CHAPTER FOUR

SHEET PILE WALLS

LECTURE
DR. AHMED H. ABDULKAREEM
2017
4.1. Introduction
Connected or semi-connected sheet piles are often used to build continuous walls for waterfront structures that range from small waterfront pleasure boat launching facilities to large dock facilities. (See Figure 4.1). In contrast to the construction of other types of retaining wall, the building of sheet-pile walls does not usually require dewatering of the site. Sheet piles are also used for some temporary structures, such as braced cuts. (See Chapter 5). The principles of sheet-pile wall design are discussed in the current chapter.

Several types of sheet pile are commonly used in construction:
(a) wooden sheet piles,
(b) precast concrete sheet piles, and
(c) steel sheet piles.
Aluminum sheet piles are also marketed.

(a) Wooden sheet piles are used only for temporary, light structures that are above the water table. The most common types are ordinary wooden planks and Wakefield piles. The wooden planks are about 50 mm × 300 mm (2 in. × 12 in.) in cross section and are driven edge to edge (Figure 4.2a). Wakefield piles are made by nailing three planks together, with the middle plank offset by 50 to 75 mm (2 to 3 in.) (Figure 4.2b). Wooden planks can also be milled to form tongue-and-groove piles, as shown in Figure 4.2c. Figure 4.2d shows another type of wooden sheet pile that has precut grooves. Metal splines are driven into the grooves of the adjacent sheetings to hold them together after they are sunk into the ground.

![Figure 4.1 Example of waterfront sheet-pile wall.](image-url)
Precast concrete sheet piles are heavy and are designed with reinforcements to withstand the permanent stresses to which the structure will be subjected after construction and also to handle the stresses produced during construction. In cross section, these piles are about 500 to 800 mm (20 to 32 in.) wide and 150 to 250 mm (6 to 10 in.) thick. Figure 4.2e is a schematic diagram of the elevation and the cross section of a reinforced concrete sheet pile.

Steel sheet piles in the United States are about 10 to 13 mm (0.4 to 0.5 in.) thick. European sections may be thinner and wider. Sheet-pile sections may be Z, deep arch, low arch, or straight web sections. The interlocks of the sheet-pile sections are shaped like a thumb-and-finger or ball-and-socket joint for watertight connections. Figure 4.3a is a schematic diagram of the thumb-and-finger type of interlocking for straight web sections. The ball-and-socket type of interlocking for Z section piles is shown in Figure 4.3b. Figure 4.4 shows some sheet piles at a construction site. Figure 4.5 shows a small enclosure with steel sheet piles for an excavation work. Table 4.1 lists the properties of the steel sheet-pile sections produced by Hammer & Steel, Inc. of Hazelwood, Missouri. The allowable design flexural stress for the steel sheet piles is as follows:

<table>
<thead>
<tr>
<th>Type of steel</th>
<th>Allowable stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASTM A-328</td>
<td>170 MN/m²</td>
</tr>
<tr>
<td>ASTM A-572</td>
<td>210 MN/m²</td>
</tr>
<tr>
<td>ASTM A-690</td>
<td>210 MN/m²</td>
</tr>
</tbody>
</table>
Figure 4.3 (a) Thumb-and-finger type sheet-pile connection; (b) ball-and-socket type sheet-pile connection

Figure 4.4 Some steel sheet piles at a construction site (Courtesy of N. Sivakugan, James Cook University, Australia)

Figure 4.5 A small enclosure with steel sheet piles for an excavation work (Courtesy of N. Sivakugan, James Cook University, Australia)
### Table 4.1 Properties of Some Sheet-Pile Sections Production by Bethlehem Steel Corporation

<table>
<thead>
<tr>
<th>Section designation</th>
<th>Sketch of section</th>
<th>Section modulus</th>
<th>Moment of Inertia</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>m^3/m of wall</td>
<td>in^3/ft of wall</td>
</tr>
<tr>
<td>PZ-40</td>
<td><img src="image" alt="PZ-40 Sketch" /></td>
<td>526.4 x 10^-3</td>
<td>60.7</td>
</tr>
<tr>
<td>PZ-35</td>
<td><img src="image" alt="PZ-35 Sketch" /></td>
<td>260.5 x 10^-5</td>
<td>48.5</td>
</tr>
<tr>
<td>PZ-27</td>
<td><img src="image" alt="PZ-27 Sketch" /></td>
<td>162.3 x 10^-5</td>
<td>30.2</td>
</tr>
<tr>
<td>PZ-22</td>
<td><img src="image" alt="PZ-22 Sketch" /></td>
<td>97 x 10^-5</td>
<td>18.1</td>
</tr>
<tr>
<td>PSA 31</td>
<td><img src="image" alt="PSA 31 Sketch" /></td>
<td>10.8 x 10^-5</td>
<td>2.01</td>
</tr>
<tr>
<td>PSA-23</td>
<td><img src="image" alt="PSA-23 Sketch" /></td>
<td>12.8 x 10^-5</td>
<td>2.4</td>
</tr>
</tbody>
</table>
4.2 Construction Methods

Sheet-pile walls may be divided into two basic categories:
(a) cantilever and
(b) anchored.

In the construction of sheet-pile walls, the sheet pile may be driven into the ground and then the backfill placed on the land side, or the sheet pile may first be driven into the ground and the soil in front of the sheet pile dredged. In either case, the soil used for backfill behind the sheet-pile wall is usually granular. The soil below the dredge line may be sandy or clayey. The surface of soil on the water side is referred to as the *mud line* or *dredge line*.

Thus, construction methods generally can be divided into two categories (Tsinker, 1983):

1. **Backfilled structure**
2. **Dredged structure**

The sequence of construction for a *backfilled structure* is as follows (see Figure 4.6):

*Step 1.* Dredge the *in situ* soil in front and back of the proposed structure.

*Step 2.* Drive the sheet piles.

*Step 3.* Backfill up to the level of the anchor, and place the anchor system.

*Step 4.* Backfill up to the top of the wall.

*Figure 4.6* Sequence of construction for a backfilled structure
For a cantilever type of wall, only Steps 1, 2, and 4 apply. The sequence of construction for a *dredged structure* is as follows (see Figure 4.7):

**Step 1.** Drive the sheet piles.

**Step 2.** Backfill up to the anchor level, and place the anchor system.

**Step 3.** Backfill up to the top of the wall.

**Step 4.** Dredge the front side of the wall.

With cantilever sheet-pile walls, Step 2 is not required.

*Figure 4.7* Sequence of construction for a dredged structure
4.3 Cantilever Sheet-Pile Walls

Cantilever sheet-pile walls are usually recommended for walls of moderate height—about 6 m or less, measured above the dredge line. In such walls, the sheet piles act as a wide cantilever beam above the dredge line. The basic principles for estimating net lateral pressure distribution on a cantilever sheet-pile wall can be explained with the aid of Figure 4.8. The figure shows the nature of lateral yielding of a cantilever wall penetrating sand layer below the dredge line. The wall rotates about point $O$ (Figure 4.8a). Because the hydrostatic pressures at any depth from both sides of the wall will cancel each other, we consider only the effective lateral soil pressures. In zone $A$, the lateral pressure is just the active pressure from the land side. In zone $B$, because of the nature of yielding of the wall, there will be active pressure from the land side and passive pressure from the water side. The condition is reversed in zone $C$—that is, below the point of rotation, $O$. The net actual pressure distribution on the wall is like that shown in Figure 4.8b. However, for design purposes, Figure 4.8c shows a simplified version.

Sections 4.4 through 4.7 present the mathematical formulation of the analysis of cantilever sheet-pile walls. Note that, in some waterfront structures, the water level may fluctuate as the result of tidal effects. Care should be taken in determining the water level that will affect the net pressure diagram.

![Figure 4.8 Cantilever sheet pile penetrating sand](image)
4.4 Cantilever Sheet Piling Penetrating Sandy Soils

To develop the relationships for the proper depth of embedment of sheet piles driven into a granular soil, examine Figure 4.9a. The soil retained by the sheet piling above the dredge line also is sand. The water table is at a depth \( L_1 \) below the top of the wall. Let the effective angle of friction of the sand be \( \phi' \). The intensity of the active pressure at a depth \( z = L_1 \) is

\[
\sigma_1' = \gamma L_1 K_a
\]

(4.1)

where

\( K_a = \) Rankine active pressure coefficient = \( tan^2 (45 - \frac{\phi'}{2}) \)

\( \gamma = \) unit weight of soil above the water table

Similarly, the active pressure at a depth \( z = L_1 + L_2 \) (i.e., at the level of the dredge line) is

\[
\sigma_2' = (\gamma L_1 + \gamma' L_2) K_a
\]

(4.2)

where \( \gamma' = \) effective unit weight of soil = \( \gamma_{sat} - \gamma_w \).

Note that, at the level of the dredge line, the hydrostatic pressures from both sides of

Figure 4.9 Cantilever sheet pile penetrating sand: (a) variation of net pressure diagram; (b) variation of moment
the wall are the same magnitude and cancel each other.

To determine the net lateral pressure below the dredge line up to the point of rotation, \( O \), as shown in Figure 4.8a, an engineer has to consider the passive pressure acting from the left side (the water side) toward the right side (the land side) of the wall and also the active pressure acting from the right side toward the left side of the wall. For such cases, ignoring the hydrostatic pressure from both sides of the wall, the active pressure at depth \( z \) is

\[
\sigma'_a = [\gamma L_1 + \gamma' L_2 + \gamma'(z - L_1 - L_2)]K_a \quad (4.3)
\]

Also, the passive pressure at depth \( z \) is

\[
\sigma'_p = \gamma'(z - L_1 - L_2)K_p \quad (4.4)
\]

where \( K_p = \text{Rankine passive pressure coefficient} = \tan^2(45 + \phi'/2) \)

Combining Eqs. (4.3) and (4.4) yields the net lateral pressure, namely,

\[
\sigma' = \sigma'_a - \sigma'_p = (\gamma L_1 + \gamma' L_2)K_a - \gamma'(z - L_1 - L_2) (K_p - K_a) \quad (4.5)
\]

where \( L = L_1 + L_2 \). The net pressure, \( \sigma' \) equals zero at a depth \( L_3 \) below the dredge line, so

\[
\sigma'_2 - \gamma'(z - L) (K_p - K_a) = 0
\]

or

\[
(z - L) = L_3 = \frac{\sigma'_2}{\gamma' (K_p - K_a)} \quad (4.6)
\]

Equation (4.6) indicates that the slope of the net pressure distribution line \( DEF \) is 1 vertical to \( (K_p - K_a) \gamma' \) horizontal, so, in the pressure diagram,

\[
\overline{HB} = \sigma'_3 = L_4(K_p - K_a)\gamma' \quad (4.7)
\]

At the bottom of the sheet pile, passive pressure, \( \sigma'_p \), acts from the right toward the left side, and active pressure acts from the left toward the right side of the sheet pile, so, at \( z = L + D \),

\[
\sigma'_p = (\gamma L_1 + \gamma' L_2 + \gamma'D)K_p \quad (4.8)
\]

At the same depth,
\( \sigma'_a = \gamma' D K_a \) \hspace{1cm} (4.9)

Hence, the net lateral pressure at the bottom of the sheet pile is
\[
\sigma'_p - \sigma'_a = \sigma'_4 = (\gamma L_1 + \gamma' L_2) K_p + \gamma' D (K_p - K_a) \\
= (\gamma L_1 + \gamma' L_2) K_p + \gamma' L_3 (K_p - K_a) + \gamma' L_4 (K_p - K_a) \\
= \sigma'_5 + \gamma' L_4 (K_p - K_a) \hspace{1cm} (4.10)
\]

Where
\[
\sigma'_5 = (\gamma L_1 + \gamma' L_2) K_p + \gamma' L_3 (K_p - K_a) \hspace{1cm} (4.11)
\]
\[
D = L_3 + L_4 \hspace{1cm} (4.12)
\]

For the stability of the wall, the principles of statics can now be applied:
\[ \sum \text{horizontal forces per unit length of wall} = 0 \]

and
\[ \sum \text{moment of the forces per unit length of wall about point } B = 0 \]

For the summation of the horizontal forces, we have
Area of the pressure diagram \( ACDE \) - area of \( EFHB \) + area of \( FHBG \) = 0

Or
\[
P - \frac{1}{2} \sigma'_3 L_4 + \frac{1}{2} L_5 (\sigma'_3 + \sigma'_4) = 0 \hspace{1cm} (4.13)
\]

where \( P \) = area of the pressure diagram \( ACDE \).

Summing the moment of all the forces about point \( B \) yields
\[
P(L_4 + \bar{x}) - \left( \frac{1}{2} L_3 \sigma'_3 \right) \left( \frac{L_4}{3} \right) + \frac{1}{2} L_5 (\sigma'_3 + \sigma'_4) \left( \frac{L_5}{3} \right) = 0 \hspace{1cm} (4.14)
\]

From Eq.(4.13)
\[
L_5 = \frac{\sigma'_3 L_4 - 2P}{\sigma'_3 + \sigma'_4} \hspace{1cm} (4.15)
\]

Combining Eqs. (4.7), (4.10), (4.14), and (4.15) and simplifying them further, we obtain the following fourth-degree equation in terms of \( L_4 \):
In this equation

\[ L_4^4 + A_1 L_4^3 - A_2 L_4^2 - A_3 L_4 - A_4 = 0 \]  

(4.16)

\[ A_1 = \frac{\sigma'_5}{\gamma' (K_p - K_a)} \]  

(4.17)

\[ A_2 = \frac{8P}{\gamma' (K_p - K_a)} \]  

(4.18)

\[ A_3 = \frac{6P[2\bar{z}\gamma'(K_p - K_a) + \sigma'_5]}{\gamma'^2 (K_p - K_a)^2} \]  

(4.19)

\[ A_4 = \frac{P(6\bar{z}\sigma'_4 + 4P)}{\gamma'^2 (K_p - K_a)^2} \]  

(4.20)

**Step-by-Step Procedure for Obtaining the Pressure Diagram**

Based on the preceding theory, a step-by-step procedure for obtaining the pressure diagram for a cantilever sheet-pile wall penetrating a granular soil is as follows:

*Step 1.* Calculate \( K_a \) and \( K_p \).

*Step 2.* Calculate \( \sigma_1 \) [Eq. (4.1)] and \( \sigma_2' \) [Eq. (4.2)]. (Note: \( L_1 \) and \( L_2 \) will be given.)

*Step 3.* Calculate \( L_3 \) [Eq. (4.6)].

*Step 4.* Calculate \( P \).

*Step 5.* Calculate \( z \) (i.e., the center of pressure for the area \( ACDE \)) by taking the moment about \( E \).

*Step 6.* Calculate \( \sigma'_5 \) [Eq. (4.11)].

*Step 7.* Calculate \( A_1, A_2, A_3, \) and \( A_4 \) [Eqs. (4.17) through (4.20)].

*Step 8.* Solve Eq. (4.16) by trial and error to determine \( L_4 \).

*Step 9.* Calculate \( \sigma'_4 \) [Eq. (4.10)].

*Step 10.* Calculate \( \sigma'_3 \) [Eq. (4.7)].

*Step 11.* Obtain \( L_5 \) from Eq. (4.15).

*Step 12.* Draw a pressure distribution diagram like the one shown in Figure 4.9a.
Step 13. Obtain the theoretical depth [see Eq. (4.12)] of penetration as 
\[L_3 + L_4\]. The actual depth of penetration is increased by about 20 to 30%.

Note that some designers prefer to use a factor of safety on the passive earth pressure coefficient at the beginning. In that case, in Step 1,

\[K_{p(\text{design})} = \frac{K_p}{FS}\]

where FS = factor of safety (usually between 1.5 and 2).

For this type of analysis, follow Steps 1 through 12 with the value of \(K_a = tan^2 (45 - \frac{\phi'}{2})\) and \(K_{p(\text{design})}\) (instead of \(K_p\)). The actual depth of penetration can now be determined by adding \(L_3\), obtained from Step 3, and \(L_4\), obtained from Step 8.

**Calculation of Maximum Bending Moment**

The nature of the variation of the moment diagram for a cantilever sheet-pile wall is shown in Figure 4.9b. The maximum moment will occur between points \(E\) and \(F'\). Obtaining the maximum moment \((M_{\text{max}})\) per unit length of the wall requires determining the point of zero shear. For a new axis \(z'\) (with origin at point \(E\)) for zero shear,

\[P = \frac{1}{2}(z')^2(K_p - K_a)\gamma'

Or

\[z' = \sqrt{\frac{2P}{(K_p - K_a)\gamma'}} \quad (4.21)\]

Once the point of zero shear force is determined (point \(F'\) in Figure 4.9a), the magnitude of the maximum moment can be obtained as

\[M_{\text{max}} = P(\bar{z} + z') - \left[\frac{1}{2}\gamma'z'^2(K_p - K_a)\right](\frac{1}{3})z' \quad (4.22)\]

The necessary profile of the sheet piling is then sized according to the allowable flexural stress of the sheet pile material, or

\[S = \frac{M_{\text{max}}}{\sigma_{\text{all}}} \quad (4.23)\]
where
\( S \) = section modulus of the sheet pile required per unit length of the structure
\( \sigma_{all} \) = allowable flexural stress of the sheet pile

**Example 4.1**

Figure 4.10 shows a cantilever sheet-pile wall penetrating a granular soil. Here,
\( L_1 = 2 \text{ m}, L_2 = 3 \text{ m}, \gamma = 15.9 \text{ kN/m}^3, \gamma_{sat} = 19.33 \text{ kN/m}^3, \) and \( \phi' = 32^\circ. \)

a. What is the theoretical depth of embedment, \( D? \)
b. For a 30% increase in \( D \), what should be the total length of the sheet piles?
c. What should be the minimum section modulus of the sheet piles?
Use \( \sigma_{all} = 172 \text{ MN/m}^2. \)

![Figure 4.10 Cantilever sheet-pile wall](image)

**Part a**

Using Figure 4.9a for the pressure distribution diagram, one can now prepare the following table for a step-by-step calculation.
<table>
<thead>
<tr>
<th>Quantity required</th>
<th>Eq. no.</th>
<th>Equation and calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_a$</td>
<td>9.1</td>
<td>$\gamma L_1 K_a = (15.9)(2)(0.307) = 9.763$ kN/m$^2$</td>
</tr>
<tr>
<td>$K_p$</td>
<td>9.2</td>
<td>$(\gamma L_1 + \gamma' L_2) K_a = <a href="0.307">(15.9)(2) + (19.33 - 9.81)(3)</a> = 18.53$ kN/m$^2$</td>
</tr>
<tr>
<td>$I_3$</td>
<td>9.6</td>
<td>$\gamma' (K_p - K_a) = \frac{18.53}{(19.33 - 9.81)(3.25 - 0.307)} = 0.66$ m</td>
</tr>
<tr>
<td>$P$</td>
<td></td>
<td>$\frac{1}{2} \sigma_1 I_1 + \sigma_1' I_2 + \frac{1}{2} (\sigma_2' - \sigma_1') L_2 + \frac{1}{2} \sigma_2' L_3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= (\frac{1}{2})(9.763)(2) + (9.763)(3) + (\frac{1}{2})(18.53 - 9.763)(3)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$+ (\frac{1}{2})(18.53)(0.66)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= 9.763 + 29.289 + 13.151 + 6.115 - 58.32$ kN/m</td>
</tr>
<tr>
<td>$\bar{z}$</td>
<td></td>
<td>$\frac{\Sigma M_E}{P} = \frac{1}{58.32} \left[ 9.763(0.66 + 3 + \frac{3}{3}) + 29.289(0.66 + \frac{3}{3}) + 13.151(0.66 + \frac{3}{3}) + 6.115(0.66 \times \frac{3}{3}) \right] = 2.23$ m</td>
</tr>
<tr>
<td>$\sigma_5'$</td>
<td>9.11</td>
<td>$(\gamma L_1 + \gamma' L_2) K_p + \gamma' L_3 (K_p - K_a) = <a href="3.25">(15.9)(2) + (19.33 - 9.81)(3)</a>$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$+ (19.33 - 9.81)(0.66)(3.25 - 0.307) = 214.66$ kN/m$^2$</td>
</tr>
<tr>
<td>$A_1$</td>
<td>9.17</td>
<td>$\frac{\sigma_5'}{\gamma' (K_p - K_a)} = \frac{214.66}{(19.33 - 9.81)(3.25 - 0.307)} = 7.66$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>9.18</td>
<td>$\frac{8P}{\gamma' (K_p - K_a)} = \frac{214.66}{(19.33 - 9.81)(3.25 - 0.307)} = 16.65$</td>
</tr>
</tbody>
</table>
Sheet Pile Walls

\[ A_3 = 9.19 \frac{6P[2\bar{z}\gamma'(K_p - K_a) + \sigma'_i]}{\gamma'^2(K_p - K_a)^2} \]
\[ = \frac{(6)(58.32)[(2)(2.23)(19.33 - 9.81)(3.25 - 0.307) + 214.66]}{(19.33 - 9.81)^2(3.25 - 0.307)^2} \]
\[ = 151.93 \]

\[ A_4 = 9.20 \frac{P(6\bar{z}\gamma' + 4P)}{\gamma'^2(K_p - K_a)^2} = \frac{58.32[(6)(2.23)(214.66) + (4)(58.32)]}{(19.33 - 9.81)^2(3.25 - 0.307)^2} \]
\[ = 230.72 \]

\[ L_4 = 9.16 \quad L_4^4 + A_1L_4^3 - A_2L_4^2 - A_3L_4 - A_4 = 0 \]
\[ L_4^4 + 7.66L_4^3 - 16.65L_4^2 - 151.93L_4 - 230.72 = 0; \quad L_4 \approx 4.8 \text{ m} \]

Thus,
\[ D_{\text{theory}} = L_3 + L_4 = 0.66 + 4.8 = 5.46 \text{ m} \]

Part b

The total length of the sheet piles is
\[ L_1 + L_2 + 1.3(L_3 + L_4) = 2 + 3 + 1.3(5.46) = 12.1 \text{ m} \]

Part c

Finally, we have the following table.

<table>
<thead>
<tr>
<th>Quantity required</th>
<th>Eq. no.</th>
<th>Equation and calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\bar{z}')</td>
<td>9.21</td>
<td>[\sqrt{\frac{2P}{(K_p - K_a)\gamma'}} - \sqrt{\frac{(2)(58.32)}{(3.25 - 0.307)(19.33 - 9.81)}} = 2.04 \text{ m}]</td>
</tr>
<tr>
<td>(M_{\text{max}})</td>
<td>9.22</td>
<td>[P(\bar{z} + z') - \left[\frac{1}{2}\gamma'z'^2(K_p - K_a)\right] \frac{z'}{3} = (58.32)(2.23 + 2.04)]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- \left[\left(\frac{1}{2}\right)(19.33 - 9.81)(2.04)^2(3.25 - 0.307)\right] \frac{2.04}{3} = 209.39 \text{ kN} \cdot \text{m/m}]</td>
</tr>
<tr>
<td>(S)</td>
<td>9.29</td>
<td>[\frac{M_{\text{max}}}{\sigma_{\text{all}}} = \frac{209.39 \text{ kN} \cdot \text{m}}{172 \times 10^3 \text{ kN/m}^2} = 1.217 \times 10^{-3} \text{ m}^3/\text{m of wall}]</td>
</tr>
</tbody>
</table>
4.5 Cantilever Sheet Piling Penetrating Clay

At times, cantilever sheet piles must be driven into a clay layer possessing an undrained cohesion ($\phi=0$). The net pressure diagram will be somewhat different from that shown in Figure 4.9a. Figure 4.13 shows a cantilever sheet-pile wall driven into clay with a backfill of granular soil above the level of the dredge line. The water table is at a depth $L_1$ below the top of the wall. As before, Eqs. (4.1) and (4.2) give the intensity of the net pressures $\sigma_1$ and $\sigma_2$, and the diagram for pressure distribution above the level of the dredge line can be drawn. The diagram for net pressure distribution below the dredge line can now be determined as follows.

![Figure 4.13 Cantilever sheet pile penetrating clay](image)

At any depth greater than $L_1 + L_2$, for $\phi=0$, the Rankine active earth-pressure coefficient $K_a = 1$. Similarly, for $\phi=0$, the Rankine passive earth-pressure coefficient $K_p = 1$. Thus, above the point of rotation (point $O$ in Figure 4.8a), the active pressure, from right to left is

$$\sigma_a = \left[ \gamma L_1 + \gamma' L_2 + \gamma_{sat}(z - L_1 - L_2) \right] - 2c \quad (4.24)$$

Similarly, the passive pressure from left to right may be expressed as

$$\sigma_p = \gamma_{sat}(z - L_1 - L_2) + 2c \quad (4.25)$$

Thus, the net pressure is

$$\sigma_n = \sigma_p - \sigma_a = \left[ \gamma_{sat}(z - L_1 - L_2) + 2c \right] - \left[ \gamma L_1 + \gamma' L_2 + \gamma_{sat}(z - L_1 - L_2) \right] + 2c$$

$$= 4c - (\gamma L_1 + \gamma' L_2) \quad (4.26)$$
At the bottom of the sheet pile, the passive pressure from right to left is
\[ \sigma_p = (\gamma L_1 + \gamma' L_2 + \gamma_{sat} D) + 2c \]  
(4.27)

Similarly, the active pressure from left to right is
\[ \sigma_a = \gamma_{sat} D - 2c \]  
(4.28)

Hence, the net pressure is
\[ \sigma_7 = \sigma_p - \sigma_a = 4c + (\gamma L_1 + \gamma' L_2) \]  
(4.29)

For equilibrium analysis, \( \sum F_H = 0 \); that is, the area of the pressure diagram \( ACDE \) minus the area of \( EFIB \) plus the area of \( GIH = 0 \), or
\[ P_1 - [4c - (\gamma L_1 + \gamma' L_2)]D + \frac{1}{2}L_4[4c - (\gamma L_1 + \gamma' L_2)] + 4c + (\gamma L_1 + \gamma' L_2) = 0 \]
where \( P_1 = \) area of the pressure diagram \( ACDE \).

Simplifying the preceding equation produces
\[ L_4 = \frac{D[4c - (\gamma L_1 + \gamma' L_2)] - P_1}{4c} \]  
(4.30)

Now, taking the moment about point \( B \) \( (\sum M_B = 0) \) yields
\[ P_1(D + \bar{z}_1) - [4c - (\gamma L_1 + \gamma' L_2)] \frac{D^2}{2} + \frac{1}{2}L_4(8c) \left( \frac{L_4}{3} \right) = 0 \]  
(4.31)

where \( \bar{z}_1 = \) distance of the center of pressure of the pressure diagram \( ACDE \), measured from the level of the dredge line.

Combining Eqs. (4.30) and (4.31) yields
\[ D^2[4c - (\gamma L_1 + \gamma' L_2)] - 2DP_1 - \frac{P_1(P_1 + 12c\bar{z}_1)}{(\gamma L_1 + \gamma' L_2) + 2c} = 0 \]  
(4.32)

Equation (4.32) may be solved to obtain \( D \), the theoretical depth of penetration of the clay layer by the sheet pile.

**Step-by-Step Procedure for Obtaining the Pressure Diagram**

Step 1. Calculate \( K_a \) for the granular soil (backfill).
Step 2. Obtain \( \sigma'_1 \) and \( \sigma'_2 \). [See Eqs. (4.1) and (4.2).]
Step 3. Calculate \( P_1 \) and \( z_1 \).
Step 4. Use Eq. (4.32) to obtain the theoretical value of \( D \).
Step 5. Using Eq. (4.30), calculate \( L_4 \).
Step 6. Calculate \( \sigma_6 \) and \( \sigma_7 \). [See Eqs. (4.26) and (4.29).]
Step 7. Draw the pressure distribution diagram as shown in Figure 4.13.
Step 8. The actual depth of penetration is
\[ D_{\text{actual}} = 1.4 \text{ to } 1.6 (D_{\text{theoretical}}) \]

**Maximum Bending Moment**
According to Figure 4.13, the maximum moment (zero shear) will be between \( L_1 + L_2 < z < L_1 + L_2 + L_3 \). Using a new coordinate system \( z' \) (with \( z' = 0 \) at the dredge line) for zero shear gives
\[ P_1 - \sigma_6 z' = 0 \]

or
\[ z' = \frac{P_1}{\sigma_6} \quad (4.33) \]

The magnitude of the maximum moment may now be obtained:
\[ M_{\text{max}} = P_1 (z' + \overline{z}_1) - \frac{\sigma_6 z'^2}{2} \quad (4.34) \]

Knowing the maximum bending moment, we determine the section modulus of the sheet-pile section from Eq. (4.23).

**Example 4.2:**
In Figure 4.14, for the sheet-pile wall, determine
a. The theoretical and actual depth of penetration. Use \( D_{\text{actual}} = 1.5 D_{\text{theory}} \).
b. The minimum size of sheet-pile section necessary. Use \( \sigma_{\text{all}} = 172.5 \text{ MN/m}^2 \).

![Figure 4.14 Cantilever sheet pile penetrating into saturated clay](image-url)
Solution

We will follow the step-by-step procedure given in Section 9.6:

**Step 1.**

\[ K_a = \tan^2 \left( 45 - \frac{\phi'}{2} \right) = \tan^2 \left( 45 - \frac{32}{2} \right) = 0.307 \]

**Step 2.**

\[ \sigma'_1 = \gamma L_1 K_a = (15.9)(2)(0.307) = 9.763 \text{ kN/m}^2 \]
\[ \sigma'_2 = (\gamma L_1 + \gamma' L_2) K_a = [(15.9)(2) + (19.33 - 9.81)3]0.307 \]
\[ = 18.53 \text{ kN/m}^2 \]

**Step 3.** From the net pressure distribution diagram given in Figure 9.12, we have

\[ P_1 = \frac{1}{2} \sigma'_1 L_1 + \sigma'_1 L_2 + \frac{1}{2}(\sigma'_2 - \sigma'_1) L_2 \]
\[ = 9.763 + 29.289 + 13.151 = 52.2 \text{ kN/m} \]

and

\[ \bar{z}_1 = \frac{1}{52.2} \left[ 9.763 \left( 3 + \frac{2}{3} \right) + 29.289 \left( \frac{3}{2} \right) + 13.151 \left( \frac{3}{3} \right) \right] \]
\[ = 1.78 \text{ m} \]

**Step 4.** From Eq. (9.48),

\[ D^2[4c - (\gamma L_1 + \gamma' L_2)] - 2DP_1 - \frac{P_1(P_1 + 12c\bar{z}_1)}{(\gamma L_1 + \gamma' L_2) + 2c} = 0 \]

Substituting proper values yields

\[ D^2\left\{ (4)(47) - [(2)(15.9) + (19.33 - 9.81)3] \right\} - 2D(52.2) \]
\[ - \frac{52.2[52.2 + (12)(47)(1.78)]}{[(15.9)(2) + (19.33 - 9.81)3] + (2)(47)} = 0 \]

or

\[ 127.64D^2 - 104.4D - 357.15 = 0 \]

Solving the preceding equation, we obtain \( D = 2.13 \text{ m} \).

**Step 5.** From Eq. (9.46),

\[ L_4 = \frac{D[4c - (\gamma L_1 + \gamma' L_2)] - P_1}{4c} \]

and

\[ 4c - (\gamma L_1 + \gamma' L_2) = (4)(47) - [(15.9)(2) + (19.33 - 9.81)3] \]
So,

\[ L_4 = \frac{2.13(127.64) - 52.2}{(4)(47)} = 1.17 \text{ m} \]

**Step 6.**

\[ \sigma_6 = 4c - (\gamma L_1 + \gamma' L_2) = 127.64 \text{ kN/m}^2 \]

\[ \sigma_7 = 4c + (\gamma L_1 + \gamma' L_2) = 248.36 \text{ kN/m}^2 \]

**Step 7.**  The net pressure distribution diagram can now be drawn, as shown in Figure 9.12.

**Step 8.**  \( D_{\text{actual}} \approx 1.5 D_{\text{theoretical}} = 1.5(2.13) \approx 3.2 \text{ m} \)

**Maximum-Moment Calculation**

From Eq. (9.49),

\[ z' = \frac{P_1}{\sigma_6} = \frac{52.2}{127.64} \approx 0.41 \text{ m} \]

Again, from Eq. (9.49),

\[ M_{\text{max}} = P_1(z' + \bar{z}_1) - \frac{\sigma_6 z'^2}{2} \]

So

\[ M_{\text{max}} = 52.2(0.41 + 1.78) - \frac{127.64(0.41)^2}{2} \]

\[ = 114.32 - 10.73 = 103.59 \text{ kN-m/m} \]

The minimum required section modulus (assuming that \( \sigma_{\text{all}} = 172.5 \text{ MN/m}^2 \)) is

\[ S = \frac{103.59 \text{ kN-m/m}}{172.5 \times 10^3 \text{ kN/m}^2} = 0.6 \times 10^{-3} \text{ m}^3/\text{m of the wall} \]
4.6 Anchored Sheet-Pile Walls

When the height of the backfill material behind a cantilever sheet-pile wall exceeds about 6 m, tying the wall near the top to anchor plates, anchor walls, or anchor piles becomes more economical. This type of construction is referred to as **anchored sheet-pile wall or an anchored bulkhead**. Anchors minimize the depth of penetration required by the sheet piles and also reduce the cross-sectional area and weight of the sheet piles needed for construction. However, the tie rods and anchors must be carefully designed.

The two basic methods of designing anchored sheet-pile walls are
(a) the **free earth support** method and
(b) the **fixed earth support** method.

Figure 4.17 shows the assumed nature of deflection of the sheet piles for the two methods.

The free earth support method involves a minimum penetration depth. Below the dredge line, no pivot point exists for the static system. The nature of the variation of the bending moment with depth for both methods is also shown in Figure 4.17. Note that

![Diagram](image)

Figure 4.17 Nature of variation of deflection and moment for anchored sheet piles:
(a) free earth support method (b) fixed earth support method
4.7 Free Earth Support Method for Penetration of Sandy Soil

Figure 4.18 shows an anchor sheet-pile wall with a granular soil backfill; the wall has been driven into a granular soil. The tie rod connecting the sheet pile and the anchor is located at a depth \( l_1 \) below the top of the sheet-pile wall.

The diagram of the net pressure distribution above the dredge line is similar to that shown in Figure 4.9. At depth \( z = L_1 \), \( \sigma'_1 = \gamma L_1 K_a \), and at \( z = L_1 + L_2 \), \( \sigma'_2 = (\gamma L_1 + \gamma L_2) K_a \). Below the dredge line, the net pressure will be zero at \( z + L_1 + L_2 + L_3 \). The relation for \( L_3 \) is given by Eq. (4.6), or

\[
L_3 = \frac{\sigma'_7}{\gamma' (K_p - K_a)}
\]

At \( z = L_1 + L_2 + L_3 + L_4 \), the net pressure is given by

\[
\sigma'_8 = \gamma' (K_p - K_a) L_4
\]  

(4.35)

Note that the slope of the line \( DEF \) is 1 vertical to \( \gamma' (K_p - K_a) \) horizontal. For equilibrium of the sheet pile, \( \Sigma \) horizontal forces = 0, and \( \Sigma \) moment about \( O' \) = 0. (Note: Point \( O' \) is located at the level of the tie rod.) Summing the forces in the horizontal direction (per unit length of the wall) gives

Area of the pressure diagram \( ACDE \) - area of \( EBF - F = 0 \)
where $F=5$ tension in the tie rod/unit length of the wall, or

$$ P - \frac{1}{2} \sigma' L_4 - F = 0 $$

or

$$ F = P - \frac{1}{2} [\gamma' (K_p - K_a)] L_4^2 $$

(4.36)

where $P =$ area of the pressure diagram ABCDE.

Now, taking the moment about point $O'$ gives

$$ -P[(L_1 + L_2 + L_3) - (\bar{z} + l_1)] + \frac{1}{2} [\gamma' (K_p - K_a)] L_4^2 (L_2 + L_3 + \frac{2}{3} L_4) = 0 $$

Or

$$ L_4^3 + 1.5 L_4^2 (L_2 + L_2 + L_3) - \frac{3P[(L_1 + L_2 + L_3) - (\bar{z} + l_1)]}{\gamma' (K_p - K_a)} = 0 $$

(4.37)

Equation (4.37) may be solved by trial and error to determine the theoretical depth, $L_4$:

$$ D_{\text{theoretical}} = L_3 + L_4 $$

The theoretical depth is increased by about 30 to 40% for actual construction, or

$$ D_{\text{actual}} = 1.3 \text{ to } 1.4 \times D_{\text{theoretical}} $$

(4.38)

The step-by-step procedure in Section 4.4 indicated that a factor of safety can be applied to $K_p$ at the beginning [i.e., $K_p(\text{designed}) = K_p/FS$]. If that is done, there is no need to increase the theoretical depth by 30 to 40%. This approach is often more conservative.

The maximum theoretical moment to which the sheet pile will be subjected occurs at a depth between $z = L_1$ and $z = L_1 + L_2$. The depth $z$ for zero shear and hence maximum moment may be evaluated from

$$ \frac{1}{2} \sigma' L_1 - F + \sigma' (z - L_1) + \frac{1}{2} K_a \gamma' (z - L_1)^2 = 0 $$

(4.39)

Once the value of $z$ is determined, the magnitude of the maximum moment is easily obtained.
4.8 Moment Reduction for Anchored Sheet-Pile Walls

Walls Penetrating into Sand Sheet piles are flexible, and hence sheet-pile walls yield (i.e., become displaced laterally), which redistributes the lateral earth pressure. This change tends to reduce the maximum bending moment, $M_{\text{max}}$, as calculated by the procedure outlined in Section 4.7. For that reason, Rowe (1952, 1957) suggested a procedure for reducing the maximum design moment on the sheet-pile walls obtained from the free earth support method. This section discusses the procedure of moment reduction for sheet piles penetrating into sand.

In Figure 4.25, which is valid for the case of a sheet pile penetrating sand, the following notation is used:

![Figure 4.25 Plot of log $\rho$ against $M_d/M_{\text{max}}$ for sheet-pile walls penetrating sand (after, P. W. (1952)).](image)

1. $H' = 5$ total height of pile driven (i.e., $L_1 + L_2 + D_{\text{actual}}$)

2. Relative flexibility of pile $\rho = 10.91 \times 10^{-7} \left( \frac{H'}{E} \right)$ \hfill (4.40)

where

- $H'$ is in meters
- $E = \text{modulus of elasticity of the pile material (MN/ m$^2$)}$
- $I = \text{moment of inertia of the pile section per meter of the wall (m}^4/\text{m of wall)}$

3. $M_d = \text{design moment}$

4. $M_{\text{max}} = \text{maximum theoretical moment}$

The procedure for the use of the moment reduction diagram (see Figure 4.25) is as follows:

**Step 1.** Choose a sheet-pile section (e.g., from among those given in Table 4.1).
Step 2. Find the modulus $S$ of the selected section (Step 1) per unit length of the wall.

Step 3. Determine the moment of inertia of the section (Step 1) per unit length of the wall.

Step 4. Obtain $H'$ and calculate $r$ [see Eq. (4.40)].

Step 5. Find $\log r$.

Step 6. Find the moment capacity of the pile section chosen in Step 1 as $M_d = \sigma_{all} S$.

Step 7. Determine $M_d/M_{\max}$. Note that $M_{\max}$ is the maximum theoretical moment determined before.

Step 8. Plot $\log r$ (Step 5) and $M_d/M_{\max}$ in Figure 4.25.

Step 9. Repeat Steps 1 through 8 for several sections. The points that fall above the curve (in loose sand or dense sand, as the case may be) are safe sections.

The points that fall below the curve are unsafe sections. The cheapest section may now be chosen from those points which fall above the proper curve. Note that the section chosen will have an $M_d$, $M_{\max}$.

Example 4.3:
Let $L_1 = 3.05 \, \text{m}$, $L_2 = 6.1 \, \text{m}$, $l_1 = 1.53 \, \text{m}$, $l_2 = 1.52 \, \text{m}$, $c' = 0$, $\phi' = 30^\circ$, $\gamma = 16 \, \text{kN/m}^3$, $\gamma_{sat} = 19.5 \, \text{kN/m}^3$, and $E = 207 \times 10^3 \, \text{MN/m}^2$ in Figure 9.17.

a. Determine the theoretical and actual depths of penetration. \((Note: D_{\text{actual}} = 1.3D_{\text{theory}})\)

b. Find the anchor force per unit length of the wall.

c. Determine the maximum moment, $M_{\text{max}}$.

**Solution**

Part a

We use the following table.

<table>
<thead>
<tr>
<th>Quantity required</th>
<th>Eq. no.</th>
<th>Equation and calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_a$</td>
<td></td>
<td>$\tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2\left(45 - \frac{30}{2}\right) = \frac{1}{3}$</td>
</tr>
<tr>
<td>$K_p$</td>
<td></td>
<td>$\tan^2\left(45 + \frac{\phi'}{2}\right) = \tan^2\left(45 + \frac{30}{2}\right) = 3$</td>
</tr>
<tr>
<td>$K_p - K_a$</td>
<td></td>
<td>$3 - 0.333 = 2.667$</td>
</tr>
<tr>
<td>$\gamma'$</td>
<td></td>
<td>$\gamma_{sat} - \gamma_w = 19.5 - 9.81 = 9.69 , \text{kN/m}^3$</td>
</tr>
<tr>
<td>$\sigma_1'$</td>
<td>9.1</td>
<td>$\gamma L_1 K_a = (16)(3.05)(\frac{1}{2}) = 16.27 , \text{kN/m}^2$</td>
</tr>
<tr>
<td>$\sigma_2'$</td>
<td>9.2</td>
<td>$(\gamma L_1 + \gamma' L_2) K_a = [(16)(3.05) + (9.69)(6.1)] = 35.97 , \text{kN/m}^2$</td>
</tr>
<tr>
<td>$L_3$</td>
<td>9.6</td>
<td>$\frac{\sigma_2'}{\gamma'(K_p - K_a)} = \frac{35.97}{(9.69)(2.667)} = 1.39 , \text{m}$</td>
</tr>
<tr>
<td>$P$</td>
<td></td>
<td>$\frac{\sigma_2'}{\gamma'(K_p - K_a)} + \frac{1}{2}\sigma_2' L_2 + \frac{1}{2}(\sigma_2' - \sigma_1') L_2 + \frac{1}{3}\sigma_2' L_3 = (\frac{1}{2})(16.27)(3.05) + (16.27)(6.1) + (\frac{1}{3})(35.97)(1.39)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= 24.81 + 99.25 + 60.01 + 25.0 = 209.07 , \text{kN/m}$</td>
</tr>
<tr>
<td>$\overline{\zeta}$</td>
<td></td>
<td>$\frac{\Sigma M_E}{P} = \left[(24.81 + 99.25 + \frac{3.05}{3}) + (25.0)(\frac{1.39 + 6.1}{3})\right] \frac{1}{209.07}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= 4.21 , \text{m}$</td>
</tr>
</tbody>
</table>
\[ L_a = 2.7 \text{ m} \]

*Part b*

The anchor force per unit length of the wall is

\[ F = P - \frac{1}{2} \gamma' (K_p - K_a) L_4 \]

\[ = 209.07 - \left( \frac{1}{2} \right) (9.69) (2.667) (2.7)^2 = 114.87 \text{ kN/m} \approx 115 \text{ kN/m} \]

*Part c*

From Eq. (9.69), for zero shear,

\[ \frac{1}{2} \sigma'_r L_1 - F + \sigma'_r (z - L_1) + \frac{1}{2} K_a \gamma' (z - L_1)^2 = 0 \]

Let \( z - L_1 = x \), so that

\[ \frac{1}{2} \sigma'_r l_1 - F + \sigma'_r x + \frac{1}{2} K_a \gamma' x^2 = 0 \]

or

\[ \left( \frac{1}{2} \right) (16.27) (3.05) - 115 + (16.27) (x) + \left( \frac{1}{2} \right) \left( \frac{1}{3} \right) (9.69) x^2 = 0 \]

giving \[ x^2 + 10.07x - 55.84 = 0 \]

Now, \( x = 4 \text{ m} \) and \( z - x + L_1 = 4 + 3.05 = 7.05 \text{ m} \). Taking the moment about the point of zero shear, we obtain

\[ M_{\text{max}} = -\frac{1}{2} \sigma'_r L_1 \left( x + \frac{3.05}{3} \right) + F (x + 1.52) - \frac{1}{2} \sigma'_r \frac{x^2}{2} - \frac{1}{2} K_a \gamma' x^2 \left( \frac{x}{3} \right) \]

or

\[ M_{\text{max}} = -\left( \frac{1}{2} \right) (16.27) (3.05) \left( 4 + \frac{3.05}{3} \right) + (115) (4 + 1.52) - (16.27) \left( \frac{4^2}{2} \right) \]

\[ -\left( \frac{1}{2} \right) \left( \frac{1}{3} \right) (9.69) (4)^2 \left( \frac{4}{3} \right) = 344.9 \text{ kN} \cdot \text{m/m} \]
Example 4.4:

Refer to Example 9.5. Use Rowe’s moment reduction diagram (Figure 9.24) to find an appropriate sheet pile section. For the sheet pile, use \( E = 207 \times 10^3 \) MN/m² and \( \sigma_{all} = 172,500 \) kN/m².

**Solution**

\[
H' = L_1 + L_2 + D_{actual} = 3.05 + 6.1 + 5.33 = 14.48 \text{ m}
\]

\( M_{\text{max}} = 344.9 \) kN · m/m. Now the following table can be prepared.

<table>
<thead>
<tr>
<th>Section</th>
<th>( l (\text{m}^4/\text{m}) )</th>
<th>( H' (\text{m}) )</th>
<th>( 10^{-7} \left( \frac{H^4}{EI} \right) )</th>
<th>\text{log } \rho</th>
<th>( S (\text{m}^3/\text{m}) )</th>
<th>( M_d = S\sigma_{all} ) (kN · m/m)</th>
<th>( \frac{M_d}{M_{\text{max}}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PZ-22</td>
<td>( 115.2 \times 10^{-6} )</td>
<td>14.48</td>
<td>( 20.11 \times 10^{-4} )</td>
<td>-2.7</td>
<td>( 97 \times 10^{-3} )</td>
<td>167.33</td>
<td>0.485</td>
</tr>
<tr>
<td>PZ-27</td>
<td>( 251.5 \times 10^{-6} )</td>
<td>14.48</td>
<td>( 9.21 \times 10^{-4} )</td>
<td>-3.04</td>
<td>( 162.3 \times 10^{-3} )</td>
<td>284.84</td>
<td>0.826</td>
</tr>
</tbody>
</table>

Figure 9.25 gives a plot of \( M_d/M_{\text{max}} \) versus \( \rho \). It can be seen that **PZ-27** will be sufficient.
4.9 Free Earth Support Method for Penetration of Clay

Figure 4.32 shows an anchored sheet-pile wall penetrating a clay soil and with a granular soil backfill. The diagram of pressure distribution above the dredge line is similar to that shown in Figure 4.9. From Eq. (4.26), the net pressure distribution below the dredge line (from \( z = L_1 + L_2 \) to \( z = L_1 + L + D \)) is

\[
\sigma_6 = 4c - (\gamma L_1 + \gamma' L_2)
\]

For static equilibrium, the sum of the forces in the horizontal direction is

\[
P_1 - \sigma_6 D = F \quad (4.41)
\]

where

\( P_1 \) = area of the pressure diagram \( ACD \)

\( F \) = anchor force per unit length of the sheet-pile wall

---

*Figure 4.32 Anchored sheet-pile wall penetrating clay*
Again, taking the moment about \( O \) produces

\[
P_1(L_1 + L_2 - l_1 - \bar{z}_1) - \sigma_0 D \left( l_2 + L_2 + \frac{D}{2} \right) = 0
\]

Simplification yields

\[
u_6 D^2 + 2\sigma_0 D (L_1 + L_2 - l_1) - 2P_1(L_1 + L_2 - l_1 - \bar{z}_1) = 0 \quad (4.42)
\]

Equation (4.42) gives the theoretical depth of penetration, \( D \).

As in Section 4.7, the maximum moment in this case occurs at a depth \( L_1, z, L_1 + L_2 \). The depth of zero shear (and thus the maximum moment) may be determined from Eq. (4.39).

A moment reduction technique similar to that in Section 14.11 for anchored sheet piles penetrating into clay has also been developed by Rowe (1952, 1957). This technique is presented in Figure 4.33, in which the following notation is used:

1. The stability number is

\[
S_n = 1.25 \frac{c}{(\gamma L_1 + \gamma' L_2)} \quad (4.43)
\]

where \( c = \) undrained cohesion (\( \phi=0 \)). For the definition of \( \gamma, \gamma', L_1, \) and \( L_2 \), see Figure 4.32.

2. The nondimensional wall height is

\[
\alpha = \frac{L_1 + L_2}{L_1 + L_2 + D_{\text{actual}}} \quad (4.44)
\]

3. The flexibility number is \( \rho \) [see Eq. (4.40)]

4. \( M_d = \) design moment

\( M_{\text{max}} = \) maximum theoretical moment

The procedure for moment reduction, using Figure 4.33, is as follows:

Step 1. Obtain \( H = L_1 + L_2 + D_{\text{actual}} \).

Step 2. Determine \( \alpha = (L_1 + L_2)/H \).

Step 3. Determine \( S_n \) [from Eq. (4.43)].

Step 4. For the magnitudes of \( \alpha \) and \( S_n \) obtained in Steps 2 and 3, determine \( M_d / M_{\text{max}} \) for various values of \( \log \rho \) from Figure 4.33, and plot \( M_d / M_{\text{max}} \) against \( \log \rho \).
Step 5. Follow Steps 1 through 9 as outlined for the case of moment reduction of sheet-pile walls penetrating granular soil. (See Section 4.8.)

Figure 4.33 Plot of $M_d / M_{\max}$ against stability number for sheetpile wall penetrating clay [after Rowe, (1957)].
Example 4.5:

In Figure 9.34, let \( L_1 = 3 \text{ m} \), \( L_2 = 6 \text{ m} \), and \( L_1 = 1.5 \text{ m} \). Also, let \( \gamma = 17 \text{ kN/m}^3 \), \( \gamma_{\text{sat}} = 20 \text{ kN/m}^3 \), \( \phi' = 35^\circ \), and \( c = 41 \text{ kN/m}^2 \).

a. Determine the theoretical depth of embedment of the sheet-pile wall.
b. Calculate the anchor force per unit length of the wall.

Solution

Part a

We have

\[
K_a = \tan^2 \left( 45 - \frac{\phi'}{2} \right) = \tan^2 \left( 45 - \frac{35}{2} \right) = 0.271
\]

and

\[
K_p = \tan^2 \left( 45 + \frac{\phi'}{2} \right) = \tan^2 \left( 45 + \frac{35}{2} \right) = 3.69
\]

From the pressure diagram in Figure 9.36,

\[
\sigma_1' = \gamma L_1 K_a = (17)(3)(0.271) = 13.82 \text{ kN/m}^2
\]

\[
\sigma_2' = (\gamma L_1 + \gamma' L_2) K_a = [(17)(3) + (20 - 9.81)(6)](0.271) = 30.39 \text{ kN/m}^2
\]

\[
P_1 = \text{areas } 1 + 2 + 3 = 1/2(3)(13.82) + (13.82)(6) + 1/2(30.39 - 13.82)(6)
\]

\[
= 20.73 + 82.92 + 49.71 = 153.36 \text{ kN/m}
\]

and

\[
\bar{z}_1 = \frac{(20.73) \left( 6 + \frac{3}{3} \right) + (82.92) \left( \frac{6}{2} \right) + (49.71) \left( \frac{6}{3} \right)}{153.36} = 3.2 \text{ m}
\]

From Eq. (9.85),

\[
\sigma_6 D^2 + 2\sigma_6 D(L_1 + L_2 - l_1) - 2P_1(L_1 + L_2 - l_1 - \bar{z}_1) = 0
\]

\[
\sigma_6 = 4c - (\gamma L_1 + \gamma' L_2) = (4)(41) - [(17)(3)
\]

\[
+ (20 - 9.81)(6)] = 51.86 \text{ kN/m}^2
\]

So,

\[
(51.86) D^2 + (2)(51.86)(D)(3 + 6 - 1.5) - (2)(153.36)(3 + 6 - 1.5 - 3.2) = 0
\]
or

\[ D^2 + 15D - 25.43 = 0 \]

Hence,

\[ D \approx 1.6 \text{ m} \]

Part b

From Eq. (9.84),

\[ F = P_1 - \sigma_6D = 153.36 - (51.86)(1.6) = 70.38 \text{ kN/m} \]
4.10 Anchors

Sections 4.7 through 4.9 gave an analysis of anchored sheet-pile walls and discussed how to obtain the force $F$ per unit length of the sheet-pile wall that has to be sustained by the anchors. The current section covers in more detail the various types of anchor generally used and the procedures for evaluating their ultimate holding capacities.

The general types of anchors used in sheet-pile walls are as follows:
1. Anchor plates and beams (deadman)
2. Tie backs
3. Vertical anchor piles
4. Anchor beams supported by batter (compression and tension) piles

*Anchor plates and beams* are generally made of cast concrete blocks. (See Figure 4.36a.) The anchors are attached to the sheet pile by *tie rods*. A *wale* is placed at the front or back face of a sheet pile for the purpose of conveniently attaching the tie rod to the wall. To protect the tie rod from corrosion, it is generally coated with paint or asphaltic materials.

In the construction of *tiebacks*, bars or cables are placed in predrilled holes (see Figure 4.36b) with concrete grout (cables are commonly high-strength, prestressed steel tendons). Figures 4.36c and 4.36d show a vertical anchor pile and an anchor beam with batter piles.

**Placement of Anchors**

The resistance offered by anchor plates and beams is derived primarily from the passive force of the soil located in front of them. Figure 4.36a, in which $AB$ is the sheet-pile wall, shows the best location for maximum efficiency of an anchor plate. If the anchor is placed inside wedge $ABC$, which is the Rankine active zone, it would not provide any resistance to failure. Alternatively, the anchor could be placed in zone $CFEH$. Note that line $DFG$ is the slip line for the Rankine passive pressure. If part of the passive wedge is located inside the active wedge $ABC$, full passive resistance of the anchor cannot be realized upon failure of the sheet-pile wall. However, if the anchor is placed in zone $ICH$, the Rankine passive zone in front of the anchor slab or plate is located completely outside the Rankine active zone $ABC$. In this case, full passive resistance from the anchor can be realized.

Figures 4.36b, 4.36c, and 4.36d also show the proper locations for the placement of tiebacks, vertical anchor piles, and anchor beams supported by batter piles.
Sheet Pile Walls

Figure 4.36 Various types of anchoring for sheet-pile walls: (a) anchor plate or beam; (b) tieback; (c) vertical anchor pile; (d) anchor beam with batter piles
Capacity of Deadman (After Teng, 1969)
A series of deadmen (anchor beams, anchor blocks or anchor plates) are normally placed at intervals parallel to the sheet pile walls. These anchor blocks may be constructed near the ground surface or at great depths, and in short lengths or in one continuous beam. The holding capacity of these anchorages is discussed below.

Continuous Anchor Beam Near Ground Surface (Teng, 1969)
If the length of the beam is considerably greater than its depth, it is called a continuous deadman. Fig. 4.37(a) shows a deadman. If the depth to the top of the deadman, \( h \), is less than about one-third to one-half of \( H \) (where \( H \) is depth to the bottom of the deadman), the capacity may be calculated by assuming that the top of the deadman extends to the ground surface. The ultimate capacity of a deadman may be obtained from (per unit length)

For granular soil \((c = 0)\)

\[
T_u = P_p - P_a = \frac{1}{2} \gamma H^2 K_p - \frac{1}{2} \gamma H^2 K_A
\]  

or

\[
T_u = \frac{1}{2} \gamma H^2 (K_p - K_A)
\]

For clay soil \((\phi = 0)\)

\[
T_u = P_p - P_a = q_u H + \frac{1}{2} \gamma H^2 - \frac{1}{2} \gamma H^2 - q_u H + \frac{2c^2}{\gamma} = 2q_u H - \frac{2c^2}{\gamma}
\]

where \(q_u = \) unconfined compressive strength of soil,
\(\gamma = \) effective unit weight of soil, and
\(K_p, K_a = \) Rankine's active and passive earth pressure coefficients.

It may be noted here that the active earth pressure is assumed to be zero at a depth = \(2c/\gamma\) which is the depth of the tension cracks. It is likely that the magnitude and distribution of earth pressure may change slowly with time. For lack of sufficient data on this, the design of deadmen in cohesive soils should be made with a conservative factor of safety.

Short Deadman Near Ground Surface in Granular Soil (Fig. 4.37b)
If the length of a deadman is shorter than \(3h \) (\(h = \) height of deadman) there will be an end effect with regards to the holding capacity of the anchor. The equation suggested by Teng for computing the ultimate tensile capacity \(T_u\) is
Figure 4.37 Capacity of deadmen: (a) continuous deadmen near ground surface ($\bar{h}/H < 1/3 \sim 1/2$); (b) short deadmen near ground surface; (c) deadmen at great depth below ground surface (after Teng, 1969)

$$T_u = L(P_p - P_a) + \frac{1}{3} K_o \gamma (\sqrt{K_p} + \sqrt{K_a}) H^3 \tan \phi$$  \hspace{1cm} (4.48)

where
- $h =$ height of deadman
- $\bar{h} =$ depth to the top of deadman
- $L =$ length of deadman
- $H =$ depth to the bottom of the dead man from the ground surface
- $P_p, P_a =$ total passive and active earth pressures per unit length
- $K_o =$ coefficient of earth pressures at-rest, taken equal to 0.4
- $\gamma =$ effective unit weight of soil
- $K_p, K_a =$ Rankine's coefficients of passive and active earth pressures
φ= angle of internal friction

**Anchor Capacity of Short Deadman in Cohesive Soil Near Ground Surface**

In cohesive soils, the second term of Eq. (4.48) should be replaced by the cohesive resistance

\[ T_u = L(P_p - P_a) + q_u H^2 \]  \hspace{1cm} (4.49)

where \( q_u \) = unconfmed compressive strength of soil.

**Deadman at Great Depth**

The ultimate capacity of a deadman at great depth below the ground surface as shown in Fig. (4.37c) is approximately equal to the bearing capacity of a footing whose base is located at a depth \(( \bar{h} + h/2)\), corresponding to the mid height of the deadman (Terzaghi, 1943).