Lecture 7

University of Anbar
Civil Engineering Department
MSc- Highway Engineering
Railway and Airport Engineering

Geometric Design of Railway Track

• Design of Vertical Curves
• Design of Horizontal Curves

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Design of Railway Vertical Curves

• For light rail transit main line tracks, the absolute minimum length of the vertical curve depends on the design speed of the track and the algebraic difference of the grades connected by the curve.
• Recommended criteria are given in the Track Design Handbook for Light Rail Transit for the desired length, the preferred minimum length, and the absolute minimum length as follows:
  • Desired length ($L_{mindes}$) = 60 A (m)
  • Preferred minimum length ($L_{minpref}$) = 30 A (m)
  • Absolute minimum length ($L_{minabs}$) = $A \frac{U^2}{212}$ (Crest Curve)
    ($L_{minabs}$) = $A \frac{U^2}{382}$ (Sag Curve)
Design of Railway Vertical Curves

• For freight and passenger intercity tracks, the length of the vertical curve depends on the algebraic difference between grades (A), the vertical acceleration and the speed of the train.

• The minimum length $L_{\text{min}}$ is given as:

$$L_{\text{min}} = A U^2 \frac{K}{100} a$$

Where:

$A = \text{algebraic difference between grades in percent}$

$U = \text{speed of the train km/h}$

$K = 0.077$, conversion factor to give $L_{\text{min}}$ in m

$a = \text{vertical acceleration, m/ s}^2 = 0.03 \text{ m/s}^2 \text{ for freight trains}$

$a = 0.18 \text{ m/ s}^2 \text{ for passenger trains}$

However, the length of any vertical curve cannot be less than 30 m
For light rail transit main line tracks, the absolute minimum length of the vertical curve depends on:
  the design speed of the track and the algebraic difference of the grades connected by the curve.
Recommended criteria are given in the Track Design Handbook for Light Rail Transit for the desired length,
The preferred minimum length, and the absolute minimum length as follows:
* Desired length \( (L_{\text{mindes}}) = 60A \) (m)
* Preferred minimum length \( (L_{\text{min pref}}) = 30A \) (m)
Absolute minimum length ($L_{\text{minabs}}$):

\[
L_{\text{minabs}} = \frac{Au^2}{212} \text{ (for crest vertical curves) (m)}
\]

\[
L_{\text{minabs}} = \frac{Au^2}{382} \text{ (for sag vertical curves) (m)}
\]

Where:

$A = (G2 - G1)$ algebraic difference of the grades connected by the vertical curve.

$G1 =$ percent grade of departing tangent

$G2 =$ percent grade of approaching tangent

$u =$ design speed in km/h
Design of Railway Horizontal Curves

Railway Simple Curves

• When a train is moving around a horizontal curve, it is subjected to a centrifugal force acting radially outward, similar to that discussed for highways.
• It is therefore necessary to raise the elevation of the outer rail of the track by a value $e_q$, which is the superelevation that provides an equilibrium force similar to that on highways.
• For any given equilibrium elevation, there is an equilibrium speed.
• This is the speed at which the resultant weight and the centrifugal force are perpendicular to the plane of the track.
• When this occurs, the components of the centrifugal force and the weight of the plane of the track are balanced.
Superelevation at Tracks

• It is the difference in height between the outer and the inner rail on a curve.
• It is provided by gradually lifting the outer rail above the level of the inner rail.
• The inner rail is taken as the reference rail and is normally maintained at its original level.
• The inner rail is also known as the gradient rail.
• The difference between the inclination of the car from the vertical (car angle) and that of the track from the vertical (track angle) is known as the roll angle.
• Trains traveling at a higher speed will cause more than normal wear on the outside rail, while traveling at lower speeds will cause more than normal wear on the inside track.
• Also, when the train is traveling faster than the equilibrium speed, the centrifugal force is not completely balanced by the elevation, which results in the car body tilting toward the outside of the curve.
Superelevation at Tracks

The main functions of super elevation are the following:

• To ensure a better distribution of load on both rails.
• To reduce the wear and tear of the rail and rolling stock.
• To neutralize the effect of lateral forces.
• To provide comfort to passengers.
Superelevation at Tracks

- Full equilibrium superelevation ($e_q$) is, however, rarely used in practice for two main reasons. First, the use of a full equilibrium superelevation may require long spiral transition curves. Second, full equilibrium superelevation can result in discomfort for passengers on a train traveling at a speed that is much less than the equilibrium speed or if the train is stopped along a highly superelevated curve.

- The portion of the equilibrium superelevation used in the design of the curve is known as the actual superelevation ($e_a$), and the difference between the actual superelevation and the equilibrium superelevation is known as the unbalanced superelevation ($e_u$).
Superelevation for Light Rail Transit

• For computing desired values of the actual superelevation of horizontal curves on these tracks:

\[ e_a = 0.79 \left( \frac{U^2}{R} \right) - 1.68 \]

and the desired relationship between the actual superelevation and the unbalanced superelevation is given as:

\[ e_u = \left[ 1 - \left( \frac{e_a}{2} \right) \right] \]

\( e_a = \) actual superelevation in cm
\( e_u = \) unbalanced superelevation in cm
\( U = \) curve design speed, km/h
\( R = \) radius, m
It is recommended that values obtained for using Equation should be rounded up to the nearest 0.5 cm. Also, when the sum of the unbalanced and actual elevations (ea + eu) is 2.5 cm or less, it is not necessary to provide any actual superelevation.

For tracks that are jointly used by light rail transit and freight vehicles, Equation should be used until the calculated value reaches 7.5 cm.

Higher values of up to 10 cm may be used to achieve operating speed if this is approved by both the transit authority and the railroad authorities.
Superelevation for Light Rail Transit

- The equation for the equilibrium superelevation for light rail transit track is given as:

\[ e_q = e_a + e_u = 1.184 \left( \frac{U^2}{R} \right) \]

Or

\[ e_q = e_a + e_u = 0.00068 U^2 D_c \]

Where:

- \( e_q \) = equilibrium superelevation in cm
- \( e_a \) = actual track elevation to be constructed in cm
- \( e_u \) = unbalanced superelevation in cm
- \( U \) = design speed through the curve, km/h
- \( R \) = radius of the curve, m
- \( D_c \) = degree of curve (chord definition) \( R = 1718.89 / D_c \)°
Superelevation for freight and passenger intercity tracks

AREMA gives a similar relationship for computing the equilibrium superelevation for freight and passenger intercity tracks as that given for light rail tracks.

\[ eq = ea + eu = 0.00068 \cdot U^2 \cdot Dc \]

Where:
- \( eq \) = equilibrium superelevation in cm
- \( ea \) = actual track elevation to be constructed in cm
- \( eu \) = unbalanced superelevation in cm
- \( U \) = design speed through the curve, km/h
- \( R \) = radius of the curve, m
- \( Dc \) = degree of curve (chord definition)
• However, note that the horizontal curvature on railway main line tracks should not be greater than $3^\circ$ for new tracks or the maximum on an existing track that is being realigned, and in no case should it be greater than $9^\circ\ 30"$.

• Experience has also shown that baggage cars, passenger coaches, diner cars, and Pullman cars can ride comfortably with an unbalanced superelevation of up to $7.5$ cm.

• This can be increased to $11.25$ cm if the roll angle is less than $1.5$. 
Terms Related to Superelevation

1- Equilibrium cant
   • Value of superelevation derived from the equation using equilibrium speed

2- Cant deficiency (Cd) (over balance)
   • Occurs when a train travels around a curve at a speed higher than the equilibrium speed
   • Difference between cant required at travel speed and actual cant

3- Cant excess (Ce) (under balance)
   • Occurs when a train travels around a curve at a speed lower than the equilibrium speed
   • Difference between actual cant and cant required at travel speed
Equilibrium Superelevation

- In Fig. , if \( \theta \) is the angle that the inclined plane makes with the horizontal line, then

\[
\tan \theta = \frac{\text{superelevation}}{\text{gauge}} = \frac{e}{G}
\]

Also,

\[
\tan \theta = \frac{\text{centrifugal force}}{\text{weight}} = \frac{F}{W}
\]

From these equations

\[
\frac{e}{G} = \frac{F}{W}
\]

or

\[
e = F \times \frac{G}{W}
\]

Where:

\[
F = \frac{Wv^2}{gR}
\]

\[
e = \frac{W \times V^2}{gR} \times \frac{G}{W} = \frac{GV^2}{gR}
\]
Equilibrium Superelevation

• Where $e$ is the equilibrium superelevation, $G$ is the gauge, $V$ is the velocity, $g$ is the acceleration due to gravity, and $R$ is the radius of the curve. In the metric system equilibrium superelevation is given by the formula

$$e = 1.184 \left(\frac{v^2}{R}\right)$$

Assume $G = 1524 \text{ mm}$

$$g = 9.81 \text{ m}^2/\text{sec}$$

$$R = \frac{1718.89}{D_c}$$

$$e = 0.00068 \, v^2 \, D_c$$

• Where $e$ is the superelevation in centimeters, $V$ is the speed in km/h, $R$ is the radius of the curve in meters, $Dc$ degree of curve, and $G$ is the dynamic gauge in millimeters, which is equal to the sum of the gauge and the width of the rail head in millimeters.
Railway Compound Curves

- The compound curve is seldom used in railway design.
- It is recommended that a spiral curve be used to connect the two or more legs of simple curves forming the compound curve.
Railway Spiral Curves

- It is recommended that on railways a spiral or transition curve be used to connect a tangent and a curve, unless this is not practicable.
- The design of the spiral curve for railways is similar to that for highways in that it starts with the determination of the length of the curve.
- The minimum length of the spiral curve depends on the classification of the track on which it will be constructed.
- AREMA gives two conditions that govern the length of a railway spiral curve on an entirely reconstructed or new main track for freight and passenger intercity tracks:
Condition 1:
- The unbalanced lateral acceleration acting on a passenger in a passenger car of average roll tendency should not exceed 0.03 g/s.
- To satisfy this condition, it is recommended by AREMA that the length should not be less than that obtained:

\[ L_{\text{min, spiral}} = 0.122(\text{eu}) U \]

Where:
- \( L_{\text{min, spiral}} \) = desirable length of spiral, m
- \( \text{eu} \) = unbalanced elevation, cm (usually taken as 7.5 cm for comfortable speed)
- \( U \) = maximum train speed, km/h
Railway Spiral Curves

Condition 2

• In order to limit the possible racking and torsional forces, the longitudinal slope of the outer rail with respect to the inner rail should not be greater than \(\frac{1}{744}\).

• This condition is satisfied if the length of the spiral curve is not less than \(L_{\text{min}}\) given in Equation, which is based on an 25.5-m-long car:

\[
L_{\text{min,spiral}} = 7.44 e_a
\]

Where:

\(L_{\text{min,spiral}}\) = desirable length of spiral curve (m)
\(e_a\) = actual elevation, cm
• When existing tracks are being realigned, the use of Equation b may give a spiral length for which the construction cost is excessive.

• In such cases, the unbalanced lateral acceleration acting on a passenger in a passenger car of average roll tendency may be increased to 0.04 g/s.

• This condition is satisfied if the length of the spiral curve is not less than that obtained from Equation c.

• Equation c can then be used instead of Equation a:

\[ L_{\text{min,spiral}} = 0.091 \ e_u \ U \]  

(C)

When Equation b is used to determine the length of the spiral curve, the maximum slope condition should also be satisfied, which means that the longer length obtained from Equation a and Equation b should be used.
• Passenger comfort and the rate of change in superelevation are also the two factors that influence the length of the spiral curve on a light rail transit track.
• In order to avoid excessive lateral unbalanced acceleration acting on passengers, the length of a spiral connecting a tangent to a horizontal curve on a light rail transit track should not be less than that obtained from Equation:
  \[ L_{\text{minspiral}} = 0.061 \, e_u \, U \]

Where:
\( e_u \) = unbalanced superelevation, cm, \( U \) = design speed, km/h
Railway Spiral Curves

• In order to limit the rate of change of superelevation on the spiral curve on a light rail transit track so as to avoid excessive stress on the vehicle frame, the minimum length of the spiral curve is obtained from Equations:
  \[ L_{\text{spiral}} = 0.082 \, e_a \, U \]
  \[ L_{\text{minspiral}} = 3.72 \, e_a \]

• However, the length of the spiral should not be less than 18 m. It can be reduced to 9.3 m when the geometric conditions are extremely restricted; for example, on an embedded track in a central business district (CBD).