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CHAPTER SIX

SLOPE STABILITY

LECTURE
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Introduction

An exposed ground surface that stands at an angle with the horizontal is called an *unrestrained slope*. The slope can be natural or constructed. If the ground surface is not horizontal, a component of gravity will cause the soil to move downward, as shown in Figure 6.1. If the component of gravity is large enough, slope failure can occur; that is, the soil mass in zone *abcdea* can slide downward. The driving force overcomes the resistance from the shear strength of the soil along the rupture surface.

In many cases, civil engineers are expected to make calculations to check the safety of natural slopes, slopes of excavations, and compacted embankments. This process, called *slope stability analysis*, involves determining and comparing the shear stress developed along the most likely rupture surface with the shear strength of the soil.

The stability analysis of a slope is not an easy task. Evaluating variables such as the soil stratification and its in-place shear strength parameters may prove to be a formidable task. Seepage through the slope and the choice of a potential slip surface add to the complexity of the problem. This chapter explains the basic principles involved in slope stability analysis.

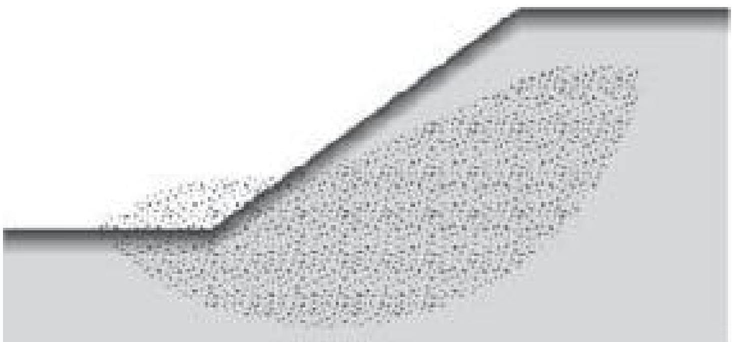

Figure 6.1 Slope failure

6.1 Factor of Safety

The task of the engineer charged with analyzing slope stability is to determine the factor of safety. Generally, the factor of safety is defined as

$$F_s = \frac{\tau_f}{\tau_d} \qquad (6\text{-}1)$$

where

FS_s = factor of safety with respect to strength

τ_f = average shear strength of the soil

τ_d = average shear stress developed along the potential failure surface

The shear strength of a soil consists of two components, cohesion and friction, and may be expressed as

$$\tau_f = c' + \sigma' \tan \phi' \qquad (6\text{-}2)$$

where
c' = cohesion
ϕ'= drained angle of friction
σ'= effective normal stress on the potential failure surface.

In a similar manner, we can also write

$$\tau_d = c'_d + \sigma' \tan \phi'_d \qquad (6\text{-}3)$$

where c'_d and ϕ'_d are, respectively, the effective cohesion and the angle of friction that develop along the potential failure surface. Substituting Eqs. (6.2) and (6.3) into Eq. (6.1), we get

$$F_s = \frac{c' + \sigma' \tan \phi'}{c'_d + \sigma' \tan \phi'_d} \qquad (6\text{-}4)$$

Now we can introduce some other aspects of the factor of safety-that is, the factor of safety with respect to cohesion, $FS_{c'}$, and the factor of safety with respect to friction, $FS_{\phi'}$ They are defined as follows:

$$F_{c'} = \frac{c'}{c'_d} \qquad (6\text{-}5)$$

and

$$F_{\phi'} = \frac{\tan \phi'}{\tan \phi'_d} \qquad (6\text{-}6\_$$

When Eqs. (6.4), (6.5), and (6.6) are compared, we see that when $FS_{c'}$ becomes equal to $FS_{\phi'}$, that is the factor of safety with respect to strength. Or, if

$$\frac{c'}{c'_d} = \frac{\tan \phi'}{\tan \phi'_d} \qquad (6\text{-}7)$$

we can write

$$F_s = F_{c'} = F_{\phi'}$$

When FS_s is equal to 1, the slope is in a state of impending failure. Generally, a value of 1.5 for the factor of safety with respect to strength is acceptable for the design of a stable slope.

6.2 Stability of Infinite Slopes

In considering the problem of slope stability, we may start with the case of an infinite slope, as shown in Figure 6.2. An infinite slope is one in which $H$is much greater than the slope height. The shear strength of the soil may be given by [Eq. (6.2)]

$$\tau_f = c' + \sigma' \tan \phi'$$

We will evaluate the factor of safety against a possible slope failure along a plane *AB* located at a depth *H* below the ground surface. The slope failure can occur by the movement of soil above the plane *AB* from right to left.

Let us consider a slope element, *abcd*, that has a unit length perpendicular to the plane of the section shown. The forces, *F*, that act on the faces *ab* and *cd* are

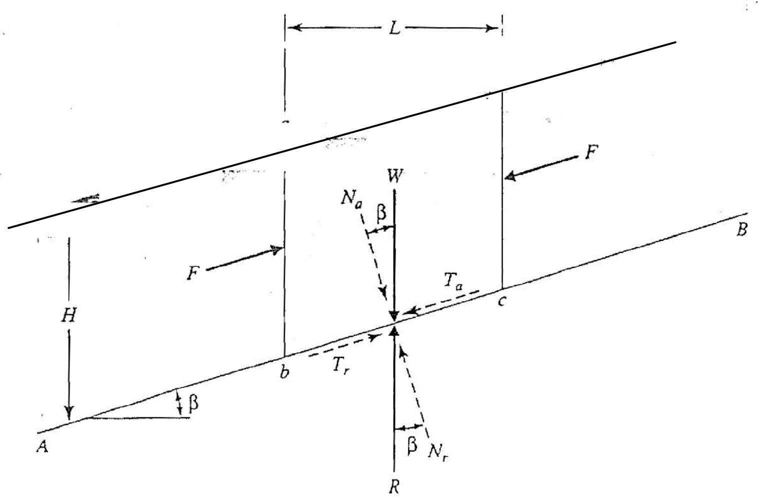

Figure 6.2 Analysis of infinite slope (without seepage)

equal and opposite and may be ignored. The effective weight of the soil element is (with pore water pressure equal to 0)

$$W = (\text{Volume of soil element}) \times (\text{Unit weight of soil}) = \gamma L H \qquad (6\text{-}8)$$

The weight W can be resolved into two components:

1. Force perpendicular to the plane $AB = N_a = W\cos\beta = \gamma LH\cos\beta$.
2. Force parallel to the plane $AB = T_a = W\sin\beta = \gamma LH\sin\beta$. Note that this is the force that tends to cause the slip along the plane.

Thus, the effective normal stress and the shear stress at the base of the slope element can be given, respectively, as

$$\sigma' = \frac{N_a}{\text{Area of base}} = \frac{\gamma LH\cos\beta}{\left(\frac{L}{\cos\beta}\right)} = \gamma H\cos^2\beta \qquad (6\text{-}9)$$

And

$$\tau = \frac{T_a}{\text{Area of base}} = \frac{\gamma LH\sin\beta}{\left(\frac{L}{\cos\beta}\right)} = \gamma H\cos\beta\sin\beta \qquad (6\text{-}10)$$

The reaction to the weight W is an equal and opposite force R. The normal and tangential components of R with respect to the plane AB are

$$N_r = R\cos\beta = W\cos\beta \qquad (6\text{-}11)$$

and

$$T_r = R\sin\beta = W\sin\beta \qquad (6\text{-}12)$$

For equilibrium, the resistive shear stress that develops at the base of the element is equal to $(T_r)/(\text{Area of base}) = \gamma H\sin\beta\cos\beta$. The resistive shear stress may also be written in the same form as Eq. (14.3):

$$\tau_d = c'_d + \sigma'\tan\phi'_d$$

The value of the effective normal stress is given by Eq. (6.9). Substitution of Eq. (6.9) into Eq. (6.3) yields

$$\tau_d = c'_d + \gamma H\cos^2\beta\tan\phi'_d \qquad (6\text{-}13)$$

This

$$\gamma H \sin\beta \cos\beta = c'_d + \gamma H \cos^2\beta \tan\phi'_d$$

Or

$$\frac{c'_d}{\gamma H} = \sin\beta\cos\beta - \cos^2\beta \tan\phi'_d$$

$$= \cos^2\beta(\tan\beta - \tan\phi'_d) \qquad (6\text{-}14)$$

The factor of safety with respect to strength was defined in Eq. (6.7), from which

$$\tan\phi'_d = \frac{\tan\phi'}{F_s} \quad \text{and} \quad c'_d = \frac{c'}{F_s}$$

Substituting the preceding relationships into Eq. (6.14), we obtain

$$F_s = \frac{c'}{\gamma H \cos^2\beta \tan\beta} + \frac{\tan\phi'}{\text{ta}} \qquad (6\text{-}15)$$

For granular soils, $c' = 0$, and the factor of safety, F_s, becomes equal to $(\tan\phi')/(\tan\beta)$. This indicates that in an infinite slope in sand, the value of F_s is independent of the height H and the slope is stable as long as $\beta < \phi'$.

If a soil possesses cohesion and friction, the depth of the plane along which critical equilibrium occurs may be determined by substituting $F_s = 1$ and $H = H_{cr}$ into Eq. (14.15). Thus,

$$H_{cr} = \frac{c'}{\gamma} \frac{1}{\cos^2\beta(\tan\beta - \tan\phi')} \qquad (6\text{-}16)$$

If there is seepage through the soil and the ground water level coincides with the ground surface as shown in Figure 6.3, the factor of safety with respect to strength can be obtained as

$$F_s = \frac{c'}{\gamma_{sat} H \cos^2\beta \tan\beta} + \frac{\gamma'}{\gamma_{sat}} \frac{\tan\phi'}{\tan\beta} \qquad (6\text{-}17)$$

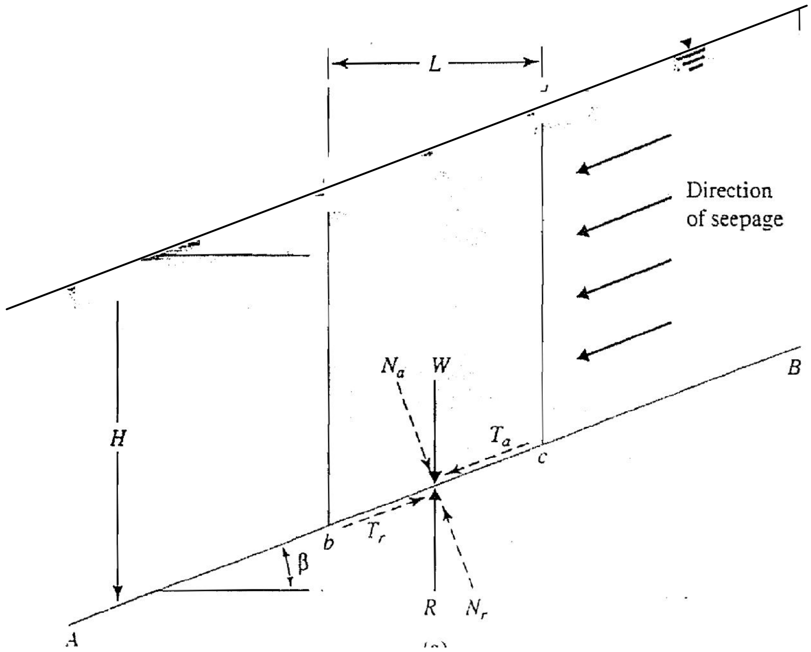

Figure 6.3Infinite slope with seepage

6.3 Finite Slopes

When the value of H_{cr} approaches the height of the slope, the slope is generally considered finite. When analyzing the stability of a finite slope in a homogeneous soil, for simplicity, we need to make an assumption about the general shape of the surface of potential failure. Although there is considerable evidence that slope failures usually occur on curved failure surfaces, Culmann (1875) approximated the surface of potential failure as a plane. The factor of safety, FS_s, calculated using Culmann's approximation gives fairly good results for near-vertical slopes only.

After extensive investigation of slope failures in the 1920s, a Swedish geotechnical commission recommended that the actual surface of sliding may be approximated to be circularly cylindrical. Since that time, most conventional stability analyses of slopes have been made by assuming that the curve of potential sliding is an arc of a circle. However, in many circumstances (for example, zoned dams and foundations on weak strata), stability analysis using plane failure of sliding is more appropriate and yields excellent results.

Analysis of Finite Slope with Plane Failure Surface (Culmann's Method)

This analysis is based on the assumption that the failure of a slope occurs along a plane when the average shearing stress that tends to cause the slip is greater than the shear strength of the soil. Also, the most critical plane is the one that has a minimum ratio of the average shearing stress that tends to cause failure to the shear strength of soil.

Figure 6.4 shows a slope of height *H*. The slope rises at an angle _ with the horizontal. *AC* is a trial failure plane. If we consider a unit length perpendicular to the section of the slope, the weight of the wedge *ABC* = *W*:

$$W = \tfrac{1}{2}(H)(\overline{BC})(1)(\gamma) = \tfrac{1}{2}H(H\cot\theta - H\cot\beta)\gamma$$

$$= \frac{1}{2}\gamma H^2\left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right] \qquad (6\text{-}19)$$

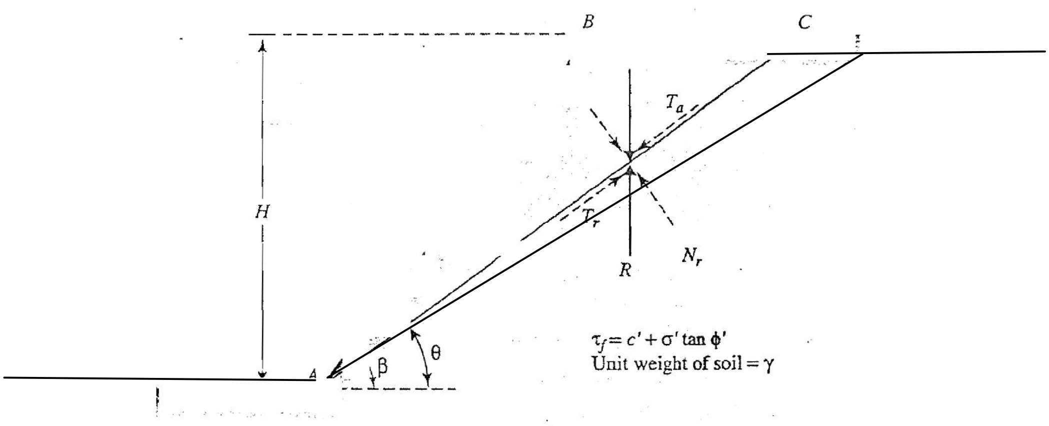

Figure 6.4 Finite slope analysis—Culmann's method

The normal and tangential components of W with respect to the plane AC are as follows:

$$N_a = \text{normal component} = W\cos\theta = \frac{1}{2}\gamma H^2\left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\cos\theta \qquad (14.30)$$

$$T_a = \text{tangential component} = W\sin\theta = \frac{1}{2}\gamma H^2\left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\sin\theta \qquad (14.31)$$

The average effective normal stress and the average shear stress on the plane AC are, respectively,

$$\sigma' = \frac{N_a}{(\overline{AC})(1)} = \frac{N_a}{\left(\dfrac{H}{\sin\theta}\right)}$$

$$= \frac{1}{2}\gamma H\left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\cos\theta\sin\theta \qquad (14.32)$$

and

$$\tau = \frac{T_a}{(\overline{AC})(1)} = \frac{T_a}{\left(\dfrac{H}{\sin\theta}\right)}$$

$$= \frac{1}{2}\gamma H\left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\sin^2\theta \qquad (14.33)$$

The average resistive shearing stress developed along the plane AC may also be expressed as

$$\tau_d = c'_d + \sigma'\tan\phi'_d$$

$$= c'_d + \frac{1}{2}\gamma H\left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\cos\theta\sin\theta\tan\phi'_d \qquad (14.34)$$

Now, from Eqs. (14.33) and (14.34),

$$\frac{1}{2}\gamma H\left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\sin^2\theta = c'_d + \frac{1}{2}\gamma H\left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\cos\theta\sin\theta\tan\phi'_d \qquad (14.35)$$

or

or

$$c_d = \frac{1}{2}\gamma H\left[\frac{\sin(\beta - \theta)(\sin\theta - \cos\theta\tan\phi'_d)}{\sin\beta}\right] \tag{14.36}$$

The expression in Eq. (14.36) is derived for the trial failure plane AC. In an effort to determine the critical failure plane, we must use the principle of maxima and minima (for a given value of ϕ'_d) to find the angle θ where the developed cohesion would be maximum. Thus, the first derivative of c_d with respect to θ is set equal to zero, or

$$\frac{\partial c'_d}{\partial\theta} = 0 \tag{14.37}$$

Because γ, H, and β are constants in Eq. (14.36), we have

$$\frac{\partial}{\partial\theta}[\sin(\beta - \theta)(\sin\theta - \cos\theta\tan\phi'_d)] = 0 \tag{14.38}$$

Solving Eq. (14.38) gives the critical value of θ, or

$$\theta_{cr} = \frac{\beta + \phi'_d}{2} \tag{14.39}$$

Substitution of the value of $\theta = \theta_{cr}$ into Eq. (14.36) yields

$$c'_d = \frac{\gamma H}{4}\left[\frac{1 - \cos(\beta - \phi'_d)}{\sin\beta\cos\phi'_d}\right] \tag{14.40}$$

The preceding equation can also be written as

$$\frac{c'_d}{\gamma H} = m = \frac{1 - \cos(\beta - \phi'_d)}{4\sin\beta\cos\phi'_d} \tag{14.41}$$

where m = stability number.

The maximum height of the slope for which critical equilibrium occurs can be obtained by substituting $c'_d = c'$ and $\phi'_d = \phi'$ into Eq. (14.40). Thus,

$$H_{cr} = \frac{4c'}{\gamma}\left[\frac{\sin\beta\cos\phi'}{1 - \cos(\beta - \phi')}\right] \tag{14.42}$$