University of Anbar
Engineering College
Department of Mechanical Engineering

ME 4309 - Engineering Control and Measurements

(3-3-1-0)

Fourth Stage

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Reference:

1. Automatic Control Engineering, First Edition 1961, by Francis H. Raven, McGraw Hill.
2. Measurement Systems Applications and Design, 5th edition 2003, by E. Doebelin, McGraw Hill.

Chapter 6: PID controller

Table of Content

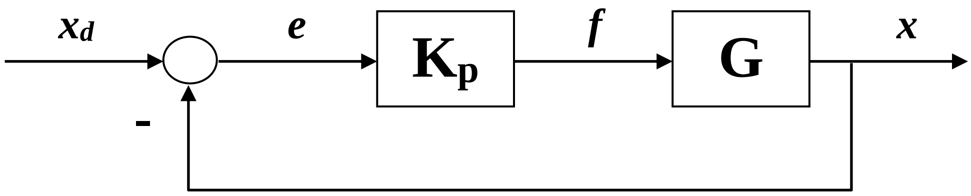
1. Introduction to PID controller

a. PID = P + I + D.
b. P – to make the system response to the input faster than the original.
   D – to make the system more stable than the original.
   I – to make the system's steady state error close to 0.
c. Gains K_p, K_i, K_d - to tune them for the best contribution from each controller respectively.

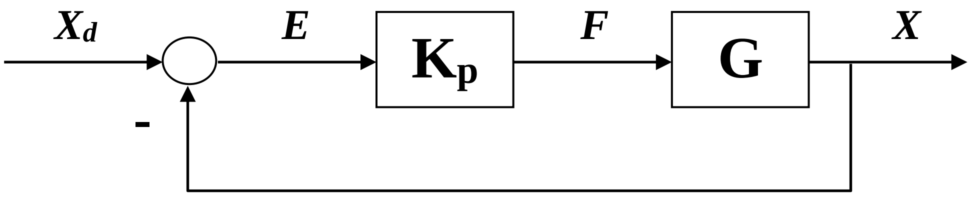
2. P-Controller

a. Aim: to make the system response to the input faster than the original by increasing ω_n .

b. Block diagram in the time domain



Block diagram for the system in the S-domain



c. Find T(s)

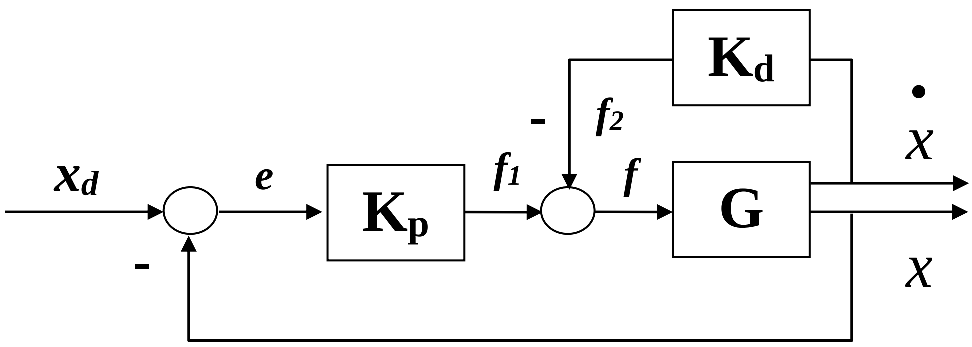
d. Determine $\omega_n, \zeta, K, e_{ss}$ (e_{ss} will be derived later.)

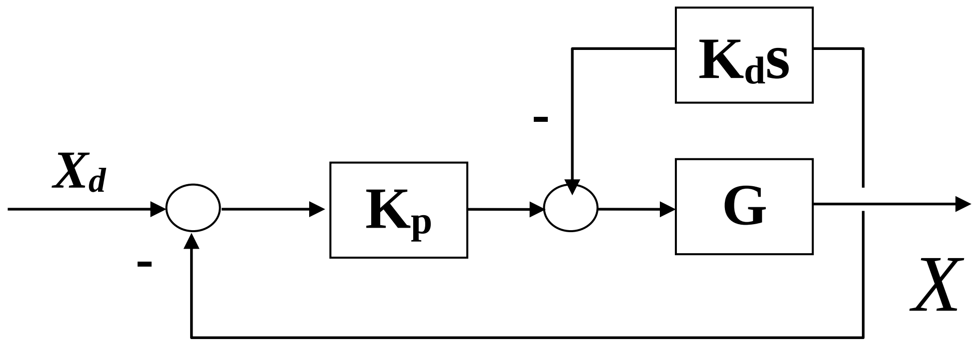
e. Summary

3. PD – Controller

a. Aim: to make the system more stable and response to the input faster than the original by reducing the value of ζ .

b. Block diagram





c. Obtain T(s)

d. Determine $\omega_n, \zeta, K, e_{ss}$ (e_{ss} will be derived later.)

e. Summary

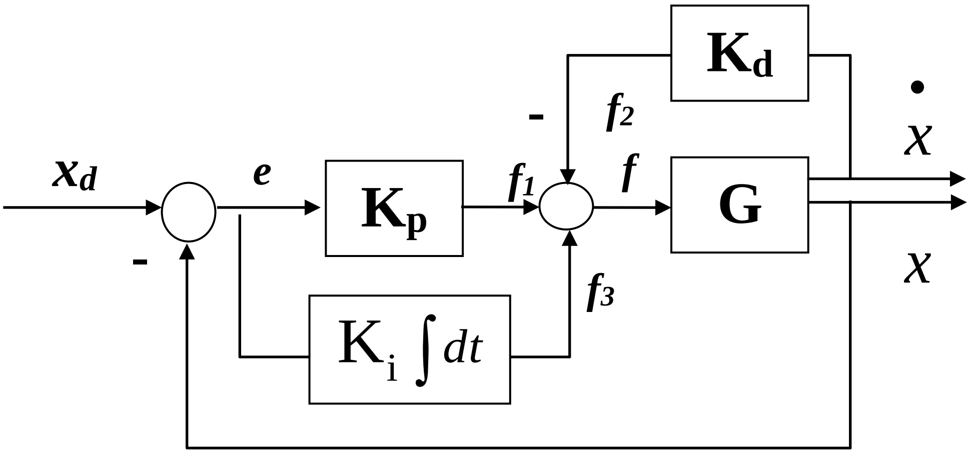
3. Summary

Performance	Original system	P added to the original	PD added to the original	PID added to the original
Response time ω_n	$\omega_n = \sqrt{\dfrac{k+1}{M}}$	$\omega_n = \sqrt{\dfrac{k+K_P}{M}}$	$\omega_n = \sqrt{\dfrac{k+K_P}{M}}$	The canonical form is not applicable here and ω_n becomes greater.
Stability ζ	$\zeta = \dfrac{B}{2\sqrt{(k+1)M}}$	$\zeta = \dfrac{B}{2\sqrt{(k+K_P)M}}$	$\zeta = \dfrac{B+K_d}{2\sqrt{(k+K_P)M}}$	ζ becomes greater.
Steady state error e_{ss}	$e_{ss} = \dfrac{k}{1+k}$	$e_{ss} = \dfrac{k}{k+K_P}$	$e_{ss} = \dfrac{k}{k+K_P}$	$e_{ss} = 0$

Conclusion: when a PID controller is used to control a system (original), the system performance specifications will be significantly improved.

4. Exercise Question

A PID controller is connected to a control system to control the plant G. The block diagram representing the system in the time domain is shown in the figure below. K_p is the gain of a proportional controller, K_d is the gain of a derivative controller and K_i is the gain of an integral controller. x_d, e, f, f_1, f_2, f_3 and x are the variables that change with t in the time domain. x_d is the input and x is the output. Assuming $f_3 = K_i \int e dt$ and the Laplace transform of $\int e(t)dt$ is $\frac{1}{s}E(s)$ where $E(s)$ is the Laplace transform of $e(t)$.



(a) Convert this block diagram for the system in the time domain to the block diagram in the S-domain.
(b) Obtain the open loop transfer function of the system.
(c) Find the closed loop transfer function of the system assuming H(s)=1.
(d) Derive a formula for the steady state error e_{ss}.
(e) Compare the e_{ss} of the PID regulated system with that of the original system and give your conclusion.