University of Anbar
College of Engineering
Dept. of Electrical Engineering

Electric physics II

Potential difference and electric potential

And capacitors

By

Assist. Lac. Yasameen Kamil

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Potential difference and electric potential

When a test charge q_0 is placed in an electric field $\mathbf{E}$ created by some source charge distribution, the electric force acting on the test charge is $q_0\mathbf{E}$. The force $q_0\mathbf{E}$ is conservative because the force between charges described by Coulomb's law is conservative. When the test charge is moved in the field by some external agent, the work done by the field on the charge is equal to the negative of the work done by the external agent causing the displacement. This is analogous to the situation of lifting an object with mass in a gravitational field—the work done by the external agent is mgh and the work done by the gravitational force is $-mgh$.

When analyzing electric and magnetic fields, it is common practice to use the notation $d\boldsymbol{l}$ to represent an infinitesimal displacement vector that is oriented tangent to a path through space. This path may be straight or curved, and an integral performed along this path is called either a *path integral* or a *line integral* (the two terms are synonymous).

For a given position of the test charge in the field, the charge–field system has a potential energy U relative to the configuration of the system that is defined as $U = 0$. Dividing the potential energy by the test charge gives a physical quantity that depends only on the source charge distribution. The potential energy per unit charge U/q_0 is independent of the value of q_0 and has a value at every point in an electric field. This quantity U/q_0 is called the **electric potential** (or simply the **potential**) V. Thus, the electric potential at any point in an electric field is

$$V = \frac{U}{q_0} \tag{25.2}$$

The fact that potential energy is a scalar quantity means that electric potential also is a scalar quantity.

Work and Potential (V)

The work done by the electric force in moving a test charge from point *a* to point *b* is given by

$$W_{a \to b} = \int_a^b \vec{F} \cdot d\vec{l} = \int_a^b q_0 \vec{E} \cdot d\vec{l}$$

Dividing through by the test charge q_0 we have

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$$

Rearranging so the order of the subscripts is the same on both sides

$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$$

Electric Potential

From this last result $$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$$

We get $$dV = -\vec{E} \cdot d\vec{l} \text{ or } \frac{dV}{dx} = -E$$

We see that the electric field points in the direction of *decreasing* potential

Work (W) = Ua – Ub = q (Va - Vb)

We are often more interested in potential differences as this relates directly to the work done in moving a charge from one point to another

Units for Energy

There is an additional unit that is used for energy in addition to that of joules

A particle having the charge of *e* (1.6×10^{-19} C) that is moved through a potential difference of 1 Volt has an increase in energy that is given by

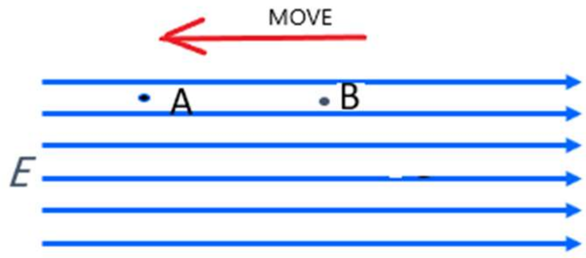
$$W = q\Delta V = 1.6\times10^{-19}\, joules = 1\, eV$$

Electric Potential

General Points for either positive or negative charges

The Potential *increases* if you move in the direction *opposite* to the electric field

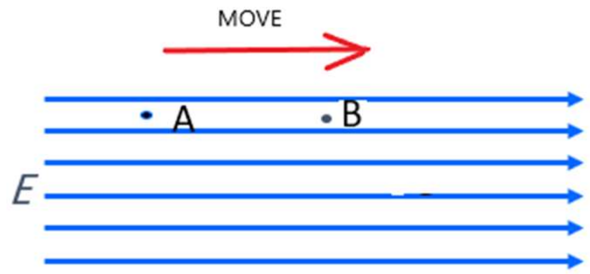
$$\Delta V = -E\ d\cos 180$$
$$\Delta V = +E\ d$$



and

The Potential *decreases* if you move in the *same* direction as the electric field

$$\Delta V = -E\ d\cos 0$$
$$\Delta V = -E\ d$$



Example 1

Points A, B, and C lie in a uniform electric field.

What is the potential difference between points A and B?

$\Delta V_{AB} = V_B - V_A$

a) $\Delta V_{AB} > 0$ b) $\Delta V_{AB} = 0$ c) $\Delta V_{AB} < 0$

The electric field, *E*, points in the direction of decreasing potential

Since points A and B are in the same relative horizontal location in the electric field there is no potential difference between them

Example 2

Points A, B, and C lie in a uniform electric field.

Point C is at a higher potential than point A.

True   False

As stated previously the electric field points in the direction of *decreasing* potential

Since point C is further to the right in the electric field and the electric field is pointing to the right, point C is at a lower potential

The statement is therefore **FALSE**

Example 3

Points A, B, and C lie in a uniform electric field.

If a negative charge is moved from point A to point B, its electric potential energy

a) Increases.

b) decreases.

c) doesn't change.

The potential energy of a charge at a location in an electric field is given by the product of the charge and the potential at the location (pE=q Δv)

As shown in Example 1, the potential at points A and B are the same

Therefore the electric potential energy also doesn't change

Units for Energy

There is an additional unit that is used for energy in addition to that of joules

A particle having the charge of *e* (1.6×10^{-19} C) that is moved through a potential difference of 1 Volt has an increase in energy that is given by

$$W = q\Delta V = 1.6\times10^{-19}\, joules = 1\, eV$$

Electric Potential (V) due to point of charge

We define the term to the right of the summation as the electric potential at point *a*

$$Electric\_Potential_a = \sum_i \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i}$$

$$V = \sum \frac{kq_i}{r_i}$$

Like energy, potential is a ***SCALAR***

<u>We define the potential of a given point charge as being</u>

$$\boldsymbol{Potential} = \boldsymbol{V} = \frac{1}{4\pi\varepsilon_0}\frac{\boldsymbol{q}}{\boldsymbol{r}}$$

This equation has the convention that the potential is zero at infinite distance

Example

Question: A particle of charge $q_1 = +6.0\,\mu C$ is located on the x-axis at the point $x_1 = 5.1$ cm. A second particle of charge $q_2 = -5.0\,\mu C$ is placed on the x-axis at $x_2 = -3.4$ cm. What is the absolute electric potential at the origin ($x = 0$)? How much work must we perform in order to slowly move a charge of $q_3 = -7.0\,\mu C$ from infinity to the origin, whilst keeping the other two charges fixed?

$$V_1 = k_e \frac{q_1}{x_1} = (8.988 \times 10^9) \frac{(6 \times 10^{-6})}{(5.1 \times 10^{-2})} = 1.06 \times 10^6\,\text{V}.$$

$$V_2 = k_e \frac{q_2}{|x_2|} = (8.988 \times 10^9) \frac{(-5 \times 10^{-6})}{(3.4 \times 10^{-2})} = -1.32 \times 10^6\,\text{V}.$$

The net potential V at the origin is simply the **algebraic** sum of the potentials due to each charge taken in isolation. Thus,

$$V = V_1 + V_2 = -2.64 \times 10^5\,\text{V}.$$

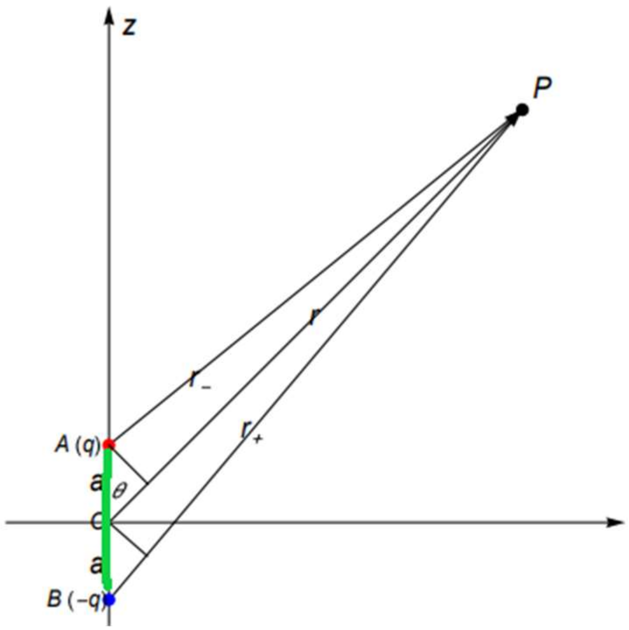
The work W which we must perform in order to slowly moving a charge q3 from infinity to the origin is simply the product of the charge and the potential difference. Thus,

$$W = q_3\,V = (-7 \times 10^{-6})\,(-2.64 \times 10^5) = 1.85\,\text{J}.$$

Electric Potential(v) due to dipole

Line BA is on the z axis. The positive charge is at (0, 0, a) and the negative charge is at (0, 0, -a)
We consider an electrical potential at the point P, due to the electric dipole moment

$$v_{AP} = \frac{kq}{r_1}$$
$$v_{BP} = \frac{kq}{r_2}$$
$$v_{total} = kq\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

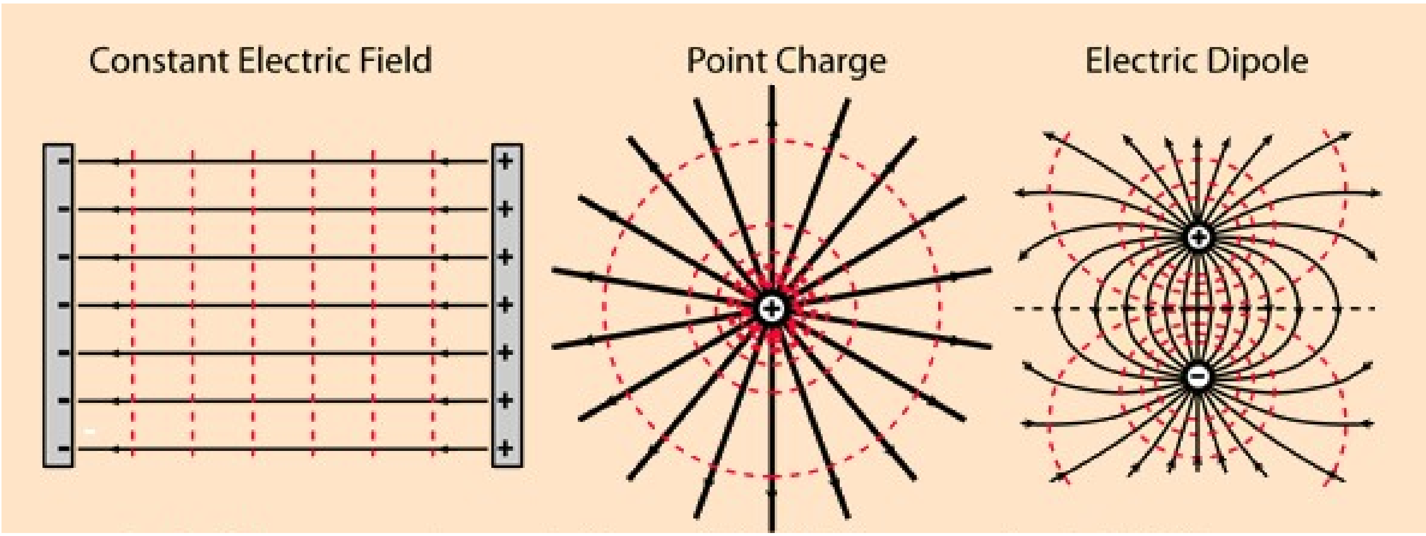


Equipotential Surfaces

- It is possible to move a test charge from one point to another without having any net work done on the charge.
- This occurs when the beginning and end points have the same potential
- It is possible to map out such points and a given set of points at the same potential form an *equipotential surface*

Equipotential Surfaces

- The electric field does no work as a charge is moved along an equipotential surface
- Since no work is done, there is no force, qE, along the direction of motion
- The electric field is *perpendicular* to the equipotential surface



- Capacitor

Capacitance and Dielectrics

Consider two conductors carrying charges of equal magnitude and opposite sign, as shown in Figure 26.1. Such a combination of two conductors is called a **capacitor.** The conductors are called *plates.* A potential difference ΔV exists between the conductors due to the presence of the charges.

What determines how much charge is on the plates of a capacitor for a given voltage? Experiments show that the quantity of charge Q on a capacitor[1] is linearly proportional to the potential difference between the conductors; that is, $Q \propto \Delta V$. The proportionality constant depends on the shape and separation of the conductors.[2] We can write this relationship as $Q = C\,\Delta V$ if we define capacitance as follows:

The **capacitance** C of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors:

$$C \equiv \frac{Q}{\Delta V} \tag{26.1}$$

Note that by definition *capacitance is always a positive quantity*. Furthermore, the charge Q and the potential difference ΔV are always expressed in Equation 26.1 as positive quantities. Because the potential difference increases linearly with the stored charge, the ratio $Q/\Delta V$ is constant for a given capacitor. Therefore, capacitance is a measure of a capacitor's ability to store charge. Because positive and negative charges are separated in the system of two conductors in a capacitor, there is electric potential energy stored in the system.

From Equation 26.1, we see that capacitance has SI units of coulombs per volt. The SI unit of capacitance is the **farad** (F), which was named in honor of Michael Faraday:

$$1\text{ F} = 1\text{ C/V}$$

Suppose that we have a capacitor rated at 4 pF. This rating means that the capacitor can store 4 pC of charge for each volt of potential difference between the two conductors. If a 9-V battery is connected across this capacitor, one of the conductors ends up with a net charge of − 36 pC and the other ends up with a net charge of + 36 pC.

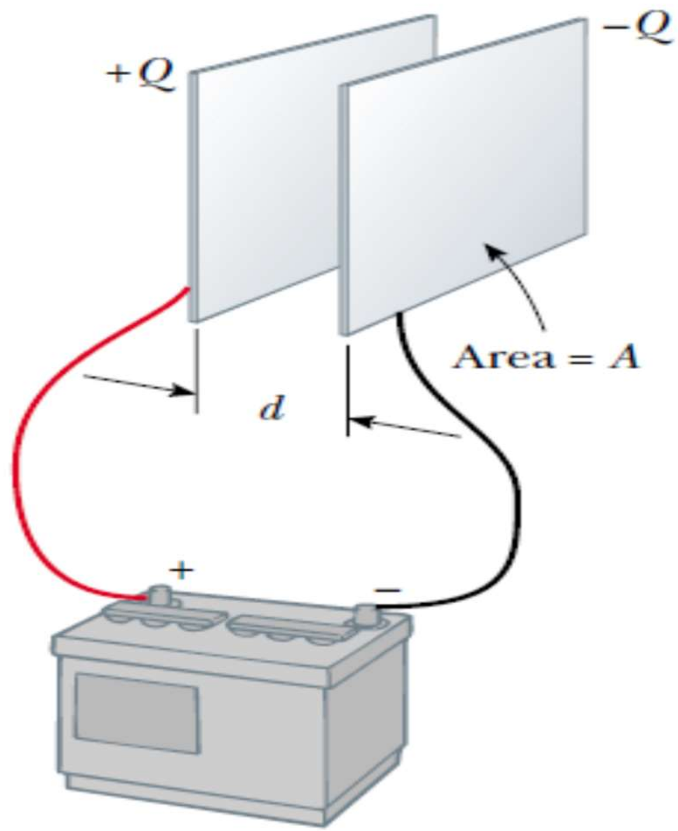
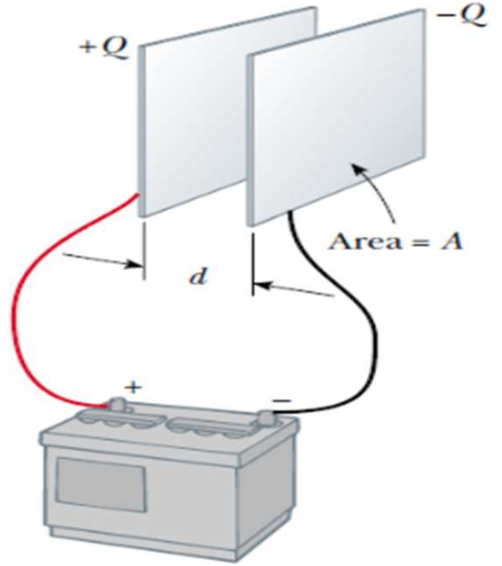


Figure 26.2 A parallel-plate capacitor consists of two parallel conducting plates, each of area *A*, separated by a distance *d*. When the capacitor is charged by connecting the plates to the terminals of a battery, the plates carry equal amounts of charge. One plate carries positive charge, and the other carries negative charge.

Types of capacitors

- 1- parallel plate capacitor



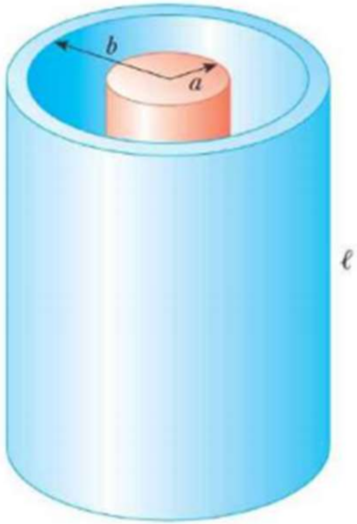
$$c = \frac{Q}{v} = \frac{\varepsilon_\circ A}{d}$$ …….(26-3) due to the geometry

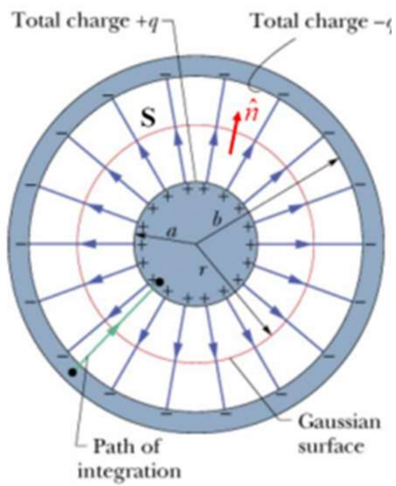
parallel plate capacitor

2- cylindrical capacitor

$$c = \frac{Q}{v} = \frac{2\pi\varepsilon_\circ l}{ln\frac{a}{b}}$$ due to the geometry

Where a= the small radios, b= the bigger radios and l: the length of the cylinder



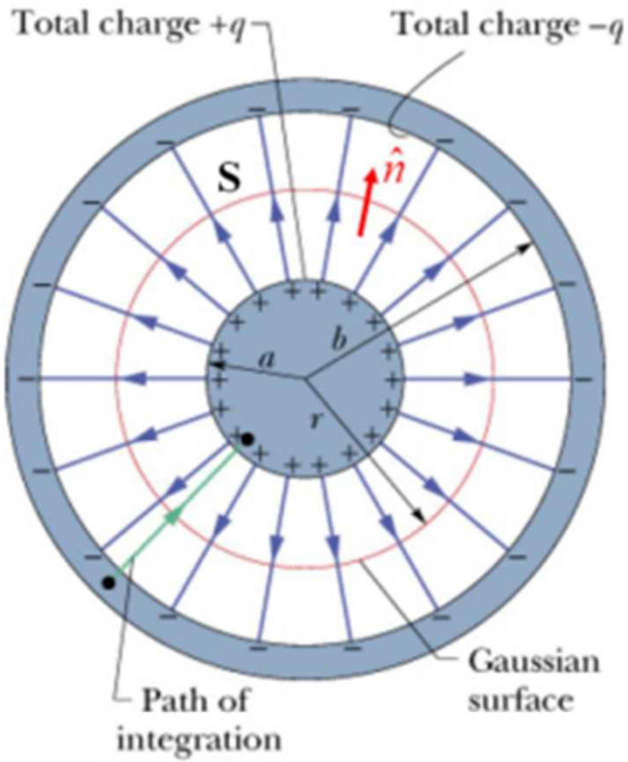


Types of capacitors

3- Spherical capacitor

$$c = \frac{Q}{v} = \frac{4\pi\varepsilon_{\circ}ab}{b-a}$$ due to the geometry

- Where a= the small radios, b= the bigger radios



Example 26.1 Parallel-Plate Capacitor

A parallel-plate capacitor with air between the plates has an area $A = 2.00 \times 10^{-4}\ \text{m}^2$ and a plate separation $d = 1.00$ mm. Find its capacitance.

Solution From Equation 26.3, we find that

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.00 \times 10^{-4}\ \text{m}^2)}{1.00 \times 10^{-3}\ \text{m}}$$

$$= 1.77 \times 10^{-12}\ \text{F} = \boxed{1.77\ \text{pF}}$$

Connection of Capacitors

1- parallel combination

. The individual potential differences across capacitors connected in parallel are the same and are equal to the potential difference applied across the combination.

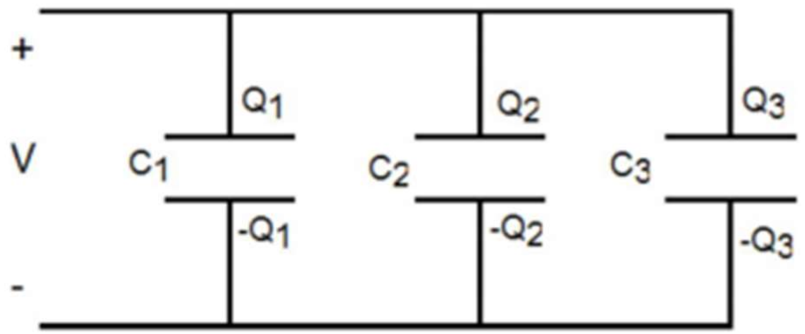
V= $v_1 = v_2 = v_3$

. The total charge on capacitors connected in parallel is the sum of the charges on the individual capacitor

Q= $Q_1 + Q_2 + Q_3$

The equivalent capacitance of parallel connected is the algebraic sum of individual capacitance

$C_{eq} = c_1 + c_2 + c_3$ parallel connection



Q1 $= c_1 v$

Q2 $= c_2 v$

Q3 $= c_3 v$

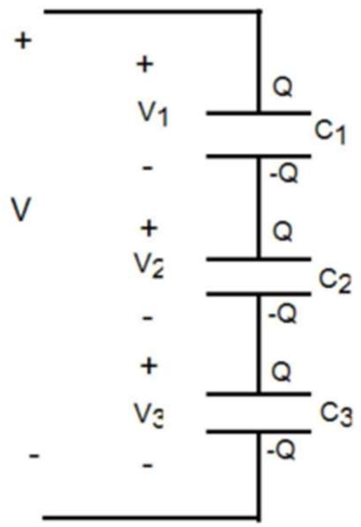
$C_{eq} v = c_1 v + c_2 v + c_3 v$

Q= $Q_1 + Q_2 + Q_3$

$C_{eq} = c_1 + c_2 + c_3$

Series combination

- The charges on capacitors connected in series are the same
- $Q = Q_1 = Q_2 = Q_3$
- The total potential differences across any number of capacitors connected in series is the sum of the potential difference across the individual capacitors
- v= $v_1 + v_2 + v_3$
- The inverse of the equivalent capacitance of series connection is the algebraic sum of inverse of the individual capacitances
- $\frac{1}{c_{eq}} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}$



$Q = c_1 v_1 = c_2 v_2 = c_3 v_3$

v= $v_1 + v_2 + v_3$

$\frac{Q}{c_{eq}} = \frac{Q}{c_1} + \frac{Q}{c_2} + \frac{Q}{c_3}$

$\frac{1}{c_{eq}} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}$

Example

Example 26.4 Equivalent Capacitance **Interactive**

Find the equivalent capacitance between a and b for the combination of capacitors shown in Figure 26.11a. All capacitances are in microfarads.

Solution Using Equations 26.8 and 26.10, we reduce the combination step by step as indicated in the figure. The 1.0-μF and 3.0-μF capacitors are in parallel and combine according to the expression $C_{eq} = C_1 + C_2 = 4.0\ \mu F$. The 2.0-$\mu$F and 6.0-$\mu$F capacitors also are in parallel and have an equivalent capacitance of 8.0 μF. Thus, the upper branch in Figure 26.11b consists of two 4.0-μF capacitors in series, which combine as follows:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.0\ \mu F} + \frac{1}{4.0\ \mu F} = \frac{1}{2.0\ \mu F}$$

$$C_{eq} = 2.0\ \mu\text{F}$$

The lower branch in Figure 26.11b consists of two 8.0-μF capacitors in series, which combine to yield an equivalent capacitance of 4.0 μF. Finally, the 2.0-μF and 4.0-μF capacitors in Figure 26.11c are in parallel and thus have an equivalent capacitance of 6.0 μF.

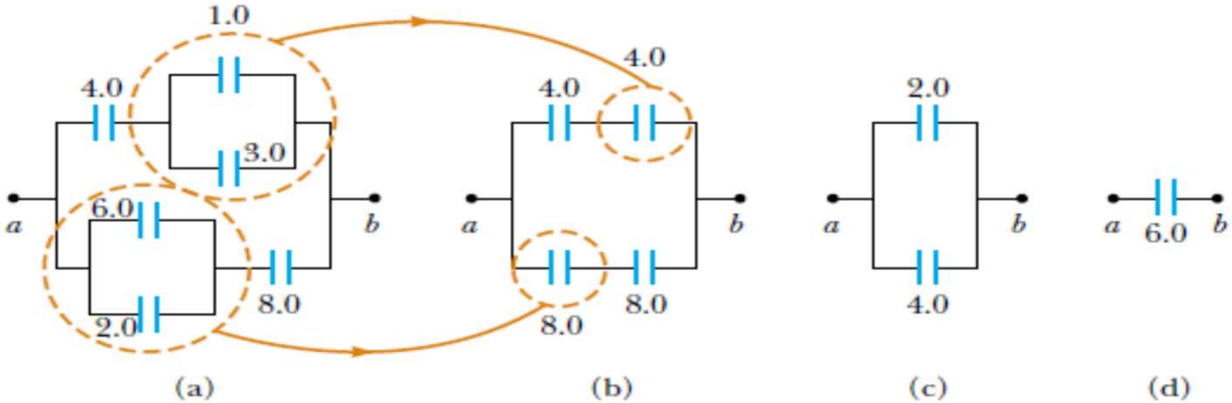


Figure 26.11 (Example 26.4) To find the equivalent capacitance of the capacitors in part (a), we reduce the various combinations in steps as indicated in parts (b), (c), and (d), using the series and parallel rules described in the text.

Energy Stored in a Charged Capacitor

- The work done in charging the capacitor appears as electric potential energy U stored in the capacitor as in the following forms

- $U = \frac{1}{2}\ cv^2$ ………1
- $U = \frac{1}{2}\ Q\ V \ldots\ldots..2$

- $U = \frac{1}{2}\ \frac{Q^2}{C}$ …………3
- The unite of energy is joule (J)

Energy density

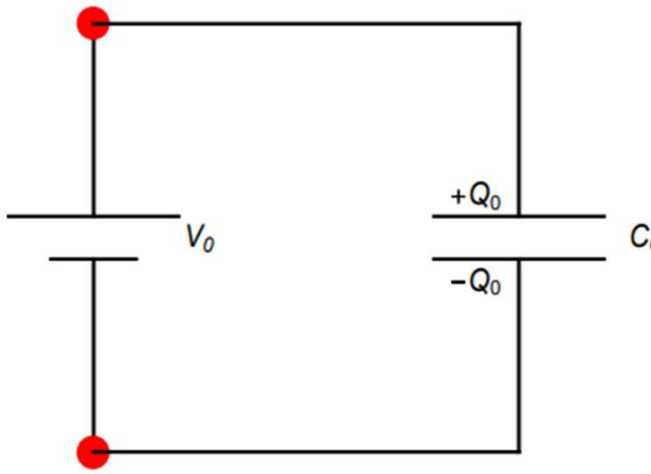
- The energy per unite volume known as the energy density
- $u_E = \frac{U}{v}$
- $u_E = \frac{\frac{1}{2}\,cv^2}{A\,d} = \frac{\frac{1}{2}\varepsilon_\circ Av^2}{dAd}$
- $u_E = \frac{1}{2}\ \varepsilon_\circ E^2$ $\quad$ *the unite is (J/m^3)*

The energy density in any electric field is proportional to the square of magnitude of the electric field at a given point.

Dielectric

- The capacitance of a set of charged parallel plates is increased by the insertion of a dielectric material.
- $c_{\circ} = \dfrac{\varepsilon_{\circ} A}{d}$
- Also $c_{\circ} = \dfrac{Q_{\circ}}{v_{\circ}}$

The dielectric is the free space (air)



- if we put the dielectric between two plates of the capacitors, then the capacitance is increased as in the following form
- c= $\dfrac{k\, \varepsilon_{\circ}\, A}{d}$ where k = the dielectric constant
- K= $\dfrac{c}{c_{\circ}}$ → $c = k\, c_{\circ}$