University of Anbar
Engineering College
Department of Mechanical Engineering

ME 4309 - Engineering Control and Measurements

(3-3-1-0)

Fourth Stage

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Reference:

1. Automatic Control Engineering, First Edition 1961, by Francis H. Raven, McGraw Hill.
2. Measurement Systems Applications and Design, 5th edition 2003, by E. Doebelin, McGraw Hill.

Chapter 5: Analysis of Control System

Table of Content

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1. Review of the Main Steps for Analysis of Control Systems

a. Main steps

Step 1: Establishing the mathematical model of a control system.

Step 2: Obtaining system performance specifications.

Step 3: Improving system performance.

b. Advanced methods to discuss in this semester

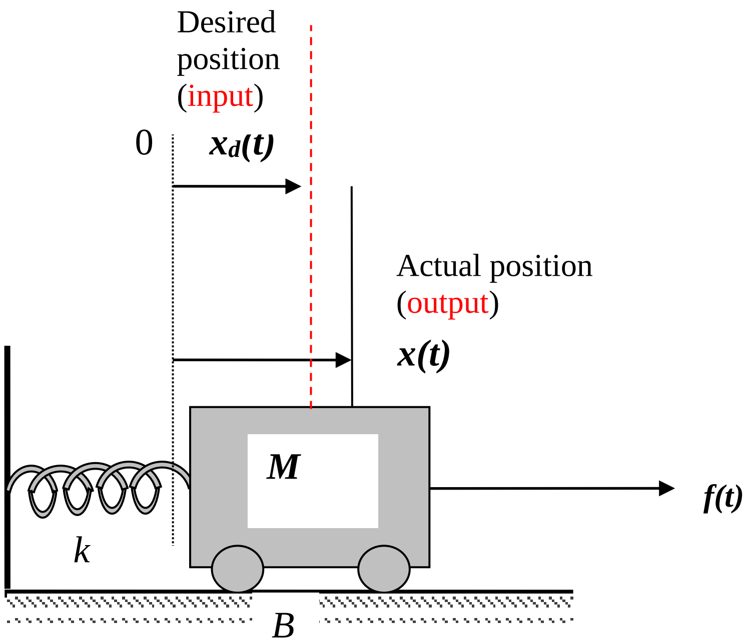
i. **A PID controller**

ii. **Feed forward control**

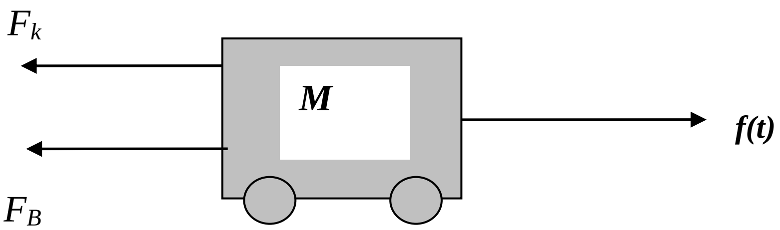
iii. **State space method**

2. Mathematical Model of a Spring-cart System

a. Diagrammatic model



b. Diagrammatic free body



$$f(t) - F_k - F_B = Ma$$

$$m\ddot{x}(t) + B\dot{x}(t) + k\,x(t) = f(t)$$

c. Laplace transform

$$ms^2 X(s) + BsX(s) + kX(s) = F(s)$$

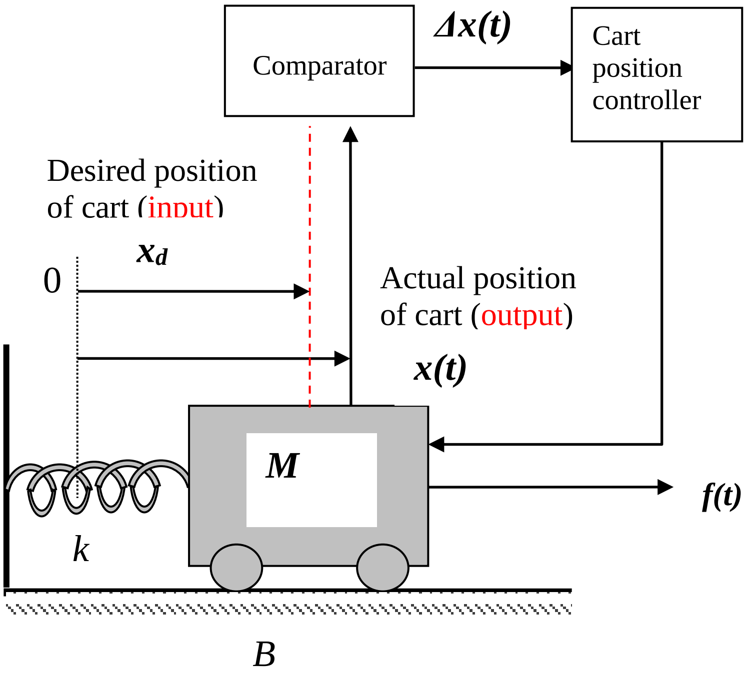
$$(ms^2 + Bs + k)X(s) = F(s)$$

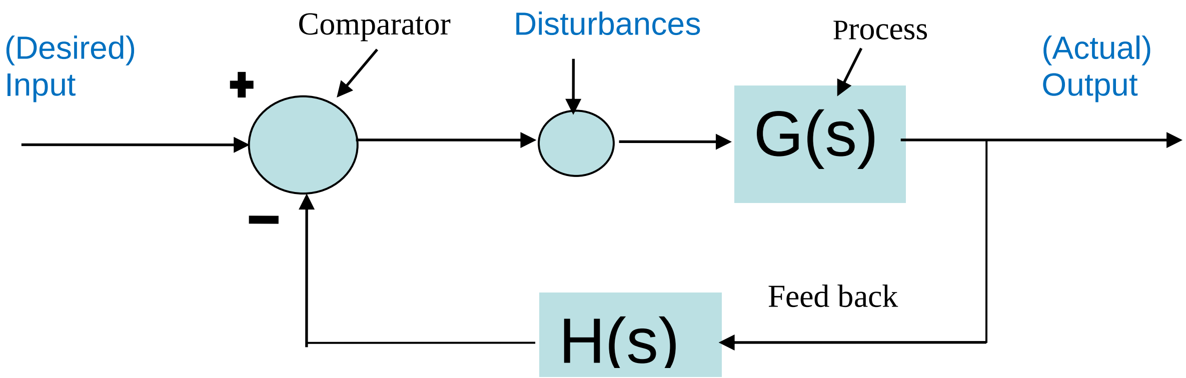
d. Transfer function

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + k} = \frac{\frac{1}{M}}{s^2 + \frac{B}{M}s + \frac{k}{M}} \quad {}^*$$

*A spring cart system is by nature a control system with an internal feedback. However in this lecture this system is treated as a control system hiding its internal feedback. In order for students to understand the principles of feedback control, a manmade feedback circuit is connected to the system to form a closed loop.

e. Closed loop





Where H(s)=1.

$$T(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{1}{M}}{s^2 + \frac{B}{M}s + \frac{k+1}{M}}$$

3. Performance specifications of the spring-cart System

a. Canonical form of *T(s)*

$$T(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

b. Performance specifications of *T(s)*

$$\omega_n = \sqrt{\frac{k+1}{M}}$$

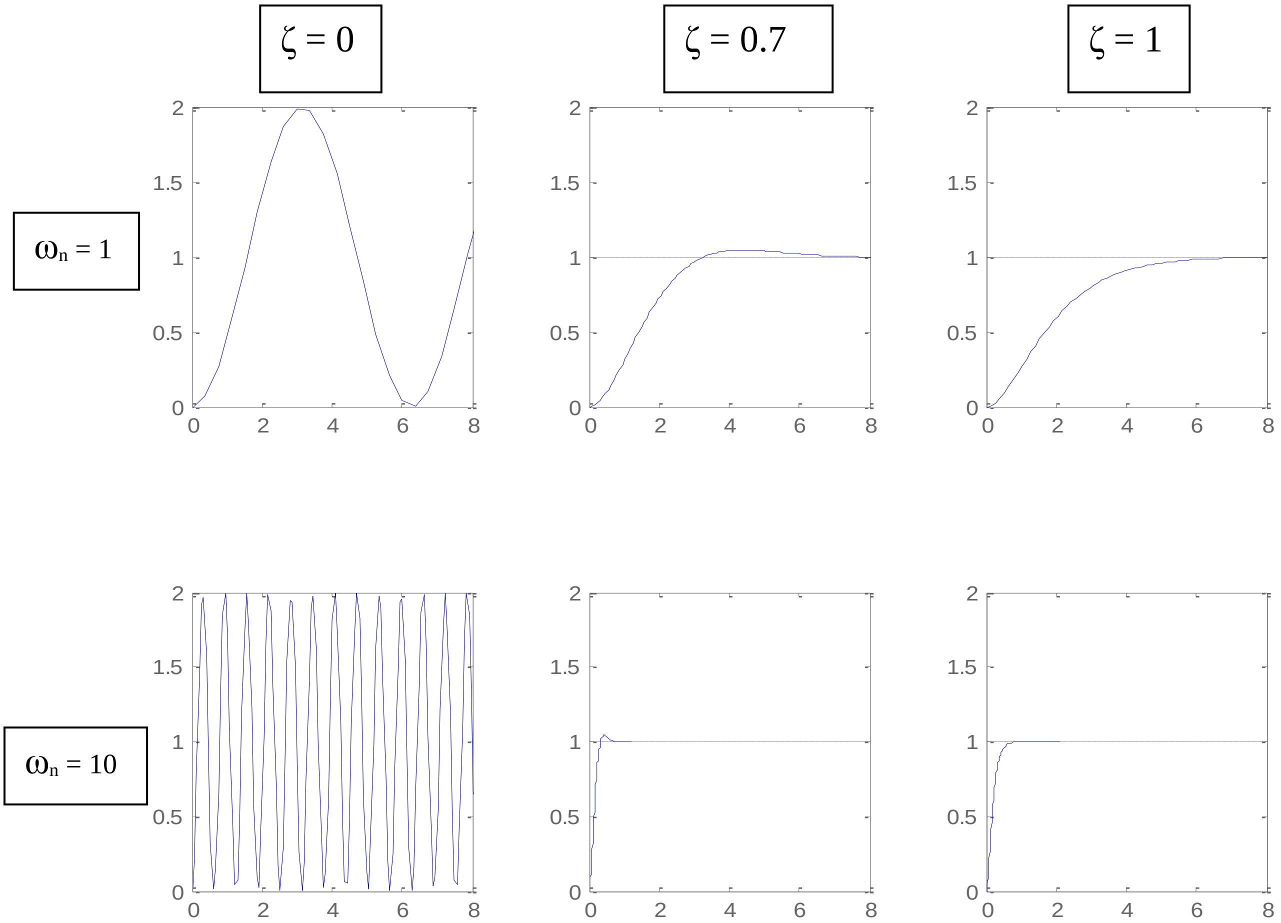
$$\zeta = \frac{B}{2\sqrt{(k+1)M}}$$

$$K = \frac{1}{k+1}$$

$$e_{ss} = \frac{k}{k+1}$$

4. Assessment Criteria for System's Performance

a. Effect of ζ *and* ω_n on the system performance



ii. Desired values of the system performance specifications

$$\zeta = 0.7$$

$$\omega_n = 2 \sim 4$$

$$e_{ss} = 0$$

5. Performance specifications and assessment of an exampled spring-cart system with M=3 kg, k=2 Nm^{-1}, B=0.6 Nm^{-1}s

a. Performance specifications

$$\omega_n = \sqrt{\frac{k+1}{M}} = \sqrt{\frac{2+1}{3}} = 1$$

$$\zeta = \frac{B}{2\sqrt{(k+1)M}} = \frac{0.6}{2\sqrt{(2+1)3}} = 0.1$$

$$K = \frac{1}{k+1}$$

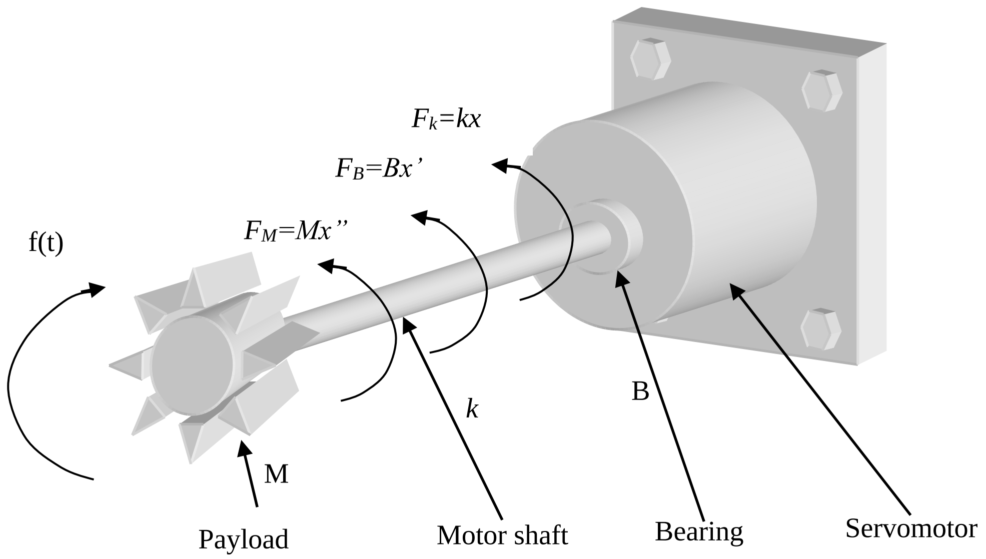
$$e_{ss} = \frac{k}{k+1} = \frac{2}{2+1} = 0.7$$

b. Assessment

Since $\omega_n = 1 << 4$, the system's response time is too long; since $\zeta = 0.1 << 0.7$, the system tends to be oscillating, and since $e_{ss} = 0.7 > 0$, the system's steady state error is too big. It is concluded that the performance specifications of this system are not satisfactory. So a measure is needed to improve them.

6. Exercise Question

A spring-servomotor system is shown in figure below, where the servomotor is used to drive the payload. The torque *f(t)* produced by the motor is applied to the payload. M is the mass of the payload, B is the coefficient of the viscosity at the bearings and *k* is the stiffness coefficient of the shaft that can be regarded as a torsion rod spring. By Newton's second law, the net torque or the difference between the torque from the motor and the torques that oppose the rotation determines the angular acceleration of the object. A torsion rod spring is viewed as a kind of spring that shows torsion when subjected to a force but it recovers its original shape when the force is removed.



(a) Obtain the differential equation of the spring-servomotor system as shown in the above figure.
(b) Find the open loop transfer function of the system.
(c) Find the closed loop transfer function of the system and its canonical form assuming H(s)=1.
(d) Find the values of the damping coefficient ζ and the undamped natural frequency ω_n of the closed loop given that M= 9 kg, B = 1.8 Nm^{-1} s and k = 8 Nm^{-1} .
(e) Analyze and assess the performance of the system.