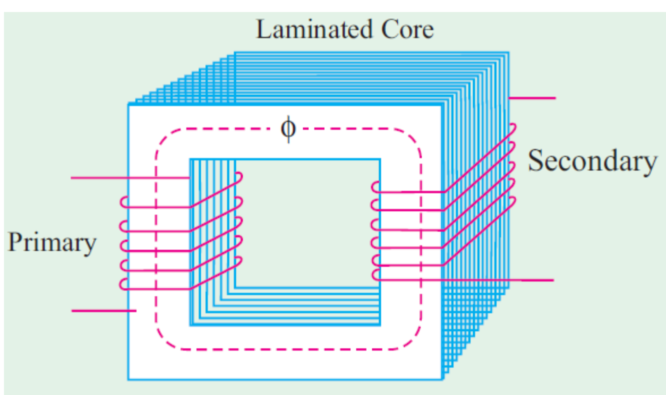
Chapter Three

Transformers

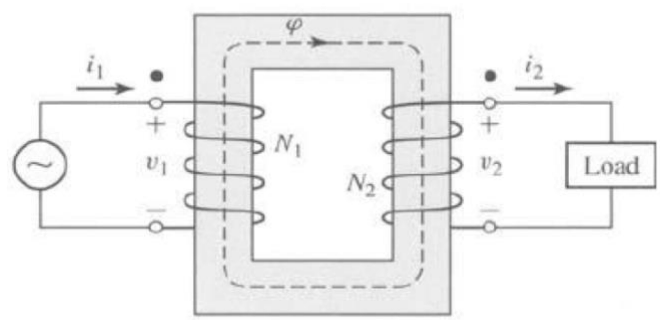
Transformer is a device which transfers electrical energy (power) from one voltage level to another voltage level with a corresponding decrease or increase in current. Unlike in rotating machines (generators and motors), there is no energy conversion. Being a static machine the efficiency of a transformer could be as high as 99%.

Transformer consists of two inductive coils which are electrically separated but magnetically linked through a path of low reluctance. The two coils possess high mutual inductance.



Transformer consists of;

- Primary windings, connected to the alternating voltage source
- Secondary windings, connected to the load;
- Iron core, which link the flux in both windings;



The primary and secondary voltages are denoted by V_1 and V_2 respectively. The current entering the primary terminals is I_1 .

Step-up transformer

If the primary coil has 3 loops and the secondary coil has 30, the voltage is stepped up 10 times.

Step-down transformer

If the primary coil has 30 loops and the secondary coil has 3, the voltage is stepped down 10 times.

E.M.F. Equation of a Transformer

Let N_1 = No. of turns in primary

N_2 = No. of turns in secondary

Φ_m = Maximum flux in the core in webers

f = Frequency of a.c. input in Hz

$$E_1 = 4.44\, f\, N_1\, \Phi_m = 4.44\, f\, N_1\, B_m A$$

Similarly, value of the e.m.f. induced in secondary is,

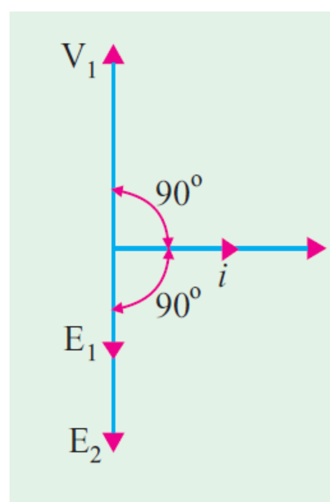
$$E_2 = 4.44\, f\, N_2\, \Phi_m = 4.44\, f\, N_2\, B_m A$$

B_m= maximum flux density in core in wb/m^2

A= area of the core

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = 4.44\, f \Phi_m$$

Ideal Transformer

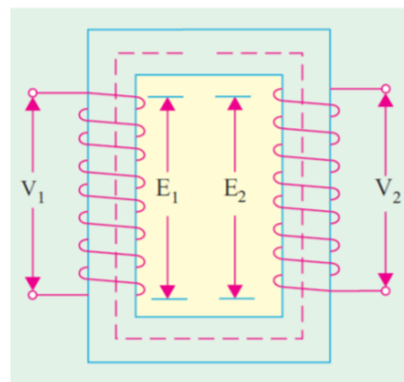


An ideal transformer is one which has no losses i.e. its windings have no ohmic resistance, there is no magnetic leakage and hence which has no I^2R and core losses. In other words, an ideal transformer consists of two purely inductive coils wound on a loss-free core.

Consider an ideal transformer whose secondary is open and whose primary is connected to sinusoidal alternating voltage V_1. The primary draws the magnetising current Iμ which produces an alternating flux φ. self-induced e.m.f. E_1 is equal to and in opposition to V_1. It is also known as counter e.m.f. or back e.m.f. of the primary

$$E_1 = V_1 \quad , \quad E_2 = V_2$$

Voltage transformation ratio



$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

This constant K is known as voltage transformation ratio.

- If K > 1, then transformer is called step-up transformer.
- If K < 1, then transformer is known as step-down transformer.

for an ideal transformer, input VA = output VA

$$V_1 I_1 = V_2 I_2$$

$$\frac{I_2}{I_1} = \frac{V_1}{V_2} = 1/K$$

Example 1 : The maximum flux density in the core of a 250/3000-volts, 50-Hz single-phase transformer is 1.2 Wb/m2. If the e.m.f. per turn is 8 volt, determine

(i) primary and secondary turns (ii) area of the core. Assume an ideal transformer.

Solution:

(i) $E_1 = N_1 \times$ e.m.f. induced/turn

$N_1 = 250/8 = 32$

$N_2 = 3000/8 = 375$

(ii) $E_2 = 4.44\ f\ N_2\ B_m A$

$3000 = 4.44 \times 50 \times 375 \times 1.2 \times A$

$A = 0.03 m^2$

Example 2: A single-phase transformer has 400 primary and 1000 secondary turns. The net cross-sectional area of the core is 60 cm^2. If the primary winding be connected to a 50-Hz supply at 520 V, calculate (i) the voltage induced in the secondary winding. (ii) the peak value of flux density in the core

Solution:

(i) $K = N_2/N_1$

$= 1000/400 = 2.5$

$E_2/E_1 = K$

$E_2 = K E_1$

= 2.5 × 520 = 1300 V

(ii)

$$E_1 = 4.44\ f\ N_1\ B_m A$$

520 = 4.44 × 50 × 400 × Bm × (60 × 10^{-4})

Bm = 0.976 Wb/m2

Example 3 : A 25-kVA ideal transformer has 500 turns on the primary and 50 turns on the secondary winding. The primary is connected to 3000-V, 50-Hz supply. Find the full-load primary and secondary currents, the secondary e.m.f. and the maximum flux in the core.

Solutions:

K= N_2/N_1 = 50/500 = 1/10

Now, full-load I_1= 25,000/3000 = 8.33 A

$$V_1\ I_1 = V_2\ I_2$$

F.L. I_2 = I_1/K = 10 × 8.33 = 83.3 A

E_2 = K E_1 = 3000 × 1/10 =300 V

Also, $E_1 = 4.44\ f\ N_1\ \Phi_m$

3000 = 4.44 × 50 × 500 × Φm

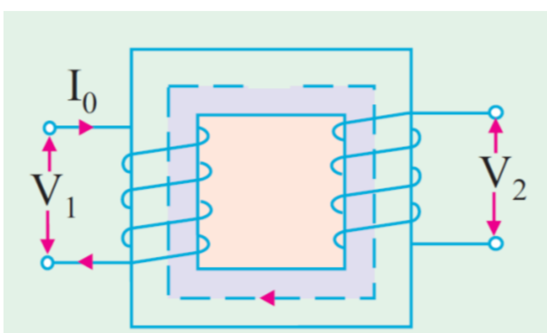
∴ Φm = 27 mWb

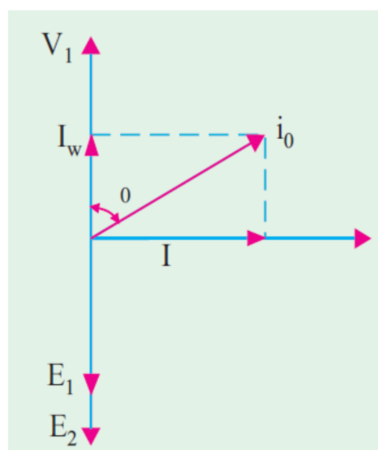
Transformer on no-load

In the above discussion, we assumed an ideal transformer i.e. one in which there were no core losses and copper losses. But practical conditions require that certain modifications be made in the foregoing theory.

When an actual transformer is put on load, there is iron loss in the core and copper loss in the windings (both primary and secondary) and these losses are not entirely negligible.

Hence, the no-load primary input current I_0 is not at 90° behind V_1 but lags it by an angle φ0 < 90°.





I_o has two components:

(i) One in phase with V_1. This is known as active or working or iron loss component Iw

$$I_w = I_o \cos \phi_o$$

(ii) The other component is is known as magnetising component Iμ

$$I\mu = I_o sin\,\phi_o$$

where $cos\,\phi_o$ is primary power factor under no-load conditions.

Hence,

$$I_o{}^2 = (I_\mu^2 + I_w^2)$$

As I_o is very small, the no-load primary Cu loss is negligibly small which means that no-load primary input is practically **equal to the iron loss in the transformer**.

No-load input power;

$$w_o = V_1 I_w = V_1 I_o\,cos\,\phi_o\ (w)$$

Example 4: (a) A 2,200/200-V transformer draws a no-load primary current of 0.6 A and absorbs 400 watts. Find the magnetizing and iron loss currents. (b) A 2,200/250-V transformer takes 0.5 A at a p.f. of 0.3 on open circuit. Find magnetizing and working components of no-load primary current.

Solution:

(a) Iron –loss current

$$= \frac{\text{no load input}}{\text{primary voltage}} = \frac{400}{2200} = 0.182\text{ A}$$

Now,

$$I_o^{\ 2} = (I_\mu^2 + I_w^2)$$

Magnetizing component

$$I\mu = \sqrt{(0.6^2 - 0.182^2)} = 0.572\ A$$

(b)

I_0 = 0.5 A

$cos\ \phi_o = 0.3$

$$I_w = I_o \cos\phi_o = 0.5 \times 0.3 = 0.15\ A$$

$$I\mu = \sqrt{(0.5^2 - 0.15^2)} = 0.476\ A$$

Example 5: A 2400 V/400V single-phase transformer takes a no-load current of 0.5A and the core loss is 400 W. Determine the values of the angle ϕ_o and the magnetising and core loss components of the no-load current.

Solution:

Iron losses =400 w

Iron losses = No-load input power =

$$w_o = V_1 I_o\ cos\ \phi_o$$

$$400 = 2400 \times 0.5 \times cos\ \phi_o$$

$$cos\ \phi_o = 0.3333$$

$$\phi_o = \cos^{-1} 0.3333 = 70.53^o$$

$$I\mu = I_o \sin \phi_o = 0.471\ A$$

$$I_w = I_o\ cos\ \phi_o = 0.167\ A$$

Transformer on-load

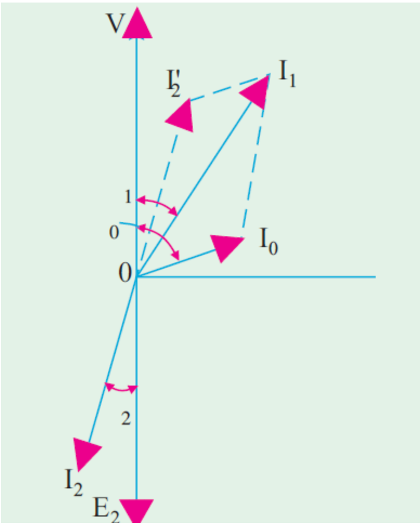
When the secondary is loaded, the secondary current I_2 is set up. The magnitude and phase of I_2 with respect to V_2 is determined by the characteristics of the load. The secondary current sets up its own m.m.f. (=$N_2 I_2$) and hence its own flux Φ2 which is in opposition to the main primary flux Φ which is due to I_o. The opposing secondary flux Φ2 weakens the primary flux Φ momentarily, hence primary back e.m.f. E1 tends to be reduced. For a moment V_1 gains the upper hand over E1 and hence causes more current to flow in primary.

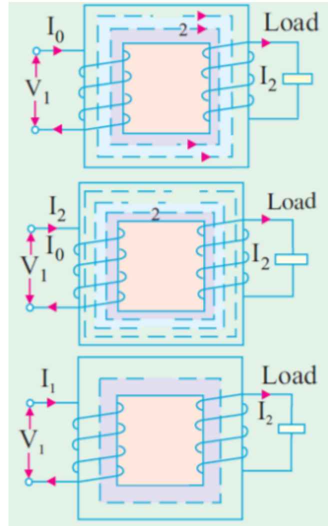
Let the additional primary current be $I_2^{`}$. This current sets up its own flux Φ2′ which is in opposition to Φ2 (but is in the same direction as Φ) and is equal to it in magnitude. Hence, the two cancel each other out. So,

$$\Phi 2 = \Phi 2'$$

$$N_2 I_2 = N_1 I_2^{`}$$

$$I_2^{`} = \frac{N_2}{N_1} \times I_2 = K I_2$$

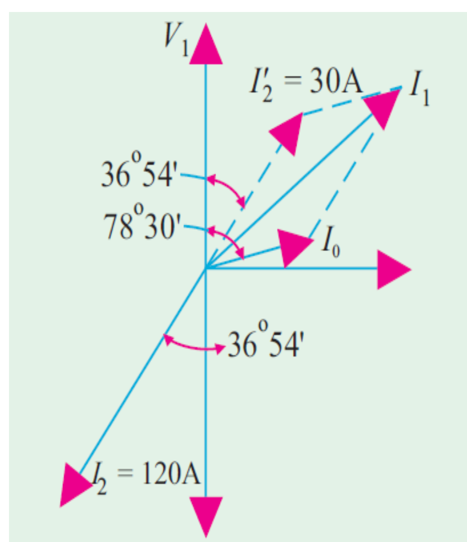




$$I_0 cos\ \phi_0 + I_2^{`} cos\ \phi_2 = I_1 cos\ \phi_1$$

$$I_0 sin\ \phi_0 + I_2^{`} sin\phi_2 = I_1 sin\ \phi_1$$

Example 6: A single-phase transformer with a ratio of 440/110-V takes a no-load current of 5A at 0.2 power factor lagging. If the secondary supplies a current of 120 A at a p.f. of 0.8 lagging, estimate the current taken by the primary.



Solution:

$cos\ \phi_0 = 0.2$

$\phi_o = \cos^{-1} 0.2 = 78.46^o$

$cos\ \phi_2 = 0.8$

$\phi_2 = \cos^{-1} 0.8 = 36.86^o$

K = V_2/V_1

= 110/440 = ¼

$I_2^` = KI_2$=30×0.25=30 A

$I_0 = 5\ A$

Angle between I_0 and $I_2^`$

ϕ_z=$78.46^o - 36.86^o = 41.6^o$

$$I_1 = \sqrt{I_2^{`2} + I_o^2 + 2I_2^`I_0\cos(\phi_z)}$$

$$I_1 = 33.9 \text{ A}$$

Example 7: A transformer has a primary winding of 800 turns and a secondary winding of 200 turns. When the load current on the secondary is 80 A at 0.8 power factor lagging, the primary current is 25 A at 0.707 power factor lagging. Determine graphically and the no-load current of the transformer and its phase with respect to the voltage.

Solution:

K = 200/800 = ¼

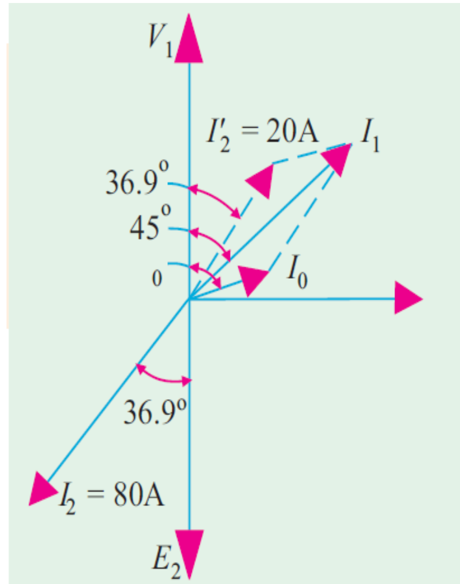
$I_2^` = (80)(¼) = 20\ A$

$\phi_2 = \cos^{-1} 0.8 = 36.9^o$

$\phi_1 = \cos^{-1} 0.707 = 45^o$

$$I_0 cos\ \phi_0 + I_2^` cos\ \phi_2 = I_1 cos\ \phi_1$$

$$I_0 cos\ \phi_0 + 20 cos\ 36.9 = 25 cos\ 45$$



$$I_0 cos\ \phi_0 = 1.675\ A$$

$$I_0 sin\ \phi_0 + I_2^{`} sin\phi_2 = I_1 sin\ \phi_1$$

$$I_0 sin\ \phi_0 = 5.675\ A$$

$$tan\ \phi_0 = \frac{5.675}{1.675} = 3.388$$

$$\phi_0 = 73.3^o$$

Now,

$$I_0 = \frac{5.675}{\sin 73.3} = 5.93\ A$$

Transformer losses and efficiency

There are broadly two sources of losses in transformers on load, these being copper losses and iron losses.

- Copper losses are variable and result in a heating of the conductors, due to the fact that they possess resistance. If R1 and R2 are the primary and secondary winding resistances then the total copper loss

$$I_1^2 R_1 + I_2^2 R_2$$

- Iron losses are constant for a given value of frequency and flux density and are of two types – hysteresis loss and eddy current loss.

$$\eta = \frac{output\ power}{input\ power} = \frac{input\ power - losses}{input\ power}$$

$$\eta = 1 - \frac{losses}{input\ power}$$

Output power=$V_2 I_2 cos\ \phi_2$

Total losses =copper loss + iron losses

input power = output power + losses

For maximum efficiency

Cu loss= Iron loss

$$w_i = I_1^2 R_1\ or\ I_2^2 R_2$$

The output current corresponding to maximum efficiency is;

$$I_2 = \sqrt{\frac{W_i}{R_2}}$$

Load KVA corresponding to maximum efficiency is given by;

$$= Full\ load\ (KVA) \times \sqrt{\frac{iron\ loss}{F.L\ cu\ loss}}$$

Example 8: A 200 kVA rated transformer has a full-load copper loss of 1.5 kW and an iron loss of 1 kW. Determine the transformer efficiency at (a) full load and 0.85 power factor. (b) a half full load and 0.85 power factor.

Solution:

(a) Full-load output power = $V_2 I_2 cos\ \phi_2$

= 200×0.85 =170 KW

Total losses= copper loss+iron loss

=1+1.5=2.5 KW

Input power= output power + total losses

= 170+2.5=172.5 KW

$$\eta = 1 - \frac{losses}{input\ power}$$

$$\eta = 1 - \frac{2.5}{172.5} = 98.55\%$$

(b) Half full load output power = 0.5×200×0.85

= 85 KW

Copper loss (or I^2R loss) is proportional to current squared. Hence the copper loss at half full-load is;

$$(0.5)^2\ (1500) = 375\ W$$

Iron loss=1000 W (constant)

Total loss = 1000+375 =1375 W

Input power = 85000+1375=86375 W

$$\eta = 1 - \frac{1375}{86375} = 98.41\%$$

Example 9: A 11000/230 V, 150-kVA, 1-phase, 50-Hz transformer has core loss of 1.4 kW and F.L. Cu loss of 1.6 kW. Determine

(i) the kVA load for max. efficiency and value of max. efficiency at unity p.f.

(ii) the efficiency at half F.L. 0.8 p.f. leading

Solution:

Load corresponding to max. efficiency is

$$= \text{F.L (KVA)} \times \sqrt{\frac{\text{iron loss}}{\text{F.L cu loss}}}$$

$$= 150 \times \sqrt{\frac{1.4}{1.6}} = 140 \text{ KVA}$$

Since Cu loss equals iron loss at maximum efficiency,

Total loss=1.4+1.4= 2.8 KW

Output = 140×1=140 KVA

$$\eta_{\max} = \frac{140}{142.8} = 98\%$$

Cu at half full load =$1.6 \times (1/2)^2 = 0.4$ KW

Total loss=0.4 +1.4=1.8 KW

Half F.L. output at 0.8 p.f.=(150/2)×0.8=60 KW

$$\eta = \frac{60}{60 + 1.8} = 97\%$$

Example 10: A 200-kVA transformer has an efficiency of 98% at full load. If the max. efficiency occurs at three quarters of full-load, calculate the efficiency at half load. Assume negligible magnetizing current and p.f. 0.8 at all loads.

Solution:

As given, the transformer has a F.L. efficiency of 98 % at 0.8 p.f.

F.L. output = 200 × 0.8 = 160 kW

F.L. input = 160/0.98 = 163.265 kW

F.L. losses = 163.265 − 160 = 3.265 kW

This loss consists of F.L. Cu loss x and iron loss y.

$$x + y = 3.265 \text{ kW}$$

It is also given that ηmax occurs at three quarters of full-load when Cu loss becomes equal to iron loss.

Cu loss at 75 % of F.L. = x $(3/4)^2$ =9 x/16

Since y remains constant, hence

$$9x/16 = y$$

By substituting the two equations, we get;

$$x + 9x/16 = 3265$$

$$x = 2090 \text{ W};\ y = 1175 \text{ W}$$

Half-load Unity p.f

Cu loss = 2090 × $(1/2)^2$= 522 W

total loss = 522 + 1175 = 1697 W

Output = 100 × 0.8 = 80 kW

$$\eta = \frac{80}{80 + 1.697} = 97.9\%$$

Cooling of Transformers

In all electrical machines, the losses produce heat and means must be provided to keep the temperature low. In generators and motors, the rotating unit serves as a fan causing air to circulate and carry away the heat. However, a transformer has no rotating parts. Therefore, some other methods of cooling must be used.

Heat is produced in a transformer by the iron losses in the core and I_2R loss in the windings. To prevent undue temperature rise, this heat is removed by cooling.

1. In small transformers (below 50 kVA), natural air cooling is employed i.e., the heat produced is carried away by the surrounding air

2. Medium size power or distribution transformers are generally cooled by housing them in tanks filled with oil. The oil serves a double purpose, carrying the heat from the windings to the surface of the tank and insulating the primary from the secondary.

3. For large transformers, external radiators are added to increase the cooling surface of the oil filled tank. The oil circulates around the transformer and moves through the radiators where the heat is released to surrounding air. Sometimes cooling fan for large transformers, external radiators are added to increase the cooling surface of the oil filled tank. The oil circulates around the transformer and moves through the radiators where the heat is released to surrounding air. Sometimes cooling fans blow air over the radiators to accelerate the cooling processes blow air over the radiators to accelerate the cooling process.

All-day transformer

The ordinary or commercial efficiency of a transformer is defined as the ratio of output power to the input power i.e.,

Commercial efficiency = Output power / Input power

There are certain types of transformers whose performance cannot be judged by this efficiency. For instance, distribution transformers used for supplying lighting loads have their primaries energized all the 24 hours in a day but the secondary supply little or no load during the major portion of the day. It means that a constant loss (i.e., iron loss) occurs during the whole day but copper loss occurs only when the transformer is loaded and would depend upon the magnitude of load.

Consequently, the copper loss varies considerably during the day and the commercial efficiency of such transformers will vary from a low value (or even zero) to a high value when the load is high. The performance of such transformers is judged on the basis of energy consumption during the whole day (i.e., 24 hours). This is known as all-day or energy efficiency.

The ratio of output in kWh to the input in kWh of a transformer over a 24-hour period is known as all-day efficiency i.e.,

$$\eta_{all-day} = \frac{\text{kWh output in 24 hours}}{kWh\ input\ in\ 24\ hours}$$

Example 11. A 100-kVA lighting transformer has a full-load loss of 3 kW, the losses being equally divided between iron and copper. During a day, the transformer operates on full-load for 3 hours, one half-load for 4 hours, the output being negligible for the remainder of the day. Calculate the all-day efficiency.

Solution:

It should be noted that lighting transformers are taken to have a load p.f. of unity.

Iron loss for 24 hour = 1.5 × 24 = 36 kWh

F.L. Cu loss = 1.5 kW

Cu loss for 3 hours on F.L. = 1.5 × 3 = 4.5 kWh

Cu loss at half full-load = 1.5/4 kW

Cu loss for 4 hours at half the load = (1.5/4) × 4 = 1.5 kWh

Total losses = 36 + 4.5 + 1.5 = 42 kWh

Total output = (100 × 3) + (50 × 4) = 500 kWh

ηall-day = 500 × 100/542 = 92.26 %

Incidentally, ordinary or commercial efficiency of the transformer is

= 100/(100 + 3) = 0.971 or 97.1 %

Example 12: A 5-kVA distribution transformer has a full-load efficiency at unity p.f. of 95 %, the copper and iron losses then being equal. Calculate its all-day efficiency if it is loaded throughout the 24 hours as follows :

No load for 10 hours — Quarter load for 7 hours

Half load for 5 hours — Full load for 2 hours

Assume load p.f. of unity.

Solution:

Output = 5 × 1 = 5 kW ; Input = 5/0.95 = 5.264 kW

Losses = (5.264 − 5.000) = 0.264 kW = 264W

Since efficiency is maximum, the losses are divided equally between Cu and iron.

Cu loss at F.L. of 5 kVA = 264/2 = 132 W ; Iron loss = 132 W

Cu loss at one-fourth F.L. = $(1/4)^2$ × 132 = 8.2 W

Cu loss at one-half F.L. = $(1/2)^2$ × 132 = 33 W

Quarter load Cu loss for 7 hours = 7 × 8.2 = 57.4 Wh

Half-load Cu loss for 5 hours = 5 × 33 = 165 Wh

F.L. Cu loss for 2 hours = 2 × 132 = 264 Wh

Total Cu loss during one day = 57.4 + 165 + 264 = 486.4 Wh = 0.486 kWh

Iron loss in 24 hours = 24 × 132 = 3168 Wh = 3.168 kWh

Total losses in 24 hours = 3.168 + 0.486 = 3.654 kWh

Since load p.f. is to be assumed as unity.

F.L. output = 5 × 1 = 5 kW ; Half F.L. output = (5/2) × 1 = 2.5 kW

Quarter load output = (5/4) × 1 = 1.25 kW

Transformer output in a day of 24 hours = (7 × 1.25) + (5 × 2.5) + (2 × 5) = 31.25 kWh

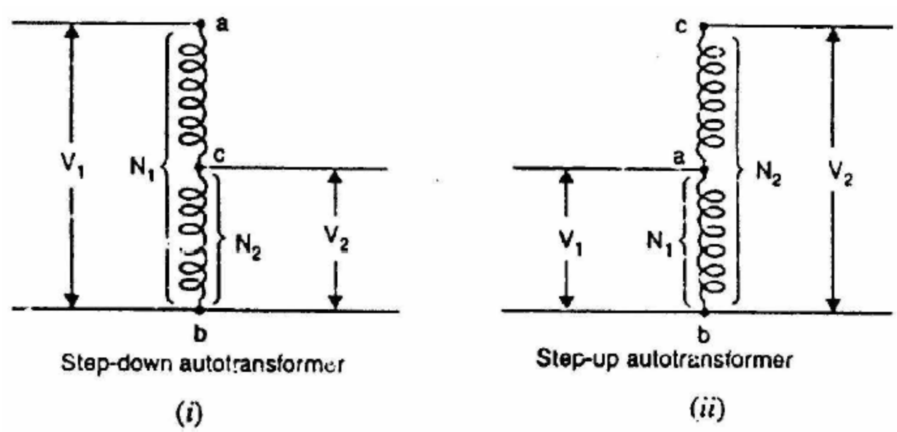
Efficiency all day = 89.53 %

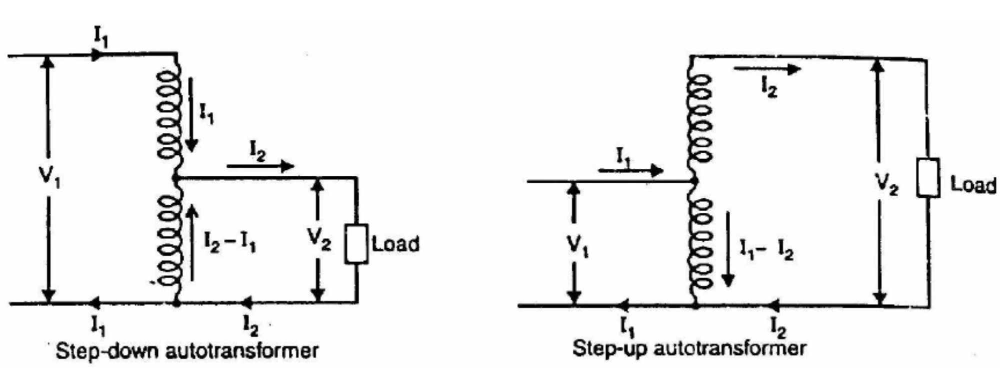
Auto Transformers

An autotransformer has a single winding on an iron core and a part of winding is common to both the primary and secondary circuits. Fig (i) below shows the connections of a step-down autotransformer whereas Fig. (ii) Shows the connections of a step-up autotransformer. In either case, the winding ab having

N1 turns is the primary winding and winding be having N2 turns is the secondary winding. Note that the primary and secondary windings are connected electrically as well as magnetically. Therefore, power from the primary is transferred to the secondary conductively as well as inductively ((transformer action). The voltage transformation ratio K of an ideal autotransformer is

$$K = \frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$$



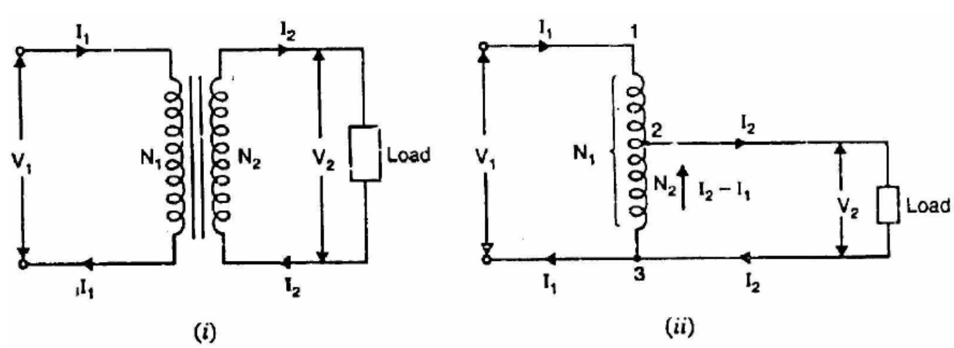


Saving of Copper in Autotransformer

For the same output and voltage transformation ratio K(N2/N1), an autotransformer requires less copper than an ordinary 2-winding transformer.

The length of copper required in a winding is proportional to the number of turns and the area of cross-section of the winding wire is proportional to the current rating. Therefore, the volume and hence weight of copper required in a winding is proportional to current x turns i.e.,

$$\text{Weight of Cu required in a winding} \propto \text{current} \times \text{turns}$$



Winding transformer

Weight of Cu required $\propto (I_1N_1 + I_2N_2)$

Autotransformer

Weight of Cu required in section 1-2 $\propto I_1(N_1 - N_2)$

Weight of Cu required in section 2-3 $\propto (I_2 - I_1)N_2$

$\therefore$ Total weight of Cu required $\propto I_1(N_1 - N_2) + (I_2 - I_1)N_2$

$$\frac{\text{Weight of Cu in autotransformer}}{\text{Weight of Cu in ordinary transformer}} = \frac{I_1(N_1 - N_2) + (I_2 - I_1)N_2}{I_1N_1 + I_2N_2}$$

$$= \frac{N_1I_1 - N_2I_1 + N_2I_2 - N_2I_1}{N_1I_1 + N_2I_2}$$

$$= \frac{N_1I_1 + N_2I_2 - 2N_2I_1}{N_1I_1 + N_2I_2}$$

$$= 1 - \frac{2N_2I_1}{N_1I_1 + N_2I_2}$$

$$= 1 - \frac{2N_2I_1}{2N_1I_1} \qquad (\because\ N_2I_2 = N_1I_1)$$

$$= 1 - \frac{N_2}{N_1} = 1 - K$$

$\therefore$ Wt. of Cu in autotransformer (W_a)

= (1 − K) × Wt. in ordinary transformer (W_o)

or $\quad W_a = (1 - K) \times W_o$

$\therefore \quad$ Saving in Cu $= W_o - W_a = W_a - (1 - K)W_o = K\ W_o$

or $\quad$ Saving in Cu = K × Wt. ofCu in ordinary transformer

Thus if K = 0.1, the saving of Cu is only 10% but if K = 0.9, saving of Cu is 90%. Therefore, the nearer the value of K of autotransformer is to 1, the greater is the saving of Cu.

Advantages of autotransformers

(i) An autotransformer requires less Cu than a two-winding transformer of similar rating.

(ii) An autotransformer operates at a higher efficiency than a two-winding transformer of similar rating.

(iii) An autotransformer has better voltage regulation than a two-winding transformer of the same rating.

(iv) An autotransformer has smaller size than a two-winding transformer of the same rating.

(v) An autotransformer requires smaller exciting current than a two-winding transformer of the same rating.