

University of Anbar  
Engineering College  
Civil Engineering Department

## **CHAPTER TWO**

# **LATERAL EARTH PRESSURE AND RETAINING WALLS**

**LECTURE**  
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## 2.1 Introduction

A retaining wall is a wall that provides lateral support for a vertical or near-vertical slope of soil. It is a common structure used in many construction projects. The most common types of retaining wall may be classified as follows:

1. Gravity retaining walls
2. Semigravity retaining walls
3. Cantilever retaining walls
4. Counterfort retaining walls

**Gravity retaining walls (Figure 1.1a)** are constructed with plain concrete or stone masonry. They depend for stability on their own weight and any soil resting on the masonry. This type of construction is not economical for high walls.

In many cases, a small amount of steel may be used for the construction of gravity walls, thereby minimizing the size of wall sections. Such walls are generally referred to as **semigravity walls (Figure 1.1b)**.

**Cantilever retaining walls (Figure 1.1c)** are made of reinforced concrete that consists of a thin stem and a base slab. This type of wall is economical to a height of about 8 m.

**Counterfort retaining walls (Figure 1.1d)** are similar to cantilever walls. At regular intervals, however, they have thin vertical concrete slabs known as *counterforts* that tie the wall and the base slab together. The purpose of the counterforts is to reduce the shear and the bending moments.

To design retaining walls properly, an engineer must know the basic parameters—the *unit weight*, *angle of friction*, and *cohesion*—of the soil retained behind the wall and the soil below the base slab. Knowing the properties of the soil behind the wall enables the engineer to determine the lateral pressure distribution that has to be designed for.

There are two phases in the design of a conventional retaining wall. First, with the lateral earth pressure known, the structure as a whole is checked for *stability*. The structure is examined for possible *overturning*, *sliding*, and *bearing capacity* failures. Second, each component of the structure is checked for *strength*, and the *steel reinforcement* of each component is determined.

This chapter presents the procedures for determination of lateral earth pressure and retaining-wall stability.

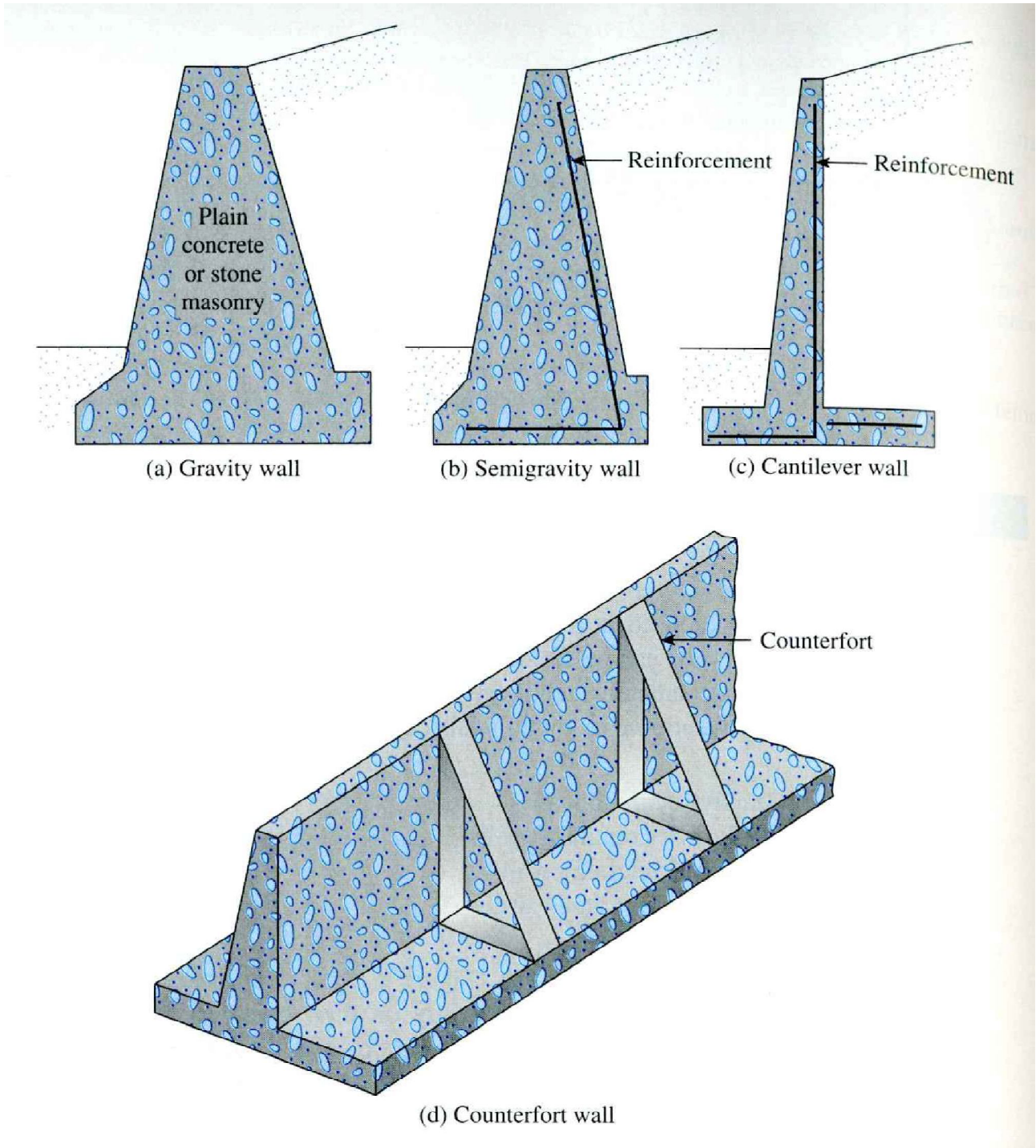


Figure 2.1 Types of retaining wall

## 2.2 Lateral Earth Pressure at Rest

Consider a vertical wall of height  $H$ , as shown in Figure 2.2, retaining a soil having a unit weight of  $\gamma$ . A uniformly distributed load,  $q$ /unit area, is also applied at the ground surface. The shear strength of the soil is

$$s = c' + \sigma' \tan \phi'$$

Where

$c'$  = cohesion

$\phi'$  = effective angle of friction

$\sigma'$  = effective normal stress

At any depth  $z$  below the ground surface, the vertical subsurface stress is

$$\sigma'_o = q + \gamma z \quad (2.1)$$

If the wall is at rest and is not allowed to move at all, either away from the soil mass or into the soil mass (i.e., there is zero horizontal strain), the lateral pressure at a depth  $z$  is

$$\sigma_h = K_o \sigma'_o + u \quad (2.2)$$

Where

$u$  = pore water pressure

$K_o$  = coefficient of at-rest earth pressure

For normally consolidated soil, the relation for  $K_o$  (Jaky, 1944) is

$$K_o \approx 1 - \sin \phi' \quad (2.3)$$

Equation (2.3) is an empirical approximation.

For overconsolidated soil, the at-rest earth pressure coefficient may be expressed as (Mayne and Kulhawy, 1982)

$$K_o = (1 - \sin \phi') OCR^{\sin \phi'} \quad (2.4)$$

where OCR = overconsolidation ratio.

With a properly selected value of the at-rest earth pressure coefficient, Eq. (2.2) can be used to determine the variation of lateral earth pressure with depth  $z$ . Figure 2.2b shows the variation of  $\sigma'_h$  with depth for the wall depicted in Figure 2.2a. Note that if

the surcharge  $q=0$  and the pore water pressure the pressure  $u=0$ , diagram will be a triangle. The total force,  $P_o$ , per unit length of the wall given in Figure 2.2a can now be obtained from the area of the pressure diagram given in Figure 2.2b and is

$$P_o = P_1 + P_2 = qK_oH + \frac{1}{2}\gamma H^2K_o \quad (2.5)$$

where

$P_1$ = area of rectangle 1

$P_2$ = area of triangle 2

The location of the line of action of the resultant force,  $P_o$ , can be obtained by taking the moment about the bottom of the wall. Thus,

$$\bar{z} = \frac{P_1\left(\frac{H}{2}\right) + P_2\left(\frac{H}{3}\right)}{P_o} \quad (2.6)$$

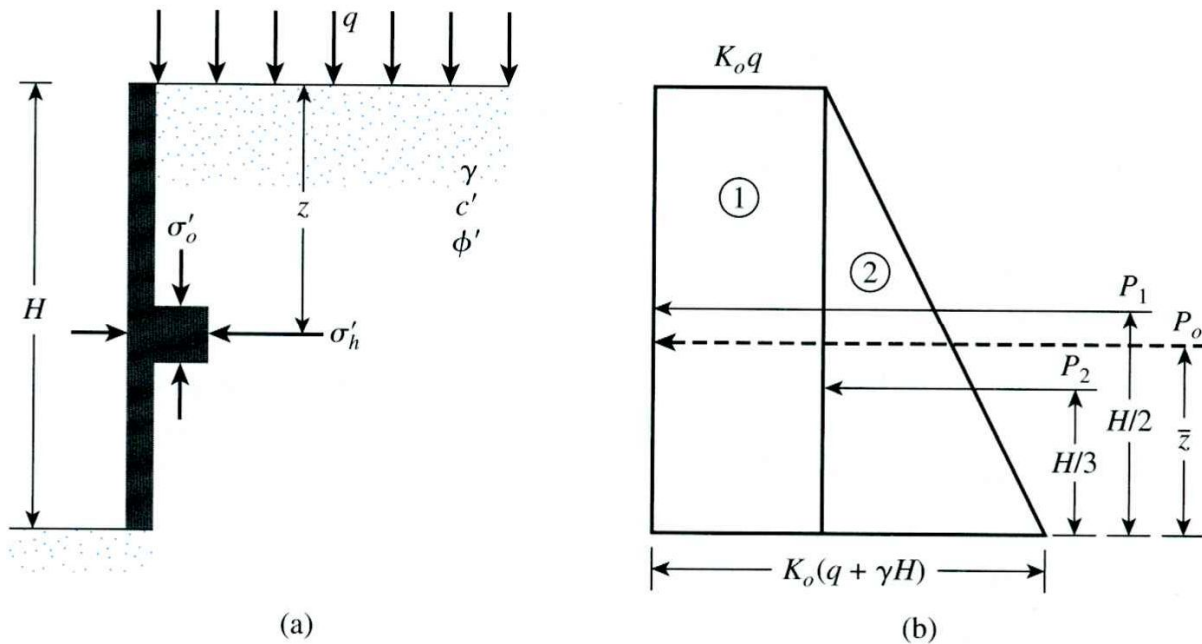


Figure 2.2 At-rest earth pressure

If the water table is located at a depth  $z < H$ , the at-rest pressure diagram shown in Figure 2.2b will have to be somewhat modified, as shown in Figure 2.3. If the effective unit weight of soil below the water table equals  $\gamma$  (i.e.,  $\gamma_{\text{sat}} - \gamma_w$ ), then

$$\text{At } z = 0 : \sigma'_h = K_o \sigma'_o = K_o q$$

$$\text{At } z = H_1 : \sigma'_h = K_o \sigma'_o = K_o (q + \gamma H_1)$$

And

$$\text{At } z = H_2, \sigma'_h = K_o \sigma'_o = K_o (q + \gamma H_1 + \gamma' H_2)$$

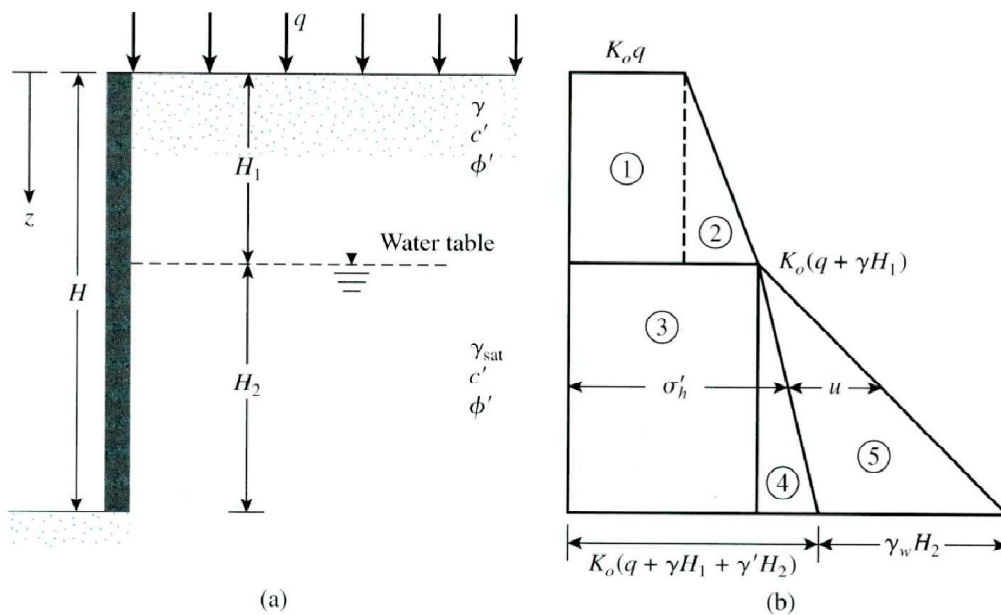
Note that in the preceding equations,  $\sigma'_o$  and  $\sigma'_h$  are effective vertical and horizontal pressures, respectively. Determining the total pressure distribution on the wall requires adding the hydrostatic pressure,  $u$ , which is zero from  $z=0$  to  $z=H_1$  and is  $\gamma_w H_2$  at  $z=H_2$ . The variation of  $\sigma'_h$  and  $u$  with depth is shown in Figure 2.3b. Hence, the total force per unit length of the wall can be determined from the area of the pressure diagram. Specifically,

$$P_o = A_1 + A_2 + A_3 + A_4 + A_5$$

where  $A$  = area of the pressure diagram.

So,

$$P_o = K_o q H_1 + \frac{1}{2} K_o \gamma H_1^2 + K_o (q + \gamma H_1) H_2 + \frac{1}{2} K_o \gamma' H_2^2 + \frac{1}{2} \gamma_w H_2^2$$



**Figure 2.3** At-rest earth pressure with water table located at a depth  $z < H$

## 2.3 Active Pressure

### 2.3.1 Rankine Active Earth Pressure

The lateral earth pressure described in Section 2.2 involves walls that do not yield at all. However, if a wall tends to move away from the soil a distance  $\Delta x$  as shown in Figure 1.4a, the soil pressure on the wall at any depth will decrease. For a wall that is *frictionless*, the horizontal stress,  $\sigma'_h$ , at depth  $z$  will equal  $K_o\sigma'_o (= K_o\gamma z)$  when  $\Delta x$  is zero. However, with  $\Delta x > 0$ ,  $\sigma'_h$  will be less than  $K_o\sigma'_o$ .

The Mohr's circles corresponding to wall displacements of  $\Delta x = 0$  and  $\Delta x > 0$  are shown as circles *a* and *b*, respectively, in Figure 2.4b. If the displacement of the wall,  $\Delta x$ , continues to increase, the corresponding Mohr's circle eventually will just touch the Mohr–Coulomb failure envelope defined by the equation

$$S = c' + \sigma' \tan \phi'$$

This circle, marked *c* in the figure, represents the failure condition in the soil mass; the horizontal stress then equals  $\sigma'_a$ , referred to as the **Rankine active pressure**. The *slip lines* (failure planes) in the soil mass will then make angles of  $\pm \left(45 + \frac{\phi'}{2}\right)$  with the horizontal, as shown in Figure 2.4a.

Equation (2.7) relates the principal stresses for a Mohr's circle that touches the Mohr–Coulomb failure envelope:

$$\sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2}\right) + 2c' \tan \left(45 + \frac{\phi'}{2}\right) \quad (2.7)$$

For the Mohr's circle *c* in Figure 2.4b,

Major principle stress:  $\sigma'_1 = \sigma'_o$   
and

Minor principle stress:  $\sigma'_3 = \sigma'_a$

Thus,

$$\sigma'_o = \sigma'_a \tan^2 \left(45 + \frac{\phi'}{2}\right) + 2c' \tan \left(45 + \frac{\phi'}{2}\right)$$

$$\sigma'_a = \frac{\sigma'_o}{\tan^2 \left(45 + \frac{\phi'}{2}\right)} - \frac{2c'}{\tan \left(45 + \frac{\phi'}{2}\right)}$$

or

$$\begin{aligned}\sigma'_a &= \sigma'_o \tan^2 \left( 45 - \frac{\phi'}{2} \right) - 2c' \tan \left( 45 - \frac{\phi'}{2} \right) \\ &= \sigma'_o K_a - 2c' \sqrt{K_a}\end{aligned}\quad (2.8)$$

where  $K_a = \tan^2 \left( 45 - \frac{\phi'}{2} \right) =$  Rankine active pressure coefficient.

The variation of the active pressure with depth for the wall shown in Figure 2.4a is given in Figure 2.4c. Note that  $\sigma'_o = 0$  at  $z=0$  and  $\sigma'_o = \gamma H$  at  $z=H$ . the pressure distribution shows that at  $z=0$  the active pressure equals  $-2c' \sqrt{K_a}$ , indicating a tensile stress that decreases with depth and becomes zero at a depth  $z= z_c$ , or

$$\gamma z_c K_a - 2c' \sqrt{K_a} = 0$$

And

$$z_c = \frac{2c'}{\gamma \sqrt{K_a}} \quad (2.9)$$

The depth  $z_c$  is usually referred to as the depth of tensile crack, because the tensile stress in the soil will eventually cause a crack along the soil-wall interface.

Thus, the total Rankine active force per unit length of the wall before the tensile crack occurs is

$$\begin{aligned}P_a &= \int_0^H \sigma'_a dz = \int_0^H \gamma z K_a dz - \int_0^H 2c' \sqrt{K_a} dz \\ &= \frac{1}{2} \gamma H^2 K_a - 2c' H \sqrt{K_a}\end{aligned}\quad (2.10)$$

After the tensile crack appears, the force per unit length on the wall will be caused only by the pressure distribution between depths  $z= z_c$  and  $z= H$  as shown by the hatched area in Figure 2.4c. This force may be expressed as

$$P_a = \frac{1}{2} (H - z_c) (\gamma H K_a - 2c' \sqrt{K_a}) \quad (2.11)$$

$$P_a = \frac{1}{2} \left( H - \frac{2c'}{\gamma \sqrt{K_a}} \right) (\gamma H K_a - 2c' \sqrt{K_a}) \quad (2.12)$$



However, it is important to realize that the active earth pressure condition will be reached only if the wall is allowed to “yield” sufficiently. The necessary amount of outward displacement of the wall is about  $0.001H$  to  $0.004H$  for granular soil backfills and about  $0.01H$  to  $0.04H$  for cohesive soil backfills.

Note further that if the *total stress* shear strength parameters ( $c$ ,  $\phi$ ) were used, an equation similar to Eq. (2.9) could have been derived, namely

$$\sigma_a = \sigma_o \tan^2 \left( 45 - \frac{\phi'}{2} \right) - 2c \tan \left( 45 - \frac{\phi'}{2} \right)$$

**Example 2.1**

**Example 2.2**

**Example 2.3**

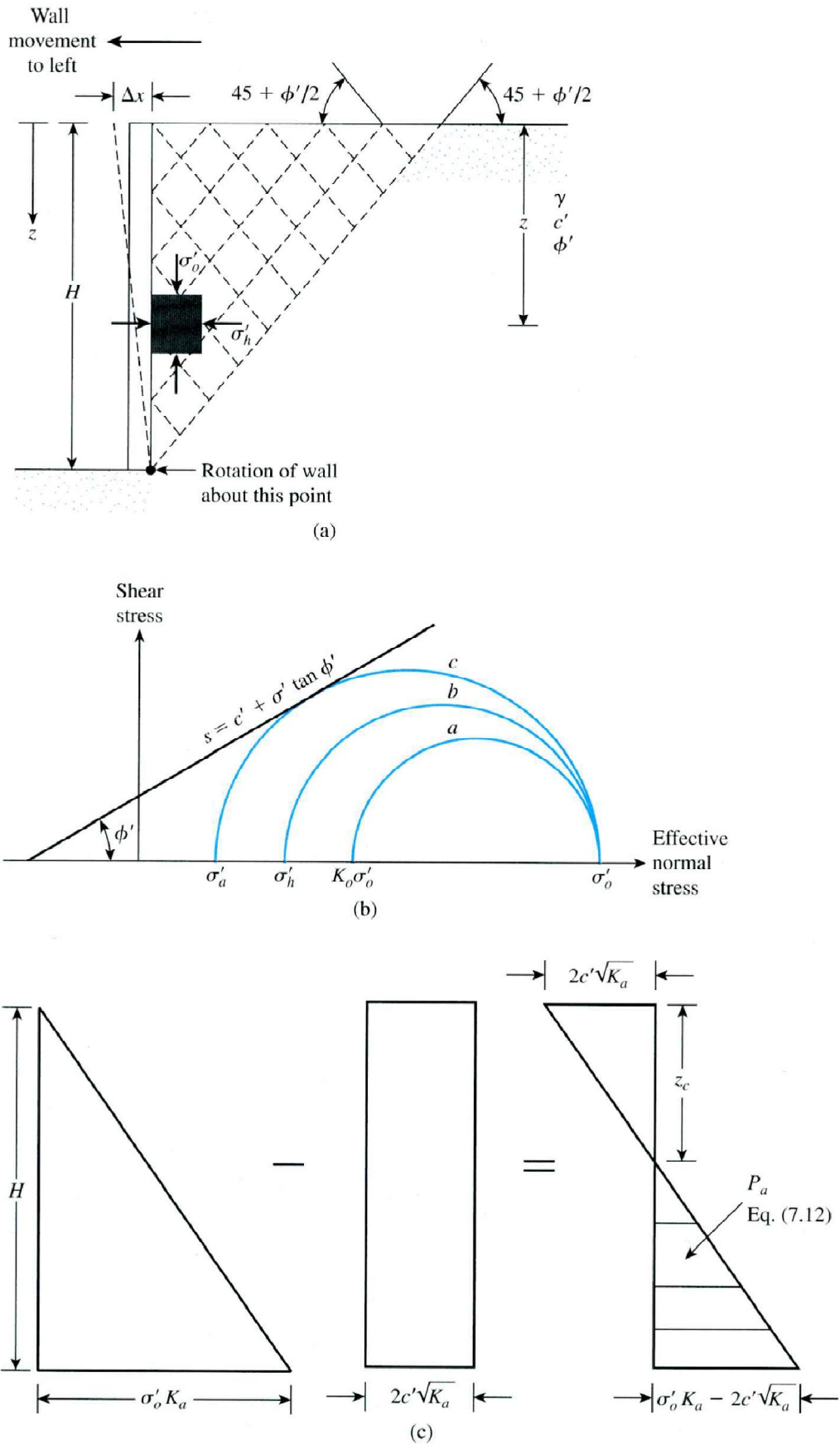


Figure 2.4 Rankine active pressure

### 2.3.2 Rankine Active Earth Pressure for Inclined Backfill

If the backfill of a frictionless retaining wall is a granular soil ( $c' = 0$ ) and rises at an angle  $\alpha$  with respect to the horizontal (see Figure 2.5), the active earth-pressure coefficient may be expressed in the form

$$K_a = \cos\alpha \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi'}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi'}} \quad (2.13)$$

where  $\phi'$  = angle of friction of soil.

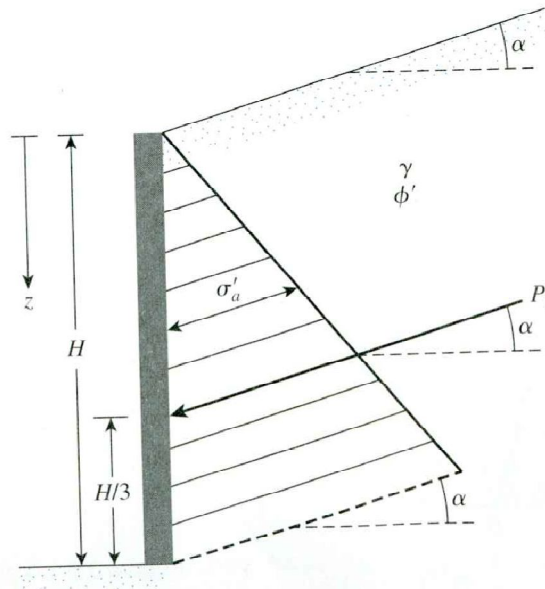
At any depth  $z$ , the Rankine active pressure may be expressed as

$$\sigma'_a = \gamma H^2 K_a \quad (2.14)$$

Also, the total force per unit length of the wall is

$$P_a = 1/2 \gamma H^2 K_a \quad (2.15)$$

Note that, in this case, the direction of the resultant force  $P_a$  is *inclined at an angle with the horizontal* and intersects the wall at a distance  $H/3$  from the base of the wall. **Table 2.1** presents the values of  $K_a$  (active earth pressure) for various values of  $\alpha$  and  $\phi'$ .



**Figure 2.5** Notations for active pressure—Eqs. (2.13), (2.14), (2.15)

**Table 2.1** Values of  $K_a$  [Eq. (2.13)]

$\alpha$ (deg)	$\phi'$ (deg) →												
	28	29	30	31	32	33	34	35	36	37	38	39	40
0	0.3610	0.3470	0.3333	0.3201	0.3073	0.2948	0.2827	0.2710	0.2596	0.2486	0.2379	0.2275	0.2174
1	0.3612	0.3471	0.3335	0.3202	0.3074	0.2949	0.2828	0.2711	0.2597	0.2487	0.2380	0.2276	0.2175
2	0.3618	0.3476	0.3339	0.3207	0.3078	0.2953	0.2832	0.2714	0.2600	0.2489	0.2382	0.2278	0.2177
3	0.3627	0.3485	0.3347	0.3214	0.3084	0.2959	0.2837	0.2719	0.2605	0.2494	0.2386	0.2282	0.2181
4	0.3639	0.3496	0.3358	0.3224	0.3094	0.2967	0.2845	0.2726	0.2611	0.2500	0.2392	0.2287	0.2186
5	0.3656	0.3512	0.3372	0.3237	0.3105	0.2978	0.2855	0.2736	0.2620	0.2508	0.2399	0.2294	0.2192
6	0.3676	0.3531	0.3389	0.3253	0.3120	0.2992	0.2868	0.2747	0.2631	0.2518	0.2409	0.2303	0.2200
7	0.3701	0.3553	0.3410	0.3272	0.3138	0.3008	0.2883	0.2761	0.2644	0.2530	0.2420	0.2313	0.2209
8	0.3730	0.3580	0.3435	0.3294	0.3159	0.3027	0.2900	0.2778	0.2659	0.2544	0.2432	0.2325	0.2220
9	0.3764	0.3611	0.3463	0.3320	0.3182	0.3049	0.2921	0.2796	0.2676	0.2560	0.2447	0.2338	0.2233
10	0.3802	0.3646	0.3495	0.3350	0.3210	0.3074	0.2944	0.2818	0.2696	0.2578	0.2464	0.2354	0.2247
11	0.3846	0.3686	0.3532	0.3383	0.3241	0.3103	0.2970	0.2841	0.2718	0.2598	0.2482	0.2371	0.2263
12	0.3896	0.3731	0.3573	0.3421	0.3275	0.3134	0.2999	0.2868	0.2742	0.2621	0.2503	0.2390	0.2281
13	0.3952	0.3782	0.3620	0.3464	0.3314	0.3170	0.3031	0.2898	0.2770	0.2646	0.2527	0.2412	0.2301
14	0.4015	0.3839	0.3671	0.3511	0.3357	0.3209	0.3068	0.2931	0.2800	0.2674	0.2552	0.2435	0.2322
15	0.4086	0.3903	0.3729	0.3564	0.3405	0.3253	0.3108	0.2968	0.2834	0.2705	0.2581	0.2461	0.2346
16	0.4165	0.3975	0.3794	0.3622	0.3458	0.3302	0.3152	0.3008	0.2871	0.2739	0.2612	0.2490	0.2373
17	0.4255	0.4056	0.3867	0.3688	0.3518	0.3356	0.3201	0.3053	0.2911	0.2776	0.2646	0.2521	0.2401
18	0.4357	0.4146	0.3948	0.3761	0.3584	0.3415	0.3255	0.3102	0.2956	0.2817	0.2683	0.2555	0.2433
19	0.4473	0.4249	0.4039	0.3842	0.3657	0.3481	0.3315	0.3156	0.3006	0.2862	0.2724	0.2593	0.2467
20	0.4605	0.4365	0.4142	0.3934	0.3739	0.3555	0.3381	0.3216	0.3060	0.2911	0.2769	0.2634	0.2504
21	0.4758	0.4498	0.4259	0.4037	0.3830	0.3637	0.3455	0.3283	0.3120	0.2965	0.2818	0.2678	0.2545
22	0.4936	0.4651	0.4392	0.4154	0.3934	0.3729	0.3537	0.3356	0.3186	0.3025	0.2872	0.2727	0.2590
23	0.5147	0.4829	0.4545	0.4287	0.4050	0.3832	0.3628	0.3438	0.3259	0.3091	0.2932	0.2781	0.2638
24	0.5404	0.5041	0.4724	0.4440	0.4183	0.3948	0.3731	0.3529	0.3341	0.3164	0.2997	0.2840	0.2692
25	0.5727	0.5299	0.4936	0.4619	0.4336	0.4081	0.3847	0.3631	0.3431	0.3245	0.3070	0.2905	0.2750

### 2.3.3 Coulomb's Active Earth Pressure

The Rankine active earth pressure calculations discussed in the preceding sections were based on the assumption that the wall is frictionless. In 1776, Coulomb proposed a theory for calculating the lateral earth pressure on a retaining wall with granular soil backfill. This theory takes wall friction into consideration.

To apply Coulomb's active earth pressure theory, let us consider a retaining wall with its back face inclined at an angle  $\beta$  with the horizontal, as shown in Figure 2.6a. The backfill is a granular soil that slopes at an angle  $\alpha$  with the horizontal. Also, let  $\delta'$  be the angle of friction between the soil and the wall (i.e., the angle of wall friction).

Under active pressure, the wall will move away from the soil mass (to the left in the figure). Coulomb assumed that, in such a case, the failure surface in the soil mass would be a plane (e.g.,  $BC_1, BC_2, \dots$ ). So, to find the active force, consider a possible soil failure wedge  $ABC_1$ . The forces acting on this wedge (per unit length at right angles to the cross section shown) are as follows:

1. The weight of the wedge,  $W$ .
2. The resultant,  $R$ , of the normal and resisting shear forces along the surface,  $BC_1$ .

The force  $R$  will be inclined at an angle to the normal drawn to  $BC_1$ .

3. The active force per unit length of the wall,  $P_a$ , which will be inclined at an angle  $\delta'$  to the normal drawn to the back face of the wall.

For equilibrium purposes, a force triangle can be drawn, as shown in Figure 2.6b. Note that  $\theta_1$  is the angle that  $BC_1$  makes with the horizontal. Because the magnitude of  $W$ , as well as the directions of all three forces, are known, the value of  $P_a$  can now be determined. Similarly, the active forces of other trial wedges, such as  $ABC_2$ ,  $ABC_3$ , ..., can be determined. The maximum value of  $P_a$  thus determined is Coulomb's active force (see top part of Figure 2.7), which may be expressed as

$$P_a = 1/2 \gamma H^2 K_a \quad (2.16)$$

Where

$K_a$  = Coulomb's active earth-pressure coefficient

$$= \frac{\sin^2(\beta - \phi')}{\sin^2 \beta \sin(\beta - \delta') \left[ 1 + \sqrt{\frac{\sin(\phi' + \delta') \sin(\phi' - \alpha)}{\sin(\beta + \delta') \sin(\alpha + \beta)}} \right]^2} \quad (2.17)$$

and  $H$  = height of the wall.

The values of the active earth pressure coefficient,  $K_a$ , for a vertical retaining wall ( $\beta = 90^\circ$ ) with horizontal backfill ( $\alpha = 0^\circ$ ) are given in Table 2.2. Note that the line of action of the resultant force ( $P_a$ ) will act at a distance  $H/3$  above the base of the wall and will be inclined at an angle  $\delta'$  to the normal drawn to the back of the wall.

In the actual design of retaining walls, the value of the wall friction angle  $\delta'$  is assumed to be between  $\phi'/2$  and  $2/3\phi'$ . The active earth pressure coefficients for various values of  $\phi'$ ,  $\alpha$ , and  $\beta$  with  $\phi'/2$  and  $2/3\phi'$  are respectively given in Tables 2.3 and 2.4. These coefficients are very useful design considerations.

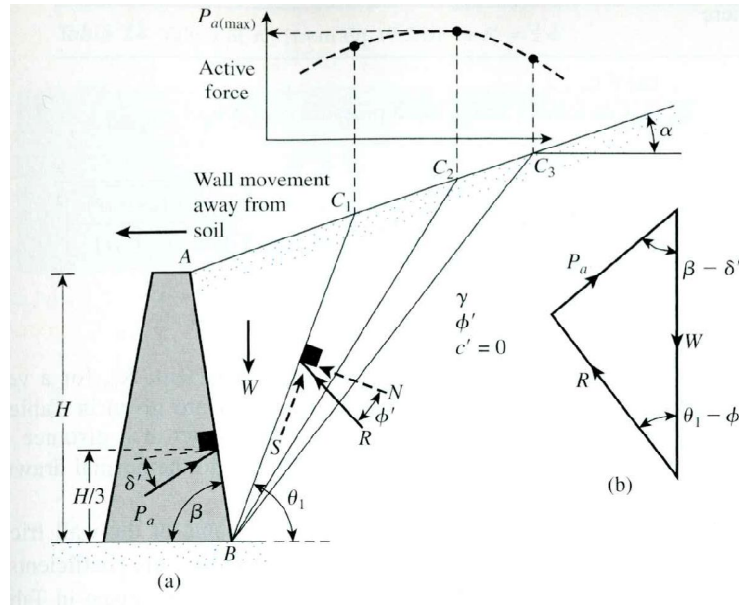


Figure 2.6 Coulomb's active pressure

Table 2.2 Values of  $K_a$  Eq(2.17) for  $\beta=90^\circ$  and  $\alpha=0^\circ$

$\phi'$ (deg)	$\delta'$ (deg)					
	0	5	10	15	20	25
28	0.3610	0.3448	0.3330	0.3251	0.3203	0.3186
30	0.3333	0.3189	0.3085	0.3014	0.2973	0.2956
32	0.3073	0.2945	0.2853	0.2791	0.2755	0.2745
34	0.2827	0.2714	0.2633	0.2579	0.2549	0.2542
36	0.2596	0.2497	0.2426	0.2379	0.2354	0.2350
38	0.2379	0.2292	0.2230	0.2190	0.2169	0.2167
40	0.2174	0.2098	0.2045	0.2011	0.1994	0.1995
42	0.1982	0.1916	0.1870	0.1841	0.1828	0.1831

Table 2.3 Values of  $K_a$  Eq(2.17) for  $\delta' = 2/3 \phi'$ 

$\alpha$ (deg)	$\phi'$ (deg)	$\beta$ (deg)					
		90	85	80	75	70	65
0	28	0.3213	0.3588	0.4007	0.4481	0.5026	0.5662
	29	0.3091	0.3467	0.3886	0.4362	0.4908	0.5547
	30	0.2973	0.3349	0.3769	0.4245	0.4794	0.5435
	31	0.2860	0.3235	0.3655	0.4133	0.4682	0.5326
	32	0.2750	0.3125	0.3545	0.4023	0.4574	0.5220
	33	0.2645	0.3019	0.3439	0.3917	0.4469	0.5117
	34	0.2543	0.2916	0.3335	0.3813	0.4367	0.5017
	35	0.2444	0.2816	0.3235	0.3713	0.4267	0.4919
	36	0.2349	0.2719	0.3137	0.3615	0.4170	0.4824
	37	0.2257	0.2626	0.3042	0.3520	0.4075	0.4732
	38	0.2168	0.2535	0.2950	0.3427	0.3983	0.4641
	39	0.2082	0.2447	0.2861	0.3337	0.3894	0.4553
	40	0.1998	0.2361	0.2774	0.3249	0.3806	0.4468
	41	0.1918	0.2278	0.2689	0.3164	0.3721	0.4384
5	28	0.1840	0.2197	0.2606	0.3080	0.3637	0.4302
	29	0.3431	0.3845	0.4311	0.4843	0.5461	0.6190
	30	0.3295	0.3709	0.4175	0.4707	0.5325	0.6056
	31	0.3165	0.3578	0.4043	0.4575	0.5194	0.5926
	32	0.3039	0.3451	0.3916	0.4447	0.5067	0.5800
	33	0.2919	0.3329	0.3792	0.4324	0.4943	0.5677
	34	0.2803	0.3211	0.3673	0.4204	0.4823	0.5558
	35	0.2691	0.3097	0.3558	0.4088	0.4707	0.5443
	36	0.2583	0.2987	0.3446	0.3975	0.4594	0.5330
	37	0.2479	0.2881	0.3338	0.3866	0.4484	0.5221
	38	0.2379	0.2778	0.3233	0.3759	0.4377	0.5115
	39	0.2282	0.2679	0.3131	0.3656	0.4273	0.5012
	40	0.2188	0.2582	0.3033	0.3556	0.4172	0.4911
	41	0.2098	0.2489	0.2937	0.3458	0.4074	0.4813
10	28	0.2011	0.2398	0.2844	0.3363	0.3978	0.4718
	29	0.1927	0.2311	0.2753	0.3271	0.3884	0.4625
	30	0.3702	0.4164	0.4686	0.5287	0.5992	0.6834
	31	0.3548	0.4007	0.4528	0.5128	0.5831	0.6672
	32	0.3400	0.3857	0.4376	0.4974	0.5676	0.6516
	33	0.3259	0.3713	0.4230	0.4826	0.5526	0.6365
	34	0.3123	0.3575	0.4089	0.4683	0.5382	0.6219
	35	0.2993	0.3442	0.3953	0.4545	0.5242	0.6078
	36	0.2868	0.3314	0.3822	0.4412	0.5107	0.5942
	37	0.2748	0.3190	0.3696	0.4283	0.4976	0.5810
	38	0.2633	0.3072	0.3574	0.4158	0.4849	0.5682
	39	0.2522	0.2957	0.3456	0.4037	0.4726	0.5558
	40	0.2415	0.2846	0.3342	0.3920	0.4607	0.5437
	41	0.2313	0.2740	0.3231	0.3807	0.4491	0.5321
15	28	0.2214	0.2636	0.3125	0.3697	0.4379	0.5207
	29	0.2119	0.2537	0.3021	0.3590	0.4270	0.5097
	30	0.2027	0.2441	0.2921	0.3487	0.4164	0.4990
	31	0.4065	0.4585	0.5179	0.5868	0.6685	0.7670

(continued)

Table 2.3 Values of  $K_a$  Eq(2.17) for  $\delta' = 2/3 \phi'$ 

$\alpha$ (deg)	$\phi'$ (deg)	$\beta$ (deg)					
		90	85	80	75	70	65
20	29	0.3881	0.4397	0.4987	0.5672	0.6483	0.7463
	30	0.3707	0.4219	0.4804	0.5484	0.6291	0.7265
	31	0.3541	0.4049	0.4629	0.5305	0.6106	0.7076
	32	0.3384	0.3887	0.4462	0.5133	0.5930	0.6895
	33	0.3234	0.3732	0.4303	0.4969	0.5761	0.6721
	34	0.3091	0.3583	0.4150	0.4811	0.5598	0.6554
	35	0.2954	0.3442	0.4003	0.4659	0.5442	0.6393
	36	0.2823	0.3306	0.3862	0.4513	0.5291	0.6238
	37	0.2698	0.3175	0.3726	0.4373	0.5146	0.6089
	38	0.2578	0.3050	0.3595	0.4237	0.5006	0.5945
	39	0.2463	0.2929	0.3470	0.4106	0.4871	0.5805
	40	0.2353	0.2813	0.3348	0.3980	0.4740	0.5671
	41	0.2247	0.2702	0.3231	0.3858	0.4613	0.5541
	42	0.2146	0.2594	0.3118	0.3740	0.4491	0.5415
	28	0.4602	0.5205	0.5900	0.6714	0.7689	0.8880
	29	0.4364	0.4958	0.5642	0.6445	0.7406	0.8581
	30	0.4142	0.4728	0.5403	0.6195	0.7144	0.8303
	31	0.3935	0.4513	0.5179	0.5961	0.6898	0.8043
	32	0.3742	0.4311	0.4968	0.5741	0.6666	0.7799
	33	0.3559	0.4121	0.4769	0.5532	0.6448	0.7569
	34	0.3388	0.3941	0.4581	0.5335	0.6241	0.7351
	35	0.3225	0.3771	0.4402	0.5148	0.6044	0.7144
	36	0.3071	0.3609	0.4233	0.4969	0.5856	0.6947
	37	0.2925	0.3455	0.4071	0.4799	0.5677	0.6759
38	0.2787	0.3308	0.3916	0.4636	0.5506	0.6579	
39	0.2654	0.3168	0.3768	0.4480	0.5342	0.6407	
40	0.2529	0.3034	0.3626	0.4331	0.5185	0.6242	
41	0.2408	0.2906	0.3490	0.4187	0.5033	0.6083	
42	0.2294	0.2784	0.3360	0.4049	0.4888	0.5930	

Table 2.4 Values of  $K_a$  Eq(2.17) for  $\delta' = 1/2 \phi'$ 

$\alpha$ (deg)	$\phi'$ (deg)	90	85	80	75	70	65
0	28	0.3264	0.3629	0.4034	0.4490	0.5011	0.5616
	29	0.3137	0.3502	0.3907	0.4363	0.4886	0.5492
	30	0.3014	0.3379	0.3784	0.4241	0.4764	0.5371
	31	0.2896	0.3260	0.3665	0.4121	0.4645	0.5253
	32	0.2782	0.3145	0.3549	0.4005	0.4529	0.5137
	33	0.2671	0.3033	0.3436	0.3892	0.4415	0.5025
	34	0.2564	0.2925	0.3327	0.3782	0.4305	0.4915
	35	0.2461	0.2820	0.3221	0.3675	0.4197	0.4807
36	0.2362	0.2718	0.3118	0.3571	0.4092	0.4702	



Table 2.4 Values of  $K_a$  Eq(2.17) for  $\delta' = 1/2 \phi'$ 

$\alpha$ (deg)	$\phi'$ (deg)	$\beta$ (deg)					
		90	85	80	75	70	65
5	37	0.2265	0.2620	0.3017	0.3469	0.3990	0.4599
	38	0.2172	0.2524	0.2920	0.3370	0.3890	0.4498
	39	0.2081	0.2431	0.2825	0.3273	0.3792	0.4400
	40	0.1994	0.2341	0.2732	0.3179	0.3696	0.4304
	41	0.1909	0.2253	0.2642	0.3087	0.3602	0.4209
	42	0.1828	0.2168	0.2554	0.2997	0.3511	0.4177
	28	0.3477	0.3879	0.4327	0.4837	0.5425	0.6115
	29	0.3337	0.3737	0.4185	0.4694	0.5282	0.5972
	30	0.3202	0.3601	0.4048	0.4556	0.5144	0.5833
	31	0.3072	0.3470	0.3915	0.4422	0.5009	0.5698
	32	0.2946	0.3342	0.3787	0.4292	0.4878	0.5566
	33	0.2825	0.3219	0.3662	0.4166	0.4750	0.5437
	34	0.2709	0.3101	0.3541	0.4043	0.4626	0.5312
	35	0.2596	0.2986	0.3424	0.3924	0.4505	0.5190
	36	0.2488	0.2874	0.3310	0.3808	0.4387	0.5070
	37	0.2383	0.2767	0.3199	0.3695	0.4272	0.4954
38	0.2282	0.2662	0.3092	0.3585	0.4160	0.4840	
39	0.2185	0.2561	0.2988	0.3478	0.4050	0.4729	
40	0.2090	0.2463	0.2887	0.3374	0.3944	0.4620	
10	41	0.1999	0.2368	0.2788	0.3273	0.3840	0.4514
	42	0.1911	0.2276	0.2693	0.3174	0.3738	0.4410
	28	0.3743	0.4187	0.4688	0.5261	0.5928	0.6719
	29	0.3584	0.4026	0.4525	0.5096	0.5761	0.6549
	30	0.3432	0.3872	0.4368	0.4936	0.5599	0.6385
	31	0.3286	0.3723	0.4217	0.4782	0.5442	0.6225
	32	0.3145	0.3580	0.4071	0.4633	0.5290	0.6071
	33	0.3011	0.3442	0.3930	0.4489	0.5143	0.5920
	34	0.2881	0.3309	0.3793	0.4350	0.5000	0.5775
	35	0.2757	0.3181	0.3662	0.4215	0.4862	0.5633
	36	0.2637	0.3058	0.3534	0.4084	0.4727	0.5495
	37	0.2522	0.2938	0.3411	0.3957	0.4597	0.5361
	38	0.2412	0.2823	0.3292	0.3833	0.4470	0.5230
	39	0.2305	0.2712	0.3176	0.3714	0.4346	0.5103
	40	0.2202	0.2604	0.3064	0.3597	0.4226	0.4979
	41	0.2103	0.2500	0.2956	0.3484	0.4109	0.4858
42	0.2007	0.2400	0.2850	0.3375	0.3995	0.4740	
15	28	0.4095	0.4594	0.5159	0.5812	0.6579	0.7498
	29	0.3908	0.4402	0.4964	0.5611	0.6373	0.7284
	30	0.3730	0.4220	0.4777	0.5419	0.6175	0.7080
	31	0.3560	0.4046	0.4598	0.5235	0.5985	0.6884
	32	0.3398	0.3880	0.4427	0.5059	0.5803	0.6695
	33	0.3244	0.3721	0.4262	0.4889	0.5627	0.6513
	34	0.3097	0.3568	0.4105	0.4726	0.5458	0.6338
	35	0.2956	0.3422	0.3953	0.4569	0.5295	0.6168

(continued)

## 2.4 Passive Pressure

### 2.4.1 Rankine Passive Earth Pressure

Figure 2.8a shows a vertical frictionless retaining wall with a horizontal backfill. At depth  $z$ , the effective vertical pressure on a soil element is  $\sigma'_o = \gamma z$ . Initially, if the wall does not yield at all, the lateral stress at that depth will be  $\sigma'_h = K_o \sigma'_o$ . This state of stress is illustrated by the Mohr's circle  $a$  in Figure 2.8b. Now, if the wall is pushed into the soil mass by an amount  $\Delta x$  as shown in Figure 2.8a, the vertical stress at depth  $z$  will stay the same; however, the horizontal stress will increase. Thus,  $\sigma'_h$  will be greater than  $K_o \sigma'_o$ . The state of stress can now be represented by the Mohr's circle  $b$  in Figure 2.8b. If the wall moves farther inward (i.e., is increased still more), the stresses at depth  $z$  will ultimately reach the state represented by Mohr's circle  $c$ . Note that this Mohr's circle touches the Mohr–Coulomb failure envelope, which implies that the soil behind the wall will fail by being pushed upward. The horizontal stress,  $\sigma'_h$ , at this point is referred to as the **Rankine passive pressure**, or  $\sigma'_h = \sigma'_p$ .

For Mohr's circle  $c$  in Figure 2.8b, the major principal stress is  $\sigma'_p$  and the minor principal stress is  $\sigma'_o$ . Substituting these quantities into Eq. (2.8) yields

$$\sigma'_p = \sigma'_o \tan^2 \left( 45 + \frac{\phi'}{2} \right) + 2c' \tan \left( 45 + \frac{\phi'}{2} \right) \quad (2.18)$$

$K_p$  = Rankine passive earth-pressure coefficient

$$K_p = \tan^2 \left( 45 + \frac{\phi'}{2} \right) \quad (2.19)$$

$$\sigma'_p = \sigma'_o K_p + 2c' \sqrt{K_p} \quad (2.20)$$

Equation (2.20) produces (Figure 2.18c), the passive pressure diagram for the wall shown in Figure 2.18a. Note that at  $z=0$

$$\sigma'_o = 0 \text{ and } \sigma'_p = 2c' \sqrt{K_p}$$

and at  $z=H$

$$\sigma'_o = \gamma H \text{ and } \sigma'_p = \gamma H K_p + 2c' \sqrt{K_p}$$

The passive force per unit length of the wall can be determined from the area of the pressure diagram, or

$$P_p = \frac{1}{2} \gamma H^2 K_p + 2c' H \sqrt{K_p} \quad (2.21)$$

The approximate magnitudes of the wall movements,  $\Delta x$ , required to develop failure under passive conditions are as follows:

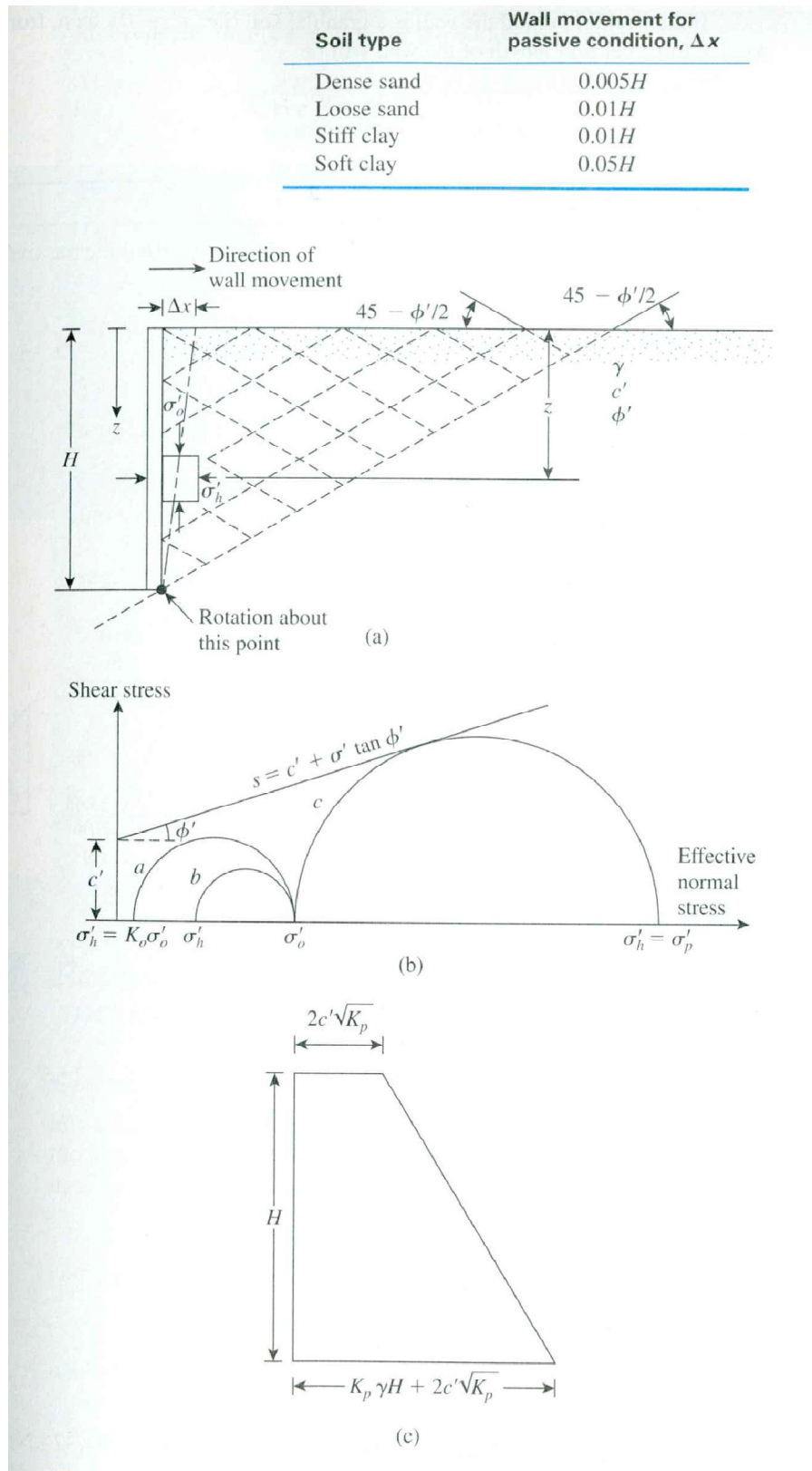


Figure 2.8 Rankine passive pressure

If the backfill behind the wall is a granular soil (i.e.,  $c'=0$ ), then, from Eq. (2.21), the passive force per unit length of the wall will be

$$P_p = \frac{1}{2} \gamma H^2 K_p \quad (2.22)$$

### 2.4.2 Rankine Passive Earth Pressure for Inclined Backfill

For a frictionless vertical retaining wall (Figure 2.5) with a *granular backfill* ( $c'=0$ ), the Rankine passive pressure at any depth can be determined in a manner similar to that done in the case of active pressure in Section 2.3.2. The pressure is

$$\sigma'_p = \gamma z K_p \quad (2.23)$$

And the passive force is

$$P_p = 1/2 \gamma H^2 K_p \quad (2.24)$$

where

$$K_p = \cos \alpha \frac{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi'}}{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi'}} \quad (2.25)$$

As in the case of the active force, the resultant force,  $P_p$ , is inclined at an angle  $\alpha$  with the horizontal and intersects the wall at a distance  $H/3$  from the bottom of the wall. The values of  $K_p$  (the passive earth pressure coefficient) for various values of  $\alpha$  and  $\phi'$  are given in **Table 2.6**.

**Table 2.6** Passive Earth Pressure Coefficient [from Eq. (2.25)]

$\downarrow \alpha$ (deg)	$\phi'$ (deg) $\rightarrow$						
	28	30	32	34	36	38	40
0	2.770	3.000	3.255	3.537	3.852	4.204	4.599
5	2.715	2.943	3.196	3.476	3.788	4.136	4.527
10	2.551	2.775	3.022	3.295	3.598	3.937	4.316
15	2.284	2.502	2.740	3.003	3.293	3.615	3.977
20	1.918	2.132	2.362	2.612	2.886	3.189	3.526
25	1.434	1.664	1.894	2.135	2.394	2.676	2.987

### 2.4.3 Coulomb's Passive Earth Pressure

Coulomb (1776) also presented an analysis for determining the passive earth pressure (i.e., when the wall moves *into* the soil mass) for walls possessing friction ( $\delta'$ =angle of wall friction) and retaining a granular backfill material similar to that discussed in Section 2.3.3.

To understand the determination of Coulomb's passive force,  $P_p$ , consider the wall shown in Figure 2.9a. As in the case of active pressure, Coulomb assumed that the potential failure surface in soil is a plane. For a trial failure wedge of soil, such as  $ABC_1$ , the forces per unit length of the wall acting on the wedge are

1. The weight of the wedge,  $W$
2. The resultant,  $R$ , of the normal and shear forces on the plane and
3. The passive force,  $P_p$

Figure 2.9b shows the force triangle at equilibrium for the trial wedge  $ABC_1$ . From this force triangle, the value of  $P_p$  can be determined, because the direction of all three forces and the magnitude of one force are known.

Similar force triangles for several trial wedges, such as  $ABC_1, ABC_2, ABC_3, \dots$  can be constructed, and the corresponding values of  $P_p$  can be determined. The top part of Figure 2.9a shows the nature of variation of  $P_p$  the values for different wedges. The *minimum value of  $P_p$*  in this diagram is *Coulomb's passive force*, mathematically expressed as

$$P_a = 1/2 \gamma H^2 K_p \quad (2.26)$$

Where

$K_a$ = Coulomb's passive earth-pressure coefficient

$$= \frac{\sin^2(\beta - \delta')}{\sin^2 \beta \sin(\beta + \delta') \left[ 1 - \sqrt{\frac{\sin(\delta' + \delta') \sin(\delta' + \alpha)}{\sin(\beta + \delta') \sin(\alpha + \beta)}} \right]^2} \quad (2.27)$$

and  $H$ = height of the wall.

The values of the passive pressure coefficient,  $K_p$ , for various values of  $\delta'$  and  $\delta'$  are given in Table 2.7 ( $\beta = 90^\circ$ ,  $\alpha = 0^\circ$ ).

