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Electric physics II

The Electric Field

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2020 - 2021

The Electric Field

- An **electric field** is said to exist in the region of space around a charged object—the source charge. When another charged object—the test charge—enters this electric field, an electric force acts on it. As an example, consider Figure 23.11, which shows a small positive test charge *q*0 placed near a second object carrying a much greater positive charge *Q*. We define the electric field due to the source charge at the location of the test charge to be the electric force on the test charge *per unit charge*, or to be more specific

the electric field vector E at a point in space is defined as

the electric force $\mathbf{F}_e$ acting on a positive test charge q_0 placed at that point divided by the test charge:

$$\mathbf{E} \equiv \frac{\mathbf{F}_e}{q_0} \qquad (23.7)$$

$$\boldsymbol{F}_e = q\,\mathbf{E} \qquad (23.8)$$

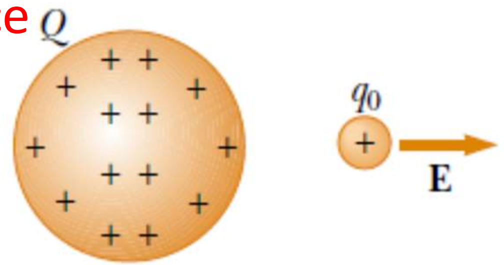


Figure 23.11 A small positive test charge q_0 placed near an object carrying a much larger positive charge Q experiences an electric field **E** directed as shown.

- Notice the similarity between Equation 23.8 and the corresponding equation for a particle with mass placed in a gravitational field, $F_g = mg$
- The vector E has the SI units of newtons per coulomb (N/C).
- The direction of E, as shown in Figure 23.11

. For a positive point charge the lines of electric field are directed outward

. For a negative charge the lines of electric field are directed inward

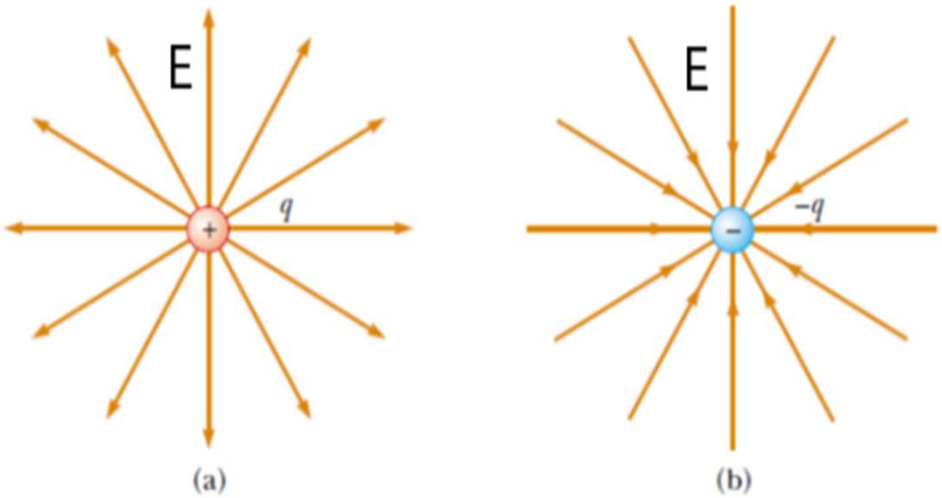


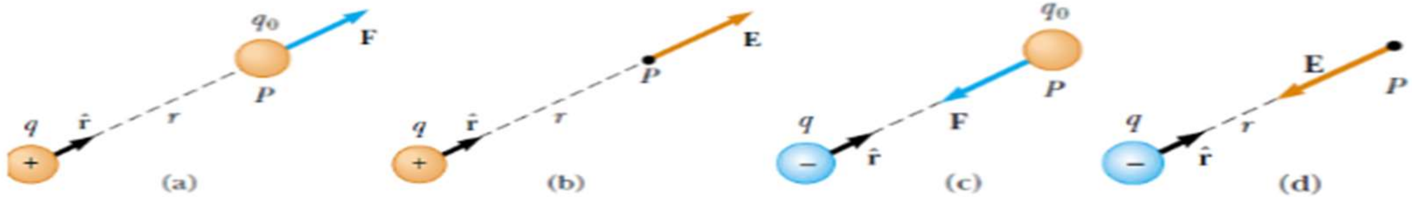
Figure (23.11)

- According to Coulomb's law, the force exerted by *q* on the test charge is

$$\mathbf{F}_e = k_e \frac{q q_0}{r^2}\, \hat{\mathbf{r}}$$

- where rˆ is a unit vector directed from *q* toward *q*0. This force in Figure 23.13a is directed away from the source charge *q*. Because the electric field at *P*, the position of the test charge, is defined by $E=F_e/q_0$, we find that at *P*, the electric field created by *q* is

$$\mathbf{E} = k_e \frac{q}{r^2}\, \hat{\mathbf{r}} \qquad (23.9)$$



Active Figure 23.13 A test charge q_0 at point *P* is a distance *r* from a point charge *q*. (a) If *q* is positive, then the force on the test charge is directed away from *q*. (b) For the positive source charge, the electric field at *P* points radially outward from *q*. (c) If *q* is negative, then the force on the test charge is directed toward *q*. (d) For the negative source charge, the electric field at *P* points radially inward toward *q*.

.To calculate the electric field at a point *P* due to a group of point charges, we first calculate the electric field vectors at *P* individually using Equation 23.9 and then add them vectorially. In other words, at any point *P*, the total electric field due to a group of source charges equals the vector sum of the electric fields of all the charges.

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2}\,\hat{\mathbf{r}}_i$$

where r_i is the distance from the *i*th source charge q_i to the point *P* and $\hat{r}_i$ is a unit vector directed from q_i toward *P*.

Example 23.5 Electric Field Due to Two Charges

A charge $q_1 = 7.0\ \mu C$ is located at the origin, and a second charge $q_2 = -5.0\ \mu C$ is located on the x axis, 0.30 m from the origin (Fig. 23.14). Find the electric field at the point P, which has coordinates (0, 0.40) m.

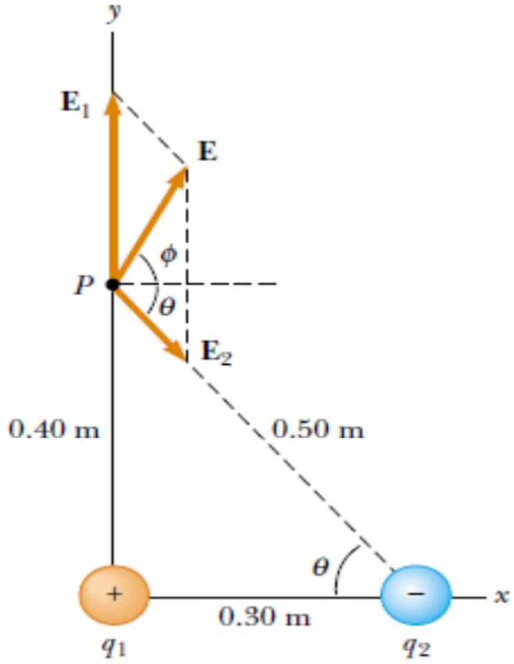


Figure 23.14 (Example 23.5) The total electric field **E** at P equals the vector sum $\mathbf{E}_1 + \mathbf{E}_2$, where $\mathbf{E}_1$ is the field due to the positive charge q_1 and $\mathbf{E}_2$ is the field due to the negative charge q_2.

Solution First, let us find the magnitude of the electric field at P due to each charge. The fields $\mathbf{E}_1$ due to the 7.0-μC charge and $\mathbf{E}_2$ due to the -5.0-μC charge are shown in Figure 23.14. Their magnitudes are

$$E_1 = k_e \frac{|q_1|}{r_1^2} = (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(7.0 \times 10^{-6}\ \text{C})}{(0.40\ \text{m})^2}$$
$$= 3.9 \times 10^5\ \text{N/C}$$

$$E_2 = k_e \frac{|q_2|}{r_2^2} = (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(5.0 \times 10^{-6}\ \text{C})}{(0.50\ \text{m})^2}$$
$$= 1.8 \times 10^5\ \text{N/C}$$

The vector $\mathbf{E}_1$ has only a y component. The vector $\mathbf{E}_2$ has an x component given by $E_2 \cos\theta = \frac{3}{5}E_2$ and a negative y component given by $-E_2 \sin\theta = -\frac{4}{5}E_2$. Hence, we can express the vectors as

$$\mathbf{E}_1 = 3.9 \times 10^5 \hat{\mathbf{j}}\ \text{N/C}$$
$$\mathbf{E}_2 = (1.1 \times 10^5 \hat{\mathbf{i}} - 1.4 \times 10^5 \hat{\mathbf{j}})\ \text{N/C}$$

The resultant field **E** at P is the superposition of $\mathbf{E}_1$ and $\mathbf{E}_2$:

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \boxed{(1.1 \times 10^5 \hat{\mathbf{i}} + 2.5 \times 10^5 \hat{\mathbf{j}})\ \text{N/C}}$$

From this result, we find that **E** makes an angle ϕ of 66° with the positive x axis and has a magnitude of 2.7×10^5 N/C.

Example 23.6 Electric Field of a Dipole

An **electric dipole** is defined as a positive charge q and a negative charge $-q$ separated by a distance $2a$. For the dipole shown in Figure 23.15, find the electric field $\mathbf{E}$ at P due to the dipole, where P is a distance $y \gg a$ from the origin.

Solution At P, the fields $\mathbf{E}_1$ and $\mathbf{E}_2$ due to the two charges are equal in magnitude because P is equidistant from the charges. The total field is $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$, where

$$E_1 = E_2 = k_e \frac{q}{r^2} = k_e \frac{q}{y^2 + a^2}$$

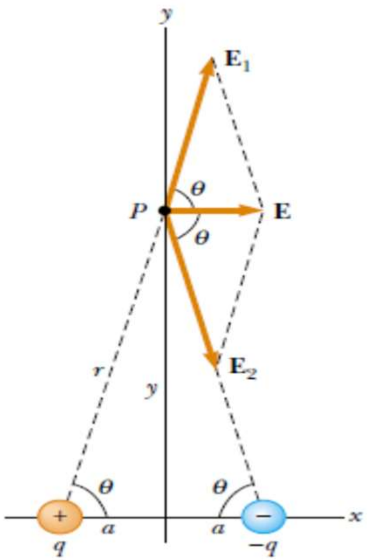


Figure 23.15 (Example 23.6) The total electric field $\mathbf{E}$ at P due to two charges of equal magnitude and opposite sign (an electric dipole) equals the vector sum $\mathbf{E}_1 + \mathbf{E}_2$. The field $\mathbf{E}_1$ is due to the positive charge q, and $\mathbf{E}_2$ is the field due to the negative charge $-q$.

The y components of $\mathbf{E}_1$ and $\mathbf{E}_2$ cancel each other, and the x components are both in the positive x direction and have the same magnitude. Therefore, $\mathbf{E}$ is parallel to the x axis and has a magnitude equal to $2E_1 \cos\theta$. From Figure 23.15 we see that $\cos\theta = a/r = a/(y^2 + a^2)^{1/2}$. Therefore,

$$E = 2E_1 \cos\theta = 2k_e \frac{q}{(y^2 + a^2)} \frac{a}{(y^2 + a^2)^{1/2}}$$

$$= k_e \frac{2qa}{(y^2 + a^2)^{3/2}}$$

Because $y \gg a$, we can neglect a^2 compared to y^2 and write

$$E \approx k_e \frac{2qa}{y^3}$$

Thus, we see that, at distances far from a dipole but along the perpendicular bisector of the line joining the two charges, the magnitude of the electric field created by the dipole varies as $1/r^3$, whereas the more slowly varying field of a point charge varies as $1/r^2$ (see Eq. 23.9). This is because at distant points, the fields of the two charges of equal magnitude and opposite sign almost cancel each other. The $1/r^3$ variation in E for the dipole also is obtained for a distant point along the x axis (see Problem 22) and for any general distant point.

The electric dipole is a good model of many molecules, such as hydrochloric acid (HCl). Neutral atoms and molecules behave as dipoles when placed in an external electric field. Furthermore, many molecules, such as HCl, are permanent dipoles. The effect of such dipoles on the behavior of materials subjected to electric fields is discussed in Chapter 26.

Electric Field of a Continuous Charge Distribution

The electric field at *P* due to one charge element carrying charge Δq is

$$\Delta \mathbf{E} = k_e \frac{\Delta q}{r^2} \hat{\mathbf{r}}$$

where *r* is the distance from the charge element to point *P* and $\hat{r}$ is a unit vector directed from the element toward *P*. The total electric field at *P* due to all elements in the charge distribution is approximately

$$\mathbf{E} \approx k_e \sum_i \frac{\Delta q_i}{r_i^{\,2}} \hat{\mathbf{r}}_i$$

where the index *i* refers to the *i*th element in the distribution. Because the charge distribution is modeled as continuous, the total field at *P* in the limit $\Delta q_i \rightarrow 0$ is

$$\mathbf{E} = k_e \lim_{\Delta q_i \rightarrow 0} \sum_i \frac{\Delta q_i}{r_i^{\,2}} \hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2} \hat{\mathbf{r}} \qquad (23.11)$$

where the integration is over the entire charge distribution. This is a vector operation and must be treated appropriately. When performing such calculations, it is convenient to use the concept of a *charge density* along with the following notations:

If a charge *Q* is uniformly distributed throughout a volume *V*, the volume charge density ρ is defined by

where ρ has units of coulombs per cubic meter (C/m³).

$$\rho \equiv \frac{Q}{V}$$

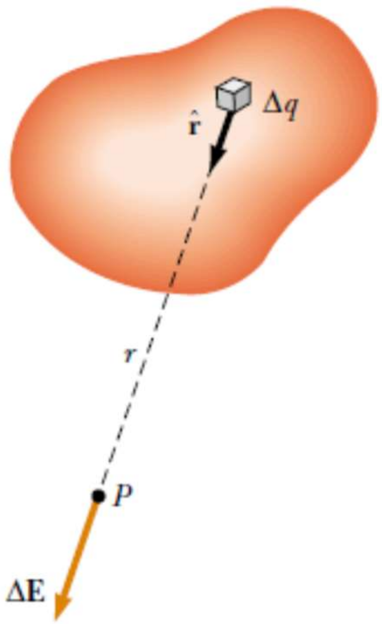


Figure 23.16 The electric field at *P* due to a continuous charge distribution is the vector sum of the fields $\Delta\mathbf{E}$ due to all the elements Δq of the charge distribution.

- If a charge *Q* is uniformly distributed on a surface of area *A*, the surface charge density σ (lowercase Greek sigma) is defined by

$$\sigma \equiv \frac{Q}{A}$$

- where σ has units of coulombs per square meter (C/m2).
- If a charge *Q* is uniformly distributed along a line of length ℓ, the linear charge density λ is defined by

$$\lambda \equiv \frac{Q}{\ell}$$

- where 3 has units of coulombs per meter (C/m).
- If the charge is nonuniformly distributed over a volume, surface, or line, the amounts of charge *dq* in a small volume, surface, or length element are $dq = \rho\, dV \qquad dq = \sigma\, dA \qquad dq = \lambda\, d\ell$

Example 23.8 The Electric Field of a Uniform Ring of Charge

A ring of radius a carries a uniformly distributed positive total charge Q. Calculate the electric field due to the ring at a point P lying a distance x from its center along the central axis perpendicular to the plane of the ring (Fig. 23.18a).

Solution The magnitude of the electric field at P due to the segment of charge dq is

$$dE = k_e \frac{dq}{r^2}$$

This field has an x component $dE_x = dE \cos\theta$ along the x axis and a component $dE_\perp$ perpendicular to the x axis. As we see in Figure 23.18b, however, the resultant field at P must lie along the x axis because the perpendicular com-

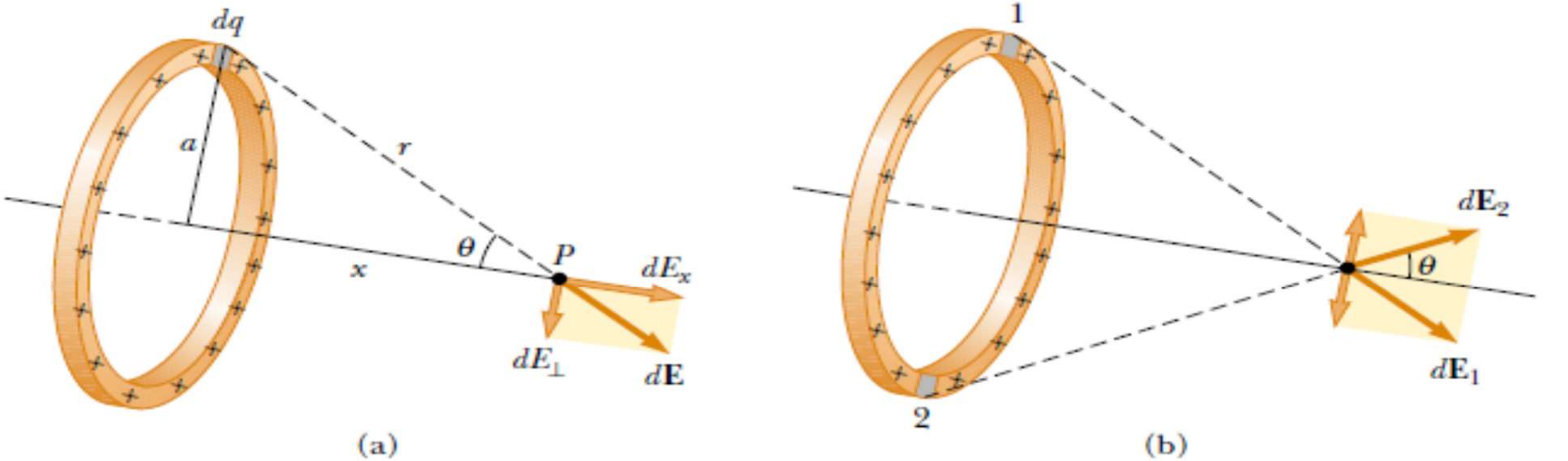


Figure 23.18 (Example 23.8) A uniformly charged ring of radius a. (a) The field at P on the x axis due to an element of charge dq. (b) The total electric field at P is along the x axis. The perpendicular component of the field at P due to segment 1 is canceled by the perpendicular component due to segment 2.

- components of all the various charge segments sum to zero. That is, the perpendicular component of the field created by any charge element is canceled by the perpendicular component created by an element on the opposite side of the ring. Because $r = (x^2 + a^2)^{1/2}$ and $\cos\theta = x/r$, we find that:

$$dE_x = dE\cos\theta = \left(k_e \frac{dq}{r^2}\right)\frac{x}{r} = \frac{k_e x}{(x^2 + a^2)^{3/2}}\, dq$$

- we can integrate to obtain the total field at *P*:

$$E_x = \int \frac{k_e x}{(x^2 + a^2)^{3/2}}\, dq = \frac{k_e x}{(x^2 + a^2)^{3/2}} \int dq$$

$$= \frac{k_e x}{(x^2 + a^2)^{3/2}}\, Q$$

- This result shows that the field is zero at $x = 0$. Does this finding surprise you?
- **What If**? Suppose a negative charge is placed at the center of the ring in Figure 23.18 and displaced slightly by a distance x **!!** a along the x axis. When released, what type of motion does it exhibit? (it will be a harmonic motion due to the different type of charges)

Example 23.9 The Electric Field of a Uniformly Charged Disk

A disk of radius R has a uniform surface charge density σ. Calculate the electric field at a point P that lies along the central perpendicular axis of the disk and a distance x from the center of the disk (Fig. 23.19).

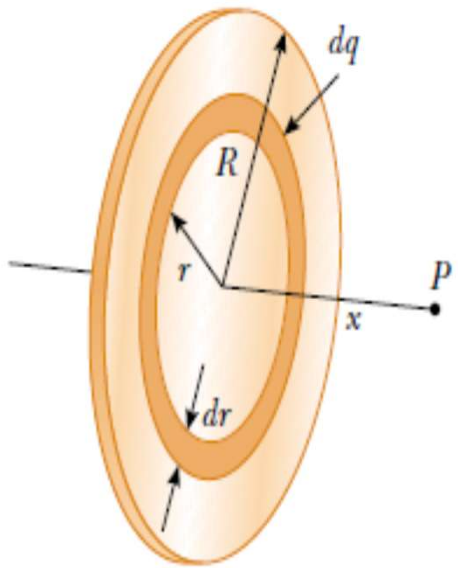


Figure 23.19 (Example 23.9) A uniformly charged disk of radius R. The electric field at an axial point P is directed along the central axis, perpendicular to the plane of the disk.

Solution If we consider the disk as a set of concentric rings, we can use our result from Example 23.8—which gives the field created by a ring of radius a—and sum the contributions of all rings making up the disk. By symmetry, the field at an axial point must be along the central axis.

The ring of radius r and width dr shown in Figure 23.19 has a surface area equal to $2\pi r\,dr$. The charge dq on this ring is equal to the area of the ring multiplied by the surface charge density: $dq = 2\pi\sigma r\,dr$. Using this result in the equation given for E_x in Example 23.8 (with a replaced by r), we have for the field due to the ring

$$dE_x = \frac{k_e x}{(x^2 + r^2)^{3/2}}(2\pi\sigma r\,dr)$$

To obtain the total field at P, we integrate this expression over the limits $r = 0$ to $r = R$, noting that x is a constant.

This gives

$$E_x = k_e x\pi\sigma \int_0^R \frac{2r\,dr}{(x^2 + r^2)^{3/2}}$$

$$= k_e x\pi\sigma \int_0^R (x^2 + r^2)^{-3/2}\, d(r^2)$$

$$= k_e x\pi\sigma \left[\frac{(x^2 + r^2)^{-1/2}}{-1/2} \right]_0^R$$

$$= 2\pi k_e \sigma \left(1 - \frac{x}{(x^2 + R^2)^{1/2}}\right)$$

This result is valid for all values of $x > 0$. We can calculate the field close to the disk along the axis by assuming that $R \gg x$; thus, the expression in parentheses reduces to unity to give us the near-field approximation:

$$E_x = 2\pi k_e \sigma = \frac{\sigma}{2\epsilon_0}$$

where ϵ_0 is the permittivity of free space. In the next chapter we shall obtain the same result for the field created by a uniformly charged infinite sheet.

A convenient way of visualizing electric field patterns is to draw curved lines that are parallel to the electric field vector at any point in space. These lines, called *electric field lines* and first introduced by Faraday, are related to the electric field in a region of space in the following manner:

• The electric field vector E is tangent to the electric field line at each point. The line has a direction, indicated by an arrowhead, that is the same as that of the electric field vector.

• The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region. Thus, the field lines are close together where the electric field is strong and far apart where the field is weak.

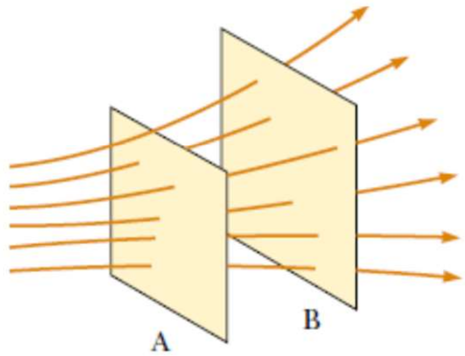


Figure 23.20 Electric field lines penetrating two surfaces. The magnitude of the field is greater on surface A than on surface B.

These properties are illustrated in Figure 23.20. The density of lines through surface A is greater than the density of lines through surface B. Therefore, the magnitude of the electric field is larger on surface A than on surface B. Furthermore, the fact that the lines at different locations point in different directions indicates that the field is nonuniform.

Is this relationship between strength of the electric field and the density of field lines consistent with Equation 23.9, the expression we obtained for E using Coulomb's law? To answer this question, consider an imaginary spherical surface of radius r concentric with a point charge. From symmetry, we see that the magnitude of the electric field is the same everywhere on the surface of the sphere. The number of lines N that emerge from the charge is equal to the number that penetrate the spherical surface. Hence, the number of lines per unit area on the sphere is $N/4\pi r^2$ (where the surface area of the sphere is $4\pi r^2$). Because E is proportional to the number of lines per unit area, we see that E varies as $1/r^2$; this finding is consistent with Equation 23.9.

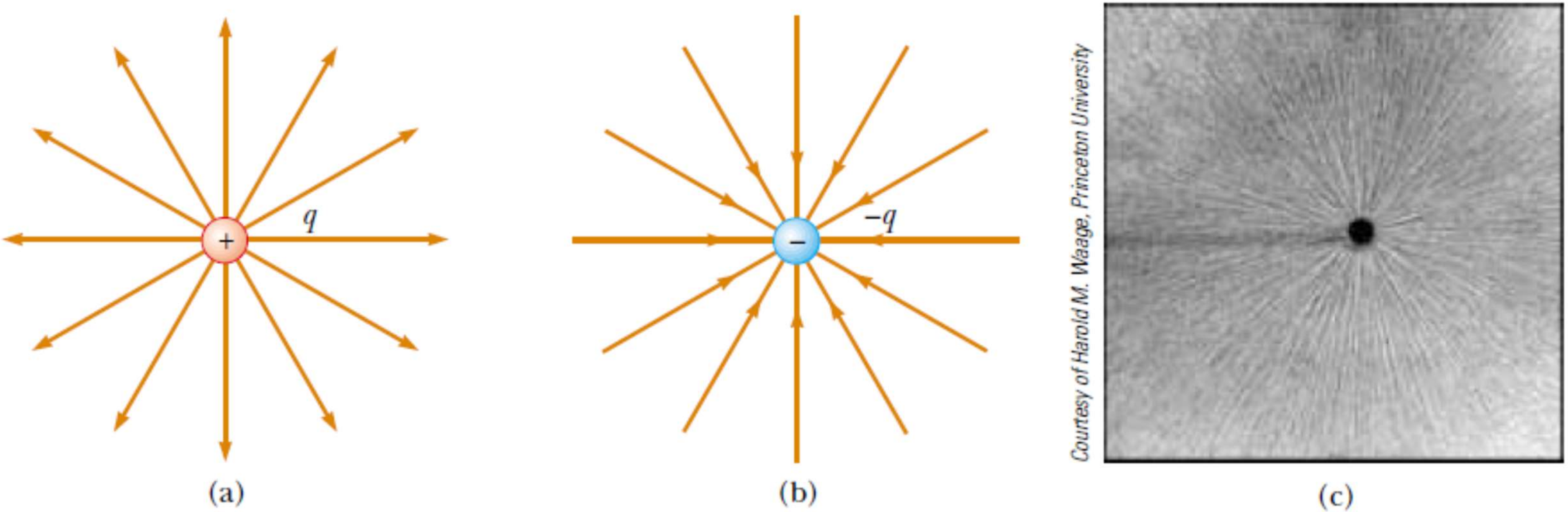
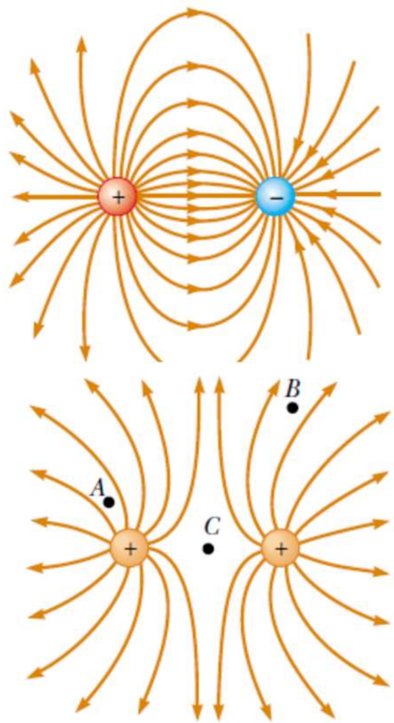


Figure 23.21 The electric field lines for a point charge. (a) For a positive point charge, the lines are directed radially outward. (b) For a negative point charge, the lines are directed radially inward. Note that the figures show only those field lines that lie in the plane of the page. (c) The dark areas are small pieces of thread suspended in oil, which align with the electric field produced by a small charged conductor at the center.



- The rules for drawing electric field lines are as follows:
- *The lines must begin on a positive charge and terminate on a negative charge. In the case of an excess of one type of charge, some lines will begin or end infinitely far away.*
- *The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.*
- *No two field lines can cross.*
- We choose the number of field lines starting from any Positively charged object to be Cq and the number of lines ending on any negatively charged object to be $C|q|$ where C is an arbitrary proportionality constant. Once C is chosen, the number of lines is fixed. For example, if object 1 has charge Q_1 and object 2 has charge Q_2, then the ratio of number of lines is $N_2/N_1 = Q_2/Q_1$. The electric field lines for two point charges of equal magnitude but opposite signs (an electric dipole)

Motion of Charged Particles in a Uniform Electric Field

- When a particle of charge *q* and mass *m* is placed in an electric field E, the electric force exerted on the charge is *q* E according to Equation 23.8. If this is the only force exerted on the particle, it must be the net force and causes the particle to accelerate according to Newton's second law. Thus,

$$\mathbf{F}_e = q\mathbf{E} = m\mathbf{a}$$

- The acceleration of the particle is therefore $$\mathbf{a} = \frac{q\mathbf{E}}{m}$$

- If E is uniform (that is, constant in magnitude and direction), then the acceleration is constant. If the particle has a positive charge, its acceleration is in the direction of the electric field. If the particle has a negative charge, its acceleration is in the direction opposite the electric field.

Example 23.10 An Accelerating Positive Charge

A positive point charge q of mass m is released from rest in a uniform electric field $\mathbf{E}$ directed along the x axis, as shown in Figure 23.25. Describe its motion.

Solution The acceleration is constant and is given by $q\mathbf{E}/m$. The motion is simple linear motion along the x axis. Therefore, we can apply the equations of kinematics in one dimension (see Chapter 2):

$$x_f = x_i + v_i t + \tfrac{1}{2}at^2$$

$$v_f = v_i + at$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

Choosing the initial position of the charge as $x_i = 0$ and assigning $v_i = 0$ because the particle starts from rest, the position of the particle as a function of time is

$$x_f = \tfrac{1}{2}at^2 = \frac{qE}{2m}\,t^2$$

The speed of the particle is given by

$$v_f = at = \frac{qE}{m}\,t$$

The third kinematic equation gives us

$$v_f^2 = 2ax_f = \left(\frac{2qE}{m}\right)x_f$$

from which we can find the kinetic energy of the charge after it has moved a distance $\Delta x = x_f - x_i$:

$$K = \tfrac{1}{2}mv_f^2 = \tfrac{1}{2}m\left(\frac{2qE}{m}\right)\Delta x = qE\Delta x$$

We can also obtain this result from the work–kinetic energy theorem because the work done by the electric force is $F_e\Delta x = qE\Delta x$ and $W = \Delta K$.

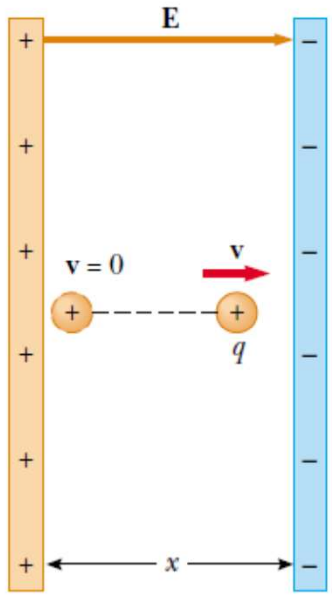
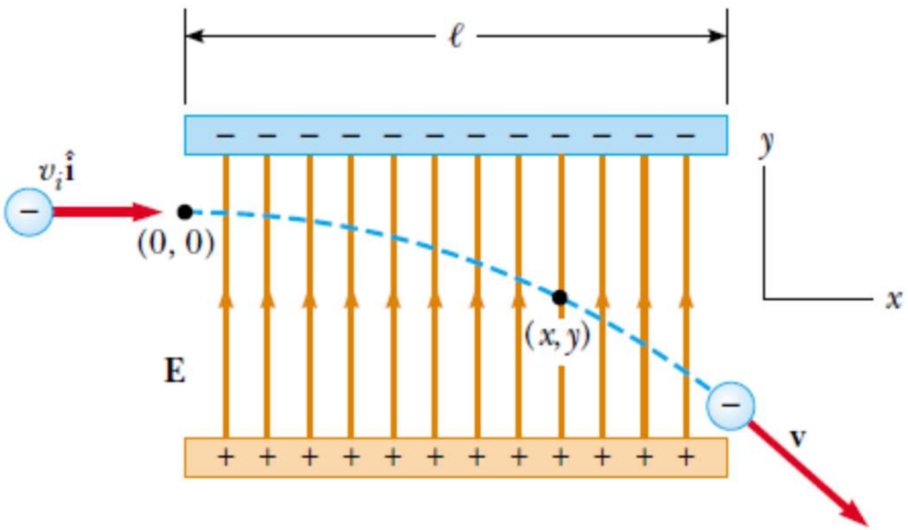


Figure 23.25 (Example 23.10) A positive point charge q in a uniform electric field $\mathbf{E}$ undergoes constant acceleration in the direction of the field.

- The electric field in the region between two oppositely charged flat metallic plates is approximately uniform (Fig. 23.26). Suppose an electron of charge $-e$ is projected horizontally into this field from the origin with an initial velocity $v_i\hat{\mathbf{i}}$ at time $t = 0$. Because the electric field E in Figure 23.26 is in the positive y direction, the acceleration of the electron is in the negative y direction. That is,

$$\mathbf{a} = -\frac{eE}{m_e}\hat{\mathbf{j}} \qquad (23.13)$$

Because the acceleration is constant, we can apply the equations of kinematics in two dimensions with $v_{xi} = v_i$ and $v_{yi} = 0$. After the electron has been in the



Active Figure 23.26 An electron is projected horizontally into a uniform electric field produced by two charged plates. The electron undergoes a downward acceleration (opposite **E**), and its motion is parabolic while it is between the plates.

- electric field for a time interval, the components of its velocity at time t are

$$v_x = v_i = \text{constant} \quad (23.14)$$

$$v_y = a_y t = -\frac{eE}{m_e} t \quad (23.15)$$

- Its position coordinates at time t are

$$x_f = v_i t \quad (23.16)$$

$$y_f = \tfrac{1}{2} a_y t^2 = -\tfrac{1}{2} \frac{eE}{m_e} t^2 \quad (23.17)$$

- Substituting the value $t = x_f / v_i$ from Equation 23.16 into Equation 23.17, we see that y_f is proportional to x_f^2. Hence, the trajectory is a parabola. This should not be a surprise—consider the analogous situation of throwing a ball horizontally in a uniform gravitational field. After the electron leaves the field, the electric force vanishes and the electron continues to move in a straight line in the direction of v in Figure 23.26 with a speed $v > v_i$.
- Note that we have neglected the gravitational force acting on the electron. This is a good approximation when we are dealing with atomic particles. For an electric field of 10^4 N/C, the ratio of the magnitude of the electric force eE to the magnitude of the gravitational force mg is on the order of 10^{14} for an electron and on the order of 10^{11} for a proton.

Example 23.11 An Accelerated Electron

Interactive

An electron enters the region of a uniform electric field as shown in Figure 23.26, with $v_i = 3.00 \times 10^6$ m/s and $E = 200$ N/C. The horizontal length of the plates is $\ell = 0.100$ m.

(A) Find the acceleration of the electron while it is in the electric field.

Solution The charge on the electron has an absolute value of 1.60×10^{-19} C, and $m_e = 9.11 \times 10^{-31}$ kg. Therefore, Equation 23.13 gives

$$\mathbf{a} = -\frac{eE}{m_e}\,\hat{\mathbf{j}} = -\frac{(1.60 \times 10^{-19}\ \mathrm{C})(200\ \mathrm{N/C})}{9.11 \times 10^{-31}\ \mathrm{kg}}\,\hat{\mathbf{j}}$$

$$= \boxed{-3.51 \times 10^{13}\,\hat{\mathbf{j}}\ \mathrm{m/s^2}}$$

(B) If the electron enters the field at time $t = 0$, find the time at which it leaves the field.

Solution The horizontal distance across the field is $\ell = 0.100$ m. Using Equation 23.16 with $x_f = \ell$, we find that the time at which the electron exits the electric field is

$$t = \frac{\ell}{v_i} = \frac{0.100\ \mathrm{m}}{3.00 \times 10^6\ \mathrm{m/s}} = \boxed{3.33 \times 10^{-8}\ \mathrm{s}}$$

(C) If the vertical position of the electron as it enters the field is $y_i = 0$, what is its vertical position when it leaves the field?

Solution Using Equation 23.17 and the results from parts (A) and (B), we find that

$$y_f = \tfrac{1}{2}a_y t^2 = -\tfrac{1}{2}(3.51 \times 10^{13}\ \mathrm{m/s^2})(3.33 \times 10^{-8}\ \mathrm{s})^2$$

$$= -0.019\,5\ \mathrm{m} = \boxed{-1.95\ \mathrm{cm}}$$

If the electron enters just below the negative plate in Figure 23.26 and the separation between the plates is less than the value we have just calculated, the electron will strike the positive plate.