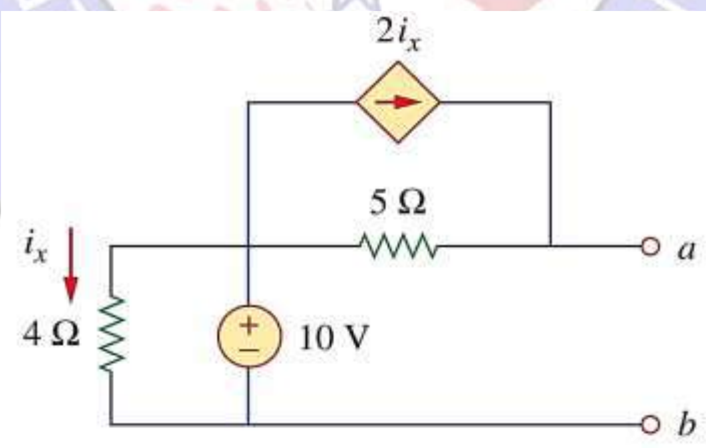


Figure 4.43
For Example 4.12.

Solution:

To find R_N, we set the independent voltage source equal to zero and connect a voltage source of $v_o = 1$ V (or any unspecified voltage v_o) to the terminals. We obtain the circuit in Fig. 4.44(a). We ignore the 4-Ω resistor because it is short-circuited. Also due to the short circuit, the 5-Ω resistor, the voltage source, and the dependent current source are all in parallel. Hence, $i_x = 0$. At node a, $i_o = \frac{1v}{5\Omega} = 0.2$ A, and

$$R_N = \frac{v_o}{i_o} = \frac{1}{0.2} = 5\ \Omega$$

To find I_N, we short-circuit terminals a and b and find the current i_{sc}, as indicated in Fig. 4.44(b). Note from this figure that the 4-Ω resistor, the 10-V voltage source, the 5-Ω resistor, and the dependent current source are all in parallel. Hence,

$$i_x = \frac{10}{4} = 2.5\ \text{A}$$

At node a, KCL gives

$$i_{sc} = \frac{10}{5} + 2i_x = 2 + 2(2.5) = 7\ \text{A}$$

Thus,

$$I_N = 7\ \text{A}$$

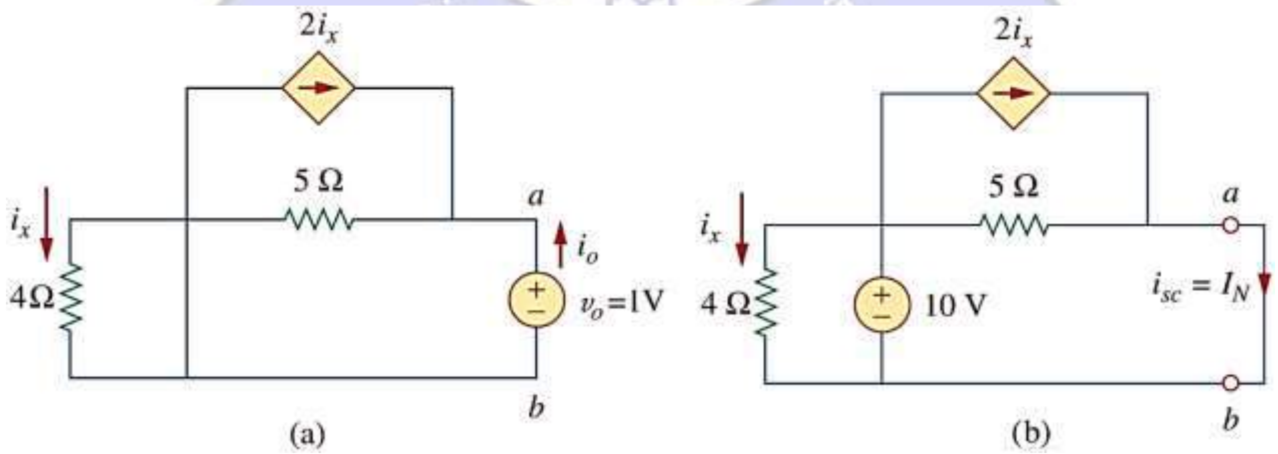


Figure 4.44
For Example 4.12: (a) finding R_N, (b) finding I_N.

1.6- Capacitors and Inductors

- A capacitor is a passive element designed to store energy in its electric field.
- The amount of charge stored, represented by q, is directly proportional to the applied voltage v as shown in figure 6.2.

$$q = Cv \tag{6.1}$$

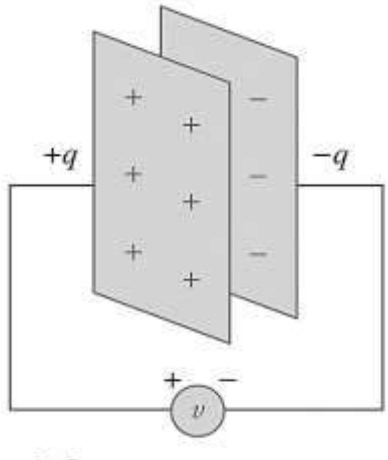


Figure 6.2
A capacitor with applied voltage v.

- To obtain the current-voltage relationship of the capacitor, we take the derivative of both sides of Eq. (6.1). Since

$$i = \frac{dq}{dt} \tag{6.3}$$

differentiating both sides of Eq. (6.1) gives

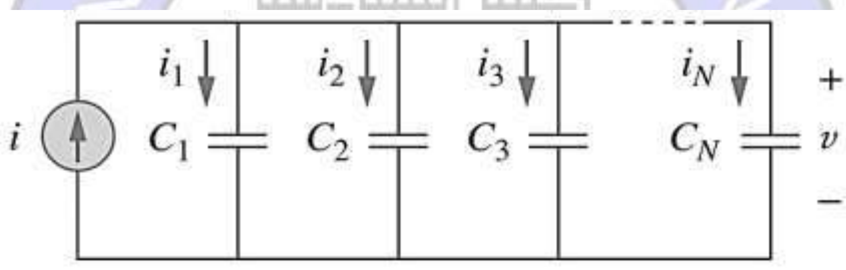
$$i = C\frac{dv}{dt} \tag{6.4}$$

- The energy stored in the capacitor is

$$w = \frac{1}{2}Cv^2 \qquad (6.9)$$

- A capacitor is an open circuit to dc.
- The equivalent capacitance of N parallel-connected capacitors is the sum of the individual capacitances.

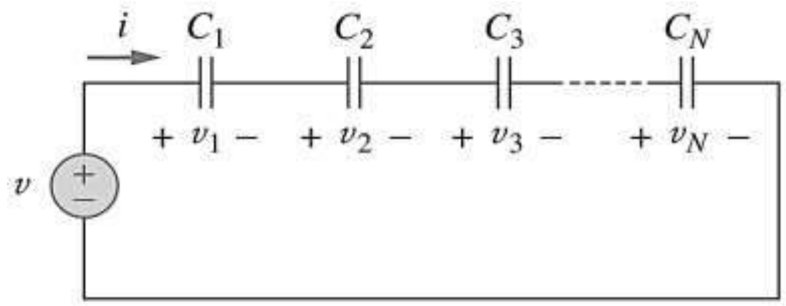
$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N \qquad (6.13)$$



(a)

- The equivalent capacitance of series-connected capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \qquad (6.17)$$



(a)

- An inductor is a passive element designed to store energy in its magnetic field.
- The voltage across the inductor is directly proportional to the time rate of change of the current.

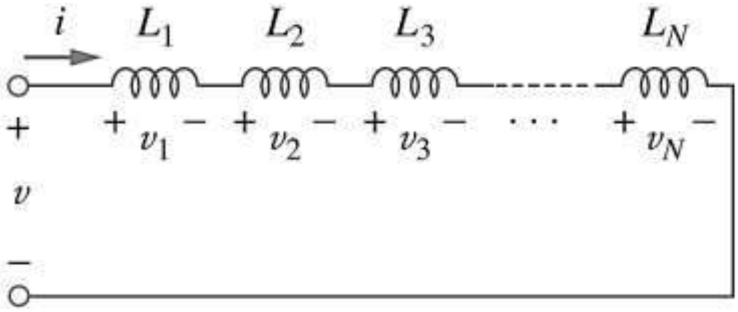
$$v = L\frac{di}{dt} \quad (6.18)$$

- The energy stored in the inductor is

$$w = \frac{1}{2}Li^2 \quad (6.24)$$

- The equivalent inductance of a series-connected is:

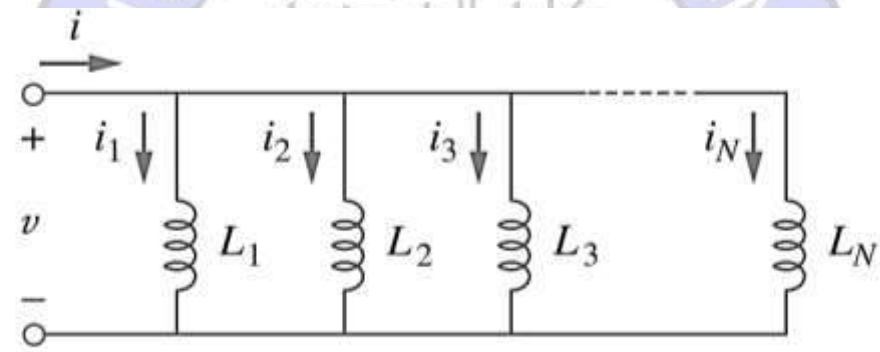
$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N \quad (6.27)$$



(a)

- The equivalent inductance of a parallel-connected is:

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2} \qquad (6.31)$$



(a)

- The summary is given in Table 6.1.

TABLE 6.1

Important characteristics of the basic elements.†

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit