From Eq. (9.37)

$$Q_p = A_p(q_p) = A_p\left[0.4p_aN_{60}\left(\frac{L}{D}\right)\right] \le A_p(4p_aN_{60})$$

$$A_p\left[0.4p_aN_{60}\left(\frac{L}{D}\right)\right] = (0.305 \times 0.305)\left[(0.4)(100)(17)\left(\frac{12}{0.305}\right)\right] = 2488.8 \text{ kN}$$

$$A_p(4p_aN_{60}) = (0.305 \times 0.305)[(4)(100)(17)] = 632.6 \text{ kN} \approx 633 \text{ kN}$$

Thus, Q_p = **633 kN**

Part b

From Eq. (9.38),

$$Q_p = A_pq_p = A_p[19.7p_a(N_{60})^{0.36}] = (0.305 \times 0.305)[(19.7)(100)(17)^{0.36}]$$

$$= \mathbf{508.2 \text{ kN}}$$

6.12 Frictional Resistance (*Qs*) in Sand

According to Eq. (9.14), the frictional resistance

$Qs = \sum p \Delta L f$

The unit frictional resistance, *f*, is hard to estimate. In making an estimation of *f*, several important factors must be kept in mind:

1. The nature of the pile installation. For driven piles in sand, the vibration caused during pile driving helps densify the soil around the pile. The zone of sand densification may be as much as 2.5 times the pile diameter, in the sand surrounding the pile.

2. It has been observed that the nature of variation of *f* in the field is approximately as shown in Figure 6.16. The unit skin friction increases with depth more or less linearly to a depth of L' and remains constant thereafter. The magnitude of the critical depth L' may be 15 to 20 pile diameters. A conservative estimate would be

$$L' < 15D \qquad (6.40)$$

3. At similar depths, the unit skin friction in loose sand is higher for a high displacement pile, compared with a low-displacement pile.

4. At similar depths, bored, or jetted, piles will have a lower unit skin friction compared with driven piles.

Taking into account the preceding factors, we can give the following approximate relationship for f (see Figure 9.16):

For $z = 0$ to L',

$$f = K\sigma'_o \tan \delta' \tag{9.41}$$

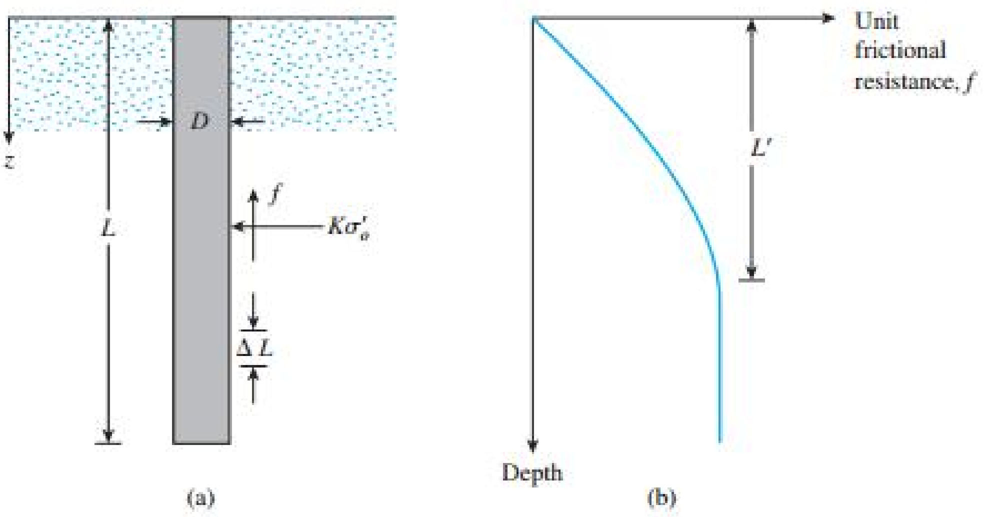

Figure 9.16 Unit frictional resistance for piles in sand

and for $z = L'$ to L,

$$f = f_{z=L'} \tag{9.42}$$

In these equations,

K = effective earth pressure coefficient
σ'_o = effective vertical stress at the depth under consideration
δ' = soil-pile friction angle

In reality, the magnitude of K varies with depth; it is approximately equal to the Rankine passive earth pressure coefficient, K_p , at the top of the pile and may be less than the at-rest pressure coefficient, K_o , at a greater depth.

Based on presently available results, the following average values of K are recommended for use in Eq. (9.41):

Pile type	K
Bored or jetted	$\approx K_o = 1 - \sin\phi'$
Low-displacement driven	$\approx K_o = 1 - \sin\phi'$ to $1.4K_o = 1.4(1 - \sin\phi')$
High-displacement driven	$\approx K_o = 1 - \sin\phi'$ to $1.8K_o = 1.8(1 - \sin\phi')$

The values of δ' from various investigations appear to be in the range from $0.5\phi'$ to $0.8\phi'$.

Based on load test results in the field, Mansur and Hunter (1970) reported the following average values of K.

H-piles. . . . . . $K = 1.65$

Steel pipe piles. . . . . . $K = 1.26$

Precast concrete piles. . . . . . $K = 1.5$

§ Coyle and Castello (1981), proposed

$$Q_s = f_{av}pL = (K\overline{\sigma}'_o \tan\delta')pL \tag{9.43}$$

where

$\overline{\sigma}'_o$ = average effective overburden pressure
δ' = soil–pile friction angle = $0.8\phi'$

The lateral earth pressure coefficient K, which was determined from field observations, is shown in Figure 9.17. Thus, if that figure is used,

$$Q_s = K\overline{\sigma}'_o \tan(0.8\phi')pL \tag{9.44}$$

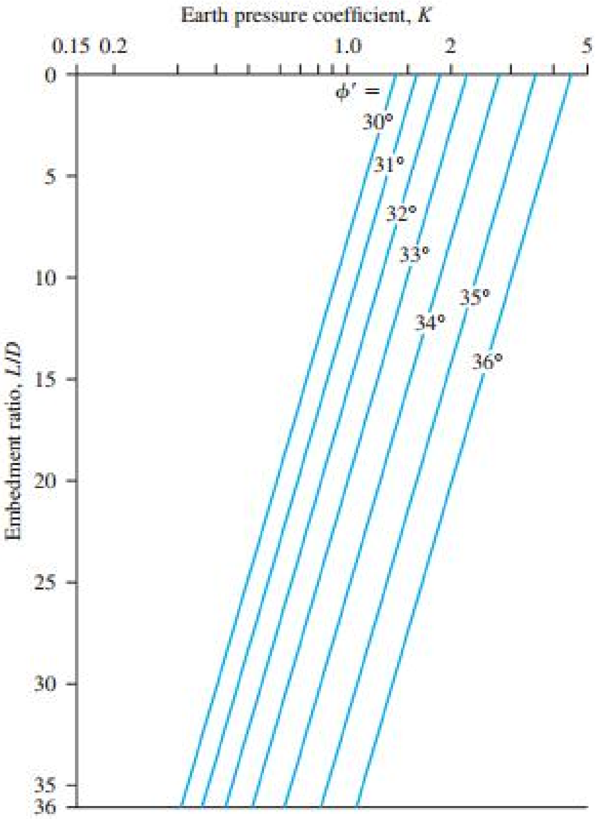

Figure 9.17 Variation of K with L/D (Based on Coyle and Castello, 1981)

Correlation with Standard Penetration Test Results

Meyerhof (1976) indicated that the average unit frictional resistance, f_{av} , for high-displacement driven piles may be obtained from average standard penetration resistance values as

$$f_{av} = 0.02 p_a (\overline{N}_{60}) \quad (9.45)$$

where

$(\overline{N}_{60})$ = average value of standard penetration resistance
p_a = atmospheric pressure ($\approx$100 kN/m^2 or 2000 lb/ft^2)

For low-displacement driven piles

$$f_{av} = 0.01 p_a (\overline{N}_{60}) \quad (9.46)$$

Briaud et al. (1985) suggested that

$$f_{av} \approx 0.224 p_a (\overline{N}_{60})^{0.29} \quad (9.47)$$

Thus,

$$Q_s = pLf_{av} \quad (9.48)$$

Example 6.4

Refer to the pile described in Example 9.3. Estimate the magnitude of Q_s for the pile.

a. Use Eq. (9.45).
b. Use Eq. (9.47).
c. Considering the results in Example 9.3, determine the allowable load-carrying capacity of the pile based on Meyerhof's method and Briaud's method. Use a factor of safety, FS = 3.

Solution

The average N_{60} value for the sand for the top 12 m is

$$\overline{N}_{60} = \frac{8 + 10 + 9 + 12 + 14 + 18 + 11 + 17}{8} = 10.25 \approx 10$$

Part a

From Eq. (9.45),

$$f_{av} = 0.02 p_a(\overline{N}_{60}) = (0.02)(100)(10) = 20 \text{ kN/m}^2$$

$$Q_s = pLf_{av} = (4 \times 0.305)(12)(20) = \mathbf{292.8 \text{ kN}}$$

Part b

From Eq. (9.47),

$$f_{av} = 0.224 p_a(\overline{N}_{60})^{0.29} = (0.224)(100)(10)^{0.29} = 43.68 \text{ kN/m}^2$$

$$Q_s = pLf_{av} = (4 \times 0.305)(12)(43.68) = \mathbf{639.5 \text{ kN}}$$

Part c

$$\text{Meyerhof's method: } Q_{all} = \frac{Q_p + Q_s}{FS} = \frac{633 + 292.8}{3} = \mathbf{308.6 \text{ kN}}$$

$$\text{Briaud's method: } Q_{all} = \frac{Q_p + Q_s}{FS} = \frac{508.2 + 639.5}{3} = \mathbf{382.6 \text{ kN}}$$

So the allowable pile capacity may be taken to be about **345 kN**. ■

Example 6.5

Refer to Example 9.1. For the pile, estimate the frictional resistance Q_s

a. Based on Eqs. (9.41) and (9.42). Use $K = 1.3$ and $\delta' = 0.8\phi'$.
b. Based on Eq. (9.44).
c. Using the results of Part d of Example 9.1, estimate the allowable bearing capacity of the pile. Use FS = 3.

Solution

Part a

From Eq. (9.40), $L' = 15D = (15)(0.407) \approx 6.1$ m. Refer to Eq. (9.41):

At $z = 0$: $\sigma'_o = 0$

$f = 0$

At $z = 6.1$ m: $\sigma'_o = (6.1)(18) = 109.8 \text{ kN/m}^2$

So

$$f = K\sigma'_o \tan\delta' = (1.3)(109.8)[\tan(0.8 \times 35)] \approx 75.9 \text{ kN/m}^2$$

Thus,

$$Q_s = \frac{(f_{z=0} + f_{z=6.1\text{m}})}{2} pL' + f_{z=6.1\text{m}}\, p(L - L')$$

$$= \left(\frac{0 + 75.9}{2}\right)(4 \times 0.407)(6.1) + (75.9)(4 \times 0.407)(20 - 6.1)$$

$$= 376.87 + 1717.56 = 2094.43 \text{ kN} \approx \mathbf{2094\ kN}$$

Part b

From Eq. (9.44),

$$Q_s = K\bar{\sigma}'_o \tan(0.8\phi')pL$$

$$\bar{\sigma}'_o = \frac{(20)(18)}{2} = 180 \text{ kN/m}^2$$

$$\frac{L}{D} = \frac{20}{0.407} = 49.1;\ \phi' = 35°$$

From Figure 9.17, $K \approx 0.41$ (by projection)

$$Q_s = (0.41)(180)\tan[(0.8 \times 35)](4 \times 0.407)(20) = \mathbf{1277.66\ kN}$$

Part c

The average value of Q_s from parts a and b is

$$Q_{s(\text{average})} = \frac{2094 + 1277.66}{2} = 1685.83 \approx 1686 \text{ kN} - \text{USE}$$

From part d of Example 9.1, $Q_p = 1280$ kN. Thus,

$$Q_{\text{all}} = \frac{Q_p + Q_s}{\text{FS}} = \frac{1280 + 1686}{3} = \mathbf{988.7\ kN}$$

6.13 Frictional (Skin) Resistance in Clay

Estimating the frictional (or skin) resistance of piles in clay is almost as difficult a task as estimating that in sand, due to the presence of several variables that cannot easily be quantified. Several methods for obtaining the unit frictional resistance of piles are described in the literature. We examine some of them next.

1. l Method

This method, proposed by Vijayvergiya and Focht (1972), is based on the assumption that the displacement of soil caused by pile driving results in a passive lateral pressure at any depth and that the average unit skin resistance is

$$f_{av} = \lambda(\overline{\sigma}'_o + 2c_u) \tag{9.51}$$

where

$\overline{\sigma}'_o$ = mean effective vertical stress for the entire embedment length
c_u = mean undrained shear strength ($\phi = 0$)

The value of l changes with the depth of penetration of the pile. (See Table 6.9.) Thus, the total frictional resistance may be calculated as

$$Q_s = pLf_{av}$$

Care should be taken in obtaining the values of $\overline{\sigma}'_o$ and c_u in layered soil. Figure 9.20 helps explain the reason. Figure 9.20a shows a pile penetrating three layers of clay. According to Figure 9.20b, the mean value of c_u is $(c_{u(1)}L_1 + c_{u(2)}L_2 + \cdots)/L$. Similarly, Figure 9.20c shows the plot of the variation of effective stress with depth. The mean effective stress is

$$\overline{\sigma}'_o = \frac{A_1 + A_2 + A_3 + \cdots}{L} \tag{9.52}$$

where $A_1, A_2, A_3, \ldots$ = areas of the vertical effective stress diagrams.

Table 6.9 Variation of λ with Pile Embedment Length, L

Embedment length, L (m)	λ
0	0.5
5	0.336
10	0.245
15	0.200
20	0.173
25	0.150
30	0.136
35	0.132
40	0.127
50	0.118
60	0.113
70	0.110
80	0.110
90	0.110

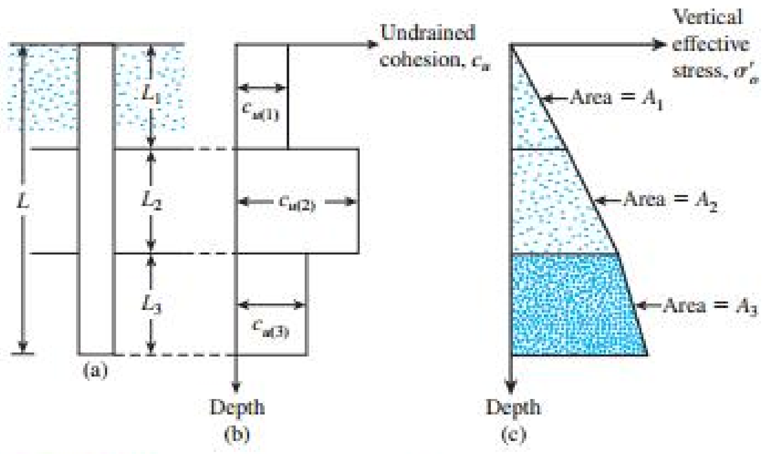

Figure 9.20 Application of λ method in layered soil

2. *a* Method

According to the *a* method, the unit skin resistance in clayey soils can be represented by the equation

$$f = \alpha c_u \qquad (9.53)$$

where *a* = empirical adhesion factor.

The approximate variation of the value of *a* is shown in Table 6.10.

The ultimate side resistance can thus be given as

$$Q_s = \Sigma f p \, \Delta L = \Sigma \alpha c_u p \, \Delta L \qquad (9.59)$$

Table 6.10 Variation of α (Interpolated Values Based on Terzaghi, Peck and Mesri, 1996)

$\frac{c_u}{p_a}$	α
≤ 0.1	1.00
0.2	0.92
0.3	0.82
0.4	0.74
0.6	0.62
0.8	0.54
1.0	0.48
1.2	0.42
1.4	0.40
1.6	0.38
1.8	0.36
2.0	0.35
2.4	0.34
2.8	0.34

Note: p_a = atmospheric pressure ≈ 100 kN/m² or 2000 lb/ft²

3- b Method

When piles are driven into saturated clays, the pore water pressure in the soil around the piles increases. The excess pore water pressure in normally consolidated clays may be four to six times c_u . However, within a month or so, this pressure gradually dissipates. Hence, the unit frictional resistance for the pile can be determined on the basis of the effective stress parameters of the clay in a remolded state *(c*$'$ =0). Thus, at any depth

$$f = \beta \sigma'_o \tag{9.60}$$

where

σ'_o = vertical effective stress

$\beta = K \tan \phi'_R$ (9.61)

ϕ'_R = drained friction angle of remolded clay

K = earth pressure coefficient

Conservatively, the magnitude of K is the earth pressure coefficient at rest, or

$$K = 1 - \sin \phi'_R \quad \text{(for normally consolidated clays)} \tag{9.62}$$

and

$$K = (1 - \sin \phi'_R)\sqrt{\text{OCR}} \quad \text{(for overconsolidated clays)} \tag{9.63}$$

where OCR = overconsolidation ratio.

Combining Eqs. (9.60), (9.61), (9.62), and (9.63), for normally consolidated clays yields

$$f = (1 - \sin \phi'_R)\tan \phi'_R \sigma'_o \tag{9.64}$$

and for overconsolidated clays,

$$f = (1 - \sin \phi'_R)\tan \phi'_R \sqrt{\text{OCR}}\, \sigma'_o \tag{9.65}$$

With the value of f determined, the total frictional resistance may be evaluated as

$$Q_s = \Sigma f p \, \Delta L$$

6.15 Point Bearing Capacity of Piles Resting on Rock

Sometimes piles are driven to an underlying layer of rock. In such cases, the engineer must evaluate the bearing capacity of the rock. The ultimate unit point resistance in rock (Goodman, 1980) is approximately

$$q_p = q_u(N_\phi + 1) \tag{9.76}$$

where

$N_\phi = \tan^2(45 + \phi'/2)$
q_u = unconfined compression strength of rock
ϕ' = drained angle of friction

Table 6.12 lists some representative values of (laboratory) unconfined compression strengths of rock. Representative values of the rock friction angle f' are given in Table 6.13.

A factor of safety of at least 3 should be used to determine the allowable point bearing capacity of piles. Thus,

$$Q_{p(\text{all})} = \frac{[q_{u(\text{design})}(N_\phi + 1)]A_p}{\text{FS}} \tag{9.78}$$

Table 6.13 Typical Values of Angle of Friction ϕ' of Rocks

Type of rock	Angle of friction, ϕ' (deg)
Sandstone	27–45
Limestone	30–40
Shale	10–20
Granite	40–50
Marble	25–30

Table 6.12 Typical Unconfined Compressive Strength of Rocks

Type of rock	q_u MN/m²	q_u lb/in²
Sandstone	70–140	10,000–20,000
Limestone	105–210	15,000–30,000
Shale	35–70	5000–10,000
Granite	140–210	20,000–30,000
Marble	60–70	8500–10,000

Example 6.7

Refer to the pipe pile in saturated clay shown in Figure 9.24. For the pile,

a. Calculate the skin resistance (Q_s) by (1) the α method, (2) the λ method, and (3) the β method. For the β method, use $\phi'_R = 30°$ for all clay layers. The top 10 m of clay is normally consolidated. The bottom clay layer has an OCR = 2. (Note: diameter of pile = 457 mm)

b. Using the results of Example 9.2, estimate the allowable pile capacity (Q_{all}). Use FS = 4.

Solution

Part a

(1) From Eq. (9.59),

$$Q_s = \Sigma \alpha c_u p \Delta L$$

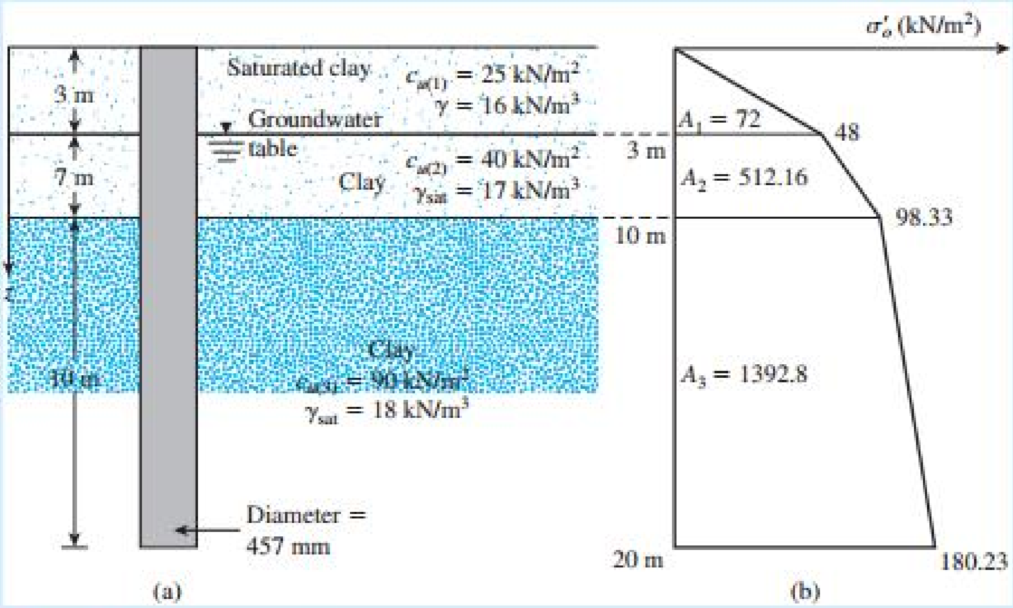

Figure 9.24 Estimation of the load bearing capacity of a driven-pipe pile

[*Note:* $p = \pi(0.457) = 1.436$ m] Now the following table can be prepared.

Depth (m)	ΔL (m)	c_u (kN/m²)	α (Table 9.10)	$\alpha c_u p \Delta L$ (kN)
0–3	3	25	0.87	93.7
3–10	7	40	0.74	297.5
10–20	10	90	0.51	659.1
				$Q_s \approx 1050$ kN

(2) From Eq. 9.51, $f_{av} = \lambda(\bar{\sigma}'_o + 2c_u)$. Now, the average value of c_u is

$$\frac{c_{u(1)}(3) + c_{u(2)}(7) + c_{u(3)}(10)}{20} = \frac{(25)(3) + (40)(7) + (90)(10)}{20} = 62.75 \text{ kN/m}^2$$

To obtain the average value of $\bar{\sigma}'_o$, the diagram for vertical effective stress variation with depth is plotted in Figure 9.24b. From Eq. (9.52),

$$\bar{\sigma}'_o = \frac{A_1 + A_2 + A_3}{L} = \frac{72 + 512.16 + 1392.8}{20} = 98.85 \text{ kN/m}^2$$

From Table 9.9, the magnitude of λ is 0.173. So

$$f_{av} = 0.173[98.85 + (2)(62.75)] = 38.81 \text{ kN/m}^2$$

Hence,

$$Q_s = pLf_{av} = \pi(0.457)(20)(38.81) = \mathbf{1114.4 \text{ kN}}$$

(3) The top layer of clay (10 m) is normally consolidated, and $\phi'_R = 30°$. For $z = 0$–3 m, from Eq. (9.64), we have

$$f_{av(1)} = (1 - \sin\phi'_R)\tan\phi'_R\,\bar{\sigma}'_o$$
$$= (1 - \sin 30°)(\tan 30°)\left(\frac{0 + 48}{2}\right) = 6.93 \text{ kN/m}^2$$

Similarly, for $z = 3$–10 m.

$$f_{av(2)} = (1 - \sin 30°)(\tan 30°)\left(\frac{48 + 98.33}{2}\right) = 21.12 \text{ kN/m}^2$$

For $z = 10$–20 m from Eq. (9.65),

$$f_{av} = (1 - \sin\phi'_R)\tan\phi'_R\sqrt{\text{OCR}}\,\bar{\sigma}'_o$$

For OCR = 2,

$$f_{av(3)} = (1 - \sin 30°)(\tan 30°)\sqrt{2}\left(\frac{98.33 + 180.23}{2}\right) = 56.86 \text{ kN/m}^2$$

So,

$$Q_s = p[f_{av(1)}(3) + f_{av(2)}(7) + f_{av(3)}(10)]$$
$$= (\pi)(0.457)[(6.93)(3) + (21.12)(7) + (56.86)(10)] = \mathbf{1058.45 \text{ kN}}$$

Part b

$$Q_u = Q_p + Q_s$$

From Example 9.2,

$$Q_p \approx 151 \text{ kN}$$

Again, the values of Q_s from the α method, λ method, and β method are close. So,

$$Q_s = \frac{1050 + 1114.4 + 1058.45}{3} \approx 1074 \text{ kN}$$

$$Q_{all} = \frac{Q_u}{FS} = \frac{151 + 1074}{4} = 306.25 \text{ kN} \approx \mathbf{306 \text{ kN}}$$

Example 6.8

A concrete pile 305 mm × 305 mm in cross section is driven to a depth of 20 m below the ground surface in a saturated clay soil. A summary of the variation of frictional resistance f_c obtained from a cone penetration test is as follows:

Depth (m)	Friction resistance, f_c (kg/cm²)
0–6	0.35
6–12	0.56
12–20	0.72

Estimate the frictional resistance Q_s for the pile.

Solution

We can prepare the following table:

Depth (m)	f_c (kN/m²)	α' (Figure 9.22)	ΔL (m)	$\alpha' f_c p(\Delta L)$ [Eq. (9.67)] (kN)
0–6	34.34	0.84	6	211.5
6–12	54.94	0.71	6	285.5
12–20	70.63	0.63	8	434.2

(*Note:* $p = (4)(0.305) = 1.22$ m)

Thus,

$$Q_s = \Sigma \alpha' f_c p(\Delta L) = \mathbf{931\ kN}$$

Example 6.9

Refer to Example 9.7 and Figure 9.24. Using Eq. (9.56), estimate the skin resistance Q_s.

Solution

For $z = 0$ to 3 m,

$$c_{u(1)} = 25\ \text{kN/m}^2;\quad \bar{\sigma}'_o = \frac{0 + 48}{2} = 24\ \text{kN/m}^2$$

$$f_{av} = 0.5(c_u \bar{\sigma}'_o)^{0.5} = 0.5[(25)(24)]^{0.5} = 12.25\ \text{kN/m}^2$$

Again,

$$f_{av} = 0.5(c_u)^{0.75}(\bar{\sigma}'_o)^{0.25} = 0.5(25)^{0.75}(24)^{0.25} = 12.37\ \text{kN/m}^2$$

Use $f_{av} = 12.37$ kN/m².

For $z = 3$ to 10 m,

$$c_{u(2)} = 40\ \text{kN/m}^2;\quad \bar{\sigma}'_o = \frac{48 + 98.33}{2} = 73.165\ \text{kN/m}^2$$

$$f_{av} = 0.5(c_u \bar{\sigma}'_o)^{0.5} = 0.5[(40)(73.165)]^{0.5} = 27.05\ \text{kN/m}^2$$

$$f_{av} = 0.5(c_u)^{0.75}(\bar{\sigma}'_o)^{0.25} = 0.5(40)^{0.75}(73.165)^{0.25} = 23.26\ \text{kN/m}^2$$

Use $f_{av} = 27.05\ \text{kN/m}^2$.

For $z = 10$ to 20 m,

$$c_{u(3)} = 90\ \text{kN/m}^2;\ \bar{\sigma}'_o = \frac{98.33 + 180.23}{2} = 139.28\ \text{kN/m}^2$$

$$f_{av} = 0.5(c_u\bar{\sigma}'_o)^{0.5} = 0.5[(90)(139.28)]^{0.5} = 55.98\ \text{kN/m}^2$$

$$f_{av} = 0.5(c_u)^{0.75}(\bar{\sigma}'_o)^{0.25} = 0.5(90)^{0.75}(139.28)^{0.25} = 50.19\ \text{kN/m}^2$$

Use $f_{av} = 55.98\ \text{kN/m}^2$.

Now,

$$Q_s = \Sigma f_{av} p \Delta L = (\pi \times 0.457)[(12.37)(3) + (27.05)(7) + (55.98)(10)] = \mathbf{1128.8\ kN}$$

\

Figure 3.5 A small enclosure with steel sheet piles for an excavation work (*Courtesy of N. Sivakugan, James Cook University, Australia*)

Table 3.1 Properties of Some Sheet-Pile Sections Production by Bethlehem Steel Corporation

Example 3.4: