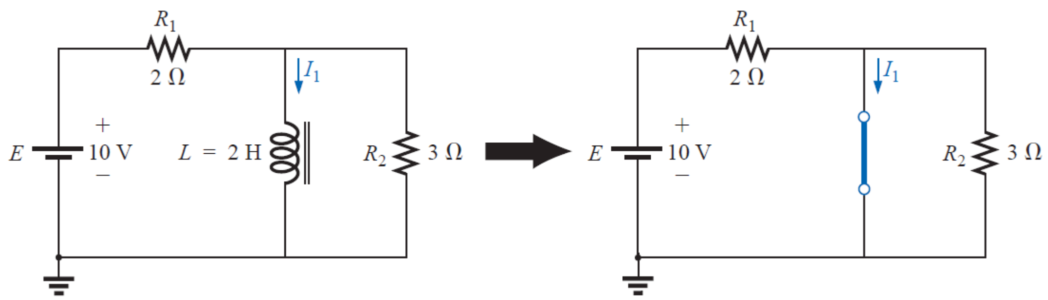
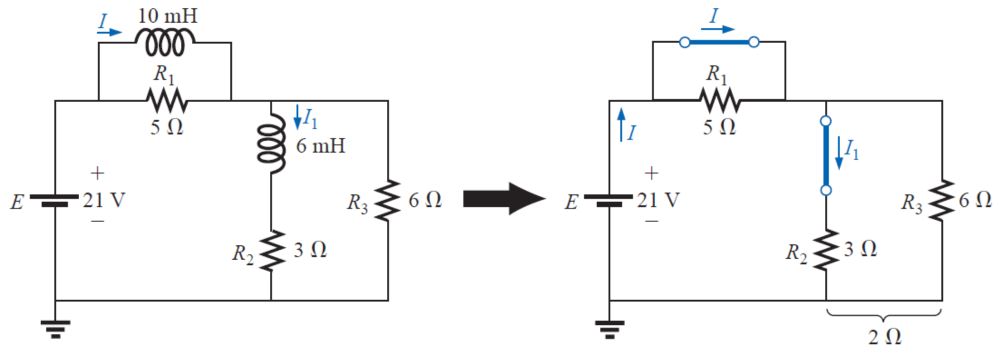
Basic RL and RC Circuits

The RL circuit with D.C (steady state)
The inductor is short time at $t = \infty$
Calculate the inductor current for circuits shown below.



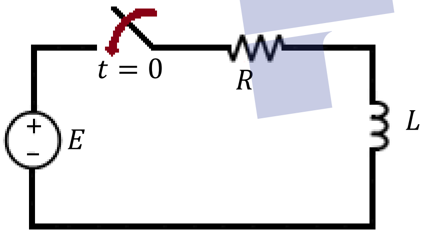
$$I_L = \frac{E}{R_1} = \frac{10}{2} = 5A$$



$$I_{L_1} = \frac{E}{\left(\frac{R_2 R_3}{R_2 + R_3}\right)}$$

$$I_{L_2} = I_{L_1} \frac{R_3}{R_2 + R_3}$$

R-L TRANSIENTS: STORAGE CYCLE



$$-E + Ri + L\frac{di}{dt} = 0$$

$$Ri + L\frac{di}{dt} = E$$

$$L\frac{di}{dt} = E - Ri$$

$$Ldi = (E - Ri)dt$$

$$\frac{Ldi}{(E - Ri)} = dt$$

$$\int \frac{Ldi}{(E - Ri)} = \int dt$$

$$-\frac{L}{R} ln(E - Ri) = t + k$$

at $t = 0, i = 0$, therefore

$$-\frac{L}{R} ln(E) = k$$

And

$$\frac{L}{R}ln(E-Ri)=t-\frac{L}{R}ln(E)$$

$$-\frac{L}{R}ln(E-Ri)+\frac{L}{R}ln(E)=t$$

$$-\frac{L}{R}\left(ln\left(\frac{E-Ri}{E}\right)\right)=t$$

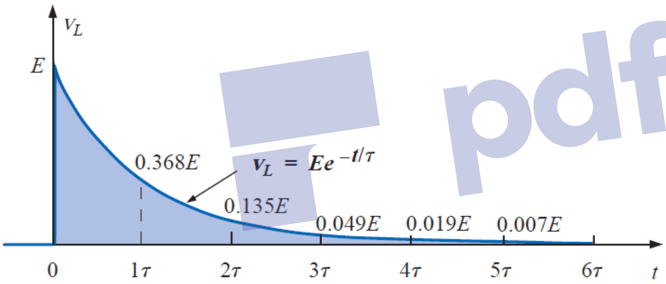
$$\frac{E-Ri}{E}=e^{-\frac{R}{L}t}$$

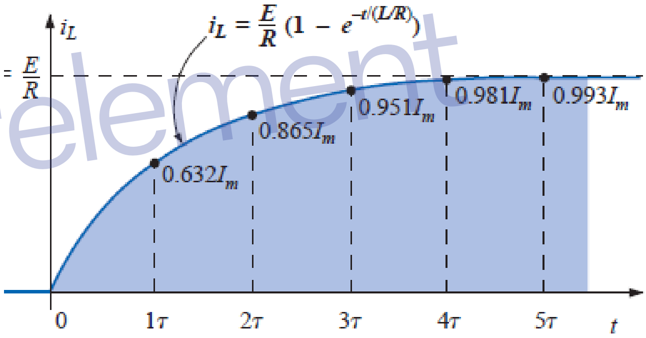
$$i=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)=\frac{E}{R}\left(1-e^{-\frac{t}{\tau}}\right)$$

$$\tau=\frac{L}{R} \qquad \text{(seconds, s)}$$

$$i_L=I_m(1-e^{-t/\tau})=\frac{E}{R}(1-e^{-t/(L/R)})$$

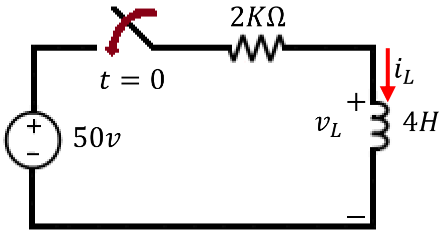
$$V_L=Ee^{-t/\tau}$$





Example:

Find the mathematical expression for the transient behaviour of i_L and v_L.



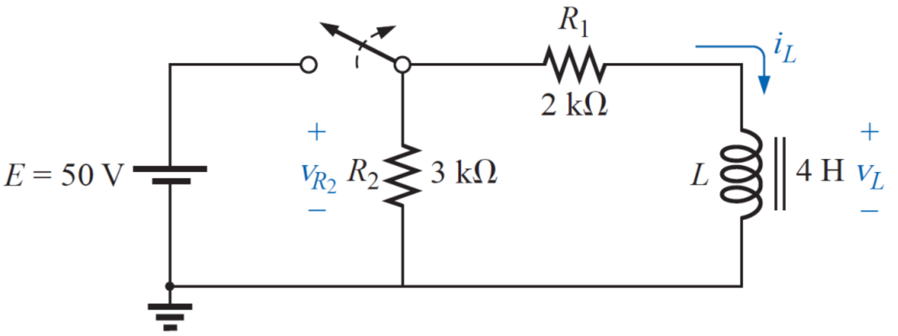
$$\tau=\frac{L}{R}=\frac{4}{2\times10^3}=2ms$$

$$i_L=\frac{E}{R}\left(1-e^{-\frac{t}{\tau}}\right)=\frac{50}{2\times10^3}(1-e^{-500t})=25(1-e^{-500t})mA$$

$$v_L=Ee^{-\frac{t}{\tau}}=50e^{-500t}\,V$$

Example:

For the circuit shown below, calculate the mathematical expression of $i_L, v_L, v_{R_1}, v_{R_2}$ before and after the storage phase has been complete ant the switch is open.



1-switch on

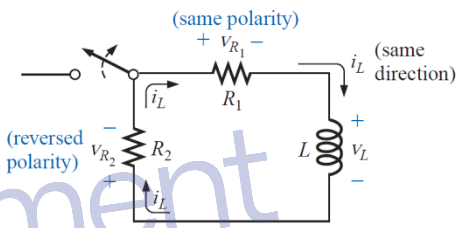
$$\tau = \frac{L}{R_{eq}} = \frac{4}{2\times10^3} = 2ms$$

$$i_L = \frac{E}{R_1}\left(1 - e^{-\frac{t}{\tau}}\right) = \frac{50}{2\times10^3}(1 - e^{-500t}) = 25(1 - e^{-500t})mA$$

$$v_L = Ee^{-\frac{t}{\tau}} = 50e^{-500t}\,V$$

$$v_{R_1} = i_L R_1 = \frac{E}{R_1}R_1\left(1 - e^{-\frac{t}{\tau}}\right) = 50(1 - e^{-500t})v$$

$$v_{R_2} = E = 50\,V$$



2-switch off

After the storage phase has passed and steady-state conditions are established, the switch can be opened without the sparking effect or rapid discharge due to the resistor R_2, which provides a complete path for the current i_L. The voltage v across the inductor will reverse polarity and have a magnitude determined by

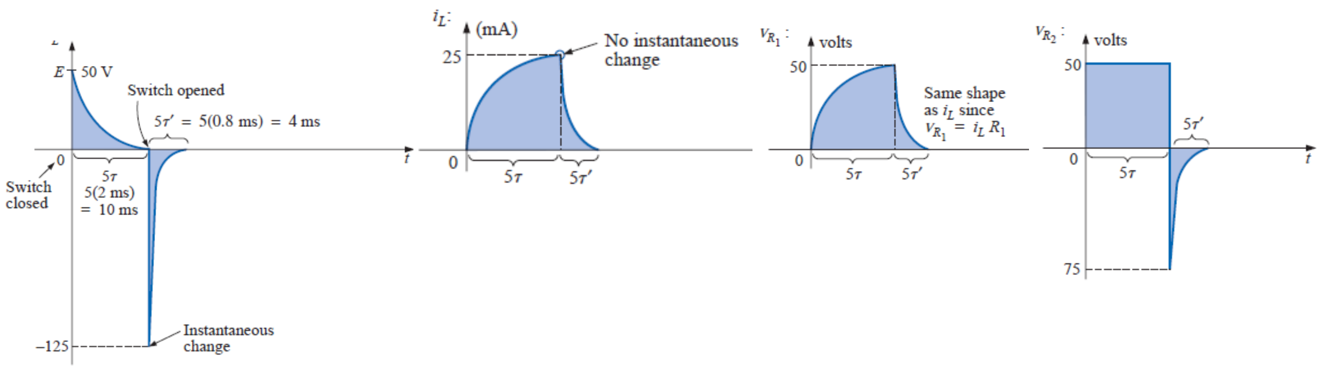
$$\acute{\tau} = \frac{L}{R_{eq}} = \frac{4}{2\times10^3 + 3\times10^3} = 0.8\,ms$$

$$i_L = \frac{E}{R_1}e^{-\frac{t}{\acute{\tau}}} = \frac{50}{2\times10^3}e^{-\frac{t}{\acute{\tau}}} = 25e^{-\frac{t}{0.8\times10^{-3}}}\,mA$$

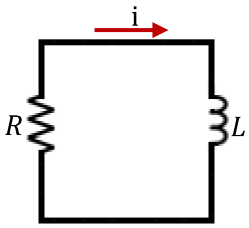
$$v_L = -i_L(R_1 + R_2) = -\frac{E}{R_1}(R_1 + R_2)e^{-\frac{t}{\acute{\tau}}} = -E\left(1 + \frac{R_2}{R_1}\right)e^{-\frac{t}{0.8\times10^{-3}}} = -50\left(1 + \frac{3}{2}\right)e^{-\frac{t}{0.8\times10^{-3}}} =$$
$$-75e^{-\frac{t}{0.8\times10^{-3}}}V$$

$$v_{R_1} = i_L R_1 = \frac{E}{R_1}R_1 e^{-\frac{t}{\acute{\tau}}} = 50e^{-\frac{t}{0.8\times10^{-3}}t}\,v$$

$$v_{R_2} = -i_L R_2 = -\frac{E}{R_1}R_2 e^{-\frac{t}{\acute{\tau}}} = -\frac{50}{2}3e^{-\frac{t}{0.8\times10^{-3}}t} = -75e^{-\frac{t}{0.8\times10^{-3}}t}\,V$$



The Source Free RL circuit



Using KVL, leads

$$Ri + L\frac{di}{dt} = 0$$

This equation represents a differential equation and can be solved by several different methods

$$\frac{di}{i} = -\frac{R}{L}dt$$

Since the current is I_0 at $t = 0$ and $i(t)$ at time t, we may equate the two definite integrals with are obtained by integrating each side between the corresponding limits

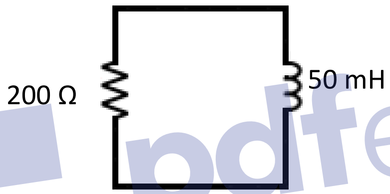
$$\int_{I_0}^{i(t)} \frac{d\acute{\imath}}{i} = -\frac{R}{L}\int_0^t d\acute{t}$$

$$i(t) = I_0\, e^{-\frac{R}{L}t}$$

$$v(t) = -v_L\, e^{-\frac{R}{L}t}$$

Example:

If the inductor has a current 2A at t=0, find an expression for $i_L(t)$ valid for $t > 0$, and its value at t=200µs.



$i(t) = I_0\, e^{-\frac{R}{L}t} = 2e^{-\frac{200}{50\times10^{-3}}t} = 2e^{-4000t}$ A

At t=200µs

$i(200\mu s) = 2e^{-4000\times200\times10^{-6}} = 2e^{-0.8} = 898.7$ mA

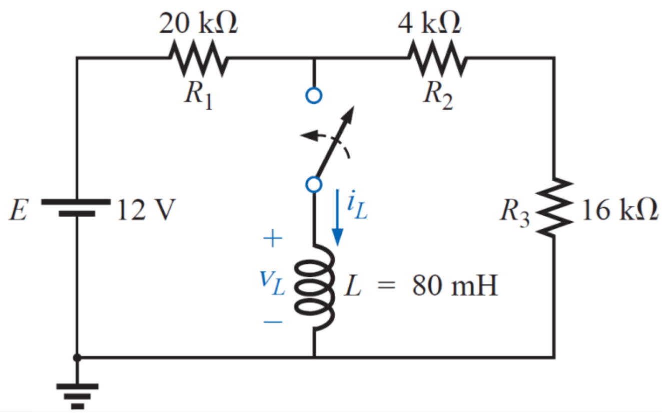
THÉVENIN EQUIVALENT:

Example:

For the network of Figure below

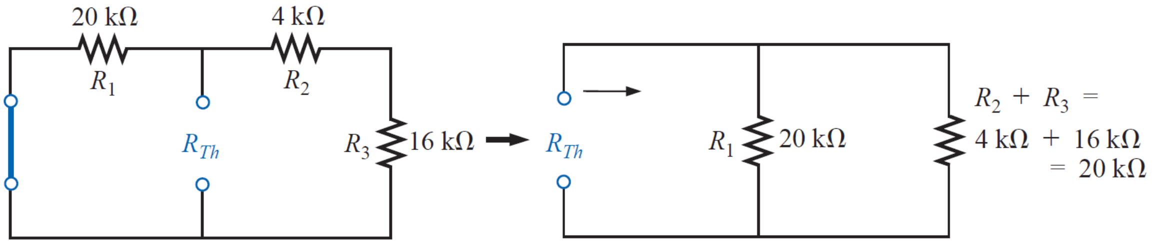
a. Find the mathematical expression for the transient behavior of the current *iL* and the voltage *vL* after the closing of the switch (*Ii* =0 mA).

b. Draw the resultant waveform for each.



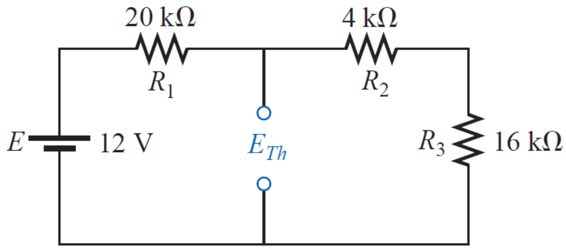
Solutions:

a. Applying Thévenin's theorem to the 80-mH inductor ,yields



$$R_{th}=\frac{(4+16)\times 20}{(4+16)+20}=10\ K\Omega$$

Applying the voltage divider rule to determine Thevinen voltage



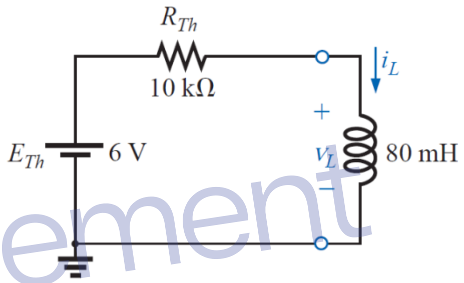
$$E_{th}=12\frac{(4+16)}{(4+16)+20}=6\ V$$

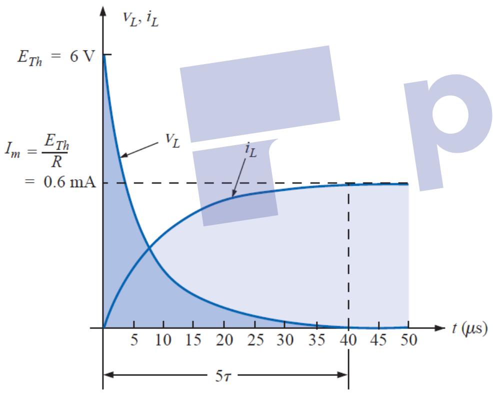
$$\tau=\frac{L}{R_{th}}=\frac{80\times10^{-3}}{10\times10^{3}}=8\ \mu s$$

$$i_L=\frac{E_{th}}{R_{th}}\left(1-e^{-\frac{t}{\tau}}\right)=\frac{6}{10\times10^{3}}\left(1-e^{-\frac{t}{8\times10^{-6}}}\right)$$

$$i_L=0.6\left(1-e^{-\frac{t}{8\times10^{-6}}}\right)mA$$

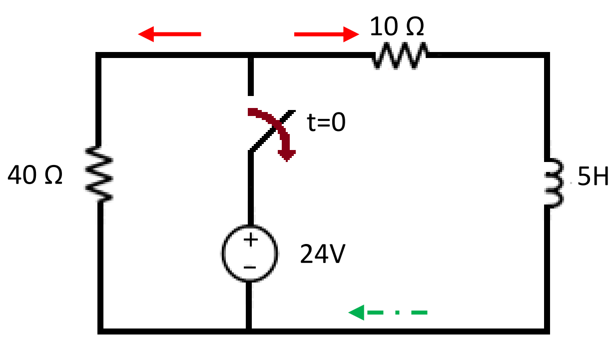
$$v_L=E_{th}e^{-\frac{t}{\tau}}=6\,e^{-\frac{t}{8\times10^{-6}}}\,V$$





Example:

Find the voltage across 40Ω resistor at t=200ms.



$$I_l = \frac{24}{10} = 2.4\ A$$

$$\tau = \frac{L}{R} = \frac{5}{10} = 0.5\ s$$

At t=0

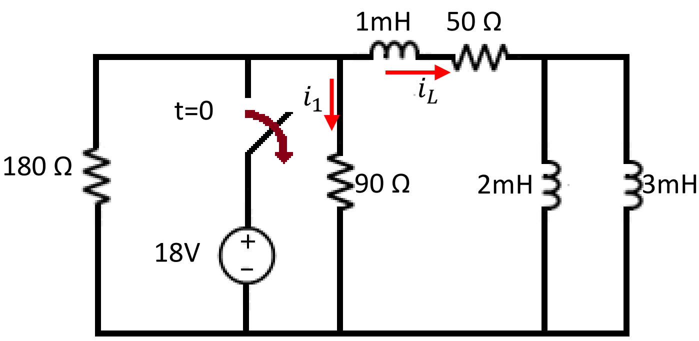
$$\tau = \frac{L}{R_{eq}} = \frac{5}{10+40} = 0.1\ s$$

$$i(t) = I_0\, e^{-\frac{R}{L}t} = 2.4\, e^{-10t}$$

$$V_{40} = i(t)R = -2.4 \times 40 \times e^{-10t} = -96e^{-10t}\ V$$

Example:

Determine both i_1 and i_L for $t > 0$.



$$L_{eq} = \frac{2 \times 3}{2+3} + 1 = 2.2mH$$

$$R_{eq} = \frac{180 \times 90}{180+90} + 50 = 110\Omega$$

$$I_L = \frac{18}{50} = 360\ mA$$

$$I_L(t) = 360\, e^{-\frac{R_{eq}}{L_{eq}}t} = 360\, e^{-\frac{110}{2.2\times10^{-3}}t} = 360\, e^{-50000t} mA$$

$$I_1(t) = -360 \frac{180}{180+90}\, e^{-50000t} = -240\, e^{-50000t}\ mA$$

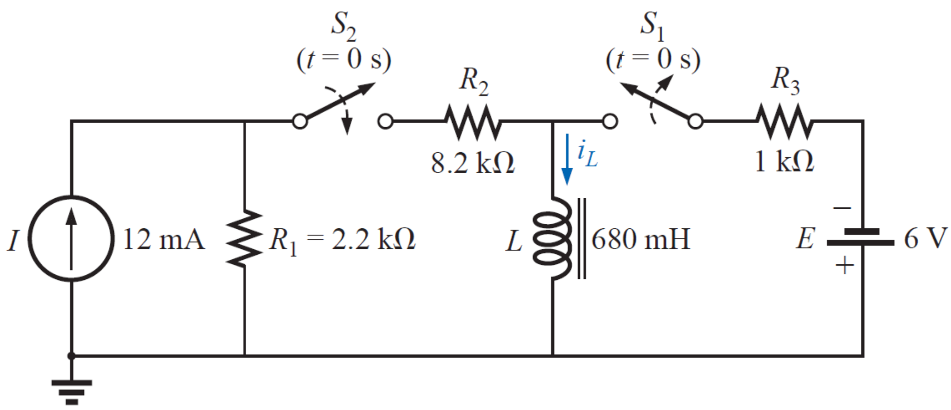
H.W

The switch *S*1 of following Figure has been closed for a long time. At *t* = 0 s, *S*1 is opened at the same instant *S*2 is closed to avoid an interruption in current through the coil.

a. Find the initial current through the coil. Pay particular attention to its direction.

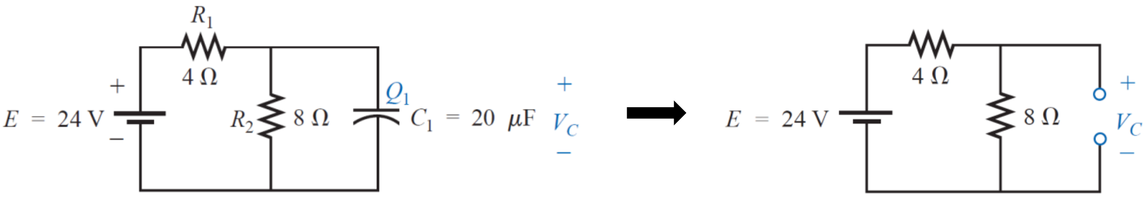
b. Find the mathematical expression for the current i_L following the closing of the switch *S*2.

c. Sketch the waveform for i_L.



RC Circuits

Find the voltage across and charge on capacitor C_1 of Figure below after it has charged up to its final value.



Solution :

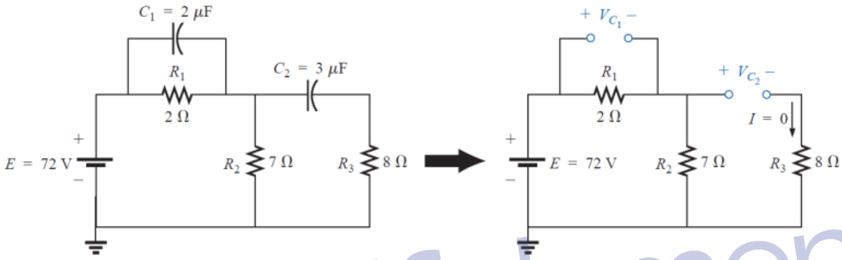
the capacitor is effectively an open circuit for dc after charging up to its final value

$$V_C = E\frac{R_2}{R_1 + R_2} = 24\frac{8}{12} = 16V$$

$$Q_1 = V_C C_1 = 16 \times 20 \times 10^{-6} = 320\ \mu C$$

Example:

Find the voltage across and charge on each capacitor of the network of Figure below after each has charged up to its final value.



Solution

$$V_{C_1} = E\frac{R_1}{R_1 + R_2} = 72\frac{2}{9} = 16V$$

$$V_{C_2} = E\frac{R_2}{R_1 + R_2} = 72\frac{7}{9} = 56V$$

$$Q_1 = V_{C_1} C_1 = 16 \times 2 \times 10^{-6} = 32\ \mu C$$

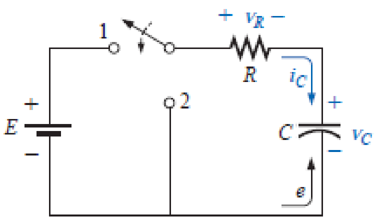
$$Q_2 = V_{C_2} C_2 = 56 \times 3 \times 10^{-6} = 168\ \mu C$$

ENERGY STORED BY A CAPACITOR

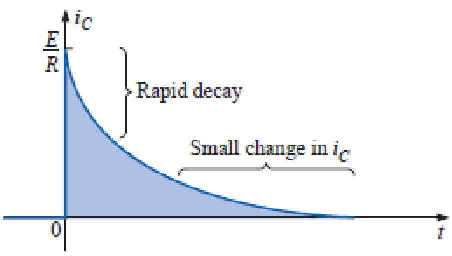
The ideal capacitor does not dissipate any of the energy supplied to it. It stores the energy in the form of an electric field between the conducting surfaces.

$$W_C = \frac{1}{2}CV^2 \quad \text{(J)}$$

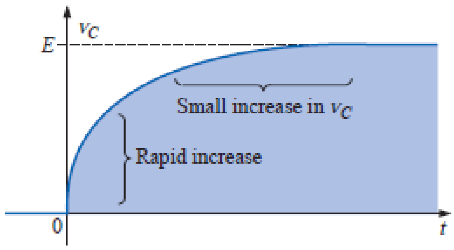
TRANSIENTS IN CAPACITIVE NETWORKS: CHARGING PHASE



Basic charging network.



i_C during the charging phase.



v_C during the charging phase.

$$-E + I_R R + V_C = 0$$
$$-E + RC\frac{dV_C}{dt} + V_C = 0$$
$$RC\frac{dV_C}{dt} = E - V_C$$
$$RC\frac{dV_C}{(E - V_C)} = dt$$
$$\int RC\frac{dV_C}{(E - V_C)} = \int dt$$
$$-RCln(E - V_C) = t + K$$

At t=0, $V_C = 0$, therefore

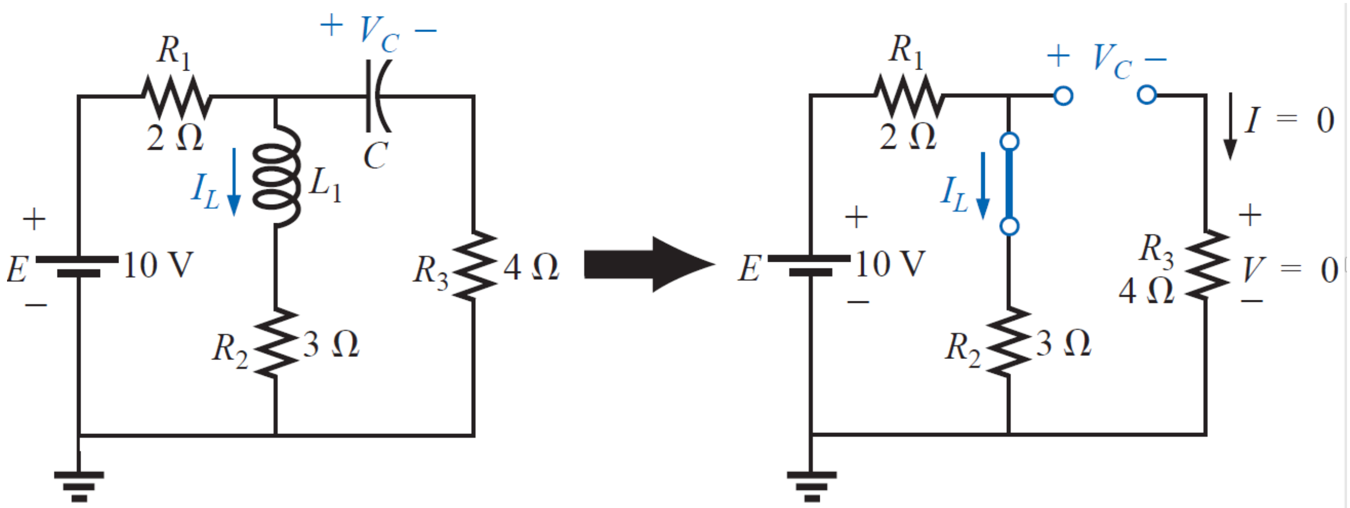
$$-RCln(E) = K$$
$$-RCln(E - V_C) = t - RCln(E)$$
$$-RCln(E - V_C) + RCln(E) = t$$
$$-RCln\left(\frac{E - V_C}{E}\right) = t$$
$$ln\left(\frac{E - V_C}{E}\right) = -\frac{t}{RC}$$
$$\frac{E - V_C}{E} = e^{-\frac{t}{RC}}$$
$$V_C = E - Ee^{-\frac{t}{RC}} = E\left(1 - e^{-\frac{t}{RC}}\right)$$
$$\tau = RC$$

And

$$I_C = I_C = I_0 e^{-\frac{t}{RC}} = \frac{E}{R}e^{-\frac{t}{RC}}$$

RLC Circuits

Example:
Find the current I_L and the voltage V_C for the network



Solution:

$$I_L = \frac{E}{R_1 + R_2} = \frac{10}{5} = 2\,A$$

$$V_L = E\frac{R_2}{R_1 + R_2} = 10\frac{3}{5} = 6\ V$$

H.W

Find the currents $I1$ and $I2$ and the voltages V_1 and V_2 for the network