Series and Parallel ac Circuits Resistive Elements

$$I_{m} = \frac{V_{m}}{R}$$

$$V_{m} = I_{m}R$$
In phaser form,

$$v = V_{m}sin\omega t = V \angle 0$$
Where $V = 0.707V_{m}$
Applying Ohm's law and using phaser algebra, we have

$$I = \frac{V \angle 0}{R \angle 0}$$
So that in the time domain,

$$i = \sqrt{2} \frac{V}{R} sin\omega t$$
Example:

Using complex algebra, find the current *i* for the circuit shown below. Sketch the waveforms of *v* and *i*.

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Inductive Reactance

The voltage leads the current by 90° and that the reactance of the coil XL is determined by ωL .

$$v = V_m sin\omega t = V \angle 0$$

By Ohm's law, $\mathbf{I} = \frac{V \angle 0}{X_L \angle 90} = \frac{V}{X_L} \angle -90$

so that in the time domain,

$$i = \sqrt{2} \frac{V}{X_L} sin(\omega t - 90)$$



 $Z_L = X_L \angle 90$

Example:

Using complex algebra, find the current *i* for the circuit shown below. Sketch the *v* and *i* curves.



Capacitive Reactance

The current leads the voltage by 90° and that the reactance of the capacitor X_C is determined by $\frac{1}{\omega C}$.

 $X_L = 4 \Omega$

$$v = V_m sin\omega t$$

In polar form
$$\mathbf{V} = V \angle 0$$
$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_c} = \frac{V \angle 0}{X_c \angle -90} = \frac{V}{X_c} \angle 90$$
$$i = \sqrt{2} \frac{V}{X_c} sin(\omega t + 90)$$
$$\mathbf{Z}_c = X_c \angle -90$$

Example:

Using complex algebra, find the current *i* for the circuit shown below. Sketch the *v* and *i* curves.



Impedance Diagram

Now that an angle is associated with resistance, inductive reactance, and capacitive reactance, each can be placed on a complex plane diagram. For any network, the resistance will *always* appear on the positive real axis, the inductive reactance on the positive imaginary axis, and the capacitive reactance on the negative imaginary axis. The result is an **impedance diagram** that can reflect the individual and total impedance levels of an ac network.

Remove Watermark No



SERIES CONFIGURATION

The overall properties of series ac circuits are the same as those for dc circuits. For instance, the total impedance of a system is the sum of the individual impedances:



Example:

Determine the input impedance to the series network shown below. Draw the impedance diagram.







Power: The total power in watts delivered to the circuit is $p_T = EIcos\theta_T = 100 \times 20 \cos 53.13^\circ = 1200 \text{ w}$ where *E* and *I* are effective values and θ_T is the phase angle between *E* and *I*, or $p_T = I^2R = 20^2 \times 3 = 1200 \text{ w}$ where *I* is the effective value, or, finally, $p_T = p_R + p_L = 60 \times 20 \cos 0 + 80 \times 20 \cos 90 = 1200 \text{ w}$

Power factor: The power factor Fp of the circuit is $\cos 53.13^\circ = 0.6$ lagging, where 53.13° is the phase angle between **E** and **I**.



Time domain: In the time domain,

 $e = \sqrt{2} \times 50 \sin \omega t = 70.7 \sin \omega t$ $V_R = \sqrt{2} \times 30 \sin(\omega t + 53.13^\circ) = 42.42 \sin(\omega t + 53.13^\circ)$ $V_C = \sqrt{2} \times 40 \sin(\omega t - 36.87) = 56.56 \sin(\omega t - 36.87)$



Power: The total power in watts delivered to the circuit is $p_T = EIcos\theta_T = 50 \times 5 \cos 53.13^\circ = 150 \text{ w}$ where *E* and *I* are effective values and θ_T is the phase angle between *E* and *I*, or $p_T = I^2R = 5^2 \times 6 = 150 \text{ w}$ $p_T = p_R + p_C = 30 \times 5 \cos 0 + 40 \times 5 \cos 90 = 150 \text{ w}$ *Power factor:* The power factor of the circuit is $F_P = \cos 53.13^\circ = 0.6 \text{ leading}$



Phasor diagram: The phasor diagram of Fig. 15.38 indicates that the current **I** is in phase with the voltage across the resistor, lags the voltage across the inductor by 90° , and leads the voltage across the capacitor by 90° .

Time domain:

 $i = \sqrt{2} \times 10 \sin(\omega t - 53.13^{\circ}) = 14.14 \sin(\omega t - 53.13^{\circ})$ $V_R = \sqrt{2} \times 30 \sin(\omega t - 53.13^{\circ}) = 42.42 \sin(\omega t - 53.13^{\circ})$ $V_L = \sqrt{2} \times 70 \sin(\omega t + 36.87) = 98.98 \sin(\omega t + 36.87)$ $V_C = \sqrt{2} \times 30 \sin(\omega t - 143.13^{\circ}) = 42.42 \sin(\omega t - 143.13^{\circ})$ Power: The total power in watts delivered to the circuit is $p_T = EIcos\theta_T = 50 \times 10 \cos 53.13^{\circ} = 300 \text{ w}$ or $p_T = I^2 R = 10^2 \times 3 = 300 \text{ w}$

 $p_T = p_R + p_L + p_C = 30 \times 10 \cos 0 + 70 \times 10 \cos 90 + 40 \times 10 \cos 90 = 300$ w *Power factor:* The power factor of the circuit is $F_P = \cos 53.13^\circ = 0.6$ leading



VOLTAGE DIVIDER RULE

The basic format for the voltage divider rule in ac circuits is exactly the same as that for dc circuits:

Example:

Find the voltage across each element of the circuit shown below



H.W

For the circuit shown below,

- 1- Calculate I, V_R , V_L , and V_C in phasor form.
- 2- Calculate the total power factor.
- 3- Calculate the average power delivered to the circuit.
- 4- Draw the phasor diagram.
- 5- Obtain the phasor sum of V_R , V_L , and V_C , and show that it equals the input voltage E.
- 6- Find V_R and V_C using the voltage divider rule.



PARALLEL ac CIRCUITS

In ac circuits, we define **admittance** (Y) as being equal to 1/Z. The unit of measure for admittance as defined by the SI system is *siemens*, which has the symbol S. Admittance is a measure of how well an ac circuit will *admit*, or allow, current to flow in the circuit. The larger its value, therefore, the heavier the current flow for the same applied potential. The total admittance of a circuit can also be found by finding the sum of the parallel admittances.



$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \dots + \frac{1}{Z_N}$$
$$Y_T = Y_1 + Y_2 + \dots + Y_N$$

For pure resistor, conductance is the reciprocal of resistance, and

$$Y_R = \frac{1}{Z_R} = \frac{1}{R \angle 0} = G \angle 0$$
 (siemens, S)

The reciprocal of reactance (1/X) is called **susceptance** and is a measure of how *susceptible* an element is to the passage of current through it. Susceptance is also measured in *siemens* and is represented by the capital letter *B*.

For the inductor,

$$Y_L = \frac{1}{Z_L} = \frac{1}{X_L \angle 90} = B_L \angle -90$$
 (siemens, S)

Note that for inductance, an increase in frequency or inductance will result in a decrease in susceptance or, correspondingly, in admittance.

For the capacitor,

 $Y_{C} = \frac{1}{Z_{C}} = \frac{1}{X_{C} \angle -90} = B_{C} \angle 90 \quad \text{(siemens, S)}$ For the capacitor, therefore, an increase in frequency or capacitance will result in an increase in its susceptibility.

For parallel ac circuits, the **admittance diagram** is used with the three admittances, represented as shown in Figure below.

Note from this figure that the conductance (like resistance) is on the positive real axis, whereas inductive and capacitive susceptances are in direct opposition on the imaginary axis.



Solution:

a.
$$\mathbf{Y}_R = G \angle 0^\circ = \frac{1}{R} \angle 0^\circ = \frac{1}{5 \Omega} \angle 0^\circ$$

= **0.2 S** $\angle 0^\circ = \mathbf{0.2 S} + \mathbf{j 0}$

$$\mathbf{Y}_{L} = B_{L} \angle -90^{\circ} = \frac{1}{X_{L}} \angle -90^{\circ} = \frac{1}{8 \Omega} \angle -90^{\circ}$$

= **0.125** S $\angle -90^{\circ} = \mathbf{0} - j \mathbf{0.125}$ S
$$\mathbf{Y}_{C} = B_{C} \angle 90^{\circ} = \frac{1}{X_{C}} \angle 90^{\circ} = \frac{1}{20 \Omega} \angle 90^{\circ}$$

= **0.050** S $\angle +90^{\circ} = \mathbf{0} + j \mathbf{0.050}$ S
b. $\mathbf{Y}_{T} = \mathbf{Y}_{R} + \mathbf{Y}_{L} + \mathbf{Y}_{C}$
= (0.2 S + j 0) + (0 - j 0.125 S) + (0 + j 0.050 S)
= 0.2 S - j 0.075 S = **0.2136** S $\angle -20.56^{\circ}$
c. $\mathbf{Z}_{T} = \frac{1}{0.2136} \frac{1}{S} \angle -20.56^{\circ} = 4.68 \Omega \angle 20.56^{\circ}$

or



Example:

For the network of Fig. shown below:

a. Find the admittance of each parallel branch.

- b. Determine the input admittance.
- c. Calculate the input impedance.
- d. Draw the admittance diagram.



PARALLEL ac NETWORKS



\mathbf{Y}_{T} and \mathbf{Z}_{T}

$$\begin{aligned} \mathbf{Y}_{T} &= \mathbf{Y}_{R} + \mathbf{Y}_{L} + \mathbf{Y}_{C} = G \angle 0^{\circ} + B_{L} \angle -90^{\circ} + B_{C} \angle 90^{\circ} \\ &= \frac{1}{3.33 \ \Omega} \angle 0^{\circ} + \frac{1}{1.43 \ \Omega} \angle -90^{\circ} + \frac{1}{3.33 \ \Omega} \angle 90^{\circ} \\ &= 0.3 \ S \angle 0^{\circ} + 0.7 \ S \angle -90^{\circ} + 0.3 \ S \angle 90^{\circ} \\ &= 0.3 \ S - j \ 0.7 \ S + j \ 0.3 \ S \\ &= 0.3 \ S - j \ 0.4 \ S = \mathbf{0.5 \ S} \angle -\mathbf{53.13^{\circ}} \\ \mathbf{Z}_{T} &= \frac{1}{\mathbf{Y}_{T}} = \frac{1}{0.5 \ S \angle -53.13^{\circ}} = \mathbf{2} \ \Omega \angle \mathbf{53.13^{\circ}} \\ \mathbf{I} &= \frac{\mathbf{E}}{\mathbf{Z}_{T}} = \mathbf{E} \mathbf{Y}_{T} = (100 \ V \angle 53.13^{\circ})(0.5 \ S \angle -53.13^{\circ}) = \mathbf{50} \ \mathbf{A} \angle \mathbf{0^{\circ}} \end{aligned}$$

 $I_{R'}$ I_L , and I_C

$$\mathbf{I}_{R} = (E \angle \theta)(G \angle 0^{\circ}) = (100 \text{ V} \angle 53.13^{\circ})(0.3 \text{ S} \angle 0^{\circ}) = \mathbf{30} \text{ A} \angle 53.13^{\circ} \mathbf{I}_{L} = (E \angle \theta)(B_{L} \angle -90^{\circ}) = (100 \text{ V} \angle 53.13^{\circ})(0.7 \text{ S} \angle -90^{\circ}) = \mathbf{70} \text{ A} \angle -\mathbf{36.87^{\circ}} \mathbf{I}_{C} = (E \angle \theta)(B_{C} \angle 90^{\circ}) = (100 \text{ V} \angle 53.13^{\circ})(0.3 \text{ S} \angle +90^{\circ}) = \mathbf{30} \text{ A} \angle \mathbf{143.13^{\circ}}$$

Kirchhoff's current law: At node a,

$$\mathbf{I} - \mathbf{I}_R - \mathbf{I}_L - \mathbf{I}_C = 0$$

Phasor diagram







Time domain:

or

 $i = \sqrt{2}(50) \sin \omega t = 70.70 \sin \omega t$ $i_R = \sqrt{2}(30) \sin(\omega t + 53.13^\circ) = 42.42 \sin(\omega t + 53.13^\circ)$ $i_L = \sqrt{2}(70) \sin(\omega t - 36.87^\circ) = 98.98 \sin(\omega t - 36.87^\circ)$ $i_C = \sqrt{2}(30) \sin(\omega t + 143.13^\circ) = 42.42 \sin(\omega t + 143.13^\circ)$

A plot of all the currents and the impressed voltages appears ain following figure



Power factor: The power factor of the circuit is

$$F_p = \cos \theta_T = \cos 53.13^\circ = 0.6 \text{ lagging}$$
$$F_p = \cos \theta_T = \frac{G}{Y_T} = \frac{0.3 \text{ S}}{0.5 \text{ S}} = 0.6 \text{ lagging}$$

Impedance approach: The input current I can also be determined by first finding the total impedance in the following manner:

$$\mathbf{Z}_{T} = \frac{\mathbf{Z}_{R}\mathbf{Z}_{L}\mathbf{Z}_{C}}{\mathbf{Z}_{R}\mathbf{Z}_{L} + \mathbf{Z}_{L}\mathbf{Z}_{C} + \mathbf{Z}_{R}\mathbf{Z}_{C}} = \mathbf{2} \ \mathbf{\Omega} \ \angle \mathbf{53.13^{\circ}}$$

and, applying Ohm's law, we obtain

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100 \text{ V} \angle 53.13^\circ}{2 \Omega \angle 53.13^\circ} = 50 \text{ A} \angle 0^\circ$$

Example:

For the circuit shown below, determine the I_R and I_L , phasor and admittance diagrams, time domain representation, power and power factor.



Example:

For the circuit shown below, determine the I_R and I_C , phasor and admittance diagrams, time domain representation, power and power factor.



CURRENT DIVIDER RULE

The basic format for the **current divider rule** in ac circuits is exactly the same as that for dc circuits; that is, for two parallel branches with impedances **Z**1 and **Z**2 as shown



Example:

Using the current divider rule, find the current through each impedance of following Figure.

$$\mathbf{I}_{R} = \frac{\mathbf{Z}_{L}\mathbf{I}_{T}}{\mathbf{Z}_{R} + \mathbf{Z}_{L}} = \frac{(4\ \Omega\ \angle90^{\circ})(20\ A\ \angle0^{\circ})}{3\ \Omega\ \angle0^{\circ} + 4\ \Omega\ \angle90^{\circ}} = \frac{80\ A\ \angle90^{\circ}}{5\ \angle53.13^{\circ}}$$
$$= \mathbf{16}\ \mathbf{A}\ \angle36.87^{\circ}$$
$$\mathbf{I}_{L} = \frac{\mathbf{Z}_{R}\mathbf{I}_{T}}{\mathbf{Z}_{R} + \mathbf{Z}_{L}} = \frac{(3\ \Omega\ \angle0^{\circ})(20\ A\ \angle0^{\circ})}{5\ \Omega\ \angle53.13^{\circ}} = \frac{60\ A\ \angle0^{\circ}}{5\ \angle53.13^{\circ}}$$
$$= \mathbf{12}\ \mathbf{A}\ \angle-\mathbf{53.13^{\circ}}$$



Example:

Using the current divider rule, find the current through each parallel branch of Figure shown below.



EQUIVALENT CIRCUITS

In a series ac circuit, the total impedance of two or more elements in series is often equivalent to an impedance that can be achieved with fewer elements of different values, the elements and their values being determined by the frequency applied. This is also true for parallel circuits.

$$\mathbf{Z}_{T} = \frac{\mathbf{Z}_{C}\mathbf{Z}_{L}}{\mathbf{Z}_{C} + \mathbf{Z}_{L}} = \frac{(5\ \Omega\ \angle -90^{\circ})(10\ \Omega\ \angle 90^{\circ})}{5\ \Omega\ \angle -90^{\circ} + 10\ \Omega\ \angle 90^{\circ}} = \frac{50\ |\angle 0^{\circ}}{5\ \angle 90^{\circ}}$$
$$= 10\ \Omega\ \angle -90^{\circ}$$



$$\mathbf{Z}_{T} = \frac{\mathbf{Z}_{L}\mathbf{Z}_{R}}{\mathbf{Z}_{L} + \mathbf{Z}_{R}} = \frac{(4 \ \Omega \ \angle 90^{\circ})(3 \ \Omega \ \angle 0^{\circ})}{4 \ \Omega \ \angle 90^{\circ} + 3 \ \Omega \ \angle 0^{\circ}}$$
$$= \frac{12 \ \angle 90^{\circ}}{5 \ \angle 53.13^{\circ}} = 2.40 \ \Omega \ \angle 36.87^{\circ}$$
$$= 1.920 \ \Omega + j \ 1.440 \ \Omega$$

Example: For the following network

- a. Calculate the total impedance \mathbf{Z}_{T} .
- b. Compute I.
- c. Find the total power factor.
- d. Calculate I₁ and I₂.
- e. Find the average power delivered to the circuit.

Example:

For the following network

- a. Compute I.
- b. Find I_1 , I_2 , and I_3 .
- c. Verify Kirchhoff's current law by showing that

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$

d. Find the total impedance of the circuit.







Tutorial:

1-Find the current *i* for the elements and sketch the waveforms for *v* and *i* on the same set of axes.



2-Calculate the total impedance and express your answer in rectangular and polar forms, and draw the impedance diagram.



3-For the circuit shown below

- **a.** Find the total impedance \mathbf{Z}_T in polar form.
- **b.** Draw the impedance diagram.
- **c.** Find the current **I** and the voltages $\overline{\mathbf{V}}_R$ and \mathbf{V}_L in phasor form.
- d. Draw the phasor diagram of the voltages E, V_R, and V_L, and the current I.
- e. Verify Kirchhoff's voltage law around the closed loop.
- f. Find the average power delivered to the circuit.
- g. Find the power factor of the circuit, and indicate whether it is leading or lagging.
- **h.** Find the sinusoidal expressions for the voltages and current if the frequency is 60 Hz.
- Plot the waveforms for the voltages and current on the same set of axes.

4-Repeat problem 3 for the following circuit, replacing V_L with V_C in parts (c) and (d).



 $R = 8 \Omega$

 \mathbf{Z}_T

 $100 \text{ V} \angle 0^\circ$

 $= 6 \Omega$

I