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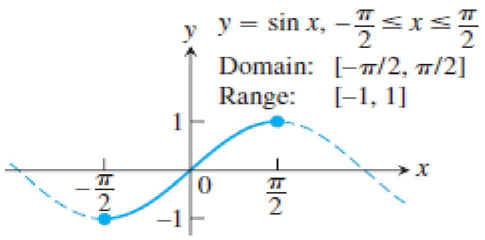
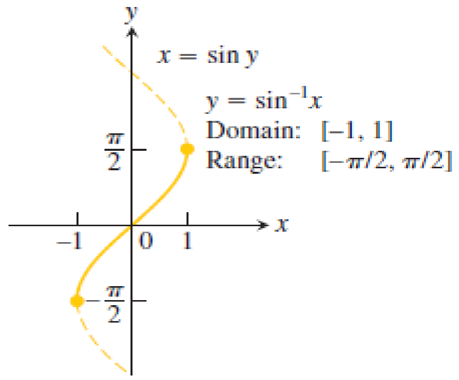
Inverse Trigonometric Functions

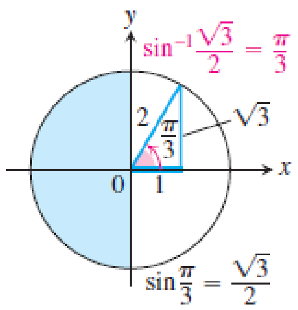
DEFINITION Arcsine and Arccosine Functions

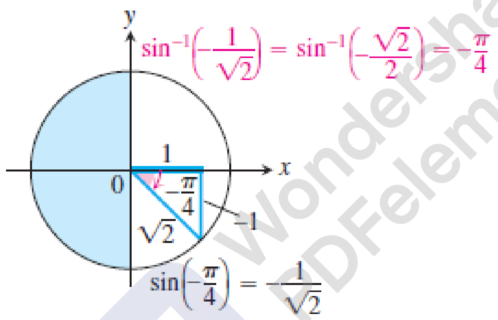
$y = \sin^{-1} x$ is the number in $[-\pi/2, \pi/2]$ for which $\sin y = x$.

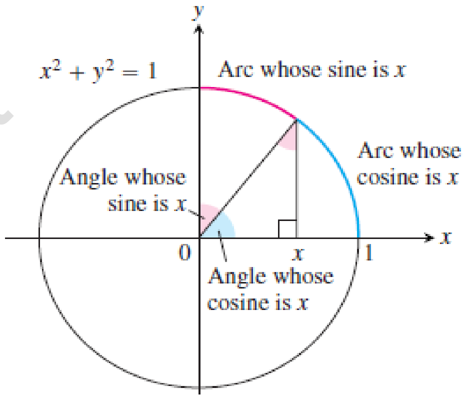
$y = \cos^{-1} x$ is the number in $[0, \pi]$ for which $\cos y = x$.











The Derivative of $y = \sin^{-1} x$

$$\sin y = x \qquad y = \sin^{-1} x \Leftrightarrow \sin y = x$$

$$\frac{d}{dx}(\sin y) = 1 \qquad \text{Derivative of both sides with respect to } x$$

$$\cos y \frac{dy}{dx} = 1 \qquad \text{Chain Rule}$$

$$\frac{dy}{dx} = \frac{1}{\cos y} \qquad \text{We can divide because } \cos y > 0 \text{ for } -\pi/2 < y < \pi/2.$$

$$= \frac{1}{\sqrt{1 - x^2}} \qquad \cos y = \sqrt{1 - \sin^2 y}$$

No matter which derivation we use, we have that the derivative of $y = \sin^{-1} x$ with re- spect to x is

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1 - x^2}}.$$

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If u is a differentiable function of x with $|u| < 1$, we apply the Chain Rule to get

$$\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \qquad |u| < 1.$$

EXAMPLE Applying the Derivative Formula

$$\frac{d}{dx}(\sin^{-1} x^2) = \frac{1}{\sqrt{1-(x^2)^2}} \cdot \frac{d}{dx}(x^2) = \frac{2x}{\sqrt{1-x^4}}$$

The Derivative of $y = \tan^{-1} u$

We find the derivative of $y = \tan^{-1} x$ by applying Theorem 1 with $f(x) = \tan x$ and $f^{-1}(x) = \tan^{-1} x$. Theorem 1 can be applied because the derivative of $\tan x$ is positive for $-\pi/2 < x < \pi/2$.

$$\begin{aligned}
(f^{-1})'(x) &= \frac{1}{f'(f^{-1}(x))} && \text{Theorem 1} \\
&= \frac{1}{\sec^2(\tan^{-1} x)} && f'(u) = \sec^2 u \\
&= \frac{1}{1+\tan^2(\tan^{-1} x)} && \sec^2 u = 1 + \tan^2 u \\
&= \frac{1}{1+x^2} && \tan(\tan^{-1} x) = x
\end{aligned}$$

The derivative is defined for all real numbers. If u is a differentiable function of x, we get the Chain Rule form:

$$\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2}\frac{du}{dx}.$$

EXAMPLE

A particle moves along the x-axis so that its position at any time $t \geq 0$ is $x(t) = \tan^{-1}\sqrt{t}$. What is the velocity of the particle when $t = 16$?

Solution

$$v(t) = \frac{d}{dt}\tan^{-1}\sqrt{t} = \frac{1}{1+(\sqrt{t})^2} \cdot \frac{d}{dt}\sqrt{t} = \frac{1}{1+t} \cdot \frac{1}{2\sqrt{t}}$$

When $t = 16$, the velocity is $$v(16) = \frac{1}{1+16} \cdot \frac{1}{2\sqrt{16}} = \frac{1}{136}.$$

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The Derivative of $y = \sec^{-1} u$

$$y = \sec^{-1} x$$

$$\sec y = x \qquad \text{Inverse function relationship}$$

$$\frac{d}{dx}(\sec y) = \frac{d}{dx} x \qquad \text{Differentiate both sides.}$$

$$\sec y \tan y \frac{dy}{dx} = 1 \qquad \text{Chain Rule}$$

$$\frac{dy}{dx} = \frac{1}{\sec y \tan y}$$

Since $|x| > 1$, y lies in $(0, \pi/2) \cup (\pi/2, \pi)$ and $\sec y \tan y \neq 0$.

$$\frac{dy}{dx} = \pm \frac{1}{x\sqrt{x^2 - 1}}.$$

$$\frac{d}{dx} \sec^{-1} x = \begin{cases} +\dfrac{1}{x\sqrt{x^2-1}} & \text{if } x > 1 \\ -\dfrac{1}{x\sqrt{x^2-1}} & \text{if } x < -1. \end{cases}$$

With the absolute value symbol, we can write a single expression that eliminates the "±" ambiguity:

$$\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2 - 1}}.$$

$$\frac{d}{dx}(\sec^{-1} u) = \frac{1}{|u|\sqrt{u^2 - 1}} \frac{du}{dx}, \qquad |u| > 1.$$

EXAMPLE Using the Formula

$$\frac{d}{dx} \sec^{-1}(5x^4) = \frac{1}{|5x^4|\sqrt{(5x^4)^2 - 1}} \frac{d}{dx}(5x^4)$$

$$= \frac{1}{5x^4\sqrt{25x^8 - 1}}(20x^3) \qquad 5x^4 > 0$$

$$= \frac{4}{x\sqrt{25x^8 - 1}}$$

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Derivatives of the Other Three

Inverse Function–Inverse Cofunction Identities

$$\cos^{-1} x = \pi/2 - \sin^{-1} x$$
$$\cot^{-1} x = \pi/2 - \tan^{-1} x$$
$$\csc^{-1} x = \pi/2 - \sec^{-1} x$$

$$\frac{d}{dx}(\cos^{-1} x) = \frac{d}{dx}\left(\frac{\pi}{2} - \sin^{-1} x\right) \qquad \text{Identity}$$
$$= -\frac{d}{dx}(\sin^{-1} x)$$
$$= -\frac{1}{\sqrt{1 - x^2}} \qquad \text{Derivative of arcsine}$$

EXAMPLE Find an equation for the line tangent to the graph of $y = \cot^{-1} x$ at $x = -1$.

Solution First we note that

$$\cot^{-1}(-1) = \pi/2 - \tan^{-1}(-1) = \pi/2 - (-\pi/4) = 3\pi/4.$$

The slope of the tangent line is

$$\left.\frac{dy}{dx}\right|_{x=-1} = -\left.\frac{1}{1 + x^2}\right|_{x=-1} = -\frac{1}{1 + (-1)^2} = -\frac{1}{2},$$

so the tangent line has equation $y - 3\pi/4 = (-1/2)(x + 1)$.

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TABLE 7.3 Derivatives of the inverse trigonometric functions

1. $\dfrac{d(\sin^{-1} u)}{dx} = \dfrac{du/dx}{\sqrt{1 - u^2}}, \quad |u| < 1$

2. $\dfrac{d(\cos^{-1} u)}{dx} = -\dfrac{du/dx}{\sqrt{1 - u^2}}, \quad |u| < 1$

3. $\dfrac{d(\tan^{-1} u)}{dx} = \dfrac{du/dx}{1 + u^2}$

4. $\dfrac{d(\cot^{-1} u)}{dx} = -\dfrac{du/dx}{1 + u^2}$

5. $\dfrac{d(\sec^{-1} u)}{dx} = \dfrac{du/dx}{|u|\sqrt{u^2 - 1}}, \quad |u| > 1$

6. $\dfrac{d(\csc^{-1} u)}{dx} = \dfrac{-du/dx}{|u|\sqrt{u^2 - 1}}, \quad |u| > 1$

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Integration Formulas

TABLE 7.4 Integrals evaluated with inverse trigonometric functions

The following formulas hold for any constant $a \neq 0$.

1. $\displaystyle\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$ (Valid for $u^2 < a^2$)
2. $\displaystyle\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$ (Valid for all u)
3. $\displaystyle\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C$ (Valid for $|u| > a > 0$)

EXAMPLE Using the Integral Formulas

(a) $$\int_{\sqrt{2}/2}^{\sqrt{3}/2} \frac{dx}{\sqrt{1 - x^2}} = \sin^{-1} x\Big]_{\sqrt{2}/2}^{\sqrt{3}/2}$$

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$

(b) $$\int_0^1 \frac{dx}{1 + x^2} = \tan^{-1} x\Big]_0^1 = \tan^{-1}(1) - \tan^{-1}(0) = \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

(c) $$\int_{2/\sqrt{3}}^{\sqrt{2}} \frac{dx}{x\sqrt{x^2 - 1}} = \sec^{-1} x\Big]_{2/\sqrt{3}}^{\sqrt{2}} = \frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}$$

EXAMPLE

(a) $$\int \frac{dx}{\sqrt{9 - x^2}} = \int \frac{dx}{\sqrt{(3)^2 - x^2}} = \sin^{-1}\left(\frac{x}{3}\right) + C$$

Table 7.4 Formula 1, with $a = 3$, $u = x$

(b) $$\int \frac{dx}{\sqrt{3 - 4x^2}} = \frac{1}{2}\int \frac{du}{\sqrt{a^2 - u^2}}$$

$a = \sqrt{3}$, $u = 2x$, and $du/2 = dx$

$$= \frac{1}{2}\sin^{-1}\left(\frac{u}{a}\right) + C \quad = \frac{1}{2}\sin^{-1}\left(\frac{2x}{\sqrt{3}}\right) + C$$

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EXAMPLE Completing the Square

Evaluate $$\int \frac{dx}{\sqrt{4x - x^2}}.$$

$$4x - x^2 = -(x^2 - 4x) = -(x^2 - 4x + 4) + 4 = 4 - (x - 2)^2.$$

Then we substitute $a = 2$, $u = x - 2$, and $du = dx$ to get

$$\int \frac{dx}{\sqrt{4x - x^2}} = \int \frac{dx}{\sqrt{4 - (x - 2)^2}}$$

$$= \int \frac{du}{\sqrt{a^2 - u^2}} \qquad a = 2, u = x - 2, \text{ and } du = dx$$

$$= \sin^{-1}\left(\frac{u}{a}\right) + C \qquad \text{Table 7.4, Formula 1}$$

$$= \sin^{-1}\left(\frac{x - 2}{2}\right) + C$$

EXAMPLE Completing the Square

Evaluate $$\int \frac{dx}{4x^2 + 4x + 2}.$$

Solution We complete the square on the binomial $4x^2 + 4x$:

$$4x^2 + 4x + 2 = 4(x^2 + x) + 2 = 4\left(x^2 + x + \frac{1}{4}\right) + 2 - \frac{4}{4}$$

$$= 4\left(x + \frac{1}{2}\right)^2 + 1 = (2x + 1)^2 + 1.$$

Then,

$$\int \frac{dx}{4x^2 + 4x + 2} = \int \frac{dx}{(2x + 1)^2 + 1} = \frac{1}{2}\int \frac{du}{u^2 + a^2} \qquad a = 1, u = 2x + 1, \text{ and } du/2 = dx$$

$$= \frac{1}{2} \cdot \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + C \qquad \text{Table 7.4, Formula 2}$$

$$= \frac{1}{2} \tan^{-1}(2x + 1) + C \qquad a = 1, u = 2x + 1$$

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EXAMPLE Using Substitution

Evaluate $\displaystyle\int \frac{dx}{\sqrt{e^{2x}-6}}$.

Solution

$$\int \frac{dx}{\sqrt{e^{2x}-6}} = \int \frac{du/u}{\sqrt{u^2-a^2}}$$

$u = e^x$, $du = e^x\,dx$, $dx = du/e^x = du/u$, $a = \sqrt{6}$

$$= \int \frac{du}{u\sqrt{u^2-a^2}}$$

$$= \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C$$

Table 7.4, Formula 3

$$= \frac{1}{\sqrt{6}}\sec^{-1}\left(\frac{e^x}{\sqrt{6}}\right) + C$$

Hyperbolic Functions

The hyperbolic functions are formed by taking combinations of the two exponential functions e^x and e^{-x}. The hyperbolic functions simplify many mathematical expressions and they are important in applications. For instance, they are used in problems such as computing the tension in a cable suspended by its two ends, as in an electric transmission line. They also play an important role in finding solutions to differential equations. In this section, we give a brief introduction to hyperbolic functions, their graphs, how their derivatives are calculated, and why they appear as important antiderivatives.

Even and Odd Parts of the Exponential Function

$$f(x) = \underbrace{\frac{f(x)+f(-x)}{2}}_{\text{even part}} + \underbrace{\frac{f(x)-f(-x)}{2}}_{\text{odd part}}.$$

If we write e^x this way, we get

$$e^x = \underbrace{\frac{e^x+e^{-x}}{2}}_{\text{even part}} + \underbrace{\frac{e^x-e^{-x}}{2}}_{\text{odd part}}.$$

Definitions and Identities

$$2\sinh x\cosh x = 2\left(\frac{e^x-e^{-x}}{2}\right)\left(\frac{e^x+e^{-x}}{2}\right) = \frac{e^{2x}-e^{-2x}}{2} = \sinh 2x.$$

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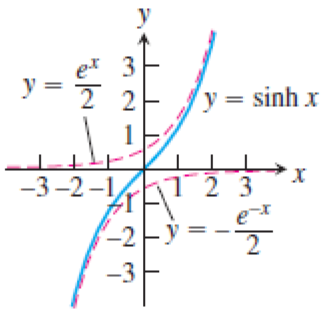
TABLE 7.5 The six basic hyperbolic functions

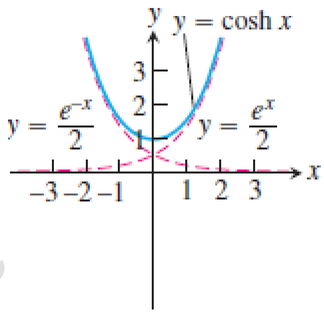
FIGURE 7.31

Hyperbolic sine of x: $\sinh x = \dfrac{e^x - e^{-x}}{2}$



(a)

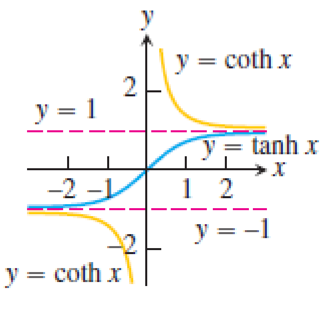
Hyperbolic cosine of x: $\cosh x = \dfrac{e^x + e^{-x}}{2}$



(b)

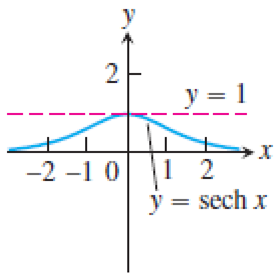
Hyperbolic tangent: $\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

Hyperbolic cotangent: $\coth x = \dfrac{\cosh x}{\sinh x} = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$



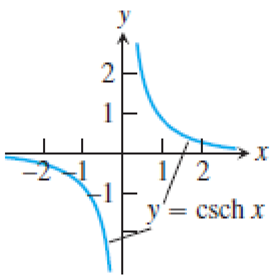
(c)

Hyperbolic secant: $\operatorname{sech} x = \dfrac{1}{\cosh x} = \dfrac{2}{e^x + e^{-x}}$



(d)

Hyperbolic cosecant: $\operatorname{csch} x = \dfrac{1}{\sinh x} = \dfrac{2}{e^x - e^{-x}}$



(e)

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TABLE 7.6 Identities for hyperbolic functions

$$\cosh^2 x - \sinh^2 x = 1$$
$$\sinh 2x = 2 \sinh x \cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$
$$\cosh^2 x = \frac{\cosh 2x + 1}{2}$$
$$\sinh^2 x = \frac{\cosh 2x - 1}{2}$$
$$\tanh^2 x = 1 - \operatorname{sech}^2 x$$
$$\coth^2 x = 1 + \operatorname{csch}^2 x$$

Derivatives and Integrals

TABLE 7.7 Derivatives of hyperbolic functions

$$\frac{d}{dx}(\sinh u) = \cosh u \frac{du}{dx}$$
$$\frac{d}{dx}(\cosh u) = \sinh u \frac{du}{dx}$$
$$\frac{d}{dx}(\tanh u) = \operatorname{sech}^2 u \frac{du}{dx}$$
$$\frac{d}{dx}(\coth u) = -\operatorname{csch}^2 u \frac{du}{dx}$$
$$\frac{d}{dx}(\operatorname{sech} u) = -\operatorname{sech} u \tanh u \frac{du}{dx}$$
$$\frac{d}{dx}(\operatorname{csch} u) = -\operatorname{csch} u \coth u \frac{du}{dx}$$

TABLE 7.8 Integral formulas for hyperbolic functions

$$\int \sinh u\, du = \cosh u + C$$
$$\int \cosh u\, du = \sinh u + C$$
$$\int \operatorname{sech}^2 u\, du = \tanh u + C$$
$$\int \operatorname{csch}^2 u\, du = -\coth u + C$$
$$\int \operatorname{sech} u \tanh u\, du = -\operatorname{sech} u + C$$
$$\int \operatorname{csch} u \coth u\, du = -\operatorname{csch} u + C$$

The derivative formulas are derived from the derivative of e^u:

$$\frac{d}{dx}(\sinh u) = \frac{d}{dx}\left(\frac{e^u - e^{-u}}{2}\right) \qquad \text{Definition of } \sinh u$$
$$= \frac{e^u\, du/dx + e^{-u}\, du/dx}{2} \qquad \text{Derivative of } e^u$$
$$= \cosh u \frac{du}{dx} \qquad \text{Definition of } \cosh u$$

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This gives the first derivative formula. The calculation

$$\frac{d}{dx}(\operatorname{csch} u) = \frac{d}{dx}\left(\frac{1}{\sinh u}\right) \qquad \text{Definition of csch } u$$

$$= -\frac{\cosh u}{\sinh^2 u}\frac{du}{dx} \qquad \text{Quotient Rule}$$

$$= -\frac{1}{\sinh u}\frac{\cosh u}{\sinh u}\frac{du}{dx} \qquad \text{Rearrange terms.}$$

$$= -\operatorname{csch} u \coth u \frac{du}{dx} \qquad \text{Definitions of csch } u \text{ and coth } u$$

gives the last formula. The others are obtained similarly.

EXAMPLE Finding Derivatives and Integrals

(a) $$\frac{d}{dt}\left(\tanh\sqrt{1+t^2}\right) = \operatorname{sech}^2\sqrt{1 = t^2}\cdot\frac{d}{dt}\left(\sqrt{1+t^2}\right)$$

$$= \frac{t}{\sqrt{1+t^2}}\operatorname{sech}^2\sqrt{1+t^2}$$

(b) $$\int \coth 5x\, dx = \int \frac{\cosh 5x}{\sinh 5x}dx = \frac{1}{5}\int\frac{du}{u}$$

$u = \sinh 5x,$
$du = 5\cosh 5x\, dx$

$$= \frac{1}{5}\ln|u| + C = \frac{1}{5}\ln|\sinh 5x| + C$$

(c) $$\int_0^1 \sinh^2 x\, dx = \int_0^1 \frac{\cosh 2x - 1}{2}dx \qquad \text{Table 7.6}$$

$$= \frac{1}{2}\int_0^1(\cosh 2x - 1)\,dx = \frac{1}{2}\left[\frac{\sinh 2x}{2} - x\right]_0^1$$

$$= \frac{\sinh 2}{4} - \frac{1}{2} \approx 0.40672$$

(d) $$\int_0^{\ln 2} 4e^x \sinh x\, dx = \int_0^{\ln 2} 4e^x\frac{e^x - e^{-x}}{2}dx = \int_0^{\ln 2}(2e^{2x} - 2)\,dx$$

$$= \left[e^{2x} - 2x\right]_0^{\ln 2} = (e^{2\ln 2} - 2\ln 2) - (1 - 0)$$

$$= 4 - 2\ln 2 - 1$$

Inverse Hyperbolic Functions

The inverses of the six basic hyperbolic functions are very useful in integration. Since $d(\sinh x)/dx = \cosh x > 0$, the hyperbolic sine is an increasing function of x. We denote its inverse by

$$y = \sinh^{-1} x.$$

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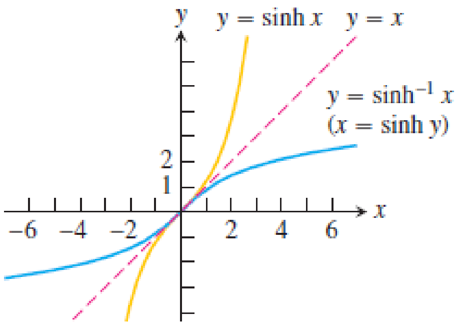
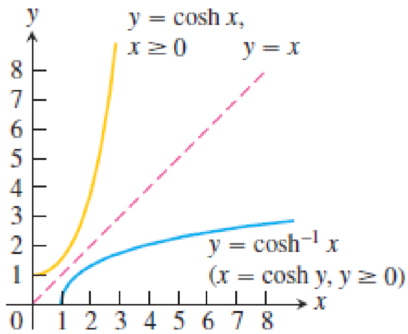
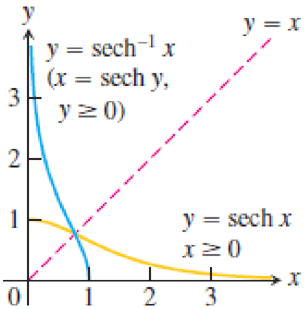
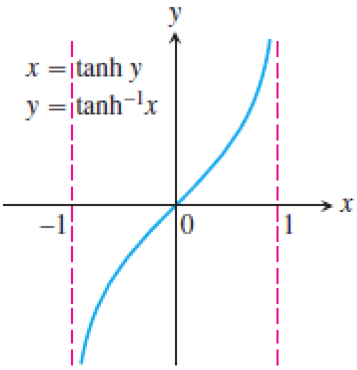




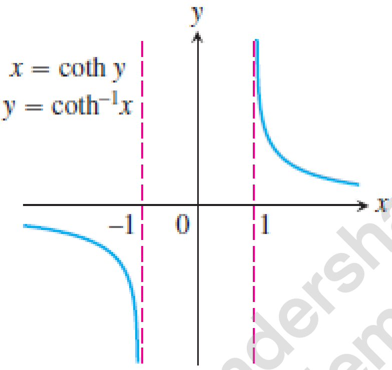


The hyperbolic tangent, cotangent, and cosecant are one-to-one on their domains and therefore have inverses, denoted by

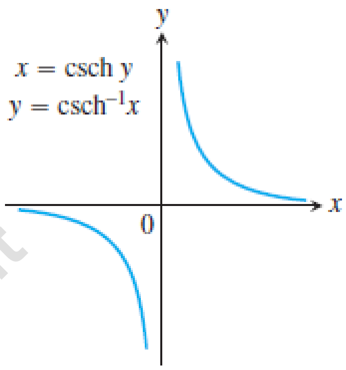
$$y = \tanh^{-1} x, \qquad y = \coth^{-1} x, \qquad y = \operatorname{csch}^{-1} x.$$



(a)



(b)



(c)

$$\operatorname{sech}\left(\cosh^{-1}\left(\frac{1}{x}\right)\right) = \frac{1}{\cosh\left(\cosh^{-1}\left(\frac{1}{x}\right)\right)} = \frac{1}{\left(\frac{1}{x}\right)} = x$$

TABLE 7.10 Derivatives of inverse hyperbolic functions

$$\frac{d(\sinh^{-1} u)}{dx} = \frac{1}{\sqrt{1 + u^2}} \frac{du}{dx}$$

$$\frac{d(\cosh^{-1} u)}{dx} = \frac{1}{\sqrt{u^2 - 1}} \frac{du}{dx}, \qquad u > 1$$

$$\frac{d(\tanh^{-1} u)}{dx} = \frac{1}{1 - u^2} \frac{du}{dx}, \qquad |u| < 1$$

$$\frac{d(\coth^{-1} u)}{dx} = \frac{1}{1 - u^2} \frac{du}{dx}, \qquad |u| > 1$$

$$\frac{d(\operatorname{sech}^{-1} u)}{dx} = \frac{-du/dx}{u\sqrt{1 - u^2}}, \qquad 0 < u < 1$$

$$\frac{d(\operatorname{csch}^{-1} u)}{dx} = \frac{-du/dx}{|u|\sqrt{1 + u^2}}, \qquad u \neq 0$$

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EXAMPLE Derivative of the Inverse Hyperbolic Cosine

$$y = \cosh^{-1} x$$

$$x = \cosh y \qquad \text{Equivalent equation}$$

$$1 = \sinh y \frac{dy}{dx} \qquad \text{Implicit differentiation with respect to } x\text{, and the Chain Rule}$$

$$\frac{dy}{dx} = \frac{1}{\sinh y} = \frac{1}{\sqrt{\cosh^2 y - 1}} \qquad \text{Since } x > 1, y > 0 \text{ and } \sinh y > 0$$

$$= \frac{1}{\sqrt{x^2 - 1}}. \qquad \cosh y = x$$

EXAMPLE : Evaluate $\displaystyle\int_0^1 \frac{2\,dx}{\sqrt{3 + 4x^2}}.$

Solution The indefinite integral is

$$\int \frac{2\,dx}{\sqrt{3 + 4x^2}} = \int \frac{du}{\sqrt{a^2 + u^2}} \qquad u = 2x,\quad du = 2\,dx,\quad a = \sqrt{3}$$

$$= \sinh^{-1}\left(\frac{u}{a}\right) + C \qquad \text{Formula from Table 7.11}$$

$$= \sinh^{-1}\left(\frac{2x}{\sqrt{3}}\right) + C.$$

Therefore,

$$\int_0^1 \frac{2\,dx}{\sqrt{3 + 4x^2}} = \sinh^{-1}\left(\frac{2x}{\sqrt{3}}\right)\Bigg]_0^1 = \sinh^{-1}\left(\frac{2}{\sqrt{3}}\right) - \sinh^{-1}(0)$$

$$= \sinh^{-1}\left(\frac{2}{\sqrt{3}}\right) - 0 \approx 0.98665.$$

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TABLE 7.11 Integrals leading to inverse hyperbolic functions

1.	$\displaystyle\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C,$	$a > 0$
2.	$\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C,$	$u > a > 0$
3.	$\displaystyle\int \frac{du}{a^2-u^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{u}{a}\right) + C & \text{if } u^2 < a^2 \\ \frac{1}{a}\coth^{-1}\left(\frac{u}{a}\right) + C, & \text{if } u^2 > a^2 \end{cases}$	
4.	$\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{u}{a}\right) + C,$	$0 < u < a$
5.	$\displaystyle\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\operatorname{csch}^{-1}\left\lvert\frac{u}{a}\right\rvert + C,$	$u \neq 0$ and $a > 0$

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