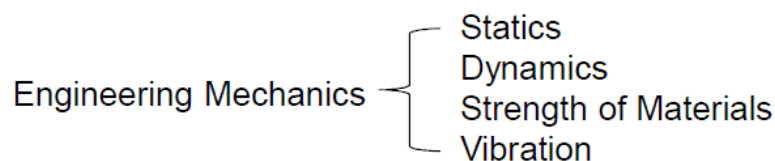


Semester I (2018-2019)

# Chapter one

## Introduction

### 1. Introduction



Statics:  $\sum (F_a + F_r) = 0$  , distribution of reaction force  $F_r$  from the applied force  $F_a$

Dynamics:  $\sum (F_a + F_r) = m\ddot{x}$  ,  $x(t) = f(F(t))$  displacement as a function of time and applied force

Strength of Materials:  $\delta = f(P)$  deflection and applied force on deformable bodies

Vibration:  $x(t) = f(F(t))$  on particles and rigid bodies

Dynamics is the branch of mechanics that deals with the motion of bodies under the action of forces (Accelerated Motion)

Two distinct parts:

–**Kinematics:**

- study of motion without reference to the forces that cause motion or are generated as a result of motion.

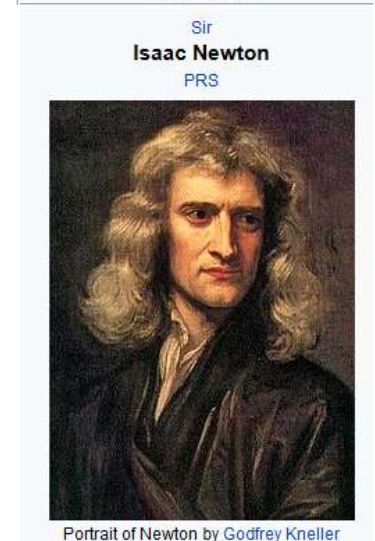
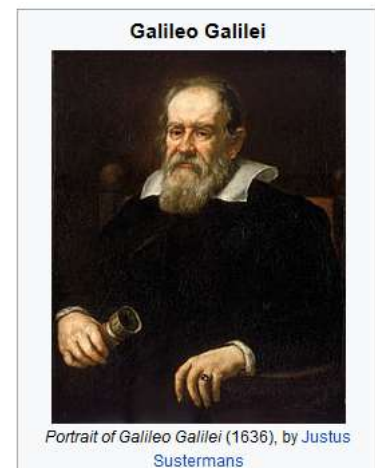
–**Kinetics:**

- relates the action of forces on bodies to their resulting motions.

## 2. History of dynamics:

Historically, the principles of dynamics developed when it was possible to make an accurate measurement of time. Galileo Galilei (1564-1642) was one of the first major contributors to this field. His work consisted of experiments using pendulums and falling bodies.

The most significant contributions in dynamics, however, were made by Isaac Newton (1642-1727), who is noted for his formulation of the three fundamental laws of motion and the law of universal gravitational attraction. Shortly after these laws were postulated, important techniques for their application were developed by Euler, D' Alembert, Lagrange, and others.

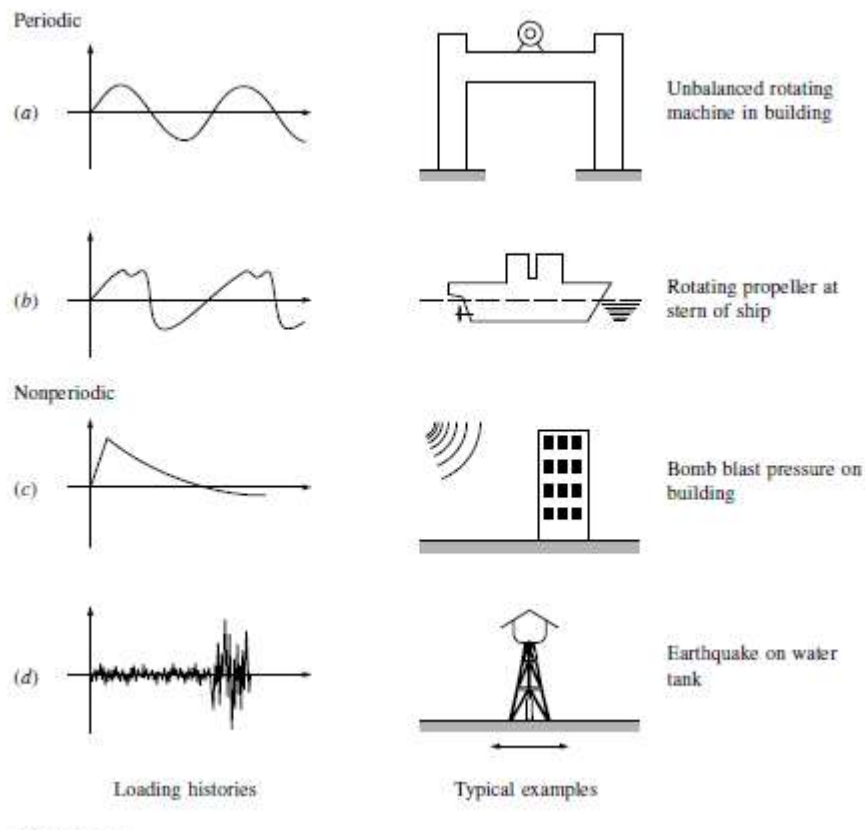


## 3. Applications of dynamics

The rapid technological developments of the present day require increasing application of the principles of mechanics, particularly dynamics. These principles are basic to the analysis and design of moving structures, to fixed

*Semester I (2018-2019)*

structures subject to shock loads, to robotic devices, to automatic control systems, to rockets, missiles, and spacecraft, to ground and air transportation vehicles, to electron ballistics of electrical devices, and to machinery of all types such as turbines, pumps, reciprocating engines, hoists, machine tools, etc. Students with interests in one or more of these and many other activities will constantly need to apply the fundamental principles of dynamics.



Q: How can we relate dynamics to hydraulic and water engineering in general?

Answer:

In fluids, two aspects are considered:

Fluid Kinematics - it deals with the study or with describing the motion of the fluids without considering the Forces and moments that cause the motion.

(2) Fluid Dynamics - it deals with the study or with describing the motion of Fluids also considering the Forces causing the motion .

## What is Fluid Kinematics ?

- ❖ Branch of fluid mechanics which deals with response of fluids in motion without considering forces and energies in them.
- ❖ The study of *kinematics* is often referred to as the *geometry of motion*.
- ❖ It is generally a continuous function in space and time.



### 4. Basic concepts

**Space** is the geometric region occupied by bodies. Position in space is determined relative to some geometric reference system by means of linear and angular measurements. The basic frame of reference for the laws of Newtonian mechanics is the primary inertial system or astronomical frame of reference, which is an imaginary set of rectangular axes assumed to have no translation or rotation in space.

**Measurements** show that the laws of Newtonian mechanics are valid for this reference system as long as any velocities involved are negligible compared with the speed of light, which is 300 000 km/s or 186,000 mi/sec. Measurements made with respect to this reference are said to be absolute, and this reference system

Semester I (2018-2019)

may be considered "fixed" in space. A reference frame attached to the surface of the earth has a some-what complicated motion in the primary system, and a correction to the basic equations of mechanics must be applied for measurements made relative to the reference frame of the earth. In the calculation of rocket and space flight trajectories. for example. the absolute motion of the earth becomes an important parameter. For most engineering problems involving machines and structures which remain on the surface of the earth, the corrections are extremely small and may be neglected. For these problems the laws of mechanics may be applied directly with measurements made relative to the earth, and in a practical sense such measurements will be considered absolute.

**Time** is a measure of the succession of events and is considered an absolute quantity in Newtonian mechanics.

**Mass** is the quantitative measure of the inertia or resistance to change in motion of a body. Mass may also be considered as the quantity of matter in a body as well as the property which gives rise to gravitational attraction. **Force** is the vector action of one body on another. The properties of forces have been thoroughly treated in Statics.

**A particle** is a body of negligible dimensions. When the dimensions of a body are irrelevant to the description of its motion or the action of forces on it, the body may be treated as a particle. An airplane, for ex-ample, may be treated as a particle for the description of its flight path.

**A rigid body** is a body whose changes in shape are negligible compared with the overall dimensions of the body or with the changes in position of the body as a whole. As an example of the assumption of rigidity, the small flexural movement of the wing tip of an airplane flying through turbulent air is clearly of no consequence to the description of the motion of the airplane as a whole along its

Semester I (2018-2019)

flight path. For this purpose, then, the treatment of the airplane as a rigid body is an acceptable approximation. On the other hand, if we need to examine the internal stresses in the wing structure due to changing dynamic loads, then the deformation characteristics of the structure would have to be examined, and for this purpose the airplane could no longer be considered a rigid body.

**Vector and scalar** quantities have been treated extensively in Statics, and their distinction should be perfectly clear by now.

## 5. Units

The basic units in Engineering mechanics are tabulated in below:

QUANTITY	DIMENSIONAL SYMBOL	SI UNITS		U.S. CUSTOMARY UNITS		
		UNIT	SYMBOL	UNIT	SYMBOL	
Mass	M	Base units	kilogram	kg	slug	—
Length	L		meter*	m	foot	ft
Time	T		second	s	second	sec
Force	F		newton	N	pound	lb

\*Also spelled *metre*.

In SI units, by definition, one newton is that force which will give a one-kilogram mass an acceleration of one meter per second squared. In the U.S. customary system a 32.1740-pound mass (1 slug) will have an acceleration of one foot per second squared when acted on by a force of one pound. Thus, for each system we have :

SI UNITS	U.S. CUSTOMARY UNITS
$(1 \text{ N}) = (1 \text{ kg})(1 \text{ m/s}^2)$	$(1 \text{ lb}) = (1 \text{ slug})(1 \text{ ft/sec}^2)$
$\text{N} = \text{kg} \cdot \text{m/s}^2$	$\text{slug} = \text{lb} \cdot \text{sec}^2/\text{ft}$

## 6. Gravitation:

*Semester I (2018-2019)*

Newton postulated a law governing the mutual attraction between any two particles. In mathematical form this law can be expressed as:

$$F = G \frac{m_1 m_2}{r^2}$$

where

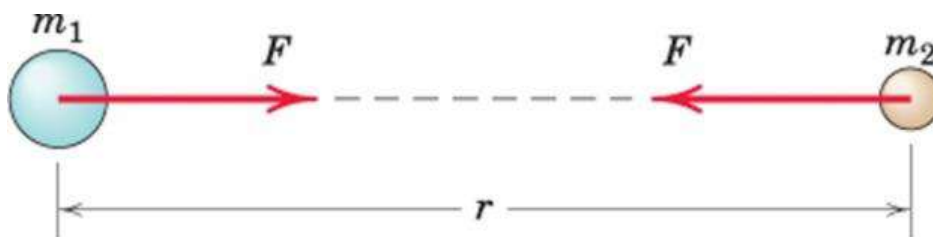
$F$  = force of attraction between the two particles

$G$  = universal constant of gravitation; according to experimental evidence

$$G = 6.673(10^{-11}) \text{ m}^3/(\text{kg} \cdot \text{s}^2)$$

$m_1, m_2$  = mass of each of the two particles

$r$  = distance between the centers of the two particles

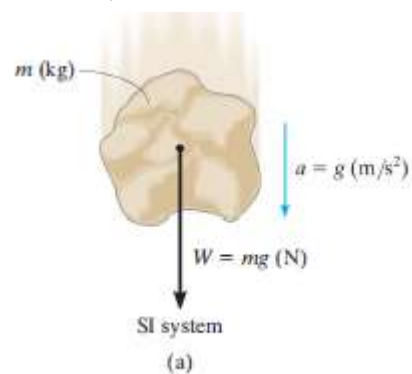


except for some spacecraft applications, the only gravitational force of appreciable magnitude in engineering is the force due to the attraction of the earth.

$$W = mg \text{ (N)} \quad (g = 9.81 \text{ m/s}^2)$$

As a result, a body of mass 1 kg has a weight of 9.81 N;

a 2-kg body weighs 19.62 N; and so on.



## 7. Effect of Altitude

The force of gravitational attraction of the earth on a body depends on the position of the body relative to the earth. If the earth were a perfect homogeneous sphere, a body with a mass of exactly 1 kg would be attracted to the earth by a force of 9.825 N on the surface of the earth. 9.822 N at an altitude of 1 km, 9.523 N at an altitude of 100 km, 7.240 N at an altitude of 1000 km. and 2.456 N at an altitude equal to the mean radius of the earth, 6371 km.

Every object which falls in a vacuum at a given height near the surface of the earth will have the same acceleration  $g$ , regardless of its mass.

From Eq., we can develop a general expression for finding the weight  $W$  of a particle having a mass  $m_1 = m$ . Let  $m_2 = m_e$  be the mass of the earth and  $R$  the distance between the earth's center and the particle. Then,

$$F = m a \quad \text{and} \quad F = G m_1 * m_2 / R^2$$

$$m a = G m_1 * m_2 / R^2$$

then

$$a = G m_e / R^2$$

or

$$g = \frac{G m_e}{R^2}$$



Semester I (2018-2019)

where  $m_e$  is the mass of the earth and  $R$  is the radius of the earth. The mass  $m_e$  and the mean radius  $R$  of the earth have been found through experimental measurements to be  $5.976 (10^{24})$  kg and  $6.371(10^6)$  m, respectively.

These values, together with the value of  $G$  already cited, when substituted into the expression for  $g$ , give a mean value of  $g = 9.825 \text{ m/s}^2$ . The variation of  $g$  with altitude is easily determined from the gravitational law.

The variation of  $g$  with altitude is easily determined from the gravitational law. If  $g_0$  represents the absolute acceleration due to gravity at sea level, the absolute value at an altitude  $h$  is

$$g = g_0 \frac{R^2}{(R + h)^2}$$

$g$  is the absolute acceleration due to gravity at altitude  $h$

$g_0$  is the absolute acceleration due to gravity at sea level

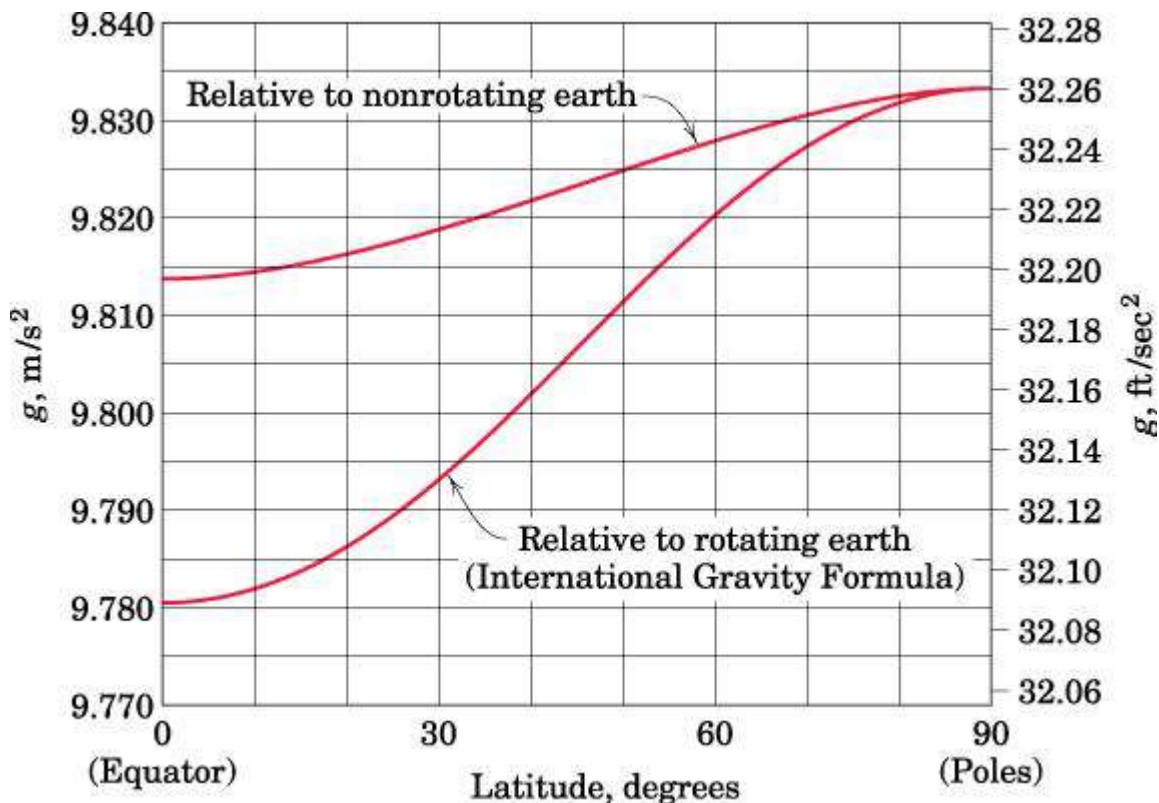
$R$  is the radius of the earth

## 8. Effect of a Rotating Earth

The acceleration due to gravity as determined from the gravitational law is the acceleration which would be measured from a set of axes whose origin is at the center of the earth but which does not rotate with the earth. With respect to these "fixed" axes, then, this value may be termed the absolute value of  $g$ . Because the earth rotates, the acceleration of a freely falling body as measured from a position at-attached to the surface of the earth is slightly less than the absolute value.

Actual acceleration of a freely falling body is less than absolute  $g$ . It measured from a position attached to the surface of the earth.

*Semester I (2018-2019)*



## 9. Standard Value of g

The standard value which has been adopted internationally for the gravitational acceleration relative to the rotating earth at sea level and at a latitude of  $45^\circ$  is  $9.80665 \text{ m/s}^2$  or  $32.1740 \text{ ft/sec}^2$ . This value differs very slightly from that obtained by evaluating the International Gravity Formula for  $\gamma = 45^\circ$ . The reason for the small difference is that the earth is not exactly ellipsoidal, as assumed in the formulation of the International Gravity Formula:

$$g = 9.78 (1 + 0.0053 \sin^2\gamma - 0.0000058 \sin^4\gamma)$$

where  $g$  (in  $\text{m/sec}^2$ ) is theoretical gravity and  $\gamma$  is latitude

*Semester I (2018-2019)*

The proximity of large land masses and the variations in the density of the crust of the earth also influence the local value of  $g$  by a small but detectable amount. In almost all engineering applications near the surface of the earth, we can neglect the difference between the absolute and relative values of the gravitational acceleration, and the effect of local variations.

Table 1.1 *SI Units of Fundamental Quantities*

Quantity	Dimension	SI Units	
		Unit	Symbol
Mass	M	kilogram - Base unit	kg
Length	L	metre - Base unit	m
Time	T	second - Base unit	s
Force	F	newton	N

Other units which are expressed in terms of the basic units are called derived units. Example: area, volume, velocity, work, power etc., Table 1.2 presents the various SI units used in mechanics.

Table 1.2 *Important SI Units used in Mechanics*

Quantity	Unit	SI Symbol
<i>(Basic Units)</i>		
Length	metre	m
Mass	kilogram	kg
Time	second	s
<i>(Derived Units)</i>		
Area	metre <sup>2</sup>	m <sup>2</sup>
Volume	metre <sup>3</sup>	m <sup>3</sup>
Moment of Inertia, area	metre <sup>4</sup>	m <sup>4</sup>
Moment of Inertia, mass	kilogram-metre <sup>2</sup>	kg-m <sup>2</sup>
Product of Inertia, area	metre <sup>4</sup>	m <sup>4</sup>
Product of Inertia, mass	kilogram-metre <sup>2</sup>	kg-m <sup>2</sup>
Force	Newton	N
Velocity, linear	metre/second	m/s
Velocity, angular	radian/second	rad/s

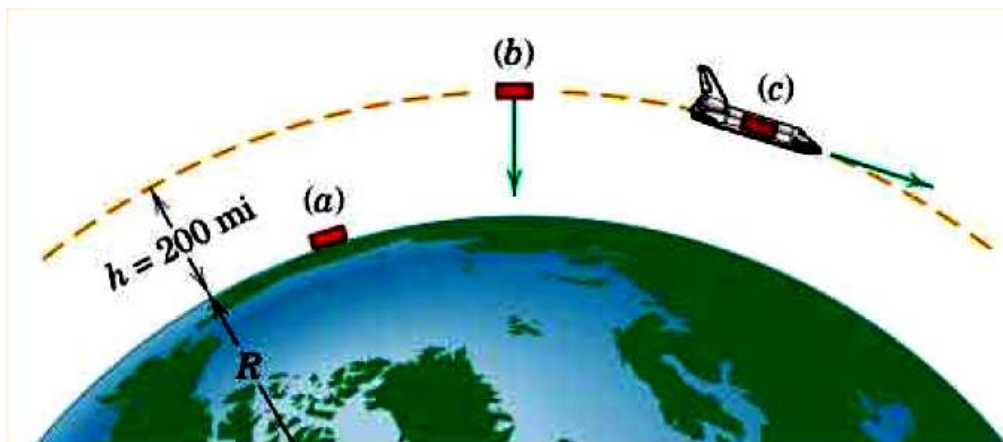
*(Contd.)*

*Semester I (2018-2019)*

Table 1.2 (Contd.) *Important SI Units used in Mechanics*

<i>Quantity</i>	<i>Unit</i>	<i>SI Symbol</i>
Acceleration, linear	metre/second <sup>2</sup>	<i>ms<sup>-2</sup></i>
Acceleration, angular	radian/second <sup>2</sup>	<i>rad/s<sup>2</sup></i>
Density	kilogram/metre <sup>3</sup>	<i>kg/m<sup>3</sup></i>
Moment of force	Newton-metre	<i>N-m</i>
Momentum, linear	kilogram-metre/second	<i>kg-m/s</i>
Momentum, angular	kilogram-metre <sup>2</sup> /second	<i>kg-m<sup>2</sup>/s</i>
Impulse, linear	Newton-second	<i>N-s</i>
Impulse, angular	Newton-metre-second	<i>N-m-s</i>
Power	Watt	<i>W(N.m/s)</i>
Pressure, stress	Pascal	<i>pa (N/m<sup>2</sup>)</i>
Spring constant	Newton/metre	<i>N/m</i>
Work, energy	Joule	<i>J (N-m)</i>

Example 1:



A space-shuttle payload module weighs 100 lb when resting on the surface of the earth at a latitude of 45° north.

(a) Determine the mass of the module in both slugs and kilograms, and its surface level weight in newtons.

(b) Now suppose the module is taken to an altitude of 200 miles above the surface of the earth and released there with no velocity relative to the center

*Semester I (2018-2019)*

of the earth Determine its weight under these conditions in both pounds and newtons.

(c) Finally, suppose the module is fixed inside the cargo bay of a space shuttle. The shuttle is in a circular orbit at an altitude of 200 miles above the surface of the earth. Determine the weight of the module in both pounds and newtons under these conditions.

**Note:** For the surface-level value of the acceleration of gravity relative to a rotating earth, use  $g = 32.1740 \text{ ft/sec}^2$  ( $9.80665 \text{ m/s}^2$ ). For the absolute value relative to a nonrotating earth, use  $g = 32.234 \text{ ft/sec}^2$  ( $9.825 \text{ m/s}^2$ ).

Solution.

$$\textcircled{1} [W = mg] \quad m = \frac{W}{g} = \frac{100 \text{ lb}}{32.1740 \text{ ft/sec}^2} = 3.11 \text{ slugs} \quad \text{Ans.}$$

From the table of conversion factors inside the front cover of the textbook, we see that 1 pound is equal to 4.4482 newtons. Thus, the weight of the module in newtons is

$$\textcircled{2} \quad W = 100 \text{ lb} \left[ \frac{4.4482 \text{ N}}{1 \text{ lb}} \right] = 445 \text{ N} \quad \text{Ans.}$$

Finally, its mass in kilograms is

$$\textcircled{3} [W = mg] \quad m = \frac{W}{g} = \frac{445 \text{ N}}{9.80665 \text{ m/s}^2} = 45.4 \text{ kg} \quad \text{Ans.}$$

As another route to the last result, we may convert from pounds mass to kilograms. Again using the table inside the front cover, we have

$$m = 100 \text{ lbm} \left[ \frac{0.45359 \text{ kg}}{1 \text{ lbm}} \right] = 45.4 \text{ kg}$$

*Semester I (2018-2019)*

(b) We begin by calculating the absolute acceleration of gravity (relative to the nonrotating earth) at an altitude of 200 miles.

$$\left[ g = g_0 \frac{R^2}{(R + h)^2} \right] \quad g_h = 32.234 \left[ \frac{3959^2}{(3959 + 200)^2} \right] = 29.2 \text{ ft/sec}^2$$

The weight at an altitude of 200 miles is then

$$W_h = mg_h = 3.11(29.2) = 90.8 \text{ lb} \quad \text{Ans.}$$

We now convert  $W_h$  to units of newtons.

$$W_h = 90.8 \text{ lb} \left[ \frac{4.4482 \text{ N}}{1 \text{ lb}} \right] = 404 \text{ N} \quad \text{Ans.}$$

As an alternative solution to part (b), we may use Newton's universal law of gravitation. In U.S. units,

$$\left[ F = \frac{Gm_1m_2}{r^2} \right] \quad W_h = \frac{Gm_e m}{(R + h)^2} = \frac{[3.439(10^{-8})][4.095(10^{23})][3.11]}{[(3959 + 200)(5280)]^2} \\ = 90.8 \text{ lb}$$

(c) The weight of an object (the force of gravitational attraction) does not depend on the motion of the object. Thus the answers for part (c) are the same as those in part (b).

$$W_h = 90.8 \text{ lb} \quad \text{or} \quad 404 \text{ N} \quad \text{Ans.}$$

**Classroom activity:** Re – solve (b) in example above using  $g_0=9.825 \text{ m/s}^2$  and  $G = 6.673(10^{-11}) \text{ m}^3/(\text{kg} \cdot \text{s}^2)$ .

Semester I (2018-2019)

**Problems**

**1/5:** Consider two iron spheres, each of diameter 100 mm, which are just touching. At what distance  $r$  from the center of the earth will the force of mutual attraction between the contacting spheres be equal to the force exerted by the earth on one of the spheres?

Solution: mass of iron sphere  $m = \rho V$        $\rho = 7210 \text{ kg/m}^3$

$$\therefore m = 7210 \times \frac{4}{3} \pi (0.05)^3 = 3.78 \text{ kg}$$

Force of mutual attraction  $\frac{G m^2}{d^2}$

Weight of each sphere  $\frac{G m_e m}{r^2}$

$$\therefore \frac{G m^2}{d^2} = \frac{G m_e m}{r^2} \Rightarrow r = d \sqrt{\frac{m_e}{m}}$$

$$r = 0.1 \sqrt{\frac{5.976 \times 10^{24}}{3.78}} \times 10^{-3} = 1.258 \times 10^8 \text{ cm}$$

**1/12:** Calculate the distance  $d$  from the center of the sun at which a particle experiences equal attractions from the earth and the sun. The particle is restricted to the line which joins the centers of the earth and the sun. Justify the two solutions physically.

Sol: Newton's law

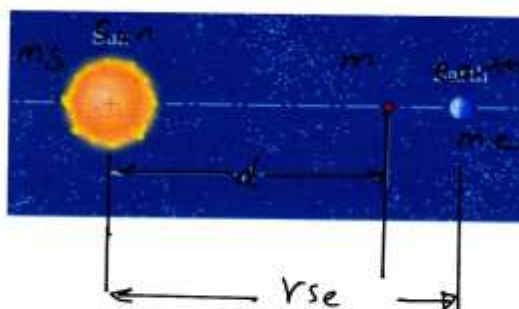
$$\frac{G m m_s}{d^2} = \frac{G m m_e}{(r_{se} - d)^2}$$

$$d^2 [m_s - m_e] - d [2 m_s r_{se}] + m_s r_{se}^2 = 0$$

$$\therefore m_e = 5.976 \times 10^{24} \text{ kg}$$

$$m_s = 333 \times 10^3 [5.976 \times 10^{24}]$$

$$r_{se} = 149.6 \times 10^9 \text{ m}$$



Semester I (2018-2019)

By solving the quadratic eq. :-

$$d = 149.3 \times 10^9 \text{ m}$$

$$\text{or } d = 149.9 \times 10^9 \text{ m}$$

which one is correct? why?