CHAPTER – 2

FORCE VECTORS

CHAPTER OBJECTIVES

- To show how to add forces and resolve them into components using the Parallelogram Law.
- To express force and position in Cartesian vector form and explain how to determine the vector's magnitude and direction.
- To introduce the dot product in order to determine the angle between two vectors or the projection of one vector onto another.

2.1 Scalars and Vectors

All physical quantities in engineering mechanics are measured using either scalars or vectors.

Scalar: **A *scalar*** is any positive or negative physical quantity that can be completely defined only by its *magnitude*. Examples of scalar quantities are: length, mass, and time.

Vector: **A *vector*** is any physical quantity that requires both a *magnitude* and a d*irection* for its complete description. Examples of vectors in statics are: force, position, and moment. A vector is shown graphically by an ***arrow***. The length of the arrow represents the *magnitude* of the vector, and the angle ***θ*** between the vector and a fixed axis defines the *direction of its line of action*. The head or tip of the arrow indicates the *sense of direction* of the vector, Fig. 2–1 .

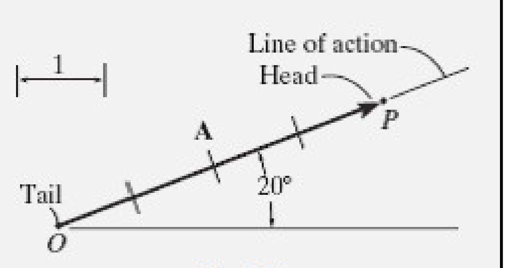
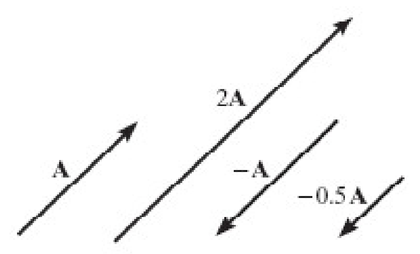


Fig. 2–1

- In print, vector quantities are represented by boldface letters such as **A** , and the magnitude of a vector is italicized, ***A*** . For handwritten work, it is often convenient to denote a vector quantity by simply drawing an arrow above it, $\vec{A}$.

2.2 Vector Operations

Multiplication and Division of a Vector by a Scalar:
If a vector is multiplied by a positive scalar, its magnitude is increased by that amount. Multiplying by a negative scalar will also change the directional sense of the vector. Graphic examples of these operations are shown in Fig. 2–2 .



Scalar multiplication and division
Fig. 2–2

Vector Addition: All vector quantities obey the *parallelogram law of addition* . To illustrate, the two " *component* " *vectors* **A** and **B** in Fig. 2–3 *a* are added to form a " *resultant* " *vector* **R** = **A** + **B** using the following procedure:

• First join the tails of the components at a point to make them concurrent, Fig. 2–3 *b* .

• From the head of **B** , draw a line parallel to **A** . Draw another line from the head of **A** that is parallel to **B** . These two lines intersect at point *P* to form the adjacent sides of a parallelogram.

• The diagonal of this parallelogram that extends to *P* forms **R** , which then represents the resultant vector **R** = **A** + **B** , Fig. 2–3 *c* .

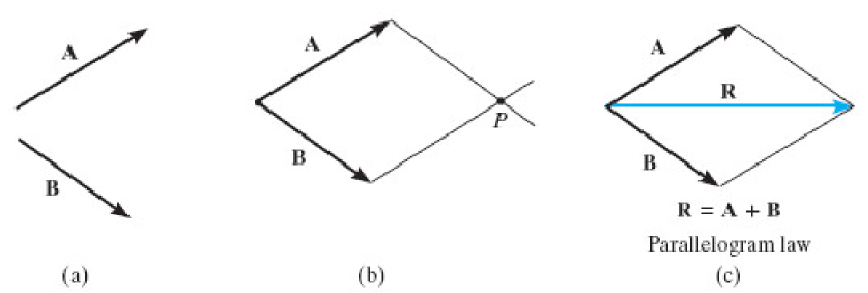
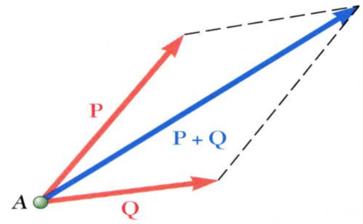
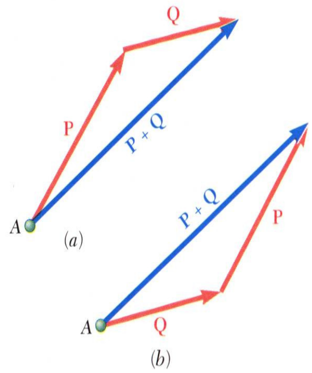


Fig. 2–3



- Trapezoid rule for vector addition.
- Triangle rule for vector addition.
- Law of cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$\vec{R} = \vec{P} + \vec{Q}$$



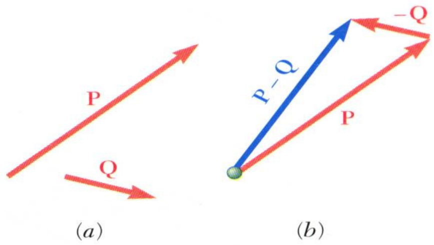
- Law of sines,

$$\frac{\sin A}{Q} = \frac{\sin B}{R} = \frac{\sin C}{A}$$

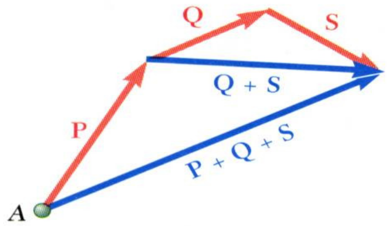
- Vector addition is commutative,

$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$

- Vector subtraction.

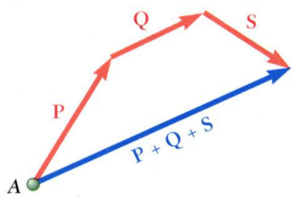


- Addition of three or more vectors through repeated application of the triangle rule.



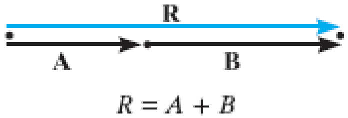
- The polygon rule for the addition of three or more vectors.
- Vector addition is associative,

$$\vec{P} + \vec{Q} + \vec{S} = \left(\vec{P} + \vec{Q}\right) + \vec{S} = \vec{P} + \left(\vec{Q} + \vec{S}\right)$$



- Multiplication of a vector by a scalar

As a special case, if the two vectors A and B are collinear , i.e., both have the same line of action, the parallelogram law reduces to an algebraic or scalar addition R = A + B .

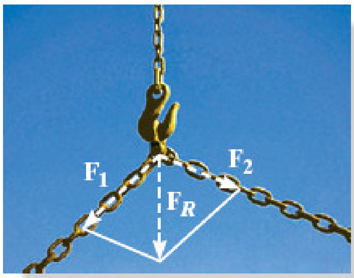


Addition of collinear vectors

2.3 Vector Addition of Forces

Force is the action of one body on another; characterized by its ***point of application, magnitude, line of action*, and *sense.*** Therefore it is a vector and it adds according to the parallelogram law. Two common problems in statics involve either finding the *resultant force*, knowing its *components,* or resolving a known force into *two components*. We will now describe how each of these problems is solved using the parallelogram law.

Finding a Resultant Force. The two component forces **F**1 and **F**2 acting on the pin in Fig. 2–7 *a* can be added together to form the resultant force **F***R* = **F**1 + **F**2 , as shown in Fig. 2–7 *b* . From this construction, or using the triangle rule, Fig. 2–7 *c* , we can apply the law of cosines or the law of sines to the triangle in order to obtain the magnitude of the resultant force and its direction.



The parallelogram law must be used to determine the resultant of the two forces acting on the hook.

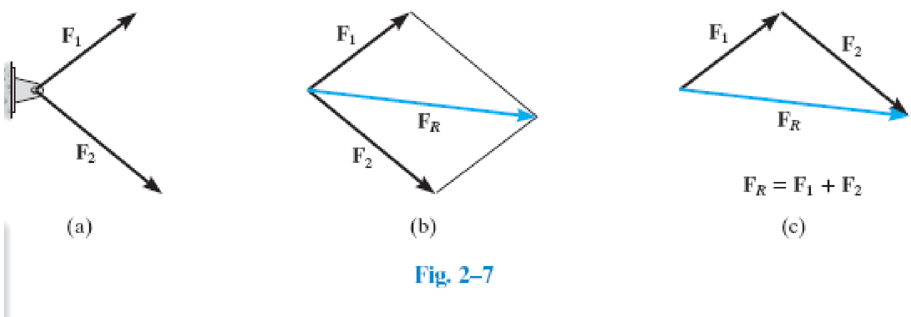
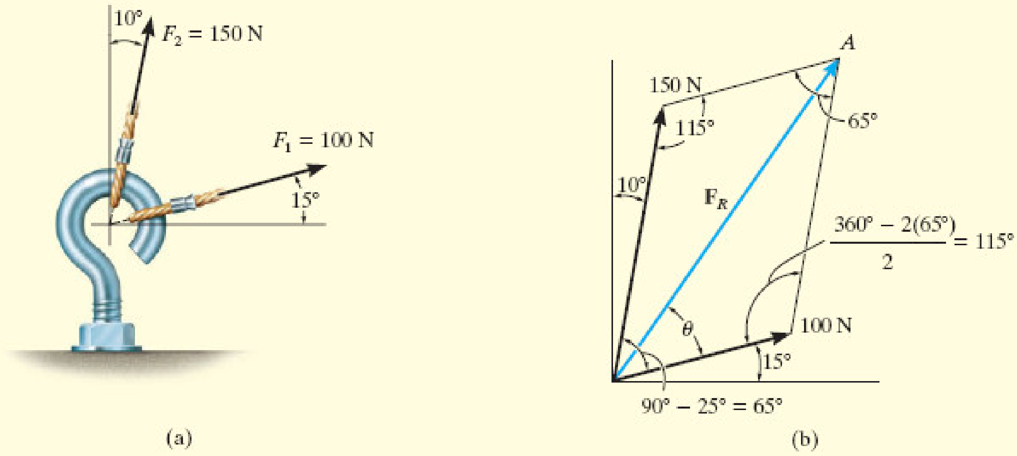


Fig. 2–7

EXAMPLE 2.1

The screw eye in Fig. 2–11*a* is subjected to two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.



SOLUTION

Parallelogram Law. The parallelogram is formed by drawing a line from the head of $\mathbf{F}_1$ that is parallel to $\mathbf{F}_2$, and another line from the head of $\mathbf{F}_2$ that is parallel to $\mathbf{F}_1$. The resultant force $\mathbf{F}_R$ extends to where these lines intersect at point *A*, Fig. 2–11*b*. The two unknowns are the magnitude of $\mathbf{F}_R$ and the angle θ (theta).

Trigonometry. From the parallelogram, the vector triangle is constructed, Fig. 2–11*c*. Using the law of cosines

$$F_R = \sqrt{(100\text{ N})^2 + (150\text{ N})^2 - 2(100\text{ N})(150\text{ N})\cos 115^\circ}$$
$$= \sqrt{10\,000 + 22\,500 - 30\,000(-0.4226)} = 212.6\text{ N}$$
$$= 213\text{ N} \qquad \textit{Ans.}$$

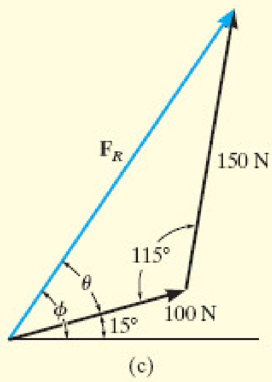


Fig. 2–11

Applying the law of sines to determine θ,

$$\frac{150\text{ N}}{\sin\theta} = \frac{212.6\text{ N}}{\sin 115^\circ} \qquad \sin\theta = \frac{150\text{ N}}{212.6\text{ N}}(\sin 115^\circ)$$
$$\theta = 39.8^\circ$$

Thus, the direction ϕ (phi) of $\mathbf{F}_R$, measured from the horizontal, is

$$\phi = 39.8^\circ + 15.0^\circ = 54.8^\circ \qquad \textit{Ans.}$$

NOTE: The results seem reasonable, since Fig. 2–11*b* shows $\mathbf{F}_R$ to have a magnitude larger than its components and a direction that is between them.

Finding the Components of a Force. Sometimes it is necessary to resolve a force into two components in order to study its pulling or pushing effect in two specific directions. For example, in Fig. 2–8 a , F is to be resolved into two components along the two members, defined by the u and v axes. In order to determine the magnitude of each component, a parallelogram is constructed first, by drawing lines starting from the tip of F , one line parallel to u , and the other line parallel to v . These lines then intersect with the v and u axes, forming a parallelogram. The force components Fu and Fv are then established by simply joining the tail of F to the intersection points on the u and v axes, Fig. 2–8 b . This parallelogram can then be reduced to a triangle, which represents the triangle rule, Fig. 2–8 c . From this, the law of sines can then be applied to determine the unknown magnitudes of the components.

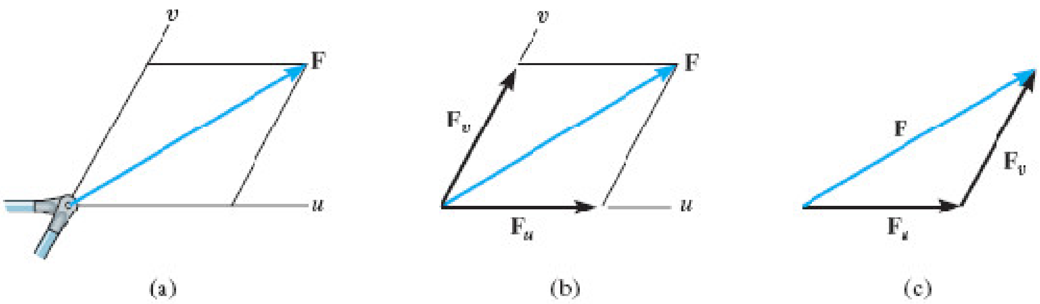
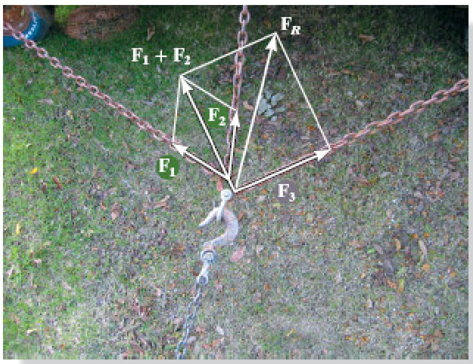


Fig. 2–8

Addition of Several Forces. If more than two forces are to be added, successive applications of the parallelogram law can be carried out in order to obtain the resultant force.



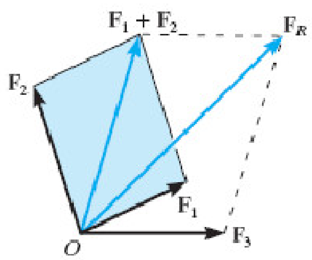
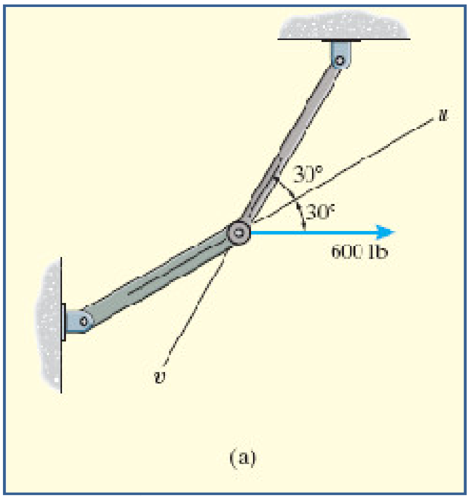


Fig. 2–9

Important Points

- A scalar is a positive or negative number.
- A vector is a quantity that has a magnitude, direction, and sense.
- Multiplication or division of a vector by a scalar will change the magnitude of the vector. The sense of the vector will change if the scalar is negative.
- As a special case, if the vectors are collinear, the resultant is formed by an algebraic or scalar addition.

EXAMPLE 2.2

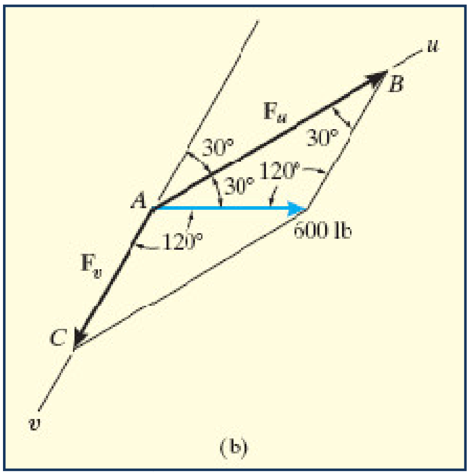


(a)

Resolve the horizontal 600-lb force in Fig. 2–12 *a* into components acting along the *u* and v axes and determine the magnitudes of these components.

Solution:

The parallelogram is constructed by extending a line from the *head* of the 600-lb force parallel to the v axis until it intersects the *u* axis at point *B* , Fig. 2–12 *b* . The arrow from *A* to *B* represents $\mathbf{F}_u$. Similarly, the line extended from the head of the 600-lb force drawn parallel to the *u* axis intersects the v axis at point *C,* which gives $\mathbf{F}_v$. The vector addition using the triangle rule is shown in Fig. 2–12 *c* . The two unknowns are the magnitudes of $\mathbf{F}_u$ and $\mathbf{F}_v$. Applying the law of sines,



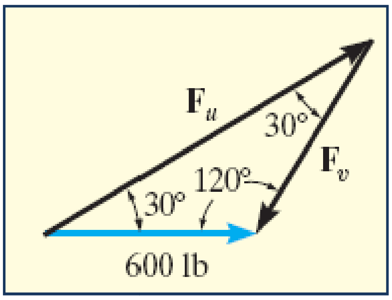
(b)

$$\frac{F_u}{\sin 120^\circ} = \frac{600\text{ lb}}{\sin 30^\circ}$$

$$F_u = 1039\text{ lb} \qquad \textit{Ans.}$$

$$\frac{F_v}{\sin 30^\circ} = \frac{600 \text{ lb}}{\sin 30^\circ}$$

$$F_v = 600 \text{ lb} \qquad \textit{Ans.}$$



(c)

NOTE: *The result for Fu shows that sometimes a component can have a greater magnitude than the resultant.*

EXAMPLE 2.3

Determine the magnitude of the component force F in Fig. 2–13*a* and the magnitude of the resultant force $\mathbf{F}_R$ if $\mathbf{F}_R$ is directed along the positive *y* axis.

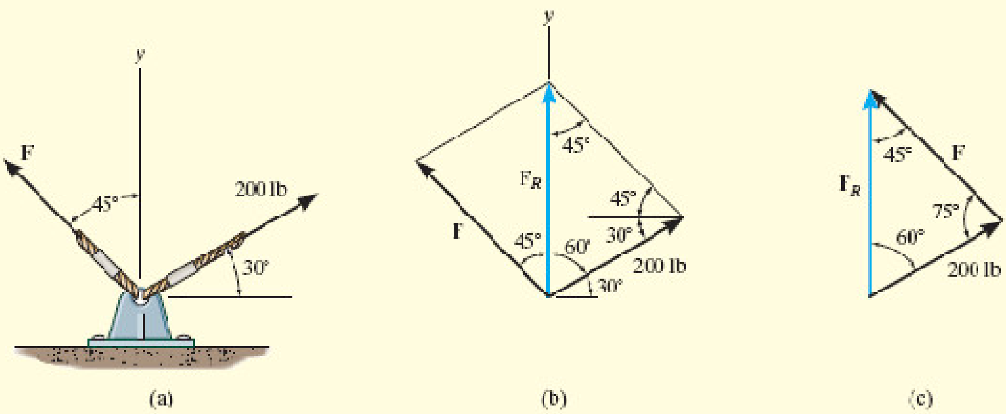


Fig. 2–13

SOLUTION: The parallelogram law of addition is shown in Fig. 2–13 b , and the triangle rule is shown in Fig. 2–13 c . The magnitudes of F_R and F are the two unknowns. They can be determined by applying the law of sines.

$$\frac{F}{\sin 60^\circ} = \frac{200 \text{ lb}}{\sin 45^\circ}$$

$$F = 245 \text{ lb}$$

$$\frac{F_R}{\sin 75^\circ} = \frac{200 \text{ lb}}{\sin 45^\circ}$$

$$F_R = 273 \text{ lb}$$

EXAMPLE 2.4

It is required that the resultant force acting on the eyebolt in Fig. 2–14*a* be directed along the positive *x* axis and that $\mathbf{F}_2$ have a *minimum* magnitude. Determine this magnitude, the angle θ, and the corresponding resultant force.

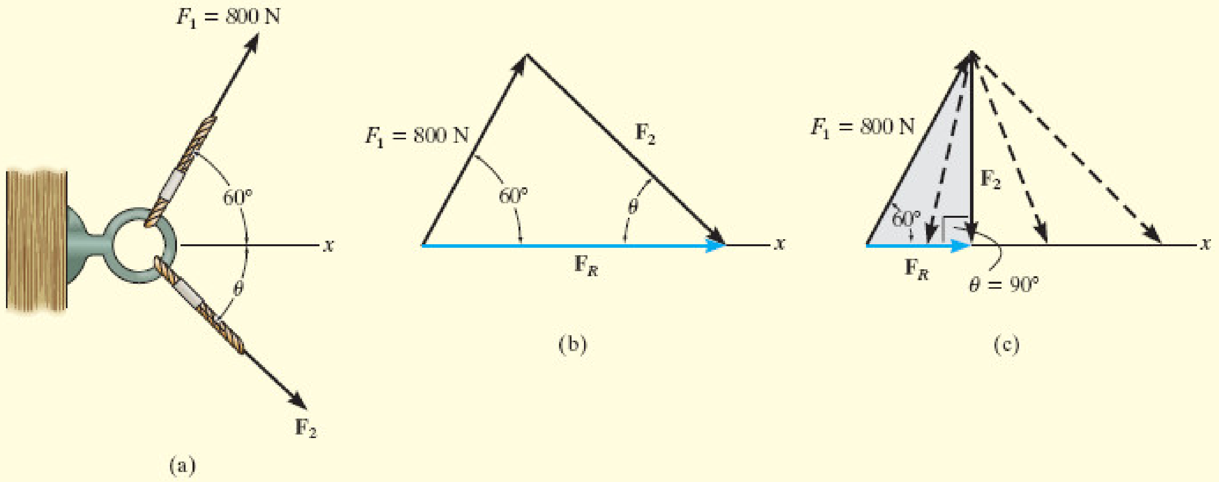
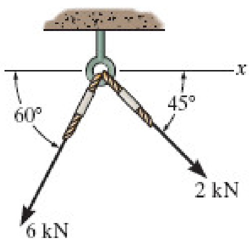


Fig. 2–14

Answers: $F_R = 400$ N , $F_2 = 693$ N

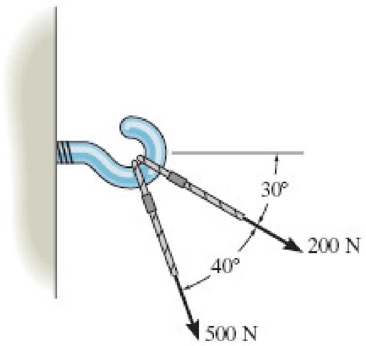
FUNDAMENTAL PROBLEMS*

F2–1. Determine the magnitude of the resultant force acting on the screw eye and its direction measured clockwise from the x axis.



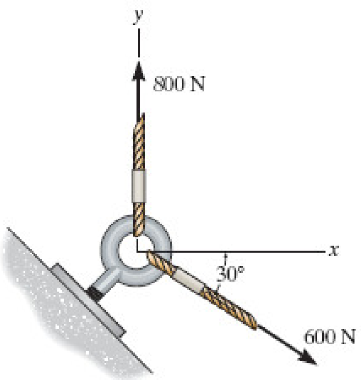
F2–1

F2–2. Two forces act on the hook. Determine the magnitude of the resultant force.



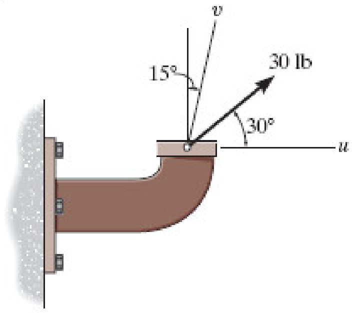
F2–2

F2–3. Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



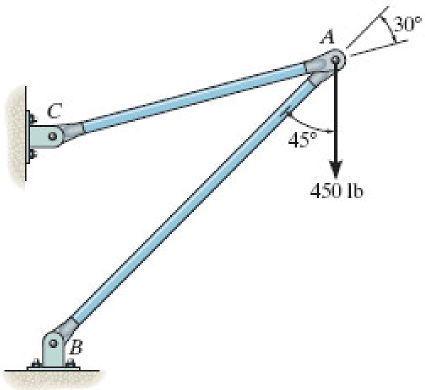
F2–3

F2–4. Resolve the 30-lb force into components along the u and v axes, and determine the magnitude of each of these components.



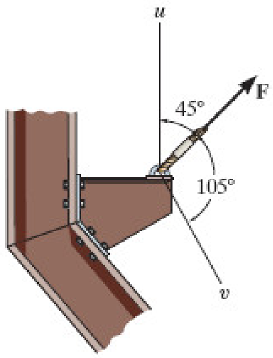
F2–4

F2–5. The force $F = 450$ lb acts on the frame. Resolve this force into components acting along members AB and AC, and determine the magnitude of each component.



F2–5

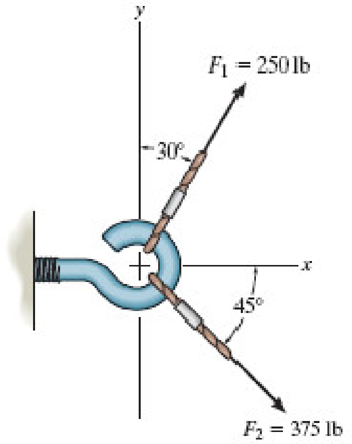
F2–6. If force **F** is to have a component along the u axis of $F_u = 6$ kN, determine the magnitude of **F** and the magnitude of its component $\mathbf{F}_v$ along the v axis.



F2–6

PROBLEMS

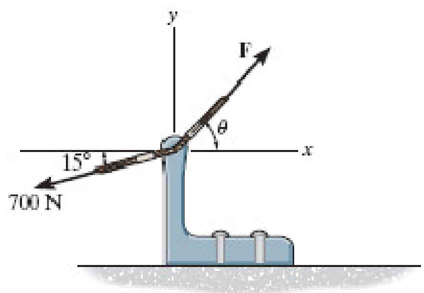
2–1. Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured counterclockwise from the positive x axis.



Prob. 2–1

2–2. If $\theta = 60°$ and $F = 450$ N, determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.

2–3. If the magnitude of the resultant force is to be 500 N, directed along the positive y axis, determine the magnitude of force **F** and its direction θ.

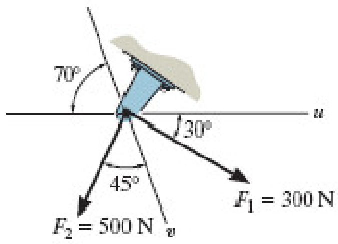


Probs. 2–2/3

***2–4.** Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured clockwise from the positive u axis.

2–5. Resolve the force $\mathbf{F}_1$ into components acting along the u and v axes and determine the magnitudes of the components.

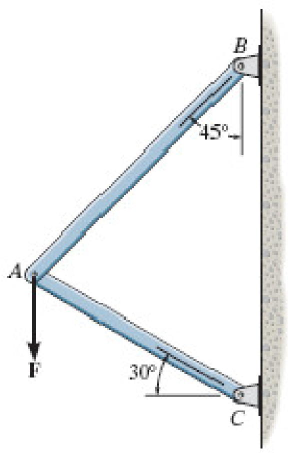
2–6. Resolve the force $\mathbf{F}_2$ into components acting along the u and v axes and determine the magnitudes of the components.



Probs. 2–4/5/6

2–7. The vertical force **F** acts downward at A on the two-membered frame. Determine the magnitudes of the two components of **F** directed along the axes of AB and AC. Set $F = 500$ N.

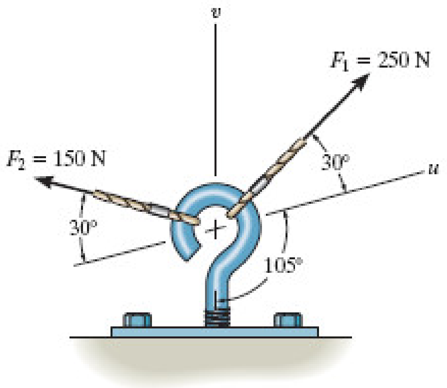
***2–8.** Solve Prob. 2–7 with $F = 350$ lb.



Probs. 2–7/8

2–9. Resolve $\mathbf{F}_1$ into components along the u and v axes and determine the magnitudes of these components.

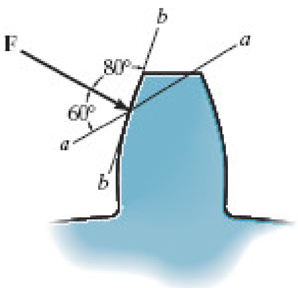
2–10. Resolve $\mathbf{F}_2$ into components along the u and v axes and determine the magnitudes of these components.



Probs. 2–9/10

2–11. The force acting on the gear tooth is $F = 20$ lb. Resolve this force into two components acting along the lines aa and bb.

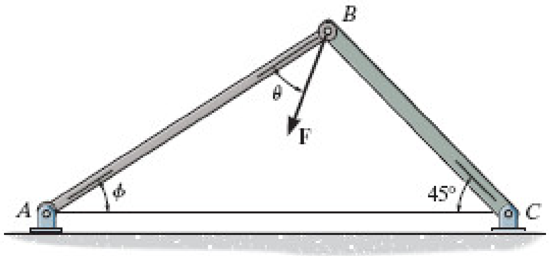
***2–12.** The component of force **F** acting along line aa is required to be 30 lb. Determine the magnitude of **F** and its component along line bb.



Probs. 2–11/12

2–13. Force **F** acts on the frame such that its component acting along member AB is 650 lb, directed from B towards A, and the component acting along member BC is 500 lb, directed from B towards C. Determine the magnitude of **F** and its direction θ. Set $\phi = 60°$.

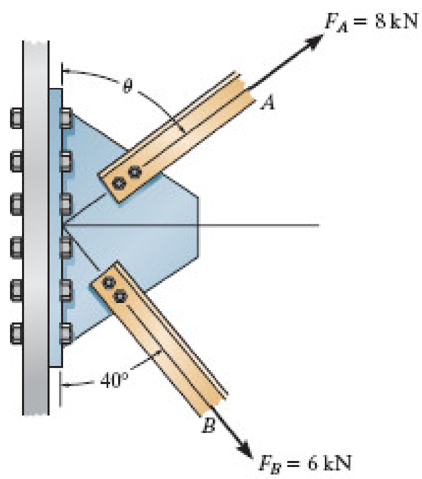
2–14. Force **F** acts on the frame such that its component acting along member AB is 650 lb, directed from B towards A. Determine the required angle ϕ ($0° \leq \phi \leq 90°$) and the component acting along member BC. Set $F = 850$ lb and $\theta = 30°$.



Probs. 2–13/14

2–15. The plate is subjected to the two forces at A and B as shown. If $\theta = 60°$, determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.

***2–16.** Determine the angle θ for connecting member A to the plate so that the resultant force of $\mathbf{F}_A$ and $\mathbf{F}_B$ is directed horizontally to the right. Also, what is the magnitude of the resultant force?



Probs. 2–15/16