University of Anbar
Engineering College
Department of Mechanical Engineering

ME 4309 - Engineering Control and Measurements

(3-3-1-0)

Fourth Stage

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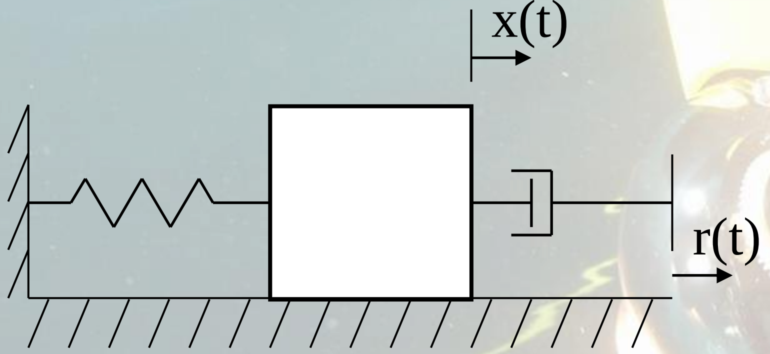
Chapter 3: Mechanical System Modelling

- Real systems are complicated
- We want to work with simple models
- We need to make some assumptions in our models

Mechanical System Modelling

Linear Mechanical



Variables: position, x
velocity, v
force, f

Mass:

$$f = ma$$

$$f = m\frac{dv}{dt} \qquad v = \frac{1}{m}\int f\,dt + C$$

Spring:

$$f = kx$$

$$f = k\int v\,dt \qquad v = \frac{1}{k}\frac{df}{dt}$$

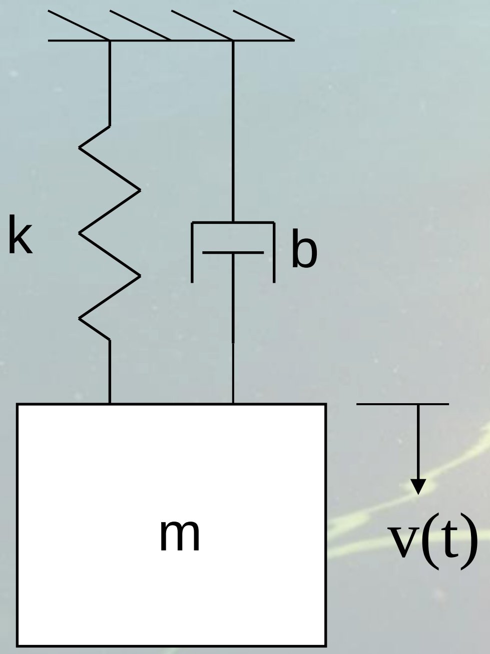
Damper:

$$f = Bv$$

Connection rules

$$\sum f_i = 0$$

Example



$$\sum forces = 0$$

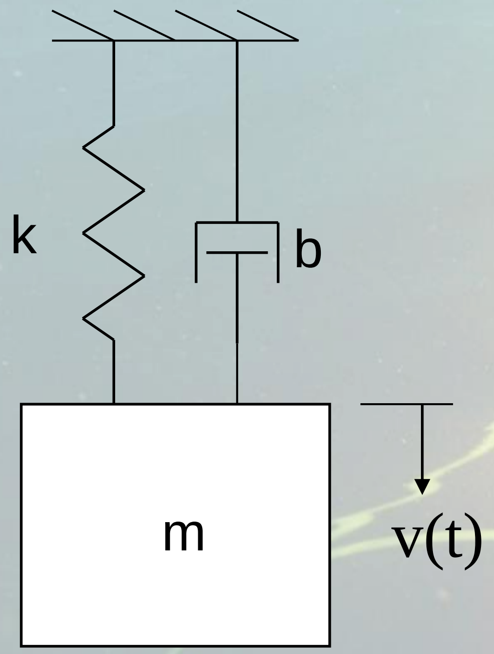
$$f_{mass} + f_{damper} + f_{spring} = 0$$

$$m\frac{dv}{dt} + bv + k\int v\, dt = 0$$

Or differentiating:

$$m\frac{d^2v}{dt^2} + b\frac{dv}{dt} + kv = 0$$

Example

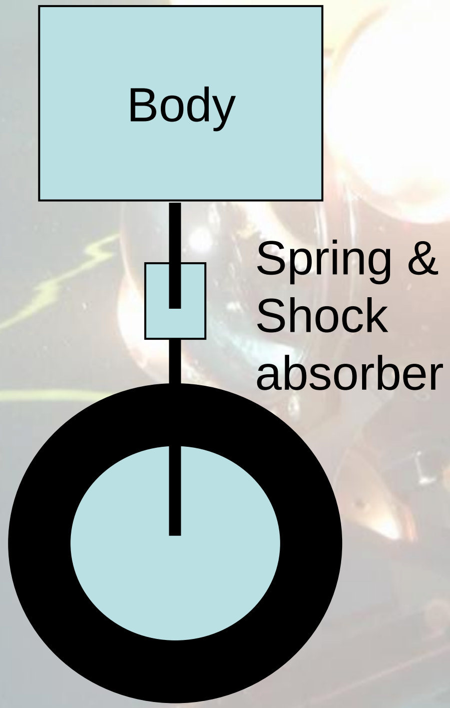


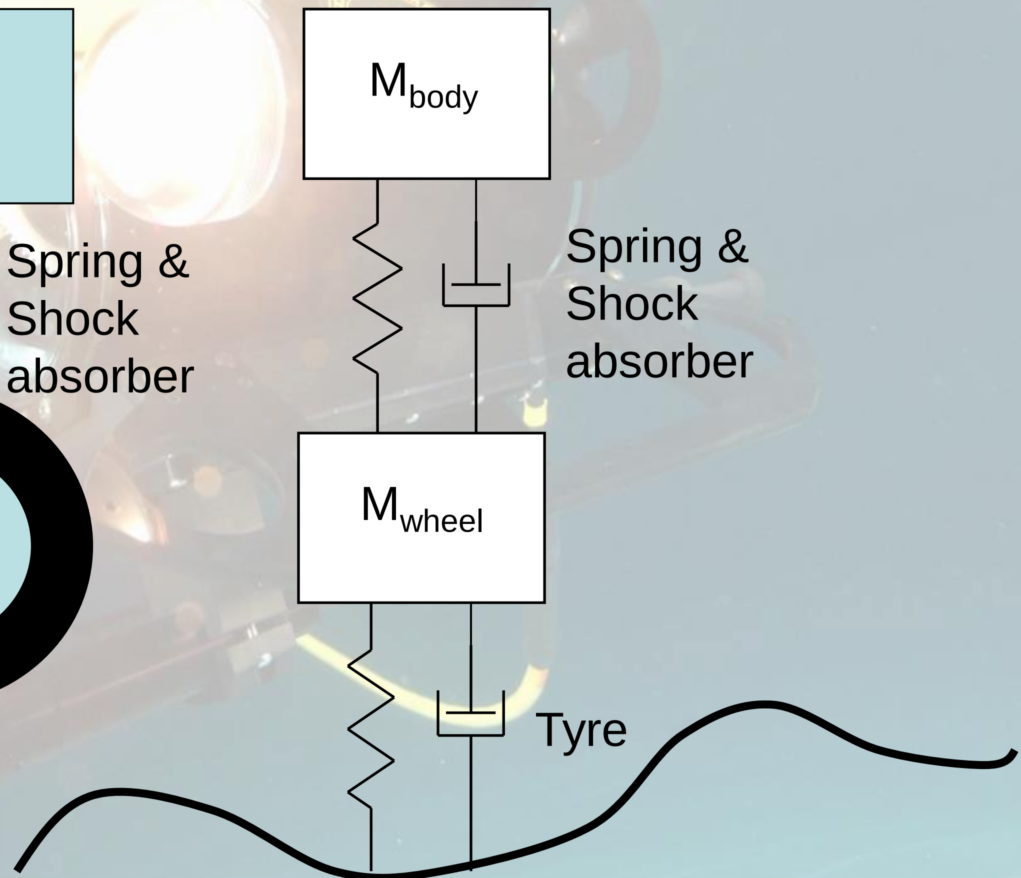
$$m\frac{d^2v}{dt^2} + b\frac{dv}{dt} + kv = 0$$

This is a 2nd order linear differential equation.

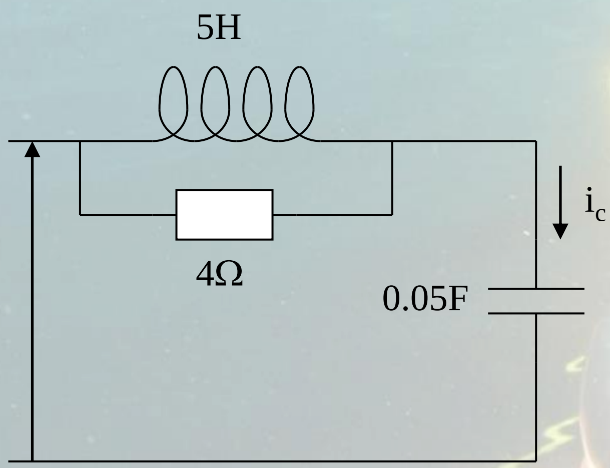
If we know m, b and k and the initial conditions, we can solve this to find v(t)

Does this model represent anything real?





Electrical System Modelling



Variables: voltage, v
current, i

Inductor:

$$v = L\frac{di}{dt} \qquad i = \frac{1}{L}\int v\,dt \;+C$$

Capacitor:

$$v = \frac{1}{C}\int i\,dt \qquad i = C\frac{dv}{dt}$$

Resistor:

$$v = iR$$

Connection rules

Kirchoff's Voltage Law

$$\sum v_i = 0$$

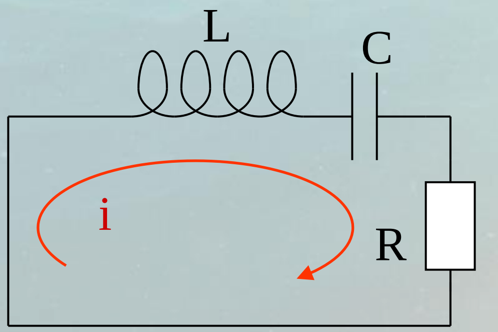
Around a loop

Kirchoff's Current Law

$$\sum i_i = 0$$

At a node

Example



$$\sum voltages = 0$$

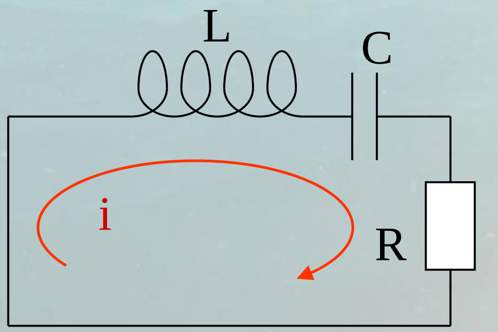
$$v_{ind} + v_{res} + v_{cap} = 0$$

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = 0$$

Or differentiating:

$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0$$

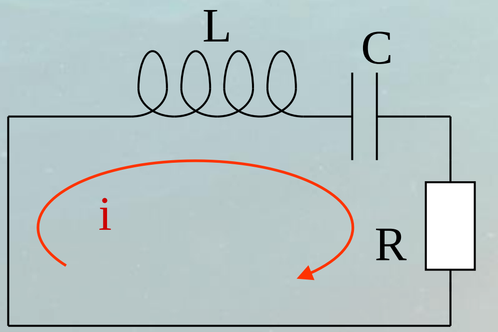
Example



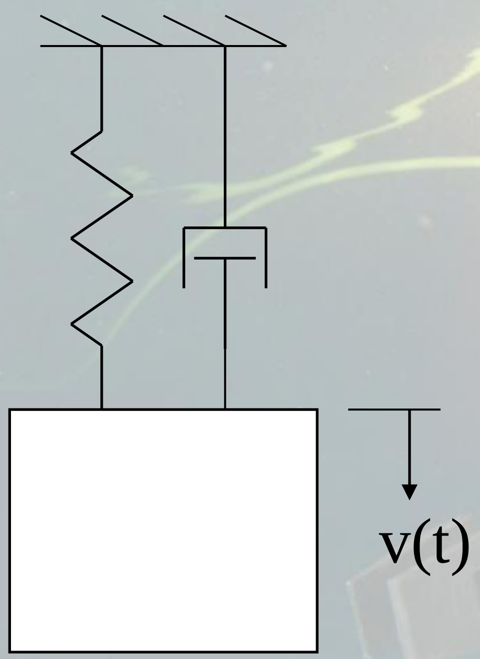
$$L\frac{d^2i}{dt^2}+R\frac{di}{dt}+\frac{i}{C}=0$$

Once again, we have a 2nd order linear differential equation which could be solved using the usual methods

Compare:



$$L\frac{d^2i}{dt^2}+R\frac{di}{dt}+\frac{i}{C}=0$$



$$m\frac{d^2v}{dt^2}+b\frac{dv}{dt}+kv=0$$

Mechanical and Electrical systems can be treated in the same way.