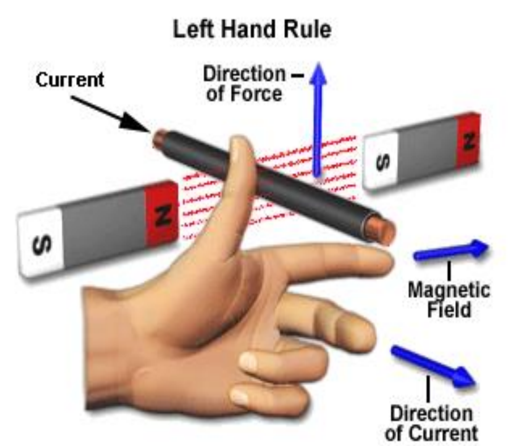
Chapter Two
D.C. Motors

A machine that converts d.c. power into mechanical power is known as a d.c. motor. Its operation is based on the principle that when a current carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force whose direction is given by Fleming's Left-hand Rule

Fleming's Left Hand Rule

Whenever a current carrying conductor is placed in a magnetic field, the conductor experiences a force which is perpendicular to both the magnetic field and the direction of current. According to **Fleming's left hand rule**, if the thumb, fore-finger and middle finger of the left hand are stretched to be perpendicular to each other as shown in the illustration at left, and if the fore finger represents the direction of magnetic field, the middle finger represents the direction of current, then the thumb represents the direction of force.



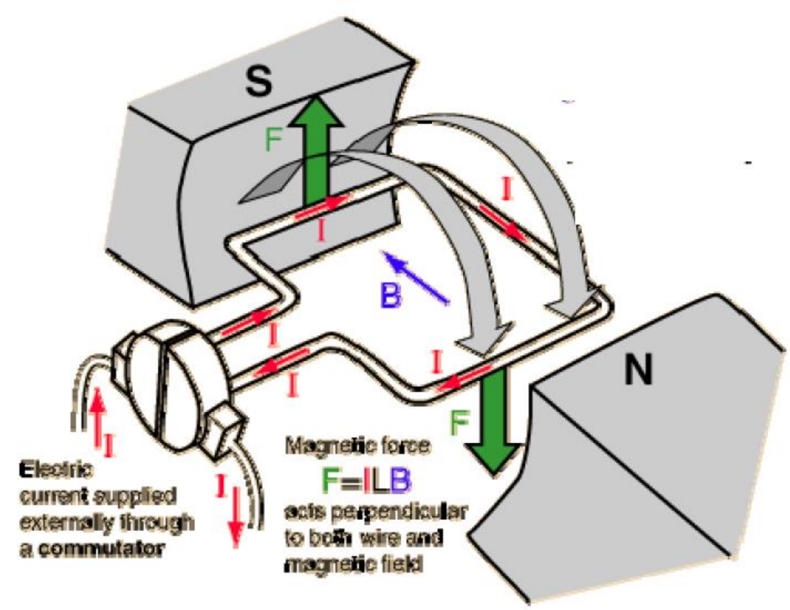
Constructionally, there is no basic difference between a d.c. generator and a d.c. motor. In fact, the same d.c. machine can be used interchangeably as a generator or as a motor. D.C. motors are also like generators, shunt-wound or series-wound or compound-wound.

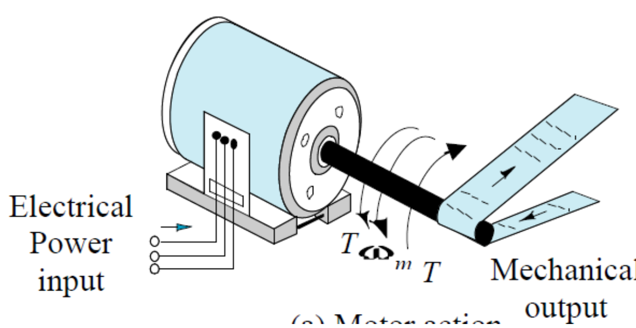
It should be noted that the function of a commutator in the motor is the same as in a generator. By reversing current in each conductor as it passes from one pole to another, it helps to develop a continuous and unidirectional torque.

When operating as a generator, it is driven by a mechanical machine and it develops voltage which in turn produces a current flow in an electric circuit. When operating as a motor, it is supplied by electric current and it develops torque which in turn produces mechanical rotation.

Principle of operation

Consider a coil in a magnetic field of flux density B .When the two ends of the coil are connected across a DC voltage source, current I flows through it. A force is exerted on the coil as a result of the interaction of magnetic field and electric current. The force on the two sides of the coil is such that the coil starts to move in the direction of force.





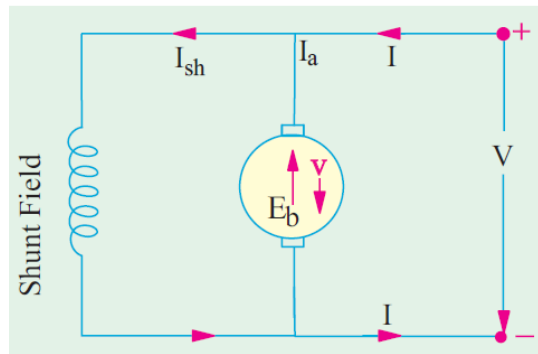
Voltage Equation of a Motor

The voltage V applied across the motor armature has to

(i) Overcome the back e.m.f. Eb and

(ii) Supply the armature ohmic drop $I_a R_a$.

$$V = E_b + I_a R_a$$



This is known as voltage equation of a motor.

Now, multiplying both sides by I_a, we get

$$V I_a = E_b I_a + I_a^2 Ra$$

VI_a = Eectrical input to the armature

$Eb\ Ia$ = Electrical equivalent of mechanical power developed in the armature

$I_a^2 R_a$= Cu loss in the armature

Hence, out of the armature input, some is wasted in I^2R loss and the rest is converted into mechanical power within the armature.

Condition for Maximum Power

The gross mechanical power developed by a motor is

$$P_m = VI_a - I_a^2 R_a$$

Differentiating both sides with respect to I_a and equating the result to zero, we get

$$V - 2 I_a R_a = 0$$

$$I_a R_a = V/2$$

As $V = E_b + I_a R_a$

$$E_b = V/2$$

Thus gross mechanical power developed by a motor is maximum when back e.m.f. is equal to half the applied voltage.

Limitations

In practice, we never aim at achieving maximum power due to the following reasons:

(i) The armature current under this condition is very large too much beyond the normal current of the motor

(ii) Half of the input power is wasted in the armature circuit in the form of heat. In fact, if we take into account other losses (iron and mechanical), the efficiency will be well below 50%.

Ex1: A 4 pole, 32 conductor, lap-wound d.c. shunt generator with terminal voltage of 200 volts delivering 12 amps to the load has ra = 2 and field circuit resistance of 200 ohms. It is driven at 1000 r.p.m. Calculate the flux per pole in the machine. If the machine has to be run as a motor with the same terminal voltage and drawing 5 amps from the mains, maintaining the same magnetic field, find the speed of the machine.

Solution:

I_a = 13 amp.

E_g = 200 + 13 x 2 = 226 V

$$\frac{\phi\, P\, N}{60}\,\frac{Z}{A} = 226$$

For a Lap-wound armature,

P = a

$$\phi = \frac{226 \times 60}{1000 \times 32} = 0.423 \text{ wb}$$

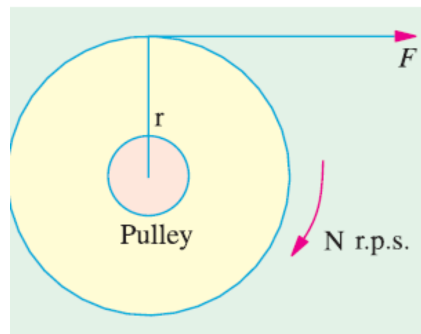
As a motor,

I_a = 4 A

E_b = 200 – 4 x 2 = 192 V

$$\text{Giving N} = \frac{60 \times 192}{0.423 \times 32} = 850\ \text{r.p.m}$$

Torque



By the term torque is meant the turning or twisting moment of a force about an axis. It is measured by the product of the force and the radius at which this force acts. Consider a pulley of radius *r* meter acted upon by a circumferential force of *F* Newton which causes it to rotate at *N* r.p.m.

Then torque $T = F \times r$ (N - m)

Work done by this force in one revolution = Force × distance = $F \times 2\pi r$ Joule

Power developed = $F \times 2\pi r \times N$ Joule/second or Watt = $(F \times r) \times 2\pi N$ Watt

Now $2\pi N$ = Angular velocity ω in radian/second and $F \times r$ = Torque T

∴ Power developed = $T \times \omega$ watt or $P = T\omega$ Watt

Moreover, if *N* is in r.p.m., then

$\omega = 2\pi N/60$ rad/*s*

$$P = \frac{2\pi N}{60} \times T$$

Armature Torque of a Motor

Let Ta be the torque developed by the armature of a motor running at N r.p.s. If Ta is in N/M, then power developed

$$= T_a \times 2\pi N \quad \text{watt}$$

We also know that electrical power converted into mechanical power in the armature

$=E_bI_a$ watt

$$T_a = \frac{E_bI_a}{2\pi N} \text{ N.m , N in r.p.s}$$

$$T_a = 9.55\frac{E_bI_a}{N} \text{ N.m , N in r.p.m}$$

And

$$E_b = \frac{\phi ZNP}{A} \text{ , N in r.p.s}$$

By substituting the three equations above we get,

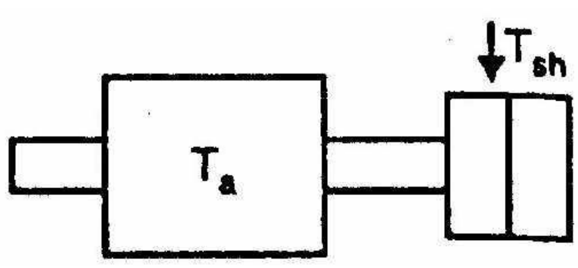
$$T_a \times 2\pi N = \frac{\phi ZNP}{A}.Ia$$

$$T_a = 0.159\frac{\phi ZP}{A}.Ia \text{ N.m}$$

Shaft Torque (Tsh)

The torque which is available at the motor shaft for doing useful work is known as shaft torque. It is represented by T_{sh}. Figure below illustrates the concept of shaft torque. The total or gross torque T_a developed in the armature of a motor is not available at the shaft because a part of it is lost in overcoming the iron and frictional losses in the motor. Therefore, shaft torque T_{sh} is somewhat less than the armature torque T_a. The difference $T_a - T_{sh}$ is called lost torque

$$T_{sh} = 9.55\frac{\text{output}}{N} \text{ N.m , N in r.p.m}$$



Ex2: Determine developed torque and shaft torque of 220-V, 4-pole series motor with 800 conductors wave-connected supplying a load of 8.2 kW by taking 45 A from the mains. The flux per pole is 25 mWb and its armature circuit resistance is 0.6 Ω.

Solution

Developed torque or gross torque is the same thing as armature torque.

$$T_a = 0.159\frac{\phi ZP}{A}.I_a$$

$$T_a = 0.159\frac{0.025\times 800\times 4}{2}\times 45 = 286.2\ \mathrm{N.m}$$

$E_b = V - I_a R_a = 220 - 45\times 0.6 = 193$ V

$$E_b = \frac{\phi\, P\, N}{60}\,\frac{Z}{A}$$

$$193 = \frac{0.025\times\ 4\times\ N}{60}\,\frac{800}{2} \rightarrow\ N = 289.5\ \mathrm{r.p.m}$$

$$\mathrm{Output} = 2\pi N T_{sh}$$

$$8200 = 2\pi\times\frac{289.5}{60}T_{sh} \rightarrow\rightarrow T_{sh} = 270.5\ \mathrm{N.m}$$

Speed of a DC motor

$$N = \frac{E_b A\times 60}{\phi ZP}\ \rightarrow\rightarrow\rightarrow N = K\frac{E_b}{\phi}$$

It shows that speed is directly proportional to back e.m.f. Eb and inversely to the flux ϕ

$$N\ \alpha\frac{E_b}{\phi}$$

For series and shunt motor,

Let N1 = Speed in the 1st case ; Ia1 = armature current in the 1st case

ϕ_1= flux/pole in the first case

N2,Ia2, ϕ_2 = corresponding quantities in the 2nd case.

Then, using the above relation, we get

$$N1 \; \alpha \frac{E_{b1}}{\phi_1} \text{ where } E_{b1} = V - I_{a1}R_a$$

$$N2 \; \alpha \frac{E_{b2}}{\phi_2} \text{ where } E_{b2} = V - I_{a2}R_a$$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

Also,

$$T_a = 0.159 \frac{\phi ZP}{A} . Ia$$

$$T_a = K \phi I_a$$

$$T_a \alpha \; \phi I_a$$

$$\frac{T_{a2}}{T_{a1}} = \frac{I_{a2}}{I_{a1}} \frac{\phi_2}{\phi_1}$$

Speed Regulation

The speed regulation of a motor is the change in speed from full-load to no-loud and is expressed as a percentage of the speed at full-load

$$\%\ \text{speed regulation} = \frac{\text{N. L speed} - \text{F. L speed}}{\text{F. L speed}}\ .\ 100$$

Ex3: A 250-V shunt motor runs at 1000 r.p.m. at no-load and takes 8A. The total armature and shunt field resistances are respectively 0.2 Ω and 250 Ω. Calculate the speed when loaded and taking 50 A. Assume the flux to be constant.

Solution:

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

$$I_{sh} = \frac{250}{250} = 1A$$

$$E_{b1} = 250 - (7 \times 0.2) = 248.6\ V$$

$$E_{b2} = 250 - (49 \times 0.2) = 240.2\ V$$

$$\frac{N_2}{1000} = \frac{240.2}{248.6} \times 1 \quad \rightarrow\rightarrow\ N_2 = 9666\ \text{r.p.m}$$

Ex4:A 220V d.c. shunt motor runs at 500 r.p.m. when the armature current is 50 A.Calculate the speed if the torque is doubled. Given that Ra = 0.2 Ω (neglect armature reaction).

Solution:

$$T_a \alpha\ \phi I_a$$

$$\text{since } \phi \text{ is constant}, T_a \alpha\ I_a$$

$T_{a1} \alpha\ I_{a1}$, $T_{a2} \alpha\ I_{a2}$

$$\frac{T_{a2}}{T_{a1}} = \frac{I_{a2}}{I_{a1}}$$

$$2 = \frac{I_{a2}}{50} \ \rightarrow\rightarrow I_{a2} = 100A$$

Now,

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \quad \text{since } \phi \text{ is constant}$$

Eb1 = 220 -50x0.2=210v

Eb2 = 220 -100x0.2=200v

$$\frac{N_2}{500} = \frac{200}{210} \ \rightarrow\rightarrow N_2 = 476 \text{ r.p.m}$$

Ex5: A 230-V d.c. shunt motor has an armature resistance of 0.5 Ω and field resistance of 115 Ω. At no load, the speed is 1,200 r.p.m. and the armature current 2.5 A. On application of rated load, the speed drops to 1,120 r.p.m. Determine the line current and power input when the motor delivers rated load.

Solution:

$Eb1 = 230 - (0.5 \times 2.5) = 228.75$ V

$Eb2 = 230 - 0.5$ Ia2

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b2}}$$

$$\frac{1120}{1200} = \frac{230 - 0.5I_{a2}}{228.75}$$

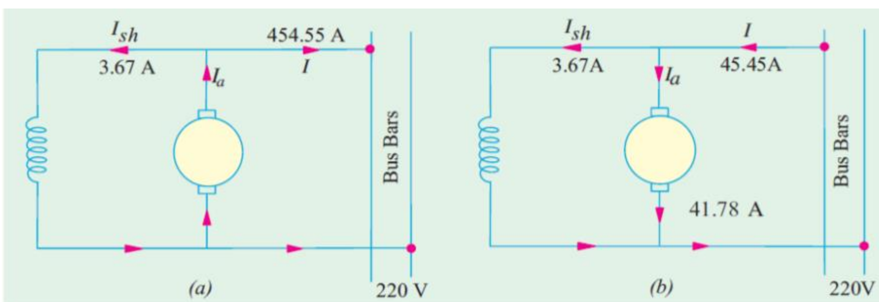
$$I_{a2} = 33 \text{ A}$$

Line current drawn by motor = $I_{a2} + I_{sh}$ = 33 + (230/115) = 35 A

Power input at rated load = 230 × 35 = 8,050 W

Ex5: belt-driven 100-kW, shunt generator running at 300 r.p.m. on 220-V busbars continues to run as a motor when the belt breaks, then taking 10 kW. What will be its speed ? Given armature resistance = 0.025 Ω, field resistance = 60 Ω and contact drop under each brush = 1 V, Ignore armature reaction.

Solution:



As a generator,

I = 100,000/220 = 454.55 A

I_{sh} = 220/60 = 3.67 A

$I_a = I + I_{sh}$ = 458.2 A

I_a Ra = 458.2 × 0.025 = 11.45

E_g = 220 + 11.45 + 2 × 1 = 233.45 V

As a motor,

Input line current = 10,000/220 = 45.45 A

I_{sh} = 220/60 = 3.67 A

$I_a = 45.45 - 3.67 = 41.78$ A

$I_a R_a = 41.78 \times 0.025 = 1.04$ V

$E_b = 220 - 1.04 - 2 = 216.96$ V

$$\frac{N_b}{N_g} = \frac{E_b}{E_g} \times \frac{\phi_g}{\phi_b} \quad , \phi_g = \phi_b$$

$$\frac{N_2}{300} = \frac{216.9}{233.4} \rightarrow\rightarrow\rightarrow N_2 = 279 \text{ r.p.m}$$

Repeat the above example considering the armature reaction (weakens the field by 3%).

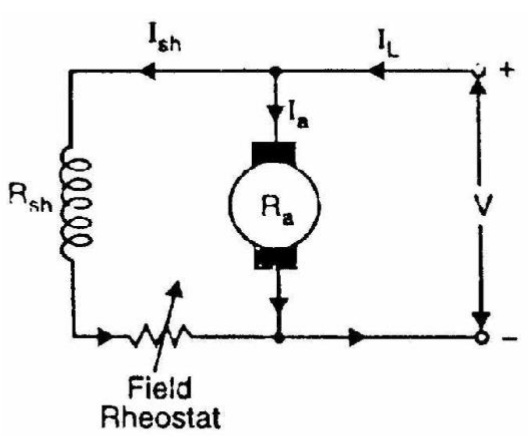
Speed Control of D.C. Shunt Motors

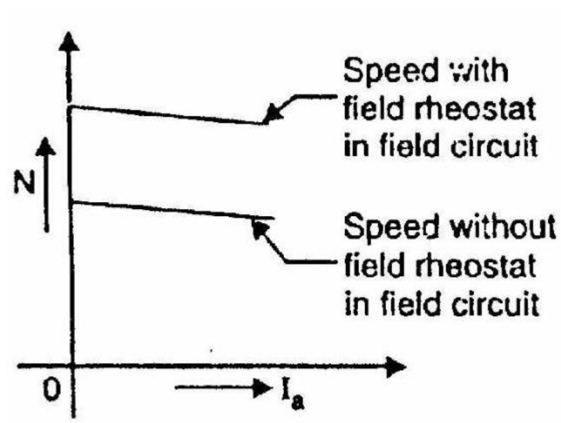
1. **Flux control method**

It is based on the fact that by varying the flux, the motor speed can be changed and hence the name flux control method. In this method, a variable resistance (known as shunt field rheostat) is placed in series with shunt field winding.

The shunt field rheostat reduces the shunt field current I_{sh} and hence the flux ϕ.

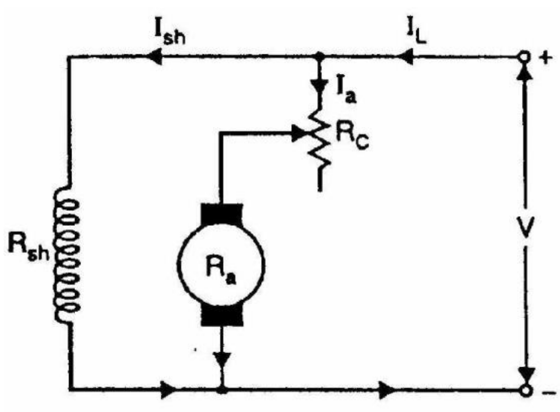
Therefore, we can only raise the speed of the motor above the normal speed. Generally, this method permits to increase the speed in the ratio 3:1.

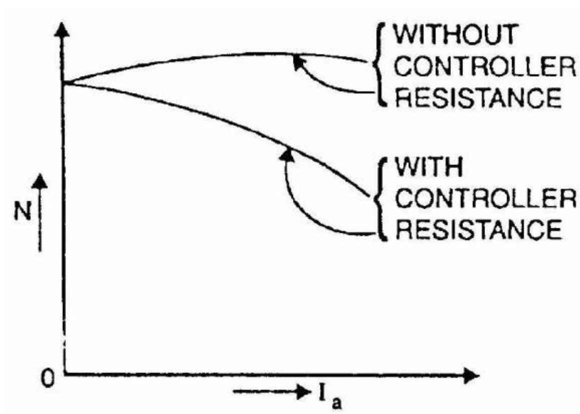




2. Armature control method

This method is based on the fact that by varying the voltage available across the armature, the back e.m.f and hence the speed of the motor can be changed. This is done by inserting a variable resistance R_C (known as controller resistance) in series with the armature.





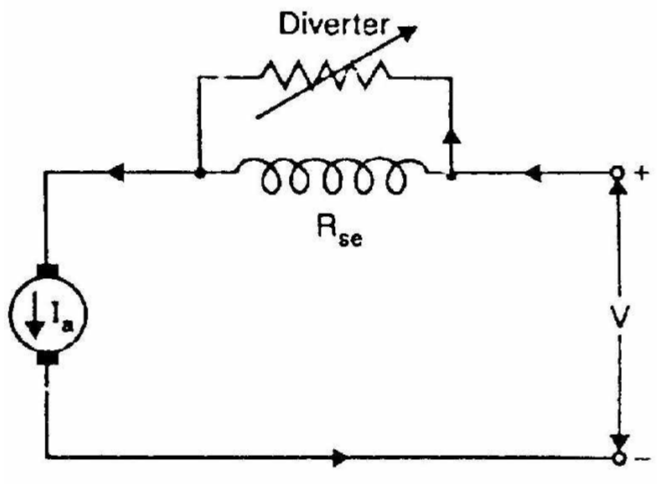
$$N \alpha \, \text{V} - I_a(R_a + R_c)$$

Due to voltage drop in the controller resistance, the back e.m.f. (E_b) is decreased. The highest speed obtainable is that corresponding to $R_C = 0$ i.e., normal speed. Hence, this method can only provide speeds below the normal speed.

Speed Control of D.C. Series Motors

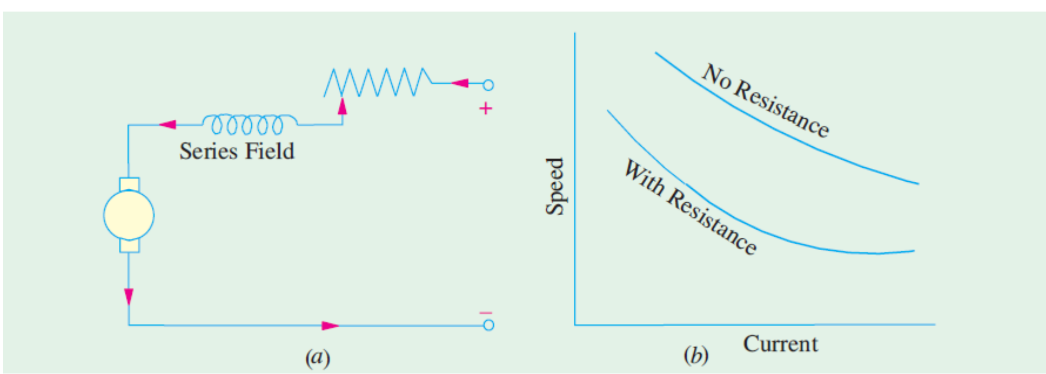
1. Flux control method (Field divertcrs)

The series winding are shunted by a variable resistance known as field diverter. Any desired amount of current can be passed through the diverter by adjusting its resistance. Hence the flux can be decreased and consequently, the speed of the motor increased.



2. Variable Resistance in Series with Motor

By increasing the resistance in series with the armature (Fig. 30.14) the voltage applied across the armature terminals can be decreased. With reduced voltage across the armature, the speed is reduced. However, it will be noted that since full motor current passes through this resistance, there is a considerable loss of power in it.

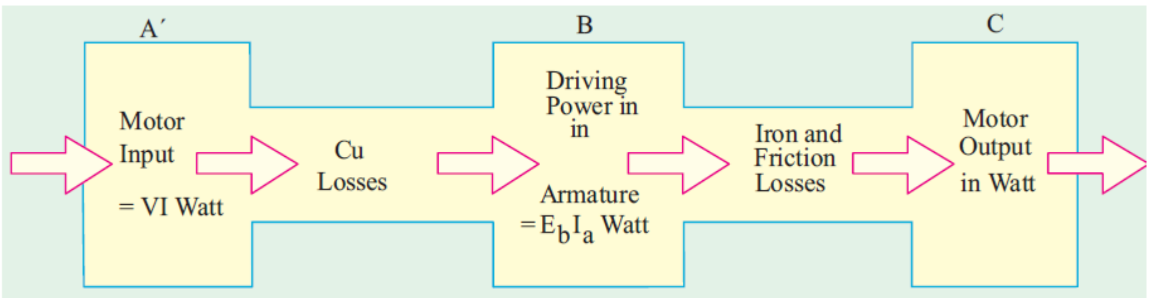


Applications of D.C. Motors

Type of motor	Characteristics	Applications
Shunt	Approximately constant speed Medium starting torque (Up to 1.5 F.L. torque)	For driving constant speed line shafting Lathes Centrifugal pumps Machine tools Blowers and fans Reciprocating pumps
Series	Variable speed Adjustable varying speed High Starting torque	For traction work i.e. Electric locomotives Rapid transit systems Trolley, cars etc. Cranes and hoists Conveyors
Compound	Variable speed Adjustable varying speed High starting torque	For intermittent high torque loads For shears and punches Elevators Conveyors Heavy planers Heavy planers Rolling mills; Ice machines; Printing presses; Air compressors

Power stages

The various stages of energy transformation in a motor and also the various losses occurring in it are shown in the following figure.



Overall or commercial efficiency η_c = C/A

Electrical efficiency η_e =B/A

Mechanical efficiency η_m=C/B

The condition for maximum efficiency is that armature Cu losses are equal to constant losses.

$$I_a^2 \mathrm{R_a} = \text{constant losses}$$

Ex6: A 4-pole, 240 V, wave connected shunt motor gives 11.19 kW when running at 1000 r.p.m. and drawing armature and field currents of 50 A and 1.0 A respectively. It has 540 conductors. Its resistance is 0.1 Ω. Assuming a drop of 1 volt per brush, find (a) total torque (b) useful torque (c) useful flux / pole and (d) efficiency.

Solution:

$E_b = V - I_a R_a$ – brush drop = 240 – (50× 0.1) – 2 = 233 V

Also, $I_a = 50$ A

$$\mathrm{T_a} = 9.55\frac{\mathrm{E_b I_a}}{\mathrm{N}}$$

$$= 9.55 \frac{233 \times 50}{1000} = 111\ N.m$$

$$T_{sh} = 9.55 \frac{\text{output}}{N} = 9.55 \times \frac{11190}{1000} = 106.7\ \text{N.m}$$

$$E_b = \frac{\phi\, P\, N}{60} \frac{Z}{A}$$

$$233 = \frac{\phi\ \times 4 \times 1000}{60} \frac{540}{2} \ \rightarrow\rightarrow \phi = 12.9\ \text{Wb}$$

Total motor input = *VI* = 240 × 51 = 12,340 W

Efficiency= 11190/12340=90.6%.

Ex7: A 200-V, d.c. shunt motor takes 4 A at no-load when running at 700 r.p.m. The field resistance is 100 Ω and the resistance of armature is 0.6 ohms. Calculate (a) speed on load (b) torque in N-m and (c) efficiency (d) armature current when efficiency is maximum. The normal input of the motor is 8 kW. Neglect armature reaction.

Solution:

(a) I_{sh} = 200/100 = 2 A

F.L. Power input = 8,000 W

F.L. line current = 8,000/200 = 40 A

I_a = 40 – 2 = 38 A

E_{bo} = 200 – 2 × 0.6= 198.8 V

E_b = 200 – 38 ×0.6 = 177.2 V

$$\frac{N}{N_0} = \frac{E_b}{E_{bo}}$$

$$\frac{N}{700} = \frac{177.2}{198.8} \rightarrow\rightarrow N = 924\ \text{r.p.m}$$

(b) $T_a = 9.55\ E_bI_a/N = 9.55 \times 177.2 \times 38/623.9 = 103$ N-m

(c) N.L. power input = 200 ×4 = 800 W

N.L Arm. Cu loss = I_a^2 Ra = $2^2 \times 0.6 = 2.4$ W

Constant losses = N.L. power input- N.L Arm. Cu loss

=800-2.4=797.6 W

F.L. arm. Cu loss = 38×38× 0.6 = 866.4 W

Total F.L. losses = Constant losses+ F.L. arm. Cu loss

=797.6 + 866.4 = 1664 W

F.L. output = F.L. Power input– Total F.L. losses

=8000-1664 = 6336 W

F.L. Motor efficiency = 6336/8,000=79.2%

(d) When efficiency is maximum,

$$I_a^2R_a = \text{constant losses}$$

$$I_a^2 \times 0.6 = 797.6 \rightarrow\rightarrow I_a = 36.5\ \text{A}$$

Ex8: A 500-V D.C. shunt motor draws a line-current of 5 A on light-load. If armature resistance is 0.15 ohm and field resistance is 200 ohms determine the efficiency of the machine running as a generator delivering a load current of 40 A. At what speed should the generator be run if the shunt-field is not changed in the above case? Assume that the motor was running at 600 r.p.m.

Solution:

(*i*) No Load, running as a motor :

Input Power = 500 × 5 = 2500 watts

Neglecting armature copper-loss at no load (since it comes out to be $2.5^2 \times 0.15 = 1$ watt)

(ii) As a Generator, delivering 40 A to load :

Output delivered = $500 \times 40 \times 10^3 = 20$ kW

Losses :

(a) Armature copper-loss = $42.5^2 \times 0.15 = 271$ watts

(b) No load losses = 2500 watts

Total losses = 2.771 kW

Generator Efficiency = (20/22.771) × 100 % = 87.83 %.

As a motor on no-load,

$E_{b0} = 500 - I_a \times R_a = 500 - 0.15 \times 2.5 = 499.625$ V

As a Generator with an armature current of 42.5 A,

$E_{g0} = 500 + 42.5 \cdot 0.15 = 506.375$ V

Since, the terminal voltage is same in both the cases, shunt field current remains as 2.5 amp. With armature reaction is ignored, the flux/pole remains same. The e.m.f. then becomes proportional to the speed. If the generator must be driven at N r.p.m.

$$N = (506.375/449.625) \times 600 = 608.1 \text{ r.p.m.}$$

Ex9: A 7.46 kW, 250-V shunt motor takes a line current of 5 A when running light. Calculate the efficiency as a motor when delivering full load output, if the armature and field resistance are 0.5 Ω and 250 Ω respectively. At what output power will the efficiency be maximum? Is it possible to obtain this output from the machine?

Solution:

$\text{Total motor input (or total no} - \text{load losses)} = 250 \times 5 = 1{,}250 \text{ W}$

$\text{Ish} = 250/250 = \text{IA} \therefore \text{Ia} = 5 - 1 = 4 \text{ A}$

$\text{Field Cu loss} = 250 \times 1 = 250 \text{ W;}$

$\text{Armature Cu loss} = 4^2 \times 0.5 = 8 \text{ W}$

Iron losses and friction losses = 1250 − 250 − 8 = 992 W

These losses would be assumed constant.

Let Ia be the full-load armature current, then armature input is = (250 × Ia) W

F.L. output = 7.46 × 1000 = 7,460 W

$\text{Armature Cu loss} = I_a^2 \times 0.05 \text{ W}$

$250 \text{ Ia} = 7{,}460 + 992 + \text{I}_a^2 \times 0.5$

$I_a = 36.5$ A

F.L. input current = 36.5 + 1 = 37.5 A

Motor input = 250 × 37.5 W

F.L. efficiency = 7460 × 100/250 × 37.5 = 79.6%

Rest of the solution is H.W.