

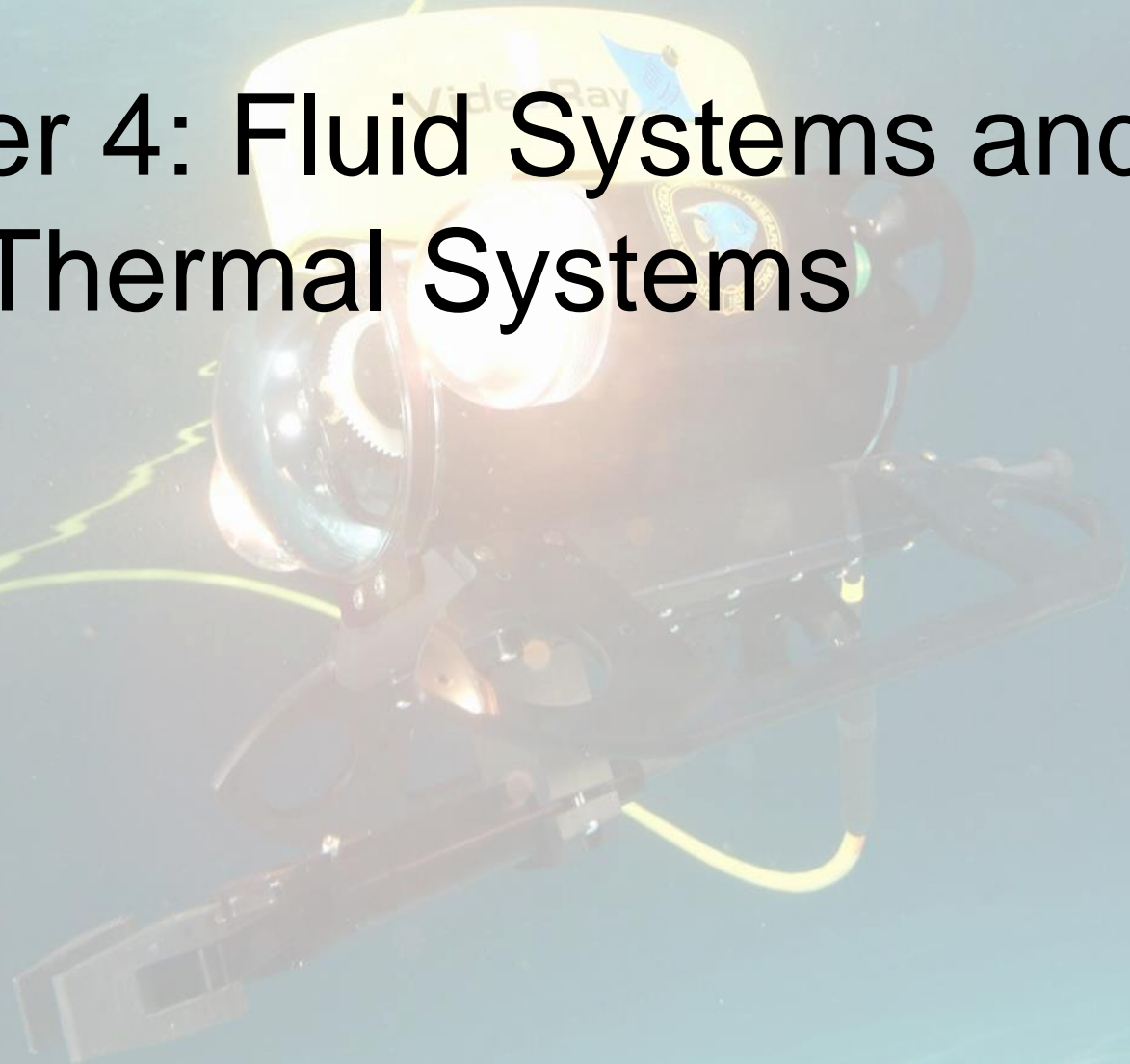
# ME 4309 - Engineering Control and Measurements

(3-3-1-0)

Fourth Stage

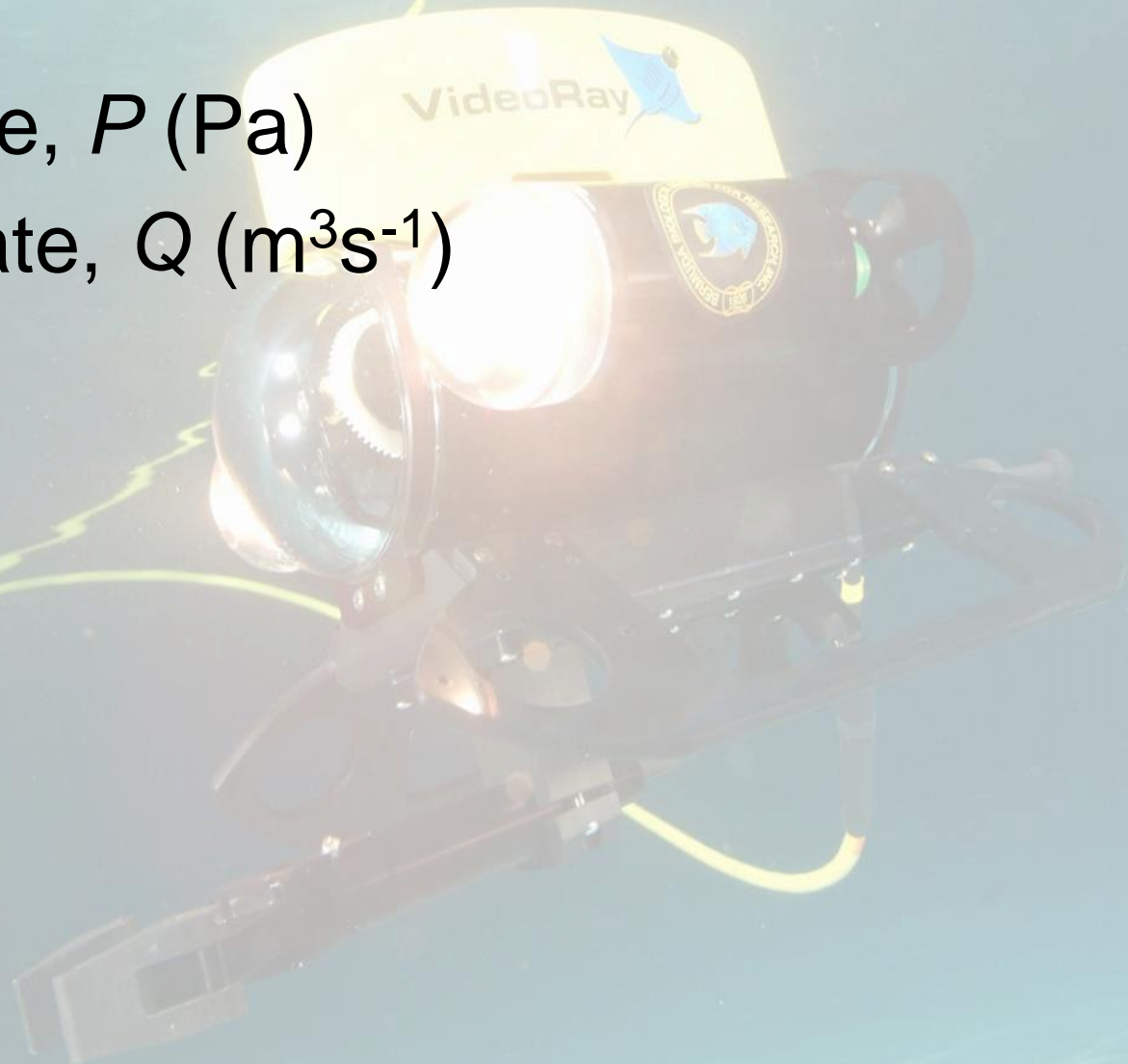
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# Chapter 4: Fluid Systems and Thermal Systems



# Fluid Systems

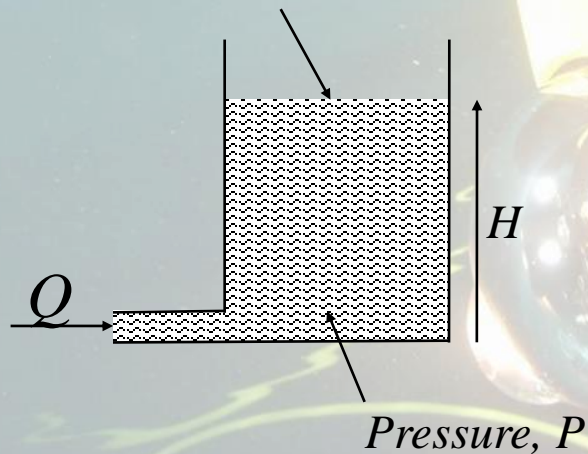
- Pressure,  $P$  (Pa)
- Flow Rate,  $Q$  ( $\text{m}^3\text{s}^{-1}$ )



# Fluid flow store

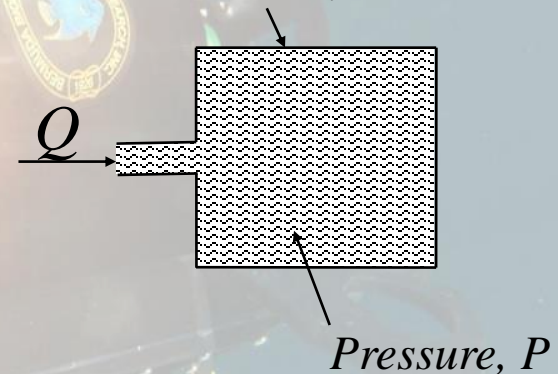
Open reservoir

*Cross sectional areas,  $A$*



Pressurised tank

*Tank Volume,  $V_t$*



$$Q = C_f \frac{dP}{dt}$$

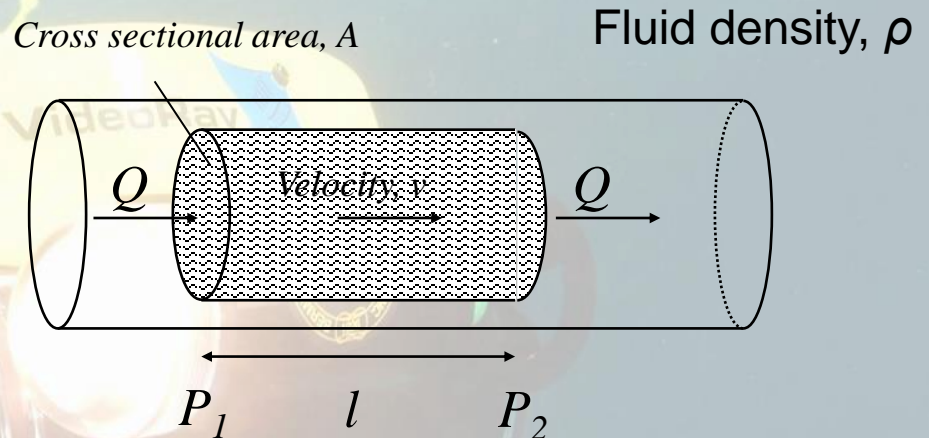
$$P = \frac{1}{C_f} \int Q dt$$

$C_f$  – fluid capacitance

# Fluid effort store

## Fluid flow in a pipe

$$\text{Force, } F = A(P_2 - P_1) = AP_{21}$$



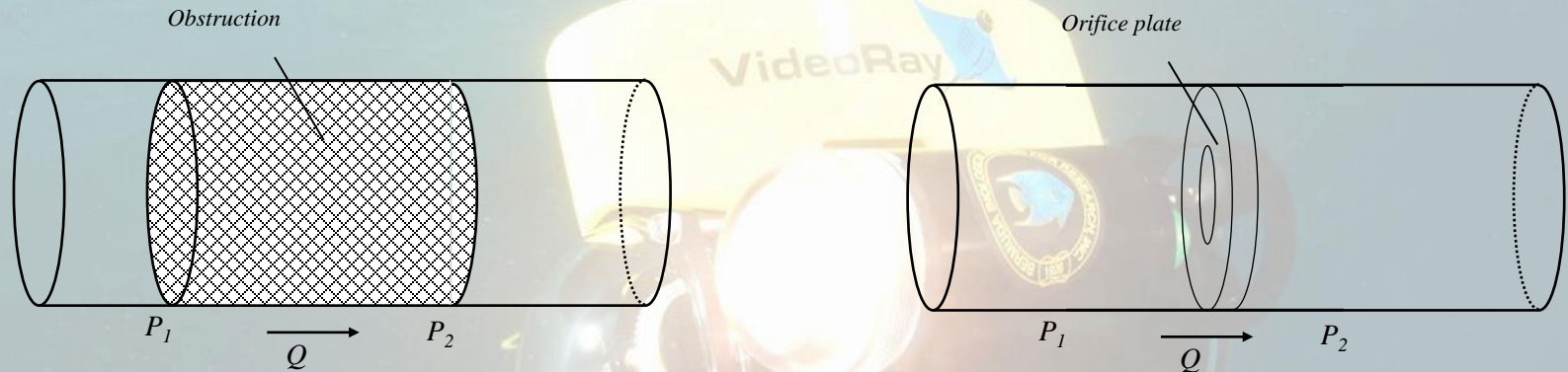
$$\text{From Newton's laws: } F = ma \quad F = \rho A l \frac{dv}{dt} \quad P = \frac{F}{A}$$

$$Q = vA$$

$$P_{12} = L_f \frac{dQ}{dt} \quad Q = \frac{1}{L_f} \int P_{12} dt$$

$$\text{Fluid inertance, } L_f = \frac{\rho l}{A}$$

# Fluid Dissipator



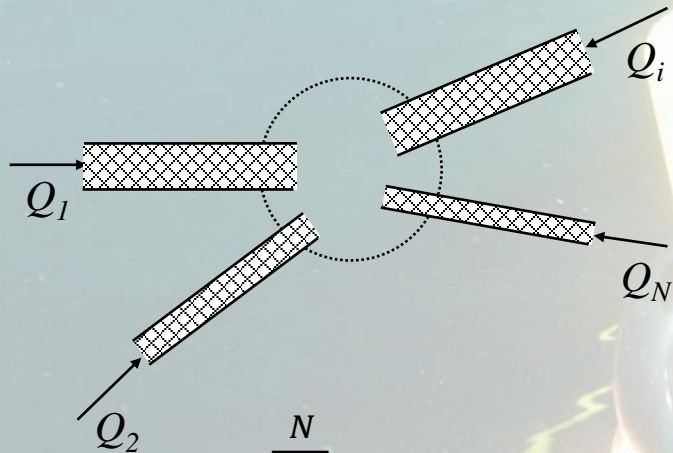
$$P_{12} = R_f Q$$

$R_f$  – fluid resistance

In reality, the fluid resistance relationship is highly nonlinear and complex. This is a linear approximation



# Fluid system interconnection rules

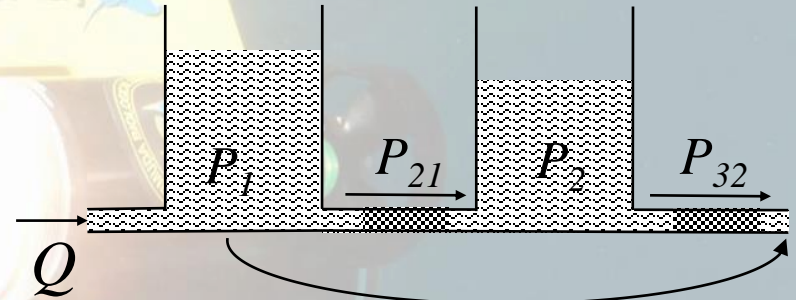


$$\sum_{j=1}^N Q_j = 0$$

Sum of flows at a junction equals zero (no leaks allowed)

For compressible fluids, sum of all mass flows at a junction equals zero

$$\sum_{j=1}^N m_j = 0$$

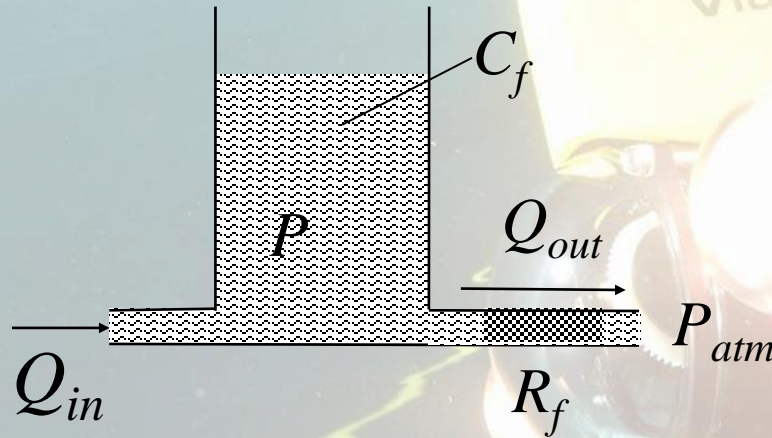


$$P_{31} = P_{21} + P_{32}$$

$$\sum_{j=1}^{N-1} P_{j,j+1} = 0$$

Sum of pressure differences around a loop equals zero

# Fluid system Example



Assume  $P_{atm}$  is zero

$$Q_{out} = \frac{P - P_{atm}}{R_f} = \frac{P}{R_f}$$

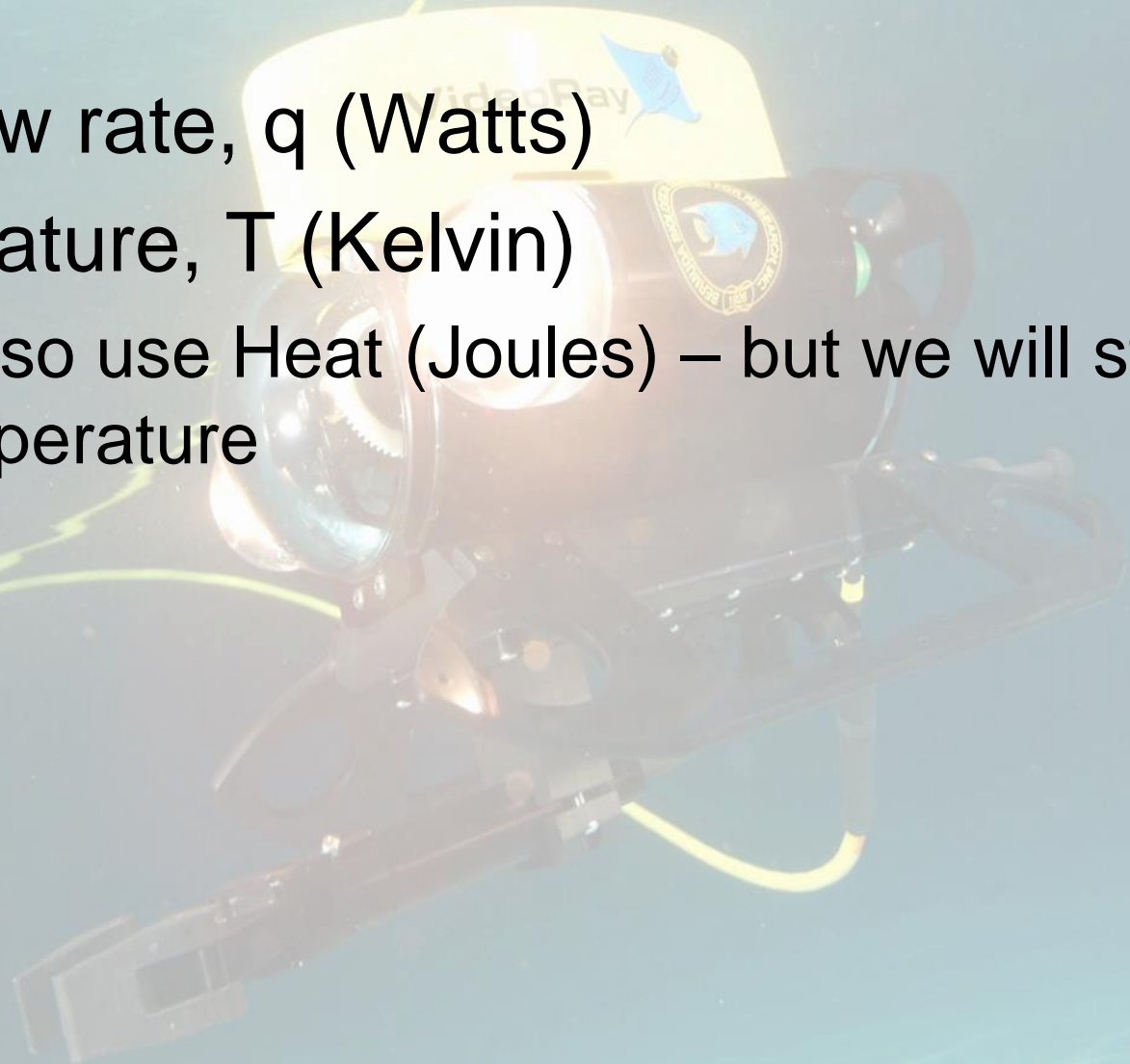
$$\frac{dP}{dt} = \frac{1}{C_f} [Q_{in} - Q_{out}]$$

$$\frac{dP}{dt} = \frac{1}{C_f} \left[ Q_{in} - \frac{P}{R_f} \right]$$

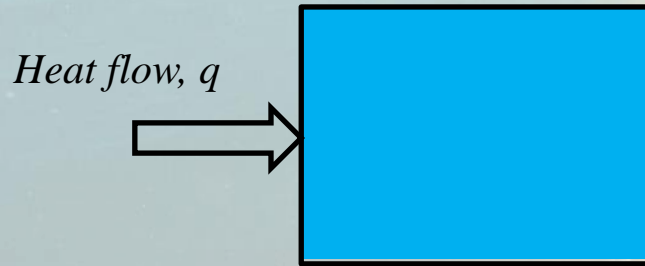


# Thermal Systems

- Heat flow rate,  $q$  (Watts)
- Temperature,  $T$  (Kelvin)
  - Can also use Heat (Joules) – but we will stick to temperature



# Thermal Flow Store



Heat stored in material

$$q = mC_p \frac{dT}{dt}$$

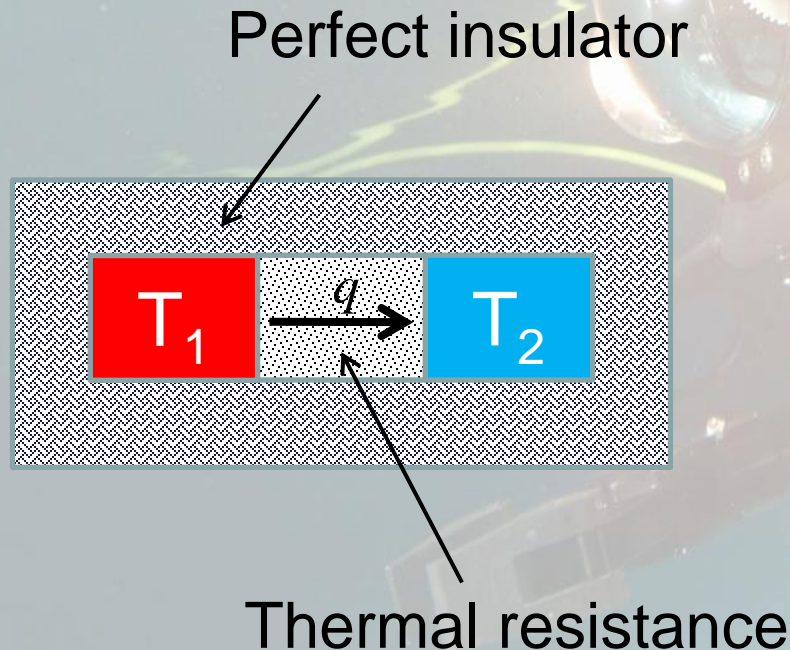
$$T = \frac{1}{mC_p} \int q dt$$

$m$  – body mass (kg),

$C_p$  – specific heat capacity of material ( $\text{Jkg}^{-1}\text{K}^{-1}$ )

# Thermal Dissipator

- Heat loss occurs through conduction, convection and radiation
- Convection and radiation are nonlinear so we will stick to conduction

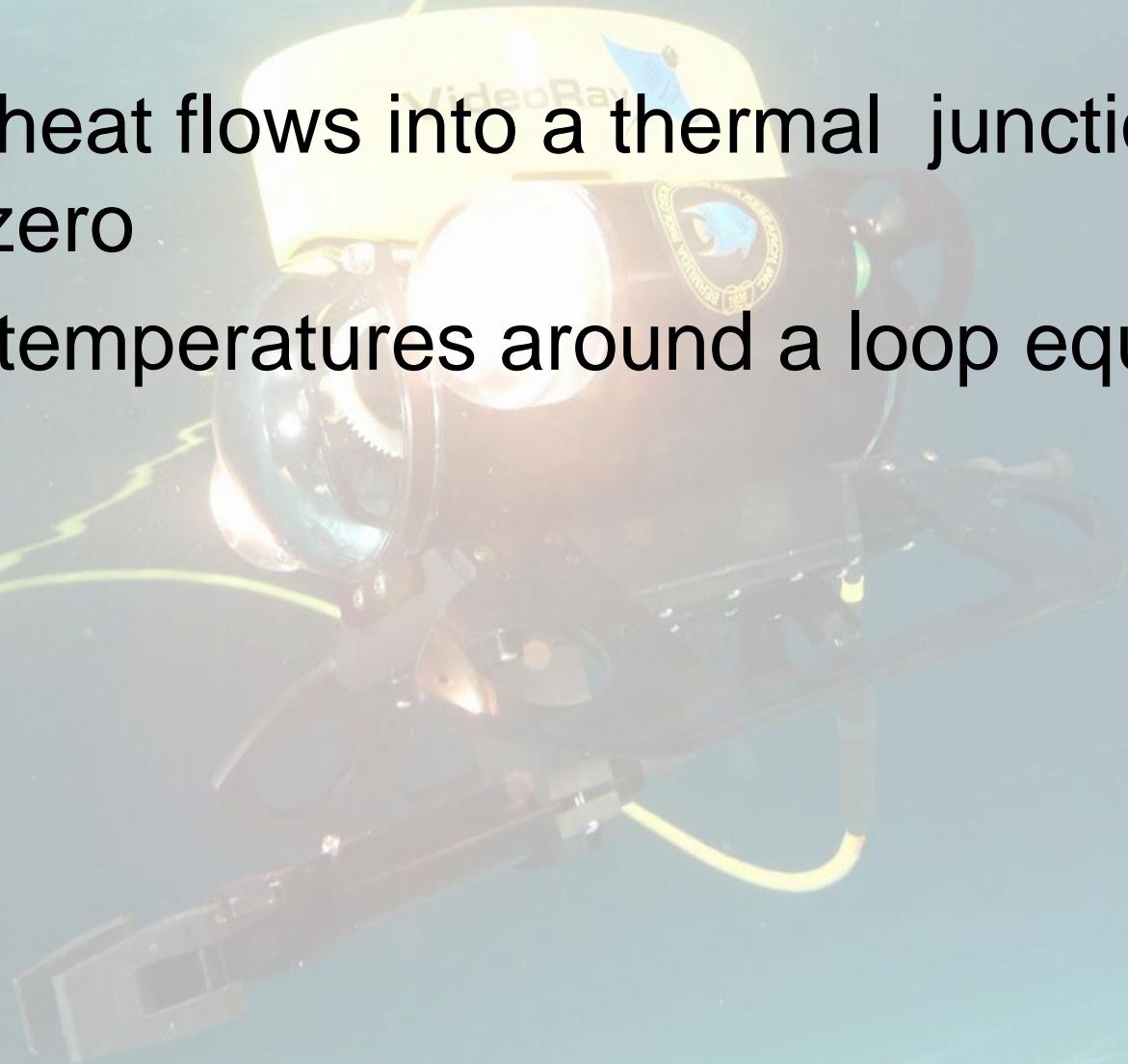


$$q = \frac{\sigma_c A}{l} (T_1 - T_2)$$

$\sigma_c$  – thermal conductivity  
 $A$  – cross sectional area  
 $l$  – length of the object

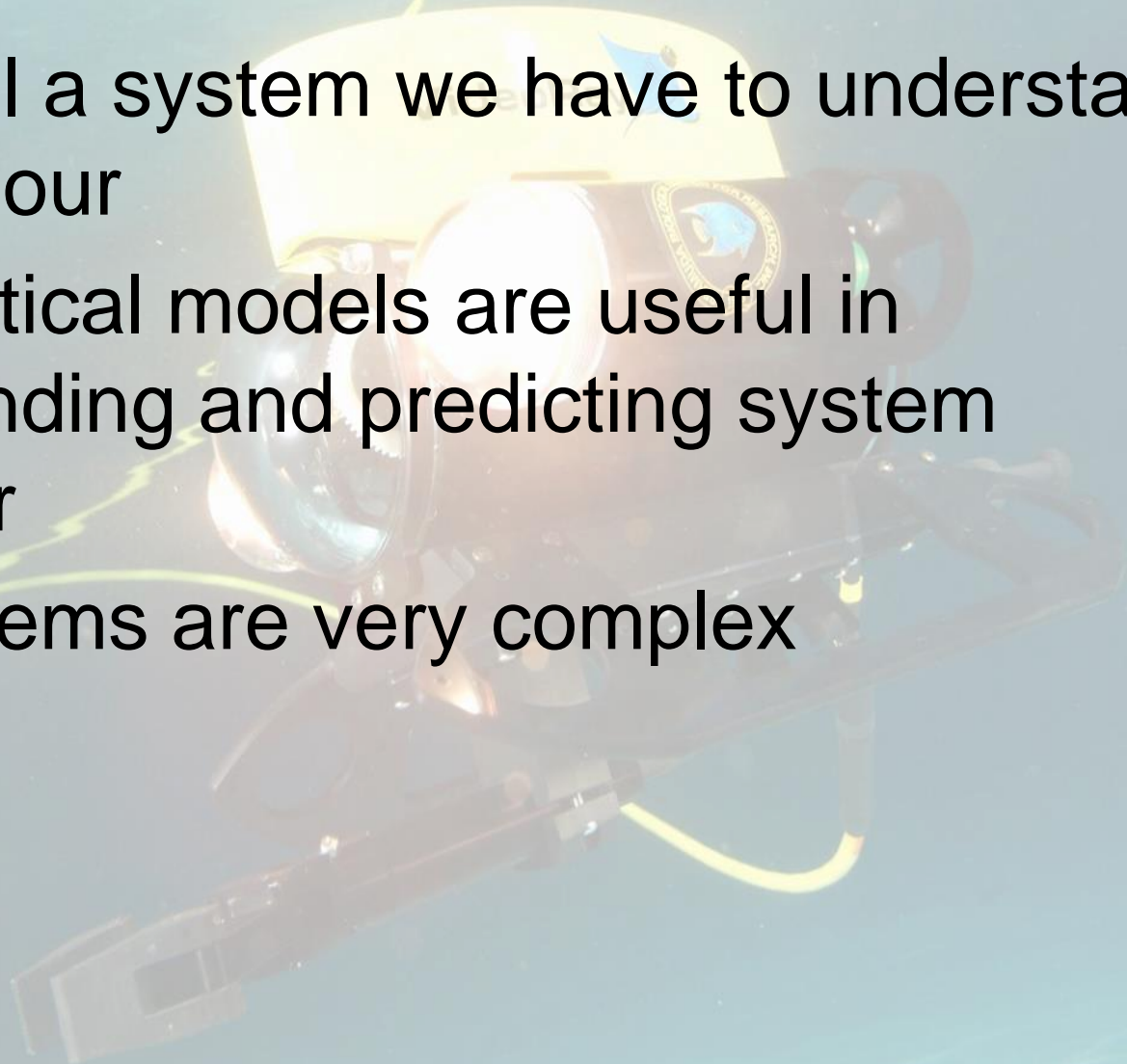
# Interconnection rules

- Sum of heat flows into a thermal junction equals zero
- Sum of temperatures around a loop equals zero



# Summary

- To control a system we have to understand its behaviour
- Mathematical models are useful in understanding and predicting system behaviour
- Real systems are very complex





# Summary

- We have to simplify them in order to get usable models
  - Linear
  - Single input , single output
  - Time invariant
  - Low order
- Take care that simplifications do not make the model invalid
- Lumped element (mass, spring, damper or RLC) models can be converted into mathematical form fairly easily. Solving the equations can be harder!