CHAPTER SIX
Structural Analysis

CHAPTER OBJECTIVES

- To show how to determine the forces in the members of a truss using the method of joints and the method of sections.
- To analyze the forces acting on the members of frames and machines composed of pin-connected members.

6.1 Simple Trusses: **A truss** is a structure composed of slender members joined together at their end points. The members commonly used in construction consist of wooden struts or metal bars. In particular, planar trusses lie in a single plane and are often used to support roofs and bridges. The truss shown in Fig. 6-1a is an example of a typical ***roof-supporting*** truss. In this figure, the roof load is transmitted to the truss at the ***joints***. Since these loading acts in the same plane as the truss, Fig. 6-1b, the analysis of the forces developed in the truss members will be two dimensional.

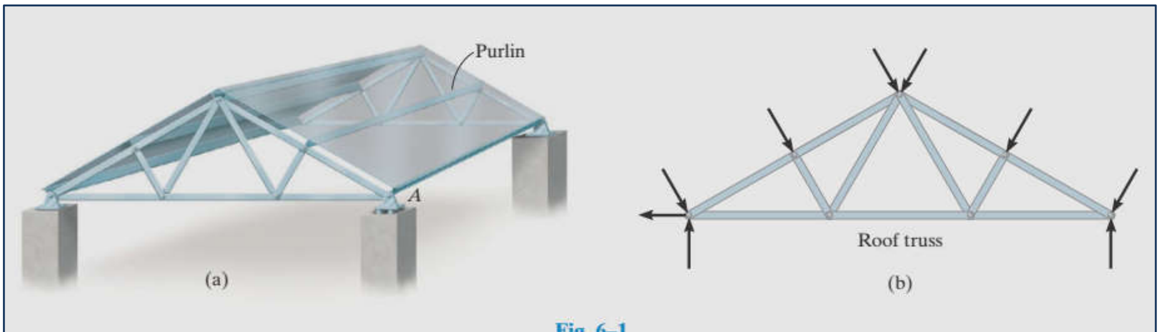


Fig. 6–1

In the case of **a bridge**, such as shown in Fig. 6-2a, the load on ***the deck*** is first transmitted to ***stringers***, then to ***floor beams***, and finally to ***the joints*** of the two supporting side trusses. Like the roof truss the bridge truss loading is also coplanar, Fig. 6-2b.

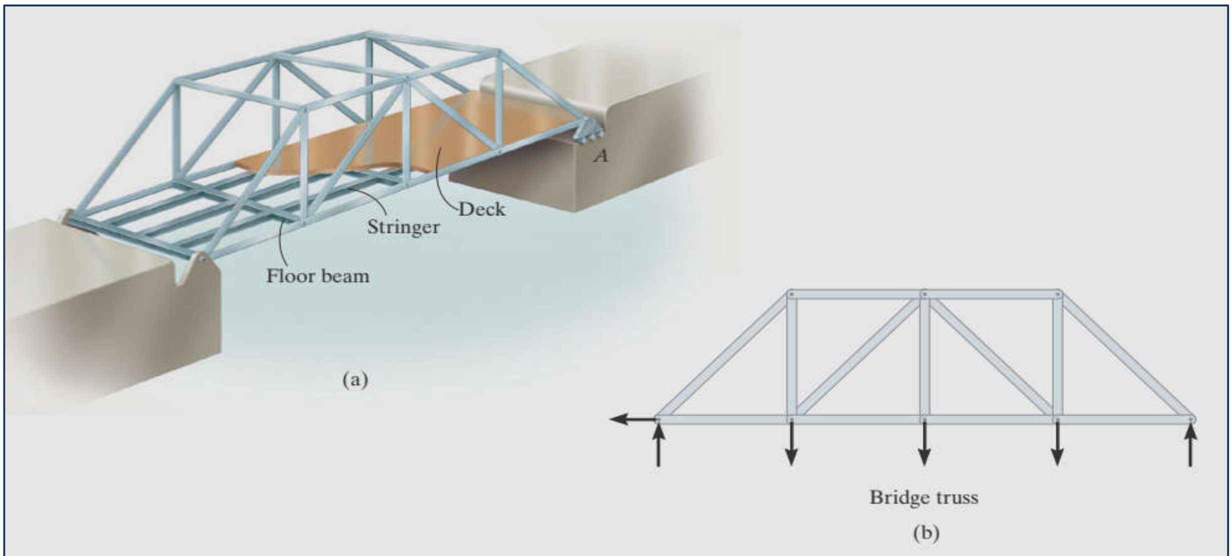


Fig. 6–2

- *When bridge or roof trusses extend over large distances, **a rocker or roller** is commonly used for supporting one end, for example, joint A in Figs. 6–1a and 6–2a. This type of support allows freedom for expansion or contraction of the members due to a change in temperature or application of loads.*

Assumptions for Design: To design both the members and the connections of a truss, it is necessary first to determine the force developed in each member when the truss is subjected to given loading. To do this we will make two important assumptions:

• ***All Loadings are applied at the joints.***

• ***The members are joined together by smooth pins.***

The joint connections are usually formed by bolting or welding the ends of the members to a common plate, called a gusset plate as shown in Fig 6-3a or by simply passing a large bolt or pin through each or the members, Fig 6-3b. If the ***force tends to elongate*** the member, it is a ***tensile*** force (**T**), Fig. 6-4a, whereas if tends ***to shorten*** the member, it is a ***compressive*** force (**C**), Fig. 6-4b.

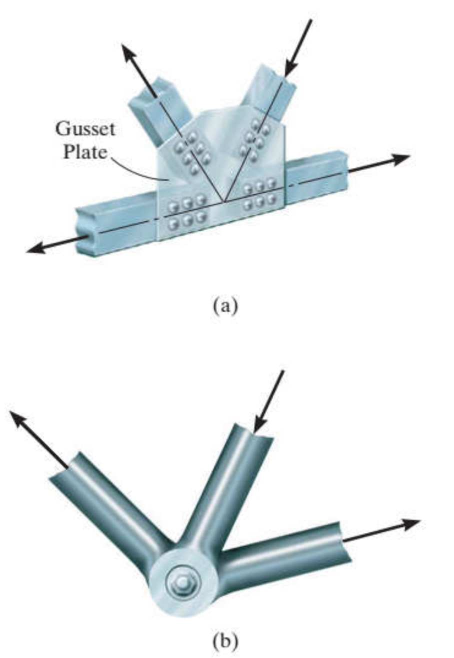


Fig. 6–3

- In the actual design of a truss it is important to state whether the nature of the force is tensile or compressive. Often, compression members must be made ***thicker*** than tension members because of the ***buckling*** or ***column*** effect that occurs when a member is in compression.

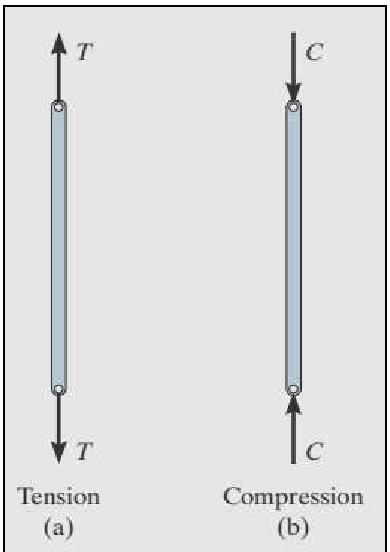


Fig. 6–4

Simple Truss: If three members are pin connected at their ends, they form a ***triangular truss*** that will be ***rigid***, Fig.6–5. Attaching two more members and connecting these members to a new joint ***D*** forms a larger truss, Fig.6–6. This procedure can be repeated as many times as desired to form an even larger truss. If a truss can be constructed by expanding the basic triangular truss in this way, it is called a ***simple truss***.

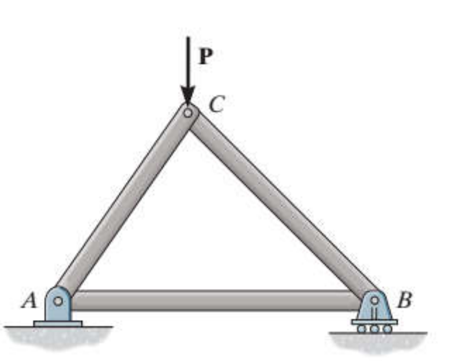


Fig. 6–5

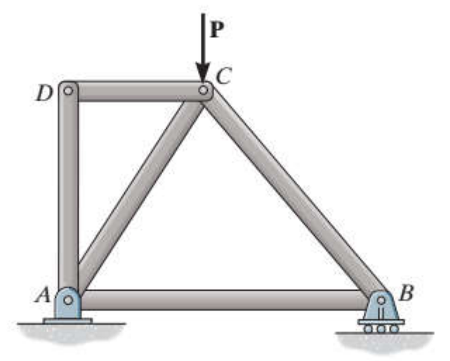


Fig. 6–6

6.2 The Method of Joints: In order to analyze or design a truss, it is necessary to determine the force in each of its members. One way to do this is to use the method of joints. This method is based on the fact that ***if the entire truss is in equilibrium, then each of its joints is also in equilibrium***. Therefore, if the free–body diagram of each joint is drawn, the force equilibrium equations can then be used to obtain the member forces acting on each joint. Since the members of a plane truss are straight two-force members lying in a single plane, each joint is subjected to a concurrent - coplanar force system.

As a result, only $\Sigma F_x = 0$ and $\Sigma F_y = 0$ need to be satisfied for equilibrium.

For example, consider the pin at joint *B* of the truss in Fig. 6–7*a*. Three forces act on the pin, namely, the 500-N force and the forces exerted by members *BA* and *BC*. The free-body diagram of the pin is shown in Fig. 6–7*b*. Here, F*BA* is "pulling" on the pin, which means that member *BA* is in *tension;* whereas F*BC* is "pushing" on the pin, and consequently member *BC* is in *compression.* These effects are clearly demonstrated by isolating the joint with small segments of the member connected to the pin, Fig. 6–7*c*. The pushing or pulling on these small segments indicates the effect of the member being either in compression or tension

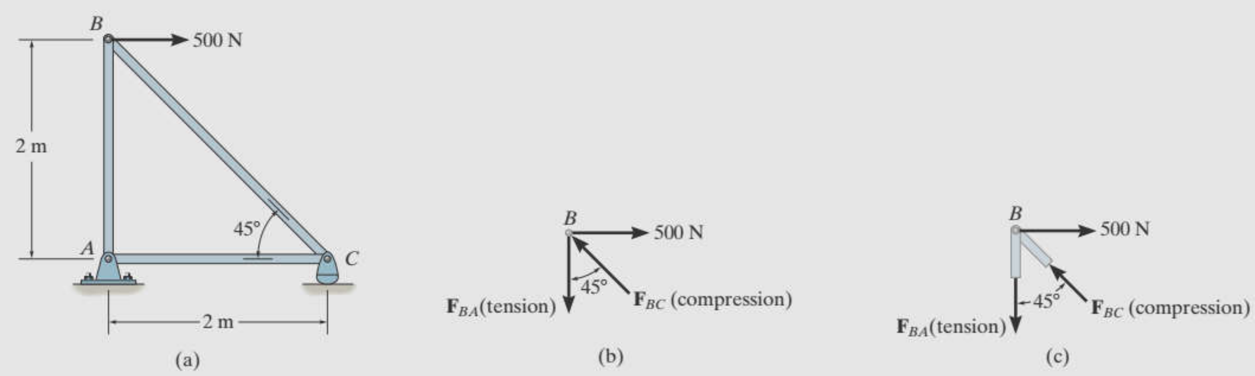


Fig. 6–7

Always assume the unknown member forces acting on the joint's free body diagram to be in tension: i.e., the forces "pull" on the pin. If this is done, then numerical solution of the equilibrium equations will yield positive scalars for members in tension and negative scalars for members in compression, Once an unknown member force is found, use its correct magnitude and sense (T or C) on subsequent joint free body diagrams.

When using the method of joints, always start at a joint having at least one known force and at ***most two unknown*** forces, as in Fig. 6–7*b*. In this way, application of $\Sigma F_x = 0$ *and* $\Sigma F_y = 0$ yields two algebraic equations which can be solved for the two unknowns. When applying these equations, the correct sense of an unknown member force can be determined using one of two possible methods.

Procedure for Analysis

The following procedure provides a means for analyzing a truss using the method of joints.

- Draw the free-body diagram of a joint having at least one known force and at most two unknown forces. (If this joint is at one of the supports, then it may be necessary first to calculate the external reactions at the support.)

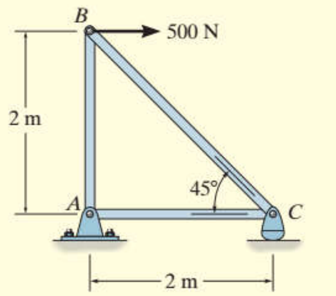
- Use one of the two methods described above for establishing the sense of an unknown force.

- Orient the x and y axes such that the forces on the free-body diagram can be easily resolved into their x and y components and then apply the two force equilibrium equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$. Solve for the two unknown member forces and verify their correct sense.

- Using the calculated results, continue to analyze each of the other joints. Remember that a member in *compression* "pushes" on the joint and a member in *tension* "pulls" on the joint. Also, be sure to choose a joint having at most two unknowns and at least one known force.

EXAMPLE 6.1

Determine the force in each member of the truss shown in Fig. 6–8*a* and indicate whether the members are in tension or compression.



(a)

SOLUTION

Since we should have no more than two unknown forces at the joint and at least one known force acting there, we will begin our analysis at joint *B*.

Joint *B*. The free-body diagram of the joint at *B* is shown in Fig. 6–8*b*. Applying the equations of equilibrium, we have

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad 500\text{ N} - F_{BC}\sin 45° = 0 \quad F_{BC} = 707.1\text{ N (C)} \quad \textit{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \quad F_{BC}\cos 45° - F_{BA} = 0 \quad F_{BA} = 500\text{ N (T)} \quad \textit{Ans.}$$

Since the force in member *BC* has been calculated, we can proceed to analyze joint *C* to determine the force in member *CA* and the support reaction at the rocker.

Joint *C*. From the free-body diagram of joint *C*, Fig. 6–8*c*, we have

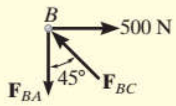
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad -F_{CA} + 707.1\cos 45°\text{ N} = 0 \quad F_{CA} = 500\text{ N (T)} \quad \textit{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \quad C_y - 707.1\sin 45°\text{ N} = 0 \quad C_y = 500\text{ N} \quad \textit{Ans.}$$

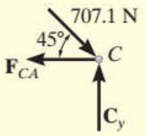
Joint *A*. Although it is not necessary, we can determine the components of the support reactions at joint *A* using the results of F_{CA} and F_{BA}. From the free-body diagram, Fig. 6–8*d*, we have

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad 500\text{ N} - A_x = 0 \quad A_x = 500\text{ N}$$

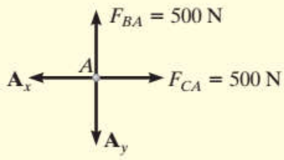
$$+\uparrow\Sigma F_y = 0; \quad 500\text{ N} - A_y = 0 \quad A_y = 500\text{ N}$$



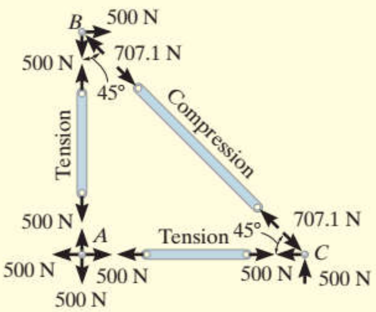
(b)



(c)



(d)



(e)

Fig. 6–8

NOTE: The results of the analysis are summarized in Fig. 6–8*e*. Note that the free-body diagram of each joint (or pin) shows the effects of all the connected members and external forces applied to the joint, whereas the free-body diagram of each member shows only the effects of the end joints on the member.

EXAMPLE 6.2

Determine the forces acting in all the members of the truss shown in Fig. 6–9*a*.

SOLUTION

By inspection, there are more than two unknowns at each joint. Consequently, the support reactions on the truss must first be determined. Show that they have been correctly calculated on the free-body diagram in Fig. 6–9*b*. We can now begin the analysis at joint *C*. Why?

Joint *C*. From the free-body diagram, Fig. 6–9*c*,

$$\xrightarrow{+}\Sigma F_x = 0; \qquad -F_{CD}\cos 30^\circ + F_{CB}\sin 45^\circ = 0$$

$$+\uparrow\Sigma F_y = 0; \qquad 1.5\text{ kN} + F_{CD}\sin 30^\circ - F_{CB}\cos 45^\circ = 0$$

These two equations must be solved *simultaneously* for each of the two unknowns. Note, however, that a *direct solution* for one of the unknown forces may be obtained by applying a force summation along an axis that is *perpendicular* to the direction of the other unknown force. For example, summing forces along the y' axis, which is perpendicular to the direction of $\mathbf{F}_{CD}$, Fig. 6–9*d*, yields a *direct solution* for F_{CB}.

$$+\nearrow\Sigma F_{y'} = 0; \qquad 1.5\cos 30^\circ\text{ kN} - F_{CB}\sin 15^\circ = 0$$

$$F_{CB} = 5.019\text{ kN} = 5.02\text{ kN (C)} \qquad \textit{Ans.}$$

Then,

$$+\searrow\Sigma F_{x'} = 0;$$

$$-F_{CD} + 5.019\cos 15^\circ - 1.5\sin 30^\circ = 0; \quad F_{CD} = 4.10\text{ kN (T)} \qquad \textit{Ans.}$$

Joint *D*. We can now proceed to analyze joint *D*. The free-body diagram is shown in Fig. 6–9*e*.

$$\xrightarrow{+}\Sigma F_x = 0; \qquad -F_{DA}\cos 30^\circ + 4.10\cos 30^\circ\text{ kN} = 0$$

$$F_{DA} = 4.10\text{ kN (T)} \qquad \textit{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \qquad F_{DB} - 2(4.10\sin 30^\circ\text{ kN}) = 0$$

$$F_{DB} = 4.10\text{ kN (T)} \qquad \textit{Ans.}$$

NOTE: The force in the last member, *BA*, can be obtained from joint *B* or joint *A*. As an exercise, draw the free-body diagram of joint *B*, sum the forces in the horizontal direction, and show that $F_{BA} = 0.776$ kN (C).

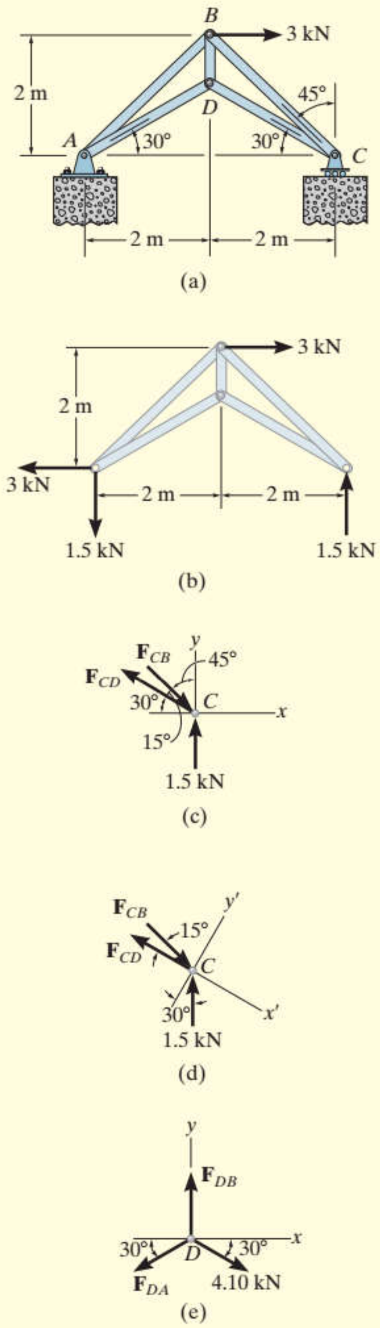


Fig. 6–9

EXAMPLE 6.3

Determine the force in each member of the truss shown in Fig. 6–10*a*. Indicate whether the members are in tension or compression.

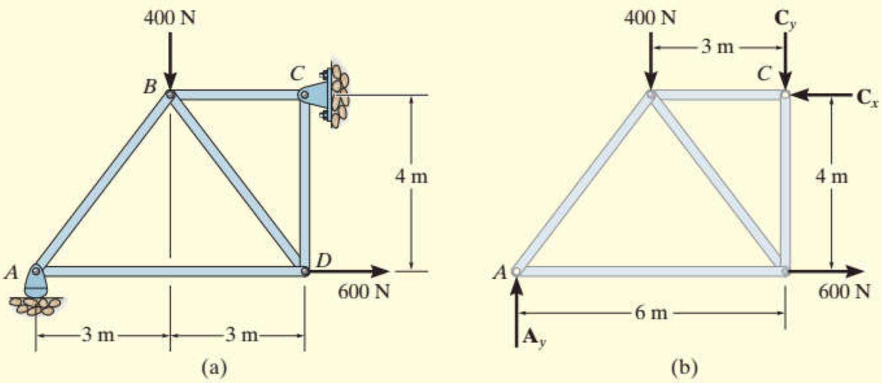


Fig. 6–10

SOLUTION

Support Reactions. No joint can be analyzed until the support reactions are determined, because each joint has at least three unknown forces acting on it. A free-body diagram of the entire truss is given in Fig. 6–10*b*. Applying the equations of equilibrium, we have

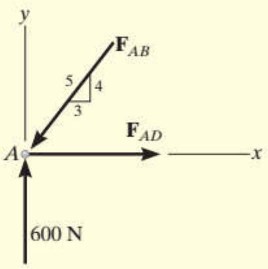
$$\xrightarrow{+} \Sigma F_x = 0; \qquad 600\text{ N} - C_x = 0 \qquad C_x = 600\text{ N}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad -A_y(6\text{ m}) + 400\text{ N}(3\text{ m}) + 600\text{ N}(4\text{ m}) = 0$$

$$A_y = 600\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad 600\text{ N} - 400\text{ N} - C_y = 0 \qquad C_y = 200\text{ N}$$

The analysis can now start at either joint *A* or *C*. The choice is arbitrary since there are one known and two unknown member forces acting on the pin at each of these joints.



(c)

Joint A. (Fig. 6–10*c*). As shown on the free-body diagram, $\mathbf{F}_{AB}$ is assumed to be compressive and $\mathbf{F}_{AD}$ is tensile. Applying the equations of equilibrium, we have

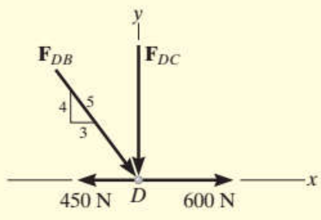
$$+\uparrow \Sigma F_y = 0; \qquad 600\text{ N} - \tfrac{4}{5}F_{AB} = 0 \qquad F_{AB} = 750\text{ N} \quad (C) \qquad \textit{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AD} - \tfrac{3}{5}(750\text{ N}) = 0 \qquad F_{AD} = 450\text{ N} \quad (T) \qquad \textit{Ans.}$$

(Example 6-3 / continued)

Joint D. (Fig. 6–10*d*). Using the result for F_{AD} and summing forces in the horizontal direction, Fig. 6–10*d*, we have

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad -450\text{ N} + \tfrac{3}{5}F_{DB} + 600\text{ N} = 0 \qquad F_{DB} = -250\text{ N}$$



(d)

The negative sign indicates that $\mathbf{F}_{DB}$ acts in the *opposite sense* to that shown in Fig. 6–10*d*.* Hence,

$$F_{DB} = 250\text{ N (T)} \qquad \textit{Ans.}$$

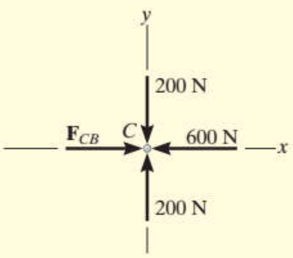
To determine $\mathbf{F}_{DC}$, we can either correct the sense of $\mathbf{F}_{DB}$ on the free-body diagram, and then apply $\Sigma F_y = 0$, or apply this equation and retain the negative sign for F_{DB}, i.e.,

$$+\uparrow\Sigma F_y = 0; \qquad -F_{DC} - \tfrac{4}{5}(-250\text{ N}) = 0 \qquad F_{DC} = 200\text{ N (C)} \qquad \textit{Ans.}$$

Joint C. (Fig. 6–10*e*).

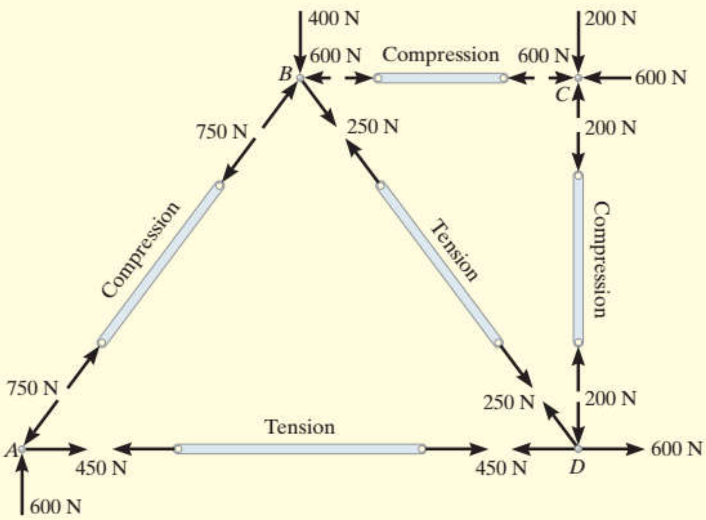
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{CB} - 600\text{ N} = 0 \qquad F_{CB} = 600\text{ N (C)} \qquad \textit{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \qquad 200\text{ N} - 200\text{ N} \equiv 0 \quad \text{(check)}$$



(e)

NOTE: The analysis is summarized in Fig. 6–10*f*, which shows the free-body diagram for each joint and member.



(f)

Fig. 6–10 (cont.)

*The proper sense could have been determined by inspection, prior to applying $\Sigma F_x = 0$.

6.3 Zero-Force Members:

Truss analysis using the method of joints is greatly simplified if we can first identify those members which support *no loading.* These *zero-force members* are used to increase the stability of the truss during construction and to provide added support if the loading is changed. The zero-force members of a truss can generally be found *by inspection* of each of the joints. For example, consider the truss shown in Fig. 6–1*a*. If a free-body diagram of the pin at joint *A* is drawn, Fig. 6–11*b*, it is seen that members *AB* and *AF* are zero-force members. (We could not have come to this conclusion if we had considered the free-body diagrams of joints *F* or *B* simply because there are five unknowns at each of these joints.) In a similar manner, consider the free-body diagram of joint *D*, Fig. 6–11*c*. Here again it is seen that *DC* and *DE* are zero-force members. From these observations, we can conclude that *if only two non-collinear members form a truss joint and no external load or support reaction is applied to the joint, the two members must be zeroforce members.* The load on the truss in Fig. 6–11*a* is therefore supported by only five members as shown in Fig. 6–11 *d*.

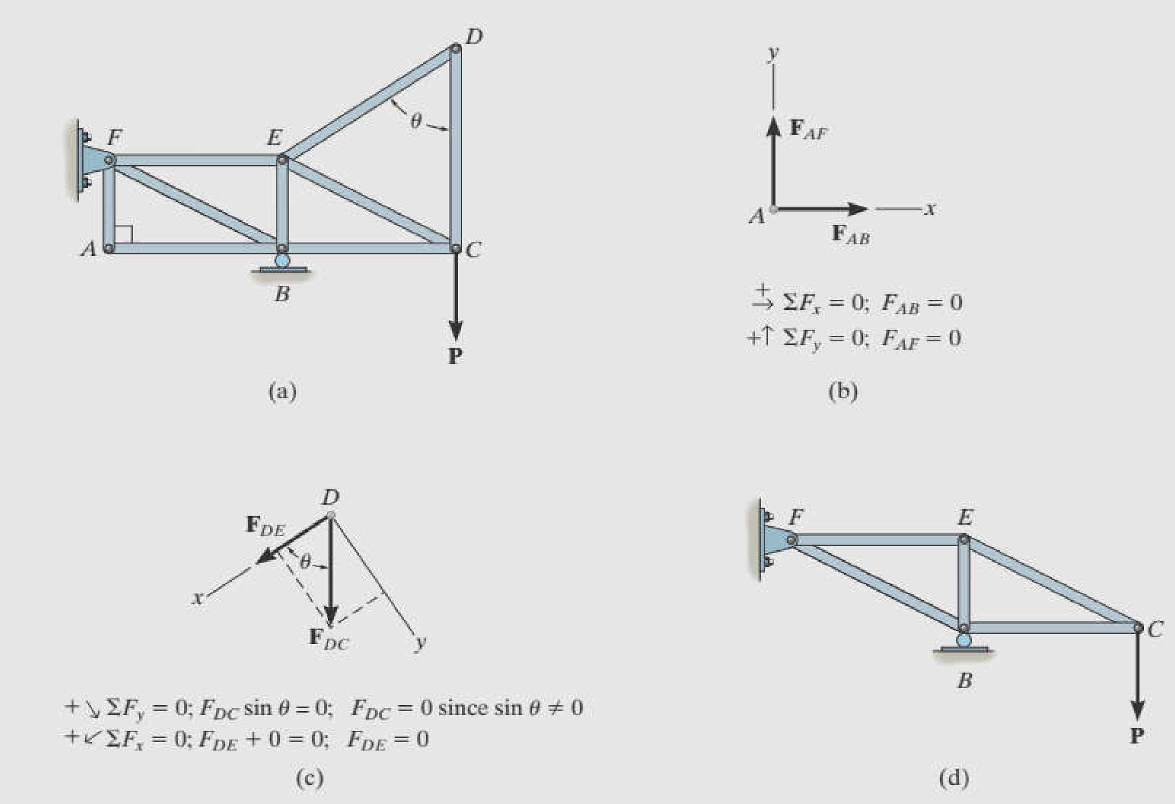
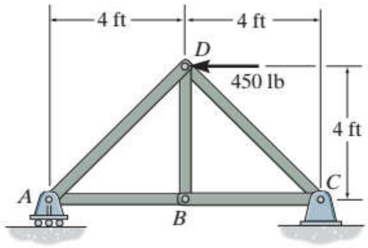


Fig. 6–11

FUNDAMENTAL PROBLEMS

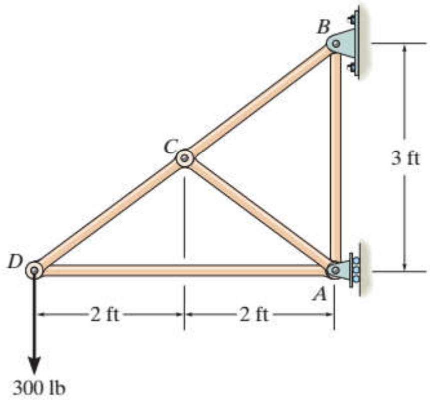
All problem solutions must include FBDs.

F6–1. Determine the force in each member of the truss. State if the members are in tension or compression.



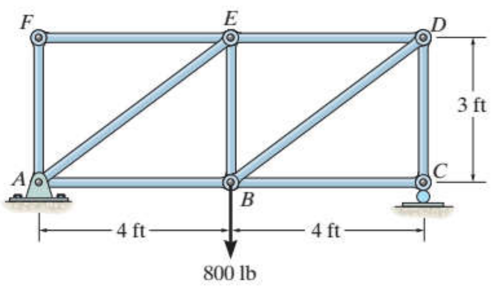
F6–1

F6–2. Determine the force in each member of the truss. State if the members are in tension or compression.



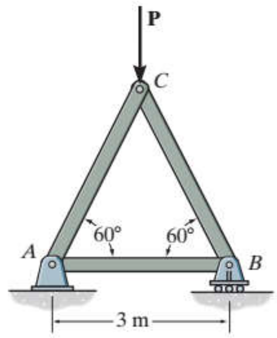
F6–2

F6–3. Determine the force in members *AE* and *DC*. State if the members are in tension or compression.



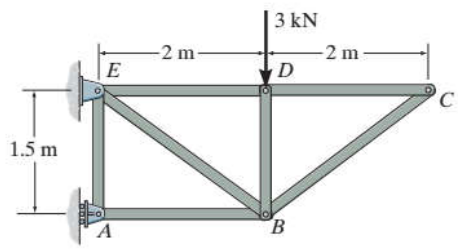
F6–3

F6–4. Determine the greatest load *P* that can be applied to the truss so that none of the members are subjected to a force exceeding either 2 kN in tension or 1.5 kN in compression.



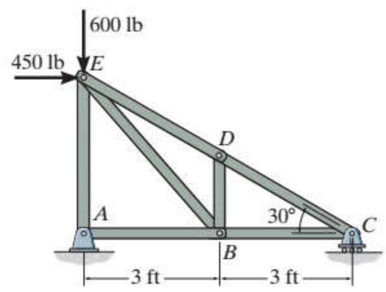
F6–4

F6–5. Identify the zero-force members in the truss.



F6–5

F6–6. Determine the force in each member of the truss. State if the members are in tension or compression.



F6–6