UNIVERSITY OF ANBAR
COLLEGE OF ENGINEERING
CIVIL ENGINEERING DEPARTMENT

FLUID MECHANICS

LECTURES
FOR
UNDERGRADUATE STUDENTS
2nd GRADE

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Contents

CHAPTER ONE 3
INTODUCTION AND FUNDAMENTAL CONCEPTS 3
1. Introduction 3
2. Civil Engineering Fluid Mechanics 3
3. System of units 4
4. Properties of Fluids 7
4.1 Mass Density (ρ) 7
4.2Weight Density; Specific Weight (γ) 7
4.3 Specific Gravity (r.d.) 7
4.4 Specific volume (∀) 8
4.5 Temperature (T) 8
4.6 Pressure (P) 8
4.7 Surface Tension (σ) 8
4.8 Compressibility (E) 9
4.9Viscosity (μ) 10
CHAPTER TWO 14
FLUID STATICS 14
1. Pressure Distribution in Fluids 14
2. Pressure at point (Pascal's Law) 15
3. Pressure Force on a Fluid Element. 17
3.1 Incompressible Fluid 19
3.2 Pressure Measurements 21
4. Manometers. 23
4.1 Piezometer Tube 23
4.2 The "U"-Tube Manometer 24
4.3Manometers to Measure Pressure Difference 25
5. Hydrostatic Forces on Plane Surface. 27
6. Hydrostatic Forces on Curved Surface. 35

CHAPTER THREE 38
FLUID DYNAMICS (BASIC EQUATIONS) 38
1. Introduction 38
2. Types of flow 38
3. Flow (Discharge) 39
3.1 Mass flow rate 39
3.2 Volume flow rate - Discharge. 39
3.3 Discharge and velocity 39
4. Basic equations 40
4.1 Continuity equation 40
4.2 Energy equation (Euler equation) 42
4.3 Conservation of Momentum (Momentum equation) 48
4.4Application of the Momentum Equation 50
4.4.1The forces due the flow around a pipe bend 50
4.4.2 Force on a pipe nozzle 52
4.4.3Impact of a Jet on a Plane 54
CHAPTER FOUR 60
FLOW IN CONDUITS 60
1. Real Fluids 60
Laminar and turbulent flow 60
2. Modified Bernoulli's Equation 63
DARCY-WEISBACH EQUATION 64
The Moody diagram for the Darcy-Weisbach friction factor f. 65
EMPIRICAL EQUATIONS 66
3. Simple Pipe Problem 67
3. Solution Procedures. 68
4. Pumps & Turbines 70
5. Minor Losses. 74
6. Pipe in Series 80
7. Pipes in Parallel. 82

CHAPTER *ONE*

INTODUCTION AND FUNDAMENTAL CONCEPTS

1. Introduction

Fluid mechanics is the study of fluids either in motion (fluid dynamics) or at rest (fluid statics) and the subsequent effects of the fluid upon the boundaries, which may be either solid surfaces or interfaces with other fluids.

There are two classes of fluids, liquids and gases. The distinction is a technical one concerning the effect of cohesive forces. A liquid, being composed of relatively close-packed molecules with strong cohesive forces, tends to retain its volume and will form a free surface in a gravitational field. Since gas molecules are widely spaced with negligible cohesive forces.

2. Civil Engineering Fluid Mechanics

Why are we studying fluid mechanics on a Civil Engineering course? The provisions of adequate water services such as the supply of potable water, drainage, sewerage are essential for the development of industrial society. It is these services which civil engineers provide.

Fluid mechanics is involved in nearly all areas of Civil Engineering either directly or indirectly. Some examples of direct involvement are those where we are concerned with manipulating the fluid:

- Sea and river (flood) defenses;
- Water distribution / sewerage (sanitation) networks;
- Hydraulic design of water/sewage treatment works;
- Dams;
- Irrigation;
- Pumps and Turbines;
- Water retaining structures.

And some examples where the primary object is construction - yet analysis of the fluid mechanics is essential:

- Flow of air in / around buildings;
- Bridge piers in rivers;
- Ground-water flow.

3. System of units

Thus far we have used familiar fluid properties such as pressure and density without definition. Before defining these and other fluid properties more precisely in the next chapter, it is worthwhile to review their dimensions and unit systems. Fluid mechanics embodies a wealth of fluid properties, many with distinctive units, and in a global economy it is important for an engineer to be able to work confidently in any customer's preferred unit system. A dimension is a physical variable used to specify some characteristic of a system. Examples include mass, length, time, and temperature. In contrast, a unit is a particular amount of a physical quantity or dimension. For example, a length can be measured in units of inches, centimeters, feet, meters, miles, furlongs, and so on. Consistent units for a variety of physical quantities can be grouped together to form a unit system.

There are three widely used systems of units in the word. These are

I. British or English system (it's not in official use now in Briton)

II. Metric system.

III. SI system (System International of Unites or International System of Units).

To avoid any confusion on this course we will always use the SI (metric) system - which you will already be familiar with. It is essential that all quantities are expressed in the same system or the wrong solutions will results.

The SI system consists of six ***primary*** units, from which all quantities may be described. For convenience ***secondary*** units are used in general practices which are made from combinations of these primary units.

In fluid mechanics there are only four primary dimensions from which all other dimensions can be derived: *mass, length, time, temperature*. These dimensions and their units in both systems are given in Table 1.1.

Notice how the term 'Dimension' of a unit has been introduced in this table. This is not a property of the individual units, rather it tells what the unit represents. For example a meter is a length which has a dimension L but also, an inch, a mile or a kilometer are all lengths so have dimension of L.(The above notation uses the MLT system of dimensions, there are other ways of writing dimensions – We will see more about this in the section of the course on dimensional analysis.)

Table 1.1: Primary dimensions in SI and BG system

Primary Dimension	**SI Unit**	**BG Unit**	**Conversion Factor**
Mass (M)	Kilogram(kg)	Slug	1 slug = 14.5939 kg
Length (L)	Meter (m)	Foot (ft)	1 ft = 0.3048 m
Time (T)	Second (sec)	Second (sec)	1 sec = 1 sec
Temperature (θ)	Kelvin (K)	Rankin (º R)	1K = 1.8 (º R)

Although a specific base dimension set may be freely chosen, other aspects of a valid dimensional system are restricted by the laws of physics. Every valid physical law can be cast in the form of a dimensionally homogeneous equation; i.e., the dimensions of the left side of the equation must be identical to those of the right side. Consider Newton's second law in the form

$F = m \times a$

We define the Newton and pound is the dimension of force

1 Newton of force = 1N = 1 kg.m/s^2

1 pound of force = 1 lb_f = 1 slug. ft/s^2 = 4.4482 N

The following table (Table 1.2) shows the dimensions of a variety of physical quantities in terms of the basic units mass, length, time (M,L,T) or force, length, time (F,L,T). The table also shows the preferred units for those quantities in both the International System (S.I.) and the British System of units. Additional units commonly used for the quantities listed are shown in the last column of the table.
A list of some important secondary variables in fluid mechanics with dimensions derived as combinations of the four primary dimensions is given in Table 1.2. A more complete list of conversion factors is given in the end of this chapter.

Table 1.2: Dimensions and units of measurement

Quantity	Dimensions (M,L,T)	Dimensions (F,L,T)	Preferred units S.I.	Preferred units E.S.	Other units
Length (L)	L	L	m	ft	in, mi
Time (T)	T	T	s	s	h, d
Mass (M)	M	FT^2L^{-1}	kg	slug	
Area (A)	L^2	L^2	m^2	ft^2	Ac
Volume (Vol)	L^3	L^3	m^3	ft^3	Ac-ft
Velocity (V)	LT^{-1}	LT^{-1}	m/s	ft/s or fps	--
Acceleration (a)	LT^{-2}	LT^{-2}	m/s^2	ft/s^2	--
Discharge (Q)	L^3T^{-1}	L^3T^{-1}	m^3/s	ft^3/s or cfs	--
Kinematic viscosity (ν)	L^2T^{-1}	L^2T^{-1}	m^2/s	ft^2/s	St
Force (F)	MLT^{-2}	F	N	lb	--
Pressure (p)	$ML^{-1}T^{-2}$	FL^{-2}	Pa	lb/ft^2	psi, atm
Shear stress (τ)	$ML^{-1}T^{-2}$	FL^{-2}	Pa	lb/ft^2	psi
Density (ρ)	ML^{-3}	FT^2L^{-4}	kg/m^3	$slug/ft^3$	--
Specific weight (ω)	$ML^{-2}T^{-2}$	FL^{-3}	N/m^3	lb/ft^3	--
Energy/Work/Heat (E)	ML^2T^{-2}	FL	J	lb ft	--
Power (P)	ML^2T^{-3}	FLT^{-1}	W	lb ft/s	hp
Dynamic viscosity (μ)	$ML^{-1}T^{-1}$	FTL^{-2}	$N\ s/m^2$	$lb\ s/ft^2$	P

The symbols for the units used in Table 1.2 are listed next:

Ac : acre, a unit of area	lb : pound
Ac-ft : acre × feet	m : meter
Atm. : atmosphere	N : newton
cfs : cubic feet per second	mi : mile
fps : feet per second	P : poise
ft : foot or feet	Pa : Pascal (N/m^2)
hp : horse power	psi : pounds per square inch
in : inch	sec : second
J : joule	St : stokes
kg : kilogram	W : watt
cfs : cubic feet per second	

4. Properties of Fluids

The properties outlines below are general properties of fluids which are of interest in engineering. The symbol usually used to represent the property is specified together with some typical values in SI units for common fluids. Values under specific conditions (temperature, pressure etc.) can be readily found in many reference books.

4.1 Mass Density (ρ)

Mass Density, ρ (rho) , is defined as the mass of substance per unit volume.

Units: Kilograms per cubic meter, kg / m^3

Dimensions: ML^{-3} (FT^2L^{-4})

Typical values:Water = 1000 kg/m^3, Mercury = 13600 kg/m^3

(At pressure =1.013×10^{-5} N/m^2 and Temperature = 288.15 º K "20 º C".)

4.2Weight Density; Specific Weight (γ)

The specific weight; weight density, γ (gamma) is defined as the weight of fluid per unit volume at the standard temperature and pressure, or the force exerted by gravity, g, upon a unit volume of the substance.

The Relationship between g and γ can be determined by Newton's 2nd Law, since

Weight per unit volume = mass per unit volume $\times$ g

$$\gamma = \rho \times g$$ Where g is the gravitational acceleration

Units: Newton's per cubic metre, N / m^3

Dimensions: $ML^{-2}T^{-2}$ (F/L^3)

Typical values: $\gamma_{water} = 9800 N/m^3$

4.3 Specific Gravity (r.d.)

The specific gravity of any substance is the ratio of the density of the substance to the density of the water. Specific density is usually represented by the symbol (sp.gr.) or sometimes by (r.d.) referring to the relative density. i.e.

(sp.gr.); r.d. = $\frac{\rho \text{ of fluid}}{\rho w}$ $r.d._{liquid} = \rho_{liquid} / \rho_w = \rho_{liquid} / 1000$

It is also described as $(\gamma / \gamma_{water})$

Units: None, since a ratio is a pure number

Dimensions: 1.

Typical values: Water = 1, Mercury = 13.6

4.4 Specific volume (V)

The specific volume v is the volume occupied by unit mass of fluid, thus

$$\forall = \frac{1}{\rho} \quad (m^3/kg)$$

4.5 Temperature (T)

The temperature is a measure of the internal energy level of a fluid.

4.6 Pressure (P)

The pressure is the stress at a point in a static fluid, and is the differences or gradients in pressure after drive a fluid flow especially in ducts.

Units: atm. , Pa, bar,

4.7 Surface Tension (σ)

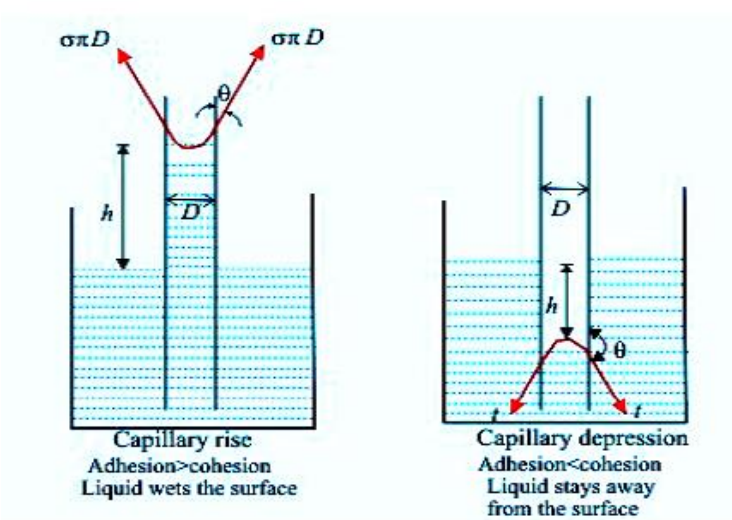
The phenomenon of surface tension, σ (sigma) arises due to the two kinds intermolecular forces.

I. Cohesion Force:- the force of attraction between the molecules of a liquid due to, they are bound to each other to remain as one assemblage of particles is known as the force of cohesion.

II. Adhesion Force:- The force of attraction between unlike molecules, i.e, between the molecules of different liquids or between the molecules of a liquid and those of solid body when they are in contact with each other.

Surface tension may also be defined as the work per unit area (N.m / m^2) or (N/m) required creating unit surface of the liquid.

When a liquid is in contact with a solid, if the forces of adhesion between the molecules of the liquid and the solid are greater than the forces of cohesion among the liquid molecules themselves, the liquid molecules crowd towards the solid surface. The area of contact between the liquid and solid increases and the liquid thus wets the solid surface



The reverse phenomenon takes place when the force of cohesion is greater than the force of adhesion. These adhesion and cohesion properties result in the phenomenon of capillarity by which a liquid either rises or falls in a tube dipped into the liquid depending upon whether the force of adhesion is more than that of cohesion or not.

Units: N/m

Dimensions: F/L.

Example

Derive an expression for the change in high (h) in a circular tube of a liquid with surface tension (σ) and contact angle (θ). As in figure below.

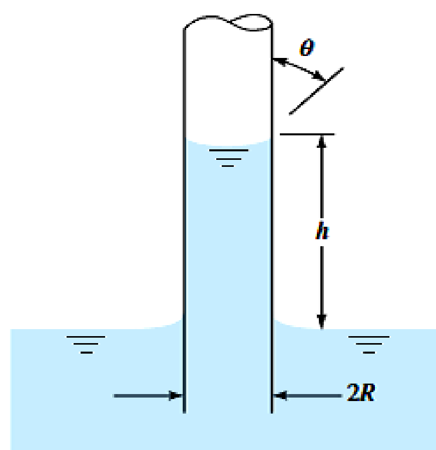
Sol.

The vertical component of the ring surface tension force at the interface in the tube must balance the weight of column of fluid of height (h).

$$2\pi.R.\sigma.\cos\theta = \rho.g.\pi.R^2.h$$

Solving for h, we have the desired result

$$h = \frac{2\sigma\cos\theta}{\gamma R} = \frac{4\sigma\cos\theta}{\gamma D}$$



Suppose that, the fluid is water having σ = 0.073 N/m, θ = 0°, ρ=1000 kg/m^3 and R=1mm, then find the capillary rise for the water-air-glass interface.

4.8 Compressibility (E)

The compressibility is the measure of its change in volume under the action of external forces. The degree of compressibility of a substance is characterized by the bulk modulus of elasticity E, defined as:

$$E = -\frac{\Delta p}{\Delta V/V}$$; the negative sign is to make (E) positive

The values of (E) for liquids are very high as compared with those of gases. Therefore the liquids are usually termed as incompressible fluids.

For example $E_{water} = 2*10^6$ kN/m^2

$E_{air} = 101$ kN/m^2

Indicates that the air is about (20000) times more compressible than water. Hence water can be treated as incompressible.

Units: N/m^2

Dimensions: F/L^2

Example

A liquid compressed in a cylinder has a volume of 1000 cm^3 at 1MN/m^2 and a volume of 995 cm^3 at 2 MN/m^2. What is its bulk modulus of elasticity (E)?

Sol.

$$E = -\frac{\Delta p}{\Delta V/V} = \frac{(2-1)\times 10^6}{(995-1000)\times 10^{-6}/\ (1000\times 10^{-6})} = 200\text{Mpa}$$

Example

If E=2.2 GPa is the bulk modulus of elasticity for water, what pressure is required to reduce a volume by 0.6 percent?

Sol.

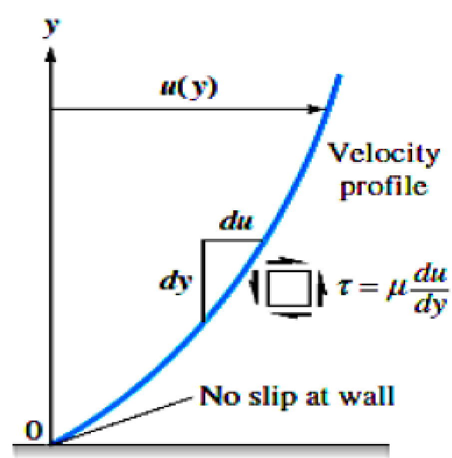
$$E = -\frac{\Delta p}{\Delta V/V} \rightarrow 2.2\times 10^9 = -\frac{p_2-0}{-0.006} \ ; \quad p_2 = 13.2\text{ MPa}$$

4.9Viscosity (μ)

Viscosity, μ (mu) , is the property of a fluid, due to cohesion and interaction between molecules, which offers resistance to shear deformation. Different fluids deform at different rates under the same shear stress. Fluid with a high viscosity deforms more slowly than fluid with a low viscosity such as water.

A fluid is defined as a material which will continue to deform with the application of a shear force. However, different fluids deform at different rates when the same shear stress (force/area) is applied.

If the force F acts over an area of contact A, then the shear stress τ (tao) is defined as $\tau = F/A$



All fluids are viscous, "Newtonian Fluids" obey the linear relationship given by Newton's law of viscosity.

$$\tau = \mu \frac{dv}{dy}$$

Where τ is the shear stress (N/m^2)

dv/dy represent the velocity gradient (1/sec.)

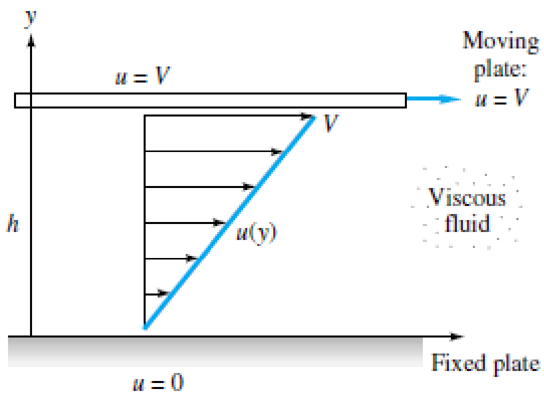
- The Coefficient of Dynamic Viscosity, μ , is defined as the shear force, per unit area, (or shear stress τ), required to drag one layer of fluid with unit velocity past another layer a unit distance away.

  Units: Newton seconds per square meter, (N.sec) / m^2 or Kilograms per meter per second, kg/(m.sec) .

  (Although note that μ is often expressed in Poise, where 10 Poise = 1 kg/(m. sec.))
- Ideal Fluid such a fluid having zero viscosity (μ=0) is called an ideal fluid and the resulting motion is called ideal fluid or in viscid flow. From this definition there is no existence of shear force.
- Real Fluid, All fluids in reality having viscosity ($\mu > 0.0$) are termed real fluid and their motion is known as viscous flow.
- A classic problem is the flow induced between a fixed lower plate and an upper plate moving steadily at velocity V, as shown in Fig. below. The clearance between plates is h, and the fluid is newtonian and does not slip at either plate.

Example

Suppose that the fluid being sheared in Fig. is SAE 30 oil at 20ºC (μ= 0.29 kg/(m.s). Compute the shear stress in the oil if the velocity (v) is 3 m/s and (h) is 2 cm.



Sol.

$$\tau = \mu \frac{dv}{dy} = \mu \frac{V}{h} = \frac{[0.29\ kg/(m.sec)](3\ m/sec)}{0.02\ m} =$$

$43 kg/(m.sec^2) = 43 N/m^2 = 43 Pa$.

- ***Kinematic Viscosity (ν)***

Kinematic Viscosity, ν (nu), is defined as the ratio of dynamic viscosity to mass density.

$$\nu = \frac{\mu}{\rho}$$

Units: square meters per second, m^2 /sec

(Although note that n is often expressed in Stokes, St, where 10^4 Stoke = 1 m^2 /sec.)

Dimensions: L^2 /T.

Multiplicative factor	Prefix
10^{12}	tera
10^9	giga
10^6	mega
10^3	kilo
10^2	hecto
10	deka
10^{-1}	deci
10^{-2}	centi
10^{-3}	milli
10^{-6}	micro
10^{-9}	nano
10^{-12}	pico
10^{-15}	femto
10^{-18}	atto

Secondary dimension	SI unit	BG unit	Conversion factor
Area $\{L^2\}$	m^2	ft^2	$1\ m^2 = 10.764\ ft^2$
Volume $\{L^3\}$	m^3	ft^3	$1\ m^3 = 35.315\ ft^3$
Velocity $\{LT^{-1}\}$	m/s	ft/s	1 ft/s = 0.3048 m/s
Acceleration $\{LT^{-2}\}$	m/s^2	ft/s^2	$1\ ft/s^2 = 0.3048\ m/s^2$
Pressure or stress $\{ML^{-1}T^{-2}\}$	$Pa = N/m^2$	lbf/ft^2	$1\ lbf/ft^2 = 47.88$ Pa
Angular velocity $\{T^{-1}\}$	s^{-1}	s^{-1}	$1\ s^{-1} = 1\ s^{-1}$
Energy, heat, work $\{ML^2T^{-2}\}$	$J = N \cdot m$	$ft \cdot lbf$	$1\ ft \cdot lbf = 1.3558$ J
Power $\{ML^2T^{-3}\}$	W = J/s	$ft \cdot lbf/s$	$1\ ft \cdot lbf/s = 1.3558$ W
Density $\{ML^{-3}\}$	kg/m^3	$slugs/ft^3$	$1\ slug/ft^3 = 515.4\ kg/m^3$
Viscosity $\{ML^{-1}T^{-1}\}$	$kg/(m \cdot s)$	$slugs/(ft \cdot s)$	$1\ slug/(ft \cdot s) = 47.88\ kg/(m \cdot s)$
Specific heat $\{L^2T^{-2}\Theta^{-1}\}$	$m^2/(s^2 \cdot K)$	$ft^2/(s^2 \cdot {}^\circ R)$	$1\ m^2/(s^2 \cdot K) = 5.980\ ft^2/(s^2 \cdot {}^\circ R)$

Volume:

	cm^3	ft^3	in^3	U.S Gallon	U.K Gallon	m^3	Liter
cm^3	1.0	3.531467e-5	0.061024	0.0002641721	0.0002199692	1e-6	0.001
ft^3	28316.85	1.0	1728	7.480519	6.228833	0.02831685	28.31685
in^3	16.38706	0.0005787037	1.0	0.004329004	0.003604649	1.6387e-5	0.016387
U.S Gallon	3785.412	0.1336806	231	1.0	0.8326738	0.00378541	3.785412
U.K Gallon	4546.092	0.1605437	277.4196	1.20095	1.0	4.5461e-3	4.5461

Length:

	cm	ft	in	yard	mile	meter	km
cm	1.0	0.0328084	0.3937008	0.01093613	6.213712E-6	0.01	1E-5
ft	30.48	1.0	12.0	0.3333333	0.0001893939	0.3048	0.0003048
in	2.54	0.08333333	1.0	0.02777778	1.578283E-5	0.0254	2.54E-5
yard	91.44	3.0	36.0	1.0	0.0005681818	0.9144	0.0009144
mile	160934.4	5280	63360	1760	1.0	1609.344	1.609344
meter	100.0	3.28084	39.37008	1.093613	0.0006213712	1.0	0.001
Km	100000	3280.84	39370.08	1093.613	0.6213712	1000	1.0

CHAPTER TWO

FLUID STATICS

1. Pressure Distribution in Fluids

Many fluid problems do not involve motion. They concern the pressure distribution in a static fluid and its effect on solid surfaces and on floating and sub- merged bodies. When the fluid velocity is zero, denoted as the hydrostatic condition, the pressure variation is due only to the weight of the fluid. Assuming a known fluid in a given gravity field, the pressure may easily be calculated by integration. Important applications in this chapter are:

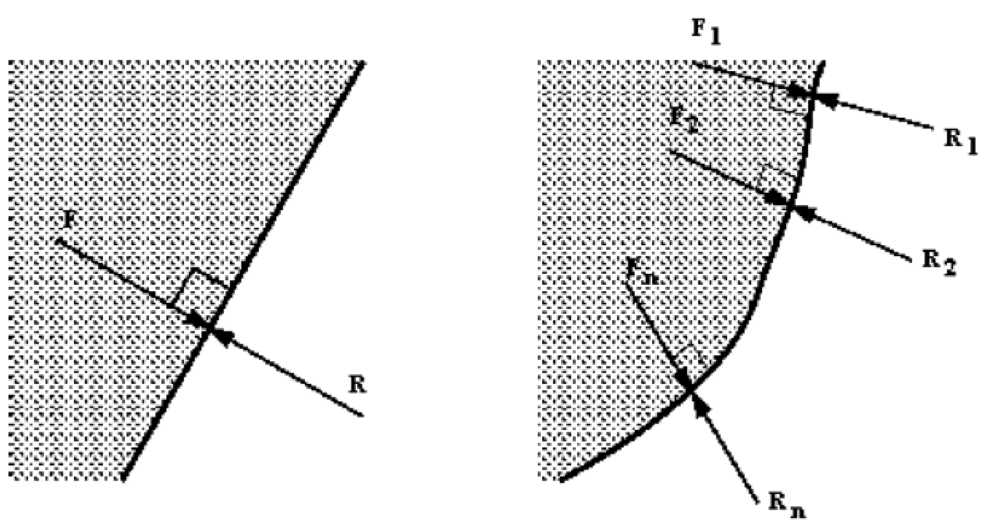
I. Pressure distribution in the atmosphere and the oceans

II. The design of manometer pressure instruments.

III. Forces on submerged flat and curved surfaces.

The general rules of statics (as applied in solid mechanics) apply to fluids at rest. From earlier we know that:

- a static fluid can have *no shearing force* acting on it, and that
- Any force between the fluid and the boundary must be acting at right angles to the boundary.



Note that this statement is also true for curved surfaces; in this case the force acting at any point is normal to the surface at that point. The statement is also true for any imaginary plane in a static fluid. We use this fact in our analysis by considering elements of fluid bounded by imaginary planes.

We also know that:

- For an element of fluid at rest, the element will be in equilibrium - the sum of the components of forces in any direction will be zero.
- The sum of the moments of forces on the element about any point must also be zero.
- If the force exerted on each unit area of a boundary is the same, the pressure is said to be *uniform*.

It is common to test equilibrium by resolving forces along three mutually perpendicular axes and also by taking moments in three mutually perpendicular planes and to equate these to zero.

$$\text{pressure} = \frac{\text{Force}}{\text{Area over which the force is applied}}$$

$$p = \frac{F}{A}$$

Units: Newton's per square meter, N/m^2, kg/ (m. sec.)

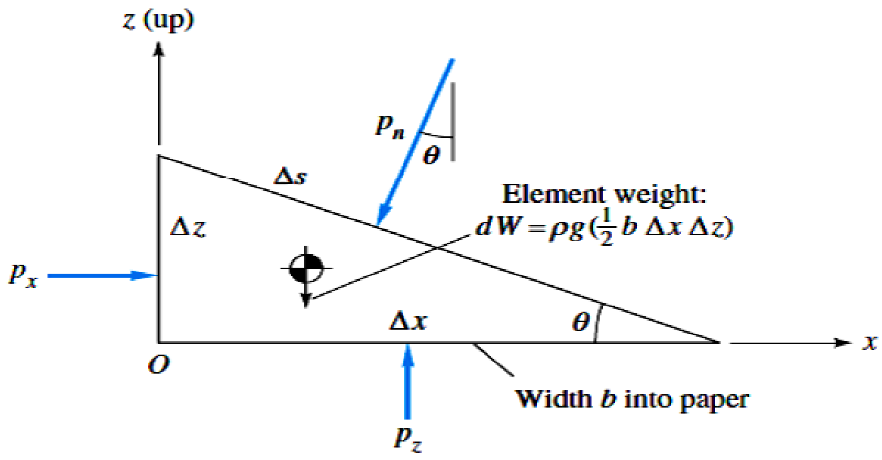
(The same unit is also known as a Pascal, Pa, i.e. $1Pa = 1\ N/m^2$)

(Also frequently used is the alternative SI unit the bar, where $1bar = 10^5\ N/m^2$).

2. Pressure at point (Pascal's Law)

(Proof that pressure acts equally in all directions.)

Consider a small wedge fluid element at rest of size (Δx , Δz by ΔS) and depth (b) into the paper by definition there is no shear stress , but we postulate that the pressures px, pz and pn as shown in Figure below.



Equilibrium of a small wedge of fluid at rest.

Summation of forces must equal zero (no acceleration) in both x &z directions

$$\sum Fx=0 = p_x\, b\ \Delta z - p_n\, b\, \Delta s \sin\theta$$

$$\sum Fz =0 = p_z\, b\, \Delta x - p_n\, b\Delta S \cos\theta - \frac{1}{2}\gamma\, b\Delta x\, \Delta z \tag{2.1}$$

But the geometry of the wedge is such that

$$\Delta s \sin\theta = \Delta z\ ,\ \Delta s \cos\theta = \Delta x \tag{2..2}$$

Substitution into Eq. (2.1) and rearrangement give

$$p_x = p_n\quad ,\ p_z = p_n + \frac{1}{2}\gamma\Delta z \tag{2.3}$$

From relation 2.3 illustrate two important principles of hydrostatic

a) There is no pressure change in the horizontal direction.

b) There is a vertical change in pressure proportional to the density, gravity and depth change.

In the limit as the fluid wedge shrinks to a (point) $\Delta z \rightarrow 0$.Then, Eq. 2.3 become

$$px = pz = pn = p \tag{2.4}$$

Since θ is arbitrary, we conclude that the pressure p at a point in a static fluid is independent of orientation.

In fluid under static conditions pressure is found to be independent of the orientation of the area. This concept is explained by Pascal's law which states that the pressure at a point in a fluid at rest is equal in magnitude in all directions.

3. Pressure Force on a Fluid Element.

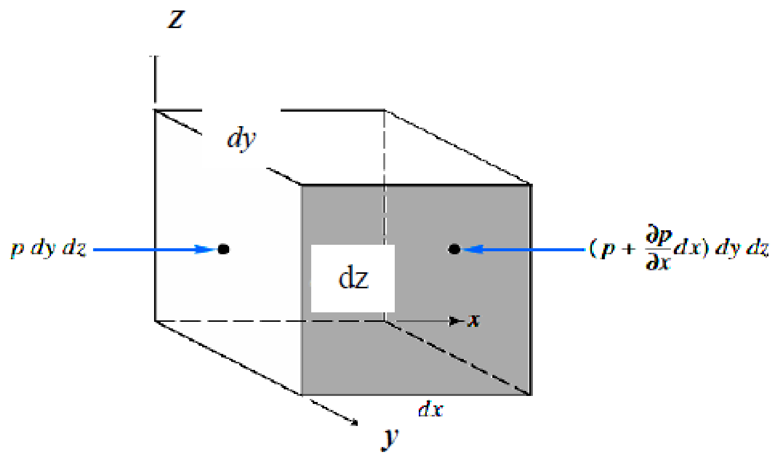
Let the pressure vary arbitrarily p = p(x,y,z,t) consider the pressure acting on the two x-faces as in Figure below . The net force in the x-direction on the element is given by

$$dF_x = pdy\,dz - (p+\frac{\partial p}{\partial x}dx)dy\,dz = -\frac{\partial p}{\partial x}\,dx\,dy\,dz \qquad (2.6)$$

In like manner the net force dF_y involves $(-\frac{\partial p}{\partial y})$, and the net force dF_z concerns $(-\frac{\partial p}{\partial z})$

the total net –force vector on the element due to pressure is

$$dF_{press} = (-i\frac{\partial p}{\partial x} - j\frac{\partial p}{\partial y} - k\frac{\partial p}{\partial z})\,dz\,dy\,dz \qquad (2.7)$$



Net x force on an element due to pressure variation.

Rewrite Eq. 2.7 as the net force per unit element volume and is denoted by (f)

$$f_{press} = -\nabla p \qquad (2.8)$$

Thus is the pressure gradient causing a net force which must be balanced by gravity or acceleration.

The pressure gradient is a surface force which acts on the sides of the element. Also, may be a body force, due to electromagnetic or gravitational potentials acting on the entire mass of the element. Consider only the gravity force or weight of element

$$\begin{Bmatrix} d\tilde{F}_{grav} = \rho g dxdydz \\ f_{grav} = \rho g \end{Bmatrix} \qquad (2.9)$$

For an incompressible fluid with constant viscosity the net viscous force is or (viscous stress)

$$f_{vs} = \mu\left(\frac{\partial^2 \bar{V}}{\partial x^2} + \frac{\partial^2 \bar{V}}{\partial y^2} + \frac{\partial^2 \bar{V}}{\partial z^2}\right) = \mu \nabla^2 \bar{V} \qquad (2.10)$$

Where the subscript (vs) stands for viscous force, note that the term (g) in Eq. 2.9 denotes the acceleration of gravity, a vector acting toward the center of the earth. On earth the average magnitude of (g) is 32.174 ft/sec^2 = 9.807 m/sec^2 in our lectures and exercises we use the approximate numerical value of g = 32.2 ft/sec^2 = 9.81m/sec^2.

The total vector resultant of these three forces which are pressure, gravity, and viscous stress must either keep the element in equilibrium or cause it to move with acceleration (a). Form Newton's law of motion per unit volume:

$$\sum f = \rho \mathbf{a} = f_{press} + f_{grav} + f_{vs} = -\nabla p + \rho g + \mu \nabla^2 V \qquad (2.11)$$

Rewrite Eq. 2.11 as follows

$$\nabla p = \rho(g - a) + \mu \nabla^2 V \qquad (2.12)$$

1- Flow at rest or at constant velocity: The acceleration and viscous terms vanishes identically, and p depends only upon gravity and density. This is the hydrostatic condition.
2- When the fluid at rest or at constant velocity, a = 0 and $\nabla^2 V$ Eq.2.12 for the pressure distribution reduces to $\nabla p = \rho g$ (2.13)

This is a hydrostatic distribution formula and is correct for all fluid at rest. Where (g) is the magnitude of local gravity, Eq. 2.13 has the pressure components are

$$\frac{\partial p}{\partial x} = 0, \quad \frac{\partial p}{\partial y} = 0, \quad \frac{\partial p}{\partial z} = -\rho g = -\gamma \qquad (2.14)$$

Where the coordinate system z is up i.e (p) is independent of x&y. Hence $\frac{\partial p}{\partial z}$ can be replaced by the total derivative $\frac{dp}{dz}$ and the hydrostatic condition reduce to

$$\frac{dp}{dz} = -\gamma \qquad (2.15)$$

Equation 2.15 is the fundamental equation for fluids at rest and can be used to determine how pressure change with elevation. This equation indicates that the pressure gradient in the vertical direction is negative; that is, the pressure decrease as we move upward in a fluid at rest.

This leads to the statement,

I. The pressure will be the same at the same level in any connected static fluid and at all points on a given horizontal plane whose density is constant or a function of pressure only.

II. The pressure increases with depth of fluid.

III. The pressure is independent of the shape of the container and the free surface of a liquid will seek a common level in any container, where the free surface is everywhere exposed to the same pressure. Equation 2.15 is the solution to the hydrostatic problem.

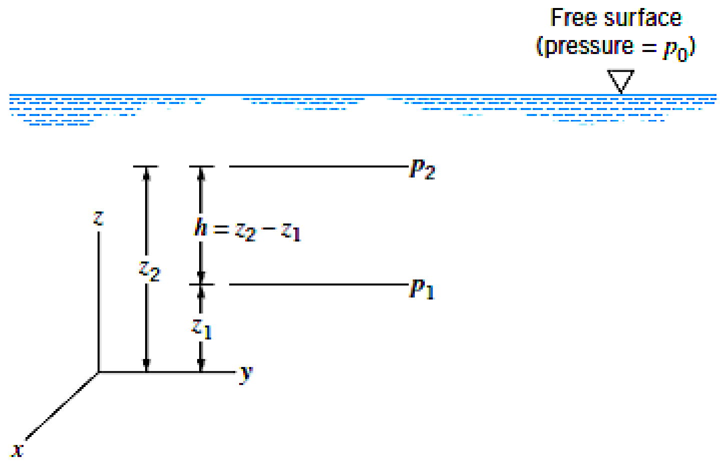
3.1 Incompressible Fluid

For liquids the variation in density is usually negligible, even over large vertical distances, so that the assumption of constant specific weight when dealing with liquids is a good one. For this instant, Eq. 2.15 can be directly integrated

$\int_{p_1}^{p_2} dp = -\int_{p_1}^{p_2} \gamma dz$ to yields $\quad p_2 - p_1 = -\gamma(z_2 - z_1)$

Or $$p_1 - p_2 = \gamma(z_2 - z_1) \qquad (2.16)$$

Where p_1 and p_2 are pressures at the vertical elevation, z_1 and z_2 as illustrated in Figure below. Eq. 2.16 can be written in compact form



$$p_1 - p_2 = \gamma \times (h) \quad \text{Or} \quad p_1 = \gamma \times (h) + p_2 \qquad (2.17)$$

Where h is the distance, z_2-z_1. This type of pressure distribution is commonly called a hydrostatic distribution. Eq. 2.17 shows that in an incompressible fluid at rest the pressure varies linearly with depth. It can also be observed from Eq. 2.17 that the pressure difference between two points can be specified by the distance h since

$$h = \frac{p_1 - p_2}{\gamma} \tag{2.18}$$

Where h is called the pressure head and is interpreted as the height of a column of fluid of specific weight γ to give a pressure difference ($p_1 - p_2$).

If p_0 is the reference pressure would be the pressure acting on the free surface, then from Eq. 2.17 the pressure at any depth h below the free surface is given by the following:

$$p = p_0 + \gamma(h) \tag{2.19}$$

Example

We can quote a pressure of 500K N/ m^2 in terms of the height of a column of water.

Sol.

$$h = \frac{p}{\rho g} = \frac{500\times10^3}{1000\times9.81} = 50.95m \text{ of water}$$

And in terms of Mercury with density, $\rho = 13.6\times10^3 kg\, m^{-3}$.

$$h = \frac{500\times10^3}{13.6\times10^3\times9.81} = 3.75m \text{ of Mercury}$$

Example

The deepest point in the ocean is (11034m) in the pacific. At this depth γ=10520 N/m^3. Estimate the absolute pressure at this depth.

Sol.

$p_{absolute} = \gamma(h) + p_{atm.}$ 10520*11034 + 101350= 116179030 N/m^2 = ***116.18 MPa***

Example

A closed tank contains 1.5 m of SAE 30 oil (γ = 8720N/m^3), 1m of water, 20 cm of mercury (γ = 133100N/m^3) and an air space on top all at 20ºC. If p_{bottom}= 60000 Pa, what is the pressure in the air space.

Sol.

Apply the hydrostatic formula down through the three layers of fluid.

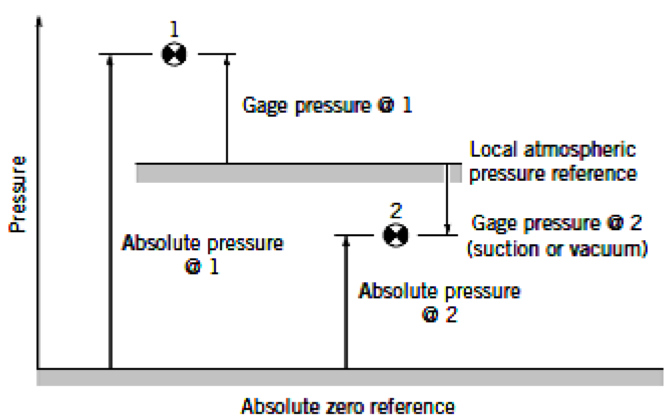
$$p_{bottom} = p_{air} + \gamma_{oil} * h_{oil} + \gamma_{water} * h_{water} + \gamma_{mercury} * h_{mercury}$$

$$60000 = p_{air} + 8720 * 1.5 + 9800 * 1 + 133100 * 0.2 \rightarrow \boldsymbol{p_{air} = 10580\ Pa}$$

3.2 Pressure Measurements

The unit of pressure in the SI system is (N/m^2) also called Pascal (Pa). The atmospheric pressure is approximately (10^5 N/m^2) is and designated as "bar". From above definition the pressure at a point within a fluid mass will be designated as either an *absolute pressure* or a *gauge pressure*. Absolute pressure is measured relative to a perfect vacuum (*absolute zero pressure*), where as gauge pressure is measured relative to the local atmospheric pressure. Thus, a gauge pressure of zero corresponds to a pressure that is equal to the local atmospheric pressure. Absolute pressures are always positive, but gage pressure can be either positive or negative depending on whether the pressure is above or below atmospheric pressure. A negative gauge pressure is also referred to as a *suction* or *vacuum* pressure. The concept of gauge and absolute pressure is illustrated graphically in figure below for two typical pressures located at points 1 and 2. Gauge pressure is the difference between the value of the pressure and the local atmospheric pressure (p_{atm}.)

$p_{gauge} = p - p_{atm}$.



The measurement of atmospheric pressure is usually accomplished with a mercury barometer, which in its simplest form consists of a glass tube closed at one end with the open end immersed in a container of mercury as shown in figure below. The tube is initially filled with mercury (inverted with its open end up) and then turned upside down (open end down) with the open end in the container of mercury. The column of mercury will come to an equilibrium position where its weight plus the force due to the vapor pressure (which develops in the space above the column) balances the force due to the atmospheric pressure. Thus,

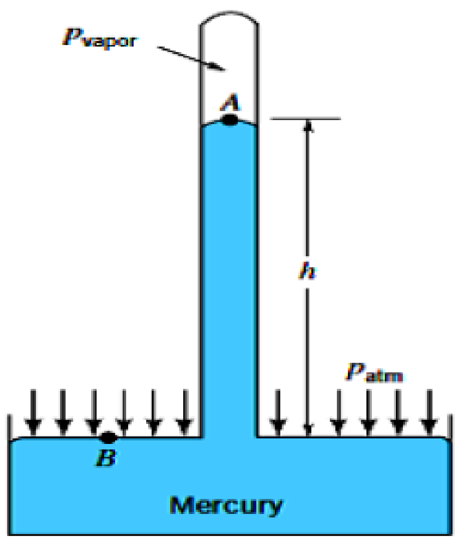
$$p_{atm.} = \gamma h + p_{vapor}$$

The vapor pressure pvapor can be neglected in most practical cases in comparison to $p_{atm.}$, since it's very small for mercury, pvapor= 0.16*$p_{atm.}$. So that,

$$p_{atm.} = \gamma h$$

$$\therefore\ h = \frac{p_{atm.}}{\rho * g} = \frac{1.0132*10^5 \left({}^{N}/_{m^2}\right)}{13560\left({}^{kg}/_{m^3}\right)*9.81\left({}^{N}/_{kg}\right)} = 0.761\ m\ of\ (Hg)$$

If water was used the value of ***h*** will be equal to ***10.32 m***



<u>Example</u>

What will be the (a) the gauge pressure , (b) the absolute pressure of water at depth 12m below the surface ?

<u>Sol.</u>

(a) $p_{gage} = \rho g h = 1000 * 9.81 * 12 = 117720 \frac{N}{m^2}, (\boldsymbol{Pa})$

(b) $p_{abs.} = p_{gage} + p_{atm.} = (117720 + 101 * 10^3) = 218720 \frac{N}{m^2} = 218.72 \frac{kN}{m^2}, (kPa)$

4. Manometers.

The manometers are the standard technique for measuring pressure involves the use of liquid columns in vertical or inclined tubes. Pressure measuring devices based on this technique are called *manometers*. Three common types of manometers include the piezometer tube, the U-tube manometer, and the inclined-tube manometer.

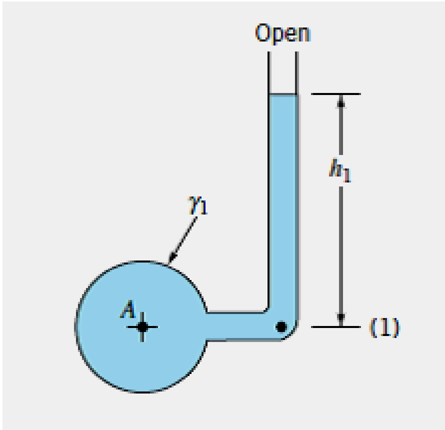
4.1 Piezometer Tube

The simplest type of manometer consists of a vertical tube, open at the top, and attached to the container in which the pressure is desired, as illustrated in figure below. Since manometers involve columns of fluids at rest, the fundamental equation describing their use is Eq. 2.19

$$p = p_0 + \gamma h$$

This gives the pressure at any elevation within a homogeneous fluid in terms of a reference pressure p_0 and the vertical distance $\boldsymbol{h}$ between $\boldsymbol{p}$ and $\boldsymbol{p_0}$. Remember that in a fluid at rest pressure will increase as we move downward and will decrease as we move upward. Application of this equation to the piezometer tube of figure shown indicates that the pressure $\boldsymbol{p_A}$ can be determined by a measurement of $\boldsymbol{h}$ through the relationship

$$p_A = \gamma_1 h_1$$

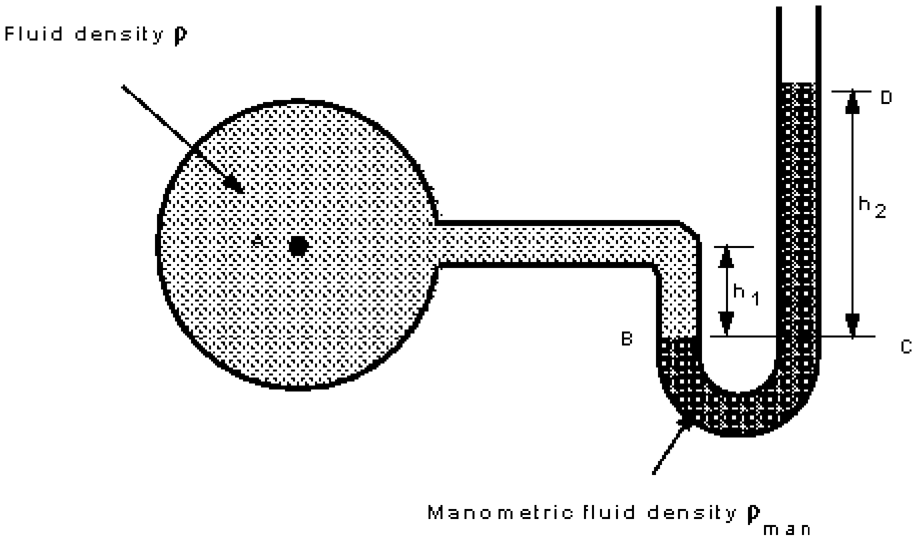


The tube is open at the top, the pressure p_0 can be set equal to *zero* as using a gauge pressure, with the height h_1 measured from the meniscus at the upper surface to point (1) then

$$h_1 = \frac{p_A}{\rho g}$$

4.2 The "U"-Tube Manometer

Manometers are devices in which columns of a suitable liquid are used to measure the difference in pressure between two points or between a certain point and the atmosphere. Manometer is needed for measuring large gauge pressures. It is basically the modified form of the piezometric tube. A common type manometer is like a transparent "U-tube" as shown in figure below.



A "U"-Tube manometer

Pressure in a continuous static fluid is the same at any horizontal level so,

Pressure at B = Pressure at C

$$p_B = p_C$$

For the *left hand* arm

Pressure at B = pressure at A + pressure due to height ***h₁*** of fluid being measured

$$p_B = p_A + \gamma h_1$$

For the *right hand* arm

Pressure at C = pressure at D + pressure due to height ***h₂*** of manometric fluid

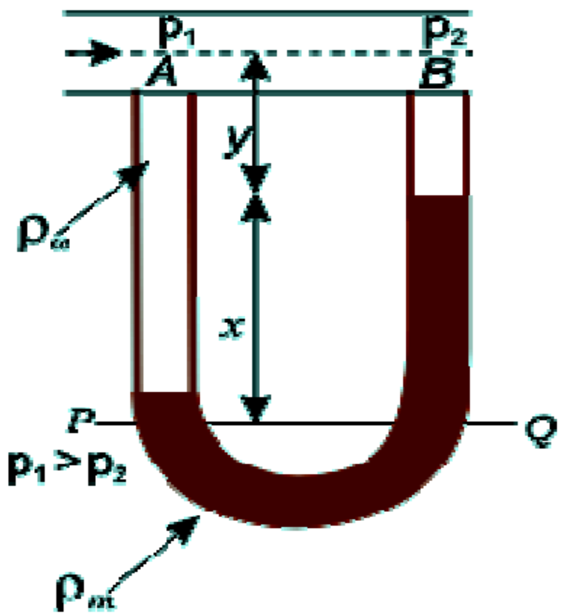
$$p_C = p_{atm.} + \gamma h_2$$

As we are measuring *gauge pressure* we can subtract $p_{atm.}$, then we can calculate p_A as

$p_A = \gamma_2 h_2 - \gamma_1 h_1$

If the fluid being measured is a gas, the density will probably be very low in comparison to the density of the manometric fluid. In this case the term $\gamma_1 h_1$ can be neglected.

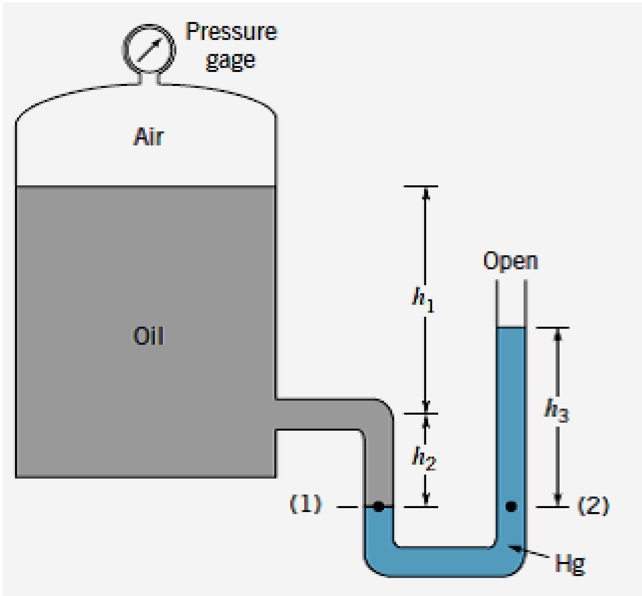
4.3Manometers to Measure Pressure Difference



$$p_1 + (y + x)\rho_w g = p_2 + y\rho_w g + \rho_m g x$$
$$p_1 - p_2 = (\rho_m - \rho_w)gx$$

Example

A closed tank contains oil and compressed air (r.d.$_{oil}$ = 0.9) as is shown in the following figure, a U-tube manometer using mercury is connected to a tank as shown. For column heights h$_1$=914.5mm, h$_2$=152.4mm and h$_3$= 228.6mm. Determine the pressure reading in Pa of the gauge.



Sol.

The pressure at level (1) is equal to the pressure at level (2), since these two points are at the same elevation in a homogeneous fluid at rest. The pressure at level (1) is

$$p_1 = p_{air} + \gamma_{oil}\,(h_1 + h_2)$$

The pressure at level (2) is

$$p_2 = \gamma_{Hg} \times h_3$$

Thus, the manometer equation can be expressed as

$$\therefore \quad p_{air} + \gamma_{oil}(h_1 + h_2) - \gamma_{Hg}\,h_3 = 0$$

Or

$$p_{air} + r.d._{oil}\gamma_w\,(h_1 + h_2) - r.d._{Hg}\gamma_w h_3 = 0$$

p_{air}= −0.9 * 9800 * (0.9145 + 0.1524) + 13.6 * 9800 * 0.2286 = =21079.23 N/m^2(Pa.)

Example

(a) At what depth below the surface of oil, specific gravity is 0.8 will produce a pressure of 120 kN/m^2?. (b) What depth of water is this equivalent to?.

Sol.

(b) Sp.gr.$= \frac{\rho_{oil}}{\rho_w}$ ρ_{oil} = Sp.gr. * ρ_w = 0.8 *1000 = 800 kg/m^3

(a) $h = \frac{p}{\rho * g} = \frac{120*10^3}{800*9.81} = 15.29\ m\ of\ oil$

(b) $h = \frac{p}{\rho * g} = \frac{120*10^3}{1000*9.81} = 12.23\ m\ of\ water$

Example

A manometer connected to a pipe indicates a negative gauge pressure of 50mm of mercury. What is the absolute pressure in the pipe in Newtons per square meter?

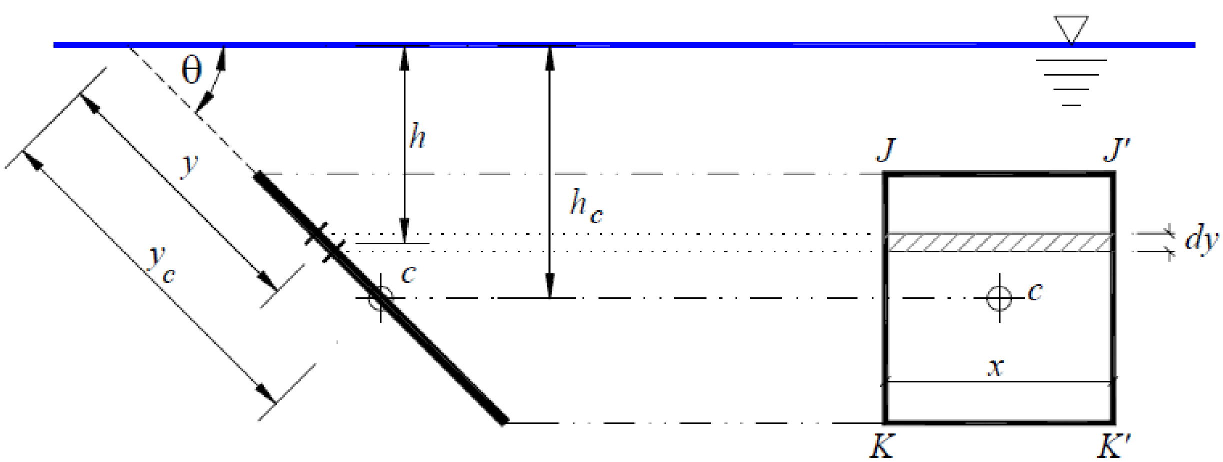
Sol.

$p_{abs.} = p_{gauge} + p_{atm.} = \gamma(h) + p_{atm.}$ = - 13.6 * 9800 *0.05 + 10^5 = 93329 (Pa)= 93.329 (kPa)

5. Hydrostatic Forces on Plane Surface.

Any hydro structure design required a computation of the hydrostatic forces on various solid surfaces contact with fluid. We wish to determine the direction, location and magnitude of the resultant force acting on one side of the surface area due to the liquid in contact.

Step 1. Find the resultant force.



Choose an element of area so that the pressure on it is uniform. Such an element is a horizontal strip with a width equal to x, so, dA = xdy . Notice that the width of the surface is not constant. Total force on the whole surface will equal to the summation of forces on all elements of dy:

Total force = F = ΣdF

Force = pressure × area

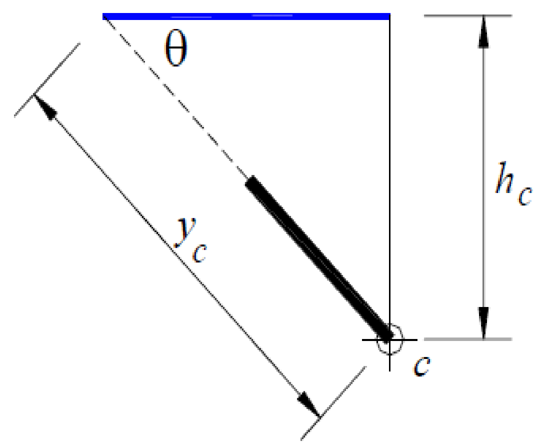
F =ΣpdA

The mathematical equivalent to the summations is integration (when the strip height "dy" is too small)

$F = \int p\, d_A$ p= $\gamma\, h$

h = y sinθ → p = γ (y sinθ)

∴ $F = \int \gamma$ (y sinθ) $d_A \rightarrow F = \gamma$sinθ $\int$ y d_A



$\int$ y d_A is the mathematical expression for the ***first moment of area***.

The first moment of area equals to the sum of the products of area times distance to the centroid.

Where:

y_c the distance from the fluid surface to the centroid of the plan surface along the inclined plan surface (in vertical surfaces, $y_c = h_c$).

→ $F = \gamma$sinθ $(y_c * A)$, but h = y sin θ

$$\therefore F = \gamma * h_c * A$$

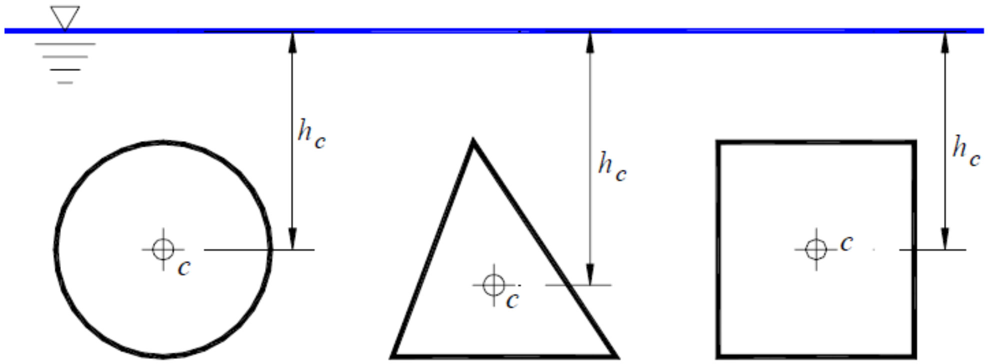
Where:

F Pressure force (normal to the surface)

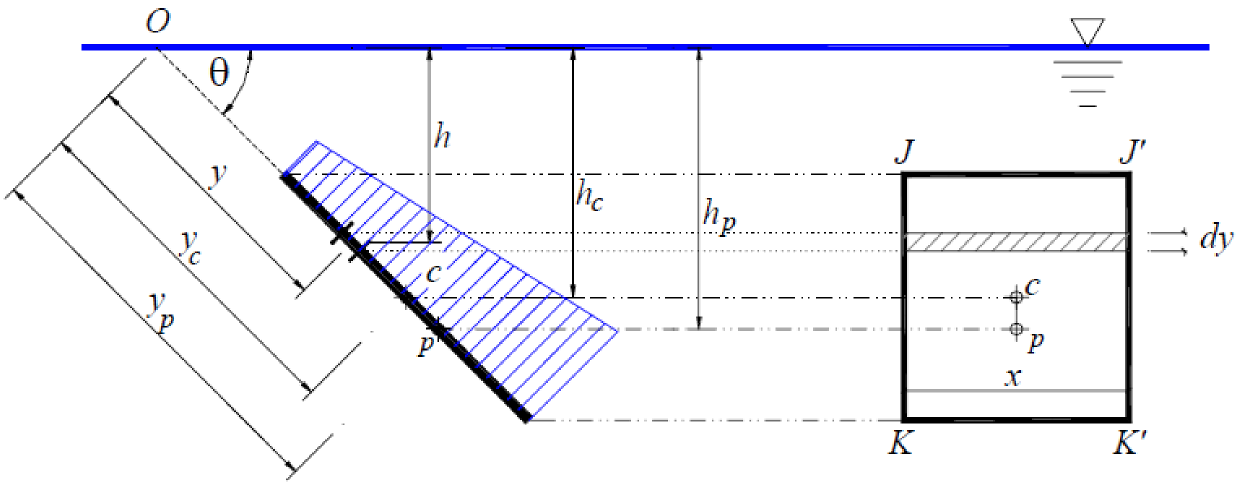
γ Specific weight of fluid (for water, γ = 9800 N/m^3 or γ = 62.4 lb/ft^3

A Cross sectional area of the plan surface.

h_c Vertical distance from the surface of fluid to the centroid of the plan surface.



Step 2.Find the location where the resultant force is applied.



Force = pressure × area

$dF = pdA \qquad p = \gamma h \qquad h = y\sin\theta \rightarrow p = \gamma(y\sin\theta)$

$dF = \gamma(y\sin\theta)dA$

To find the point of application of the pressure force, take the summation of moments of forces at O:

$\sum M_o = \sum y.dF$

Substitute for the value of dF:

$\sum M_o = \sum y[\gamma(y\sin\theta)]dA$

The summation can be replaced with integration when the step is too small:

$\sum M_o = \int \gamma(y^2\sin\theta)dA$

For the whole system to be stable, the sum of moments should equal to the moment of the pressure force F ($F \times y_p$)

Where y_p is the distance from the fluid surface to the center of pressure of the surface along the inclined plan.

$F \times y_p = \int \gamma(y^2\sin\theta)dA \qquad \rightarrow F \times y_p = \gamma\sin\theta\int y^2 dA$

$\int y^2 dA$ is the second moment of area (moment of inertia of the area) about $\boldsymbol{O} = I_o$

And $F = \gamma h_c A = \gamma(y_c\sin\theta)A$

$(\gamma \times y_c\sin\theta \times A)y_p = \gamma\sin\theta \times I_o$

$$y_p = \frac{I_o}{y_c A}$$

The above equation can be expressed in another form:

$I_o = Ay_c^{\,2} + I_c \qquad$ (***Parallel axes theorem***)

If a body has moment of inertia $\boldsymbol{I_c}$ about an axis $\boldsymbol{C}$ through its center of mass (C. G.), and if $\boldsymbol{O}$ is another axis parallel to $\boldsymbol{C}$ at a distance $\boldsymbol{y_c}$ from it, then the body's moment of inertia about $\boldsymbol{O}$ is $I_o = Ay_c^{\,2} + I_c$.

$$y_p = \frac{Ay_c^{\,2} + I_c}{y_c A}$$

Then,

$$\therefore y_p = y_c + \frac{I_c}{y_c A}$$

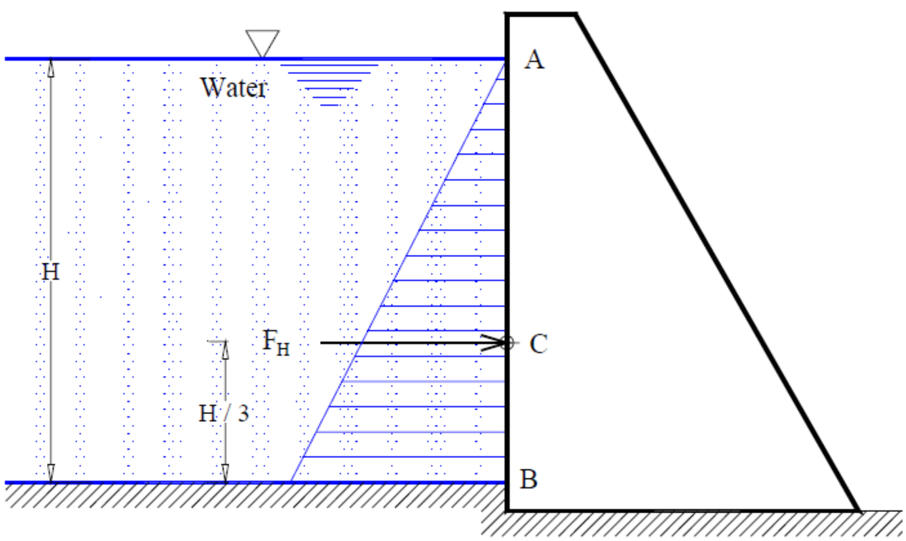
Summary of Results

- ***Hydrostatic Force:*** $F = \gamma * h_c * A$
- ***Acting at Location:*** $y_p = y_c + \frac{I_c}{y_c A}$

Supplementary Notes

1- Forces on vertical plan surfaces:

a. The surface intersects with fluid surface



Force magnitude: $F_H = \gamma * h_c * A$

Where cross sectional area of the plan surface (= distance AB × width).

$$F_H = \gamma\left(\frac{H}{2}\right)(H \times W)$$

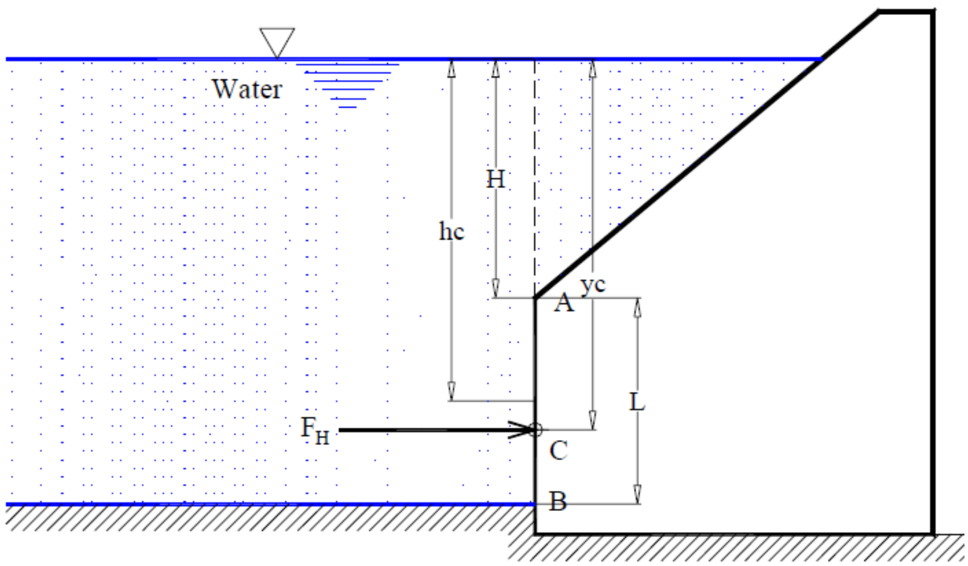
W width of the plan surface (and you can take it equal to unity, and that will give results per unit width)

$$F_H = \frac{1}{2}\gamma.H^2$$

Point of action (location):

The line of action of the force is at distance H/3 from the bottom.

b. The plan surface does not intersect with the fluid surface:



Force magnitude: $F_H = \gamma * h_c * A$

$$h_c = H + \frac{L}{2}$$

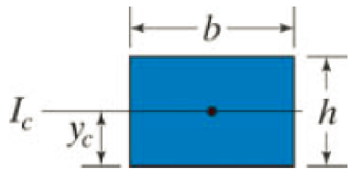
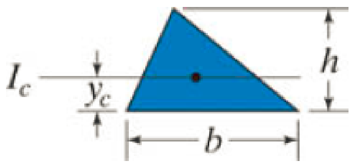
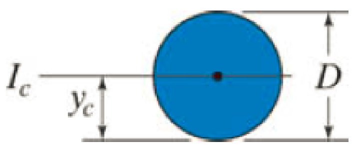
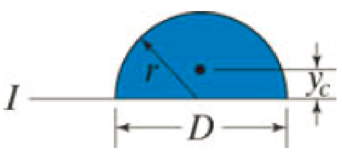
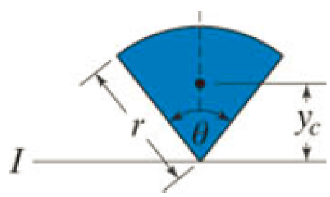
Point of action (location):

The line of action of the force is at distance = yc, from the fluid surface:

$$y_p = y_c + \frac{I_c}{y_c.A}$$

Where:

y_p	The distance from the fluid surface to the center of pressure of the surface along the inclined plan.
y_c	the distance from the fluid surface to the centroid of the plan surface along the inclined plan surface (in vertical surfaces, $y_c = h_c$).
I_c	Moment of inertia of the plan surface along its centroidal axis.

Shape	Sketch	Area	Location of Centroid	Ic
		$b \times h$	$y_c = \frac{h}{2}$	$I_c = \frac{bh^3}{12}$
		$\frac{bh}{2}$	$y_c = \frac{h}{3}$	$I_c = \frac{bh^3}{36}$
		$\frac{\pi D^2}{4}$	$y_c = \frac{D}{2}$	$I_c = \frac{\pi D^4}{64}$
		$\frac{\pi D^2}{8}$	$y_c = \frac{4r}{3\pi}$	$I = \frac{\pi D^4}{128}$
		$\frac{\theta r^2}{2}$	$y_c = \frac{4r}{3\theta}\sin\frac{\theta}{2}$	$I = \frac{r^4}{8}(\theta + \sin\theta)$

Example

Calculate the force (P) needed to maintain the gate as shown in “figure (1)”, where a layer of oil rests on top of water. The width of the gate is (6m) into the page. Neglect the weight of the gate.

Sol .

0.8 * 9800 *2 = 15680Pa.

15680 = 9800*y $\rightarrow$ y = 1.6m of water

Water = 1.6+6 = 7.6m

$F = \gamma * h_c * A$

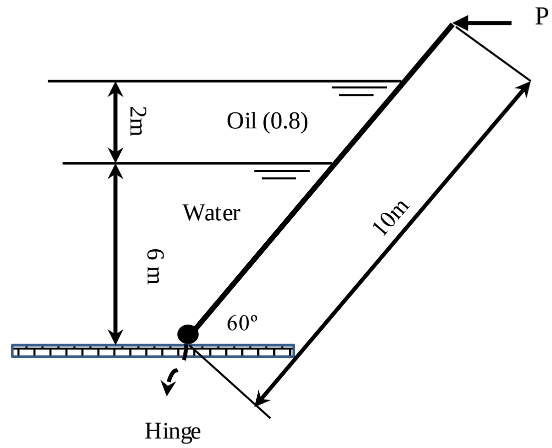
= 9800 * (7.6/2) * (8.775*6)

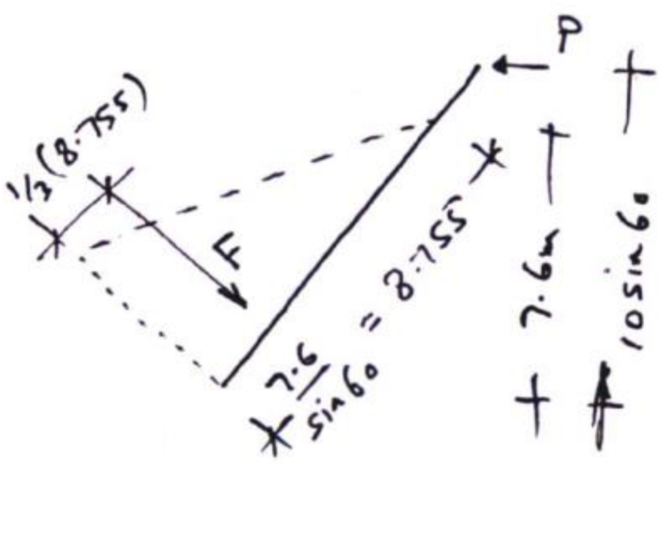
F = 1960847.8N

$\sum M_o = 0$

1960847 * (8.775/3) = P * 10 sin 60

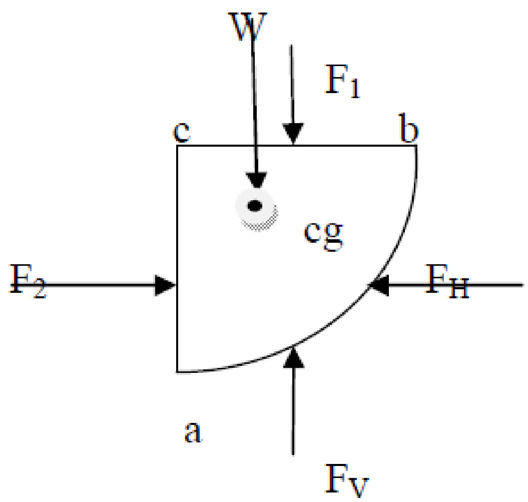
P = 660766.7N





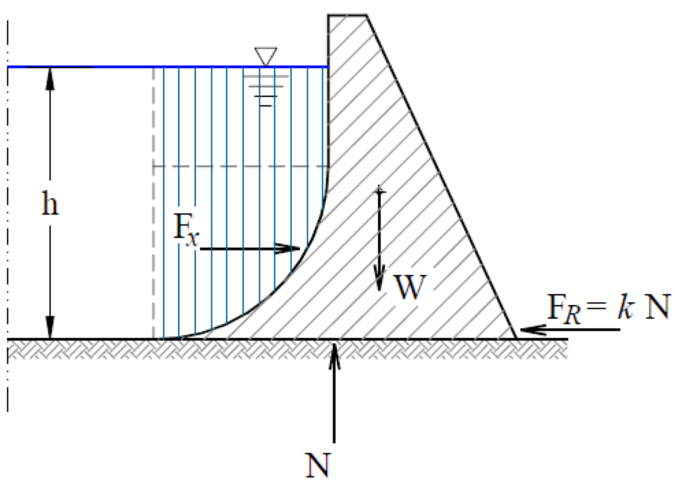
6. Hydrostatic Forces on Curved Surface.

Consider the curved section ***ab*** of the open tank as in figure shown. We wish to find the resultant force acting on this section with unit length perpendicular to the plane of the paper. The horizontal plane surface ***bc*** and the vertical plane surface ***ac*** are the projection areas of the curved surface ***ab***. F_h & F_v are the forces components that the tank exerts on the fluid. ***W*** is the specific weight (γ) of the fluid times the enclosed volume acts through (***cg***), then,

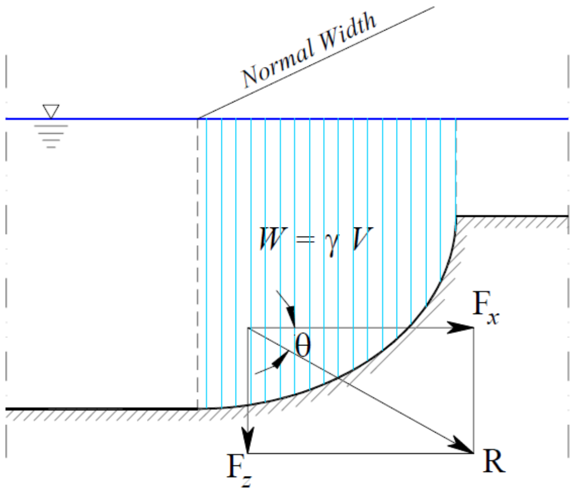


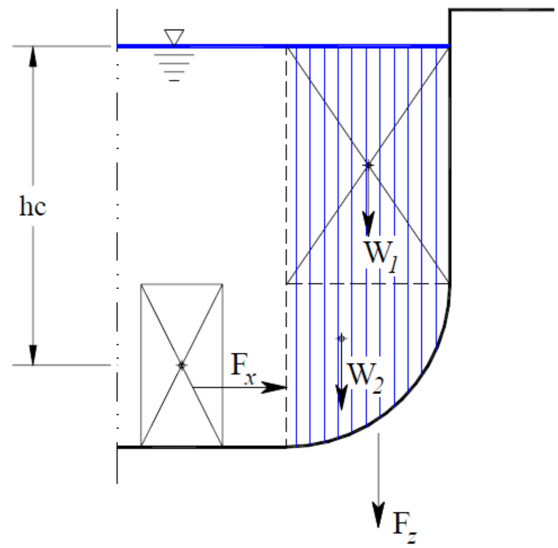
i. Vertical forces F_v: the vertical force on a curved surface is given by the weight of the liquid enclosed by the surface and the vertical force acts on horizontal free surface of the liquid. The force acts along the center of gravity of the volume.

ii. Horizontal forces F_h: the horizontal force equals the force on the projected area of the curved surface and acts at the center of pressure of the projected area.



- Because of the curvature of the surface, the direction of the resultant force is not pre-determined as in the previous case. One needs to decompose the force to its horizontal and vertical components (F_x, and F_z) and find the points of application for each.





$$F_H = \gamma * h_c * A$$

Where A is the vertical projection area of the curved surface.

$$F_V = \gamma V$$

Where V is the volume of water **over** the curved surface up to the water surface.

Resultant force

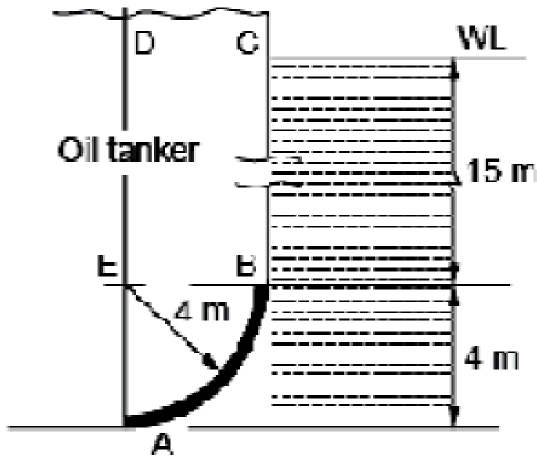
$$R = \sqrt{F_H^2 + F_V^2}$$

The angle the resultant force makes to the horizontal is

$$\theta = \tan^{-1}\left(\frac{F_V}{F_H}\right)$$

Example

Determine the resultant force exerted by sea water (sp.gr.=1.025) on the curved port in AB of an oil tanker as shown in figure. Also determine the direction of action of the force. Consider 1m width perpendicular to paper.



Sol.

$F_H = \gamma * h_c * A$ = (1.025*9800) * 17 * (4*1) = 683060N →

$F_V = \gamma V$

V = $V_{BCDE} + V_{ABE}$

F_V= (1.025*9800) { (15*4) + ($\frac{\pi * 4^2}{4}$)} = 728929.2N ↓

$$R = \sqrt{F_H^2 + F_V^2}$$

$$= \sqrt{(683060)^2 + (728929.2)^2} = 998953.823\text{N}$$

$$\theta = \tan^{-1}\left(\frac{F_V}{F_H}\right)$$

$$\theta = \tan^{-1}\left(\frac{728929.2}{683060}\right) = 46.86^{o}$$

CHAPTER THREE

FLUID DYNAMICS (BASIC EQUATIONS)

1. Introduction

This section discusses the analysis of fluid in motion - fluid dynamics. The motion of fluids can be predicted in the same way as the motion of solids are predicted using the fundamental laws of physics together with the physical properties of the fluid.

2. Types of flow

Under some circumstances the flow will not be as changeable as this. He following terms describes the states which are used to classify fluid flow:

- *Uniform flow*: If the flow velocity is the same magnitude and direction at every point in the fluid it is said to be *uniform*.
- *Non-uniform*: If at a given instant, the velocity is not the same at every point the flow is *non-uniform*.
- *Steady:* A steady flow is one in which the conditions (velocity, pressure and cross-section) may differ from point to point but DO NOT change with time.
- *Unsteady:* If at any point in the fluid, the conditions change with time, the flow is described as *unsteady*.

Combining the above we can classify any flow in to one of four types:

1. Steady Uniform flow. Conditions do not change with position in the stream or with time. An example is the flow of water in a pipe of constant diameter at constant velocity.

2. Steady Non-uniform flow. Conditions change from point to point in the stream but do not change with time. An example is flow in a tapering pipe with constant velocity at the inlet - velocity will change as you move along the length of the pipe toward the exit.

3. Unsteady Uniform flow. At a given instant in time the conditions at every point are the same, but will change with time. An example is a pipe of constant diameter connected to a pump pumping at a constant rate which is then switched off.

4. Unsteady Non-uniform flow. Every condition of the flow may change from point to point and with time at every point. For example waves in a channel.

3. Flow (Discharge)

3.1 Mass flow rate

If we want to measure the rate at which water is flowing along a pipe. A very simple way of doing this is to catch all the water coming out of the pipe in a bucket over a fixed time period. Measuring the weight of the water in the bucket and dividing this by the time taken to collect this water gives a rate of accumulation of mass. This is known as the *mass flow rate*.

For example an empty bucket weighs 2.0kg. After 7 seconds of collecting water the bucket weighs 8.0kg, then:

$$\text{mass flow rate m} = \frac{\text{mass of fluid in bucket}}{\text{time taken to collect the fluid}} = \frac{8-2}{7} = 0.857 \text{ kg/sec.}$$

Performing a similar calculation, if we know the mass flow is 1.7kg/s, how long will it take to fill a container with 8kg of fluid?

3.2 Volume flow rate - Discharge.

More commonly we need to know the volume flow rate - this is more commonly known as discharge. (It is also commonly, but inaccurately, simply called flow rate). The symbol normally used for discharge is Q. The discharge is the volume of fluid flowing per unit time. Multiplying this by the density of the fluid gives us the mass flow rate. Consequently, if the density of the fluid in the above example is 850kg/m^3 then

$$Q = \frac{\text{volume of fluid}}{\text{time}} = \frac{\text{mass of fluid}}{\text{densitytime}} = \frac{\text{mass flow rate}}{\text{density}} = \frac{0.857}{850} = 0.001008 \text{ m}^3\text{/sec} \approx 1 \text{ lps (litre/s)}$$

3.3 Discharge and velocity

If we know the size of a pipe, and we know the discharge, we can deduce the velocity

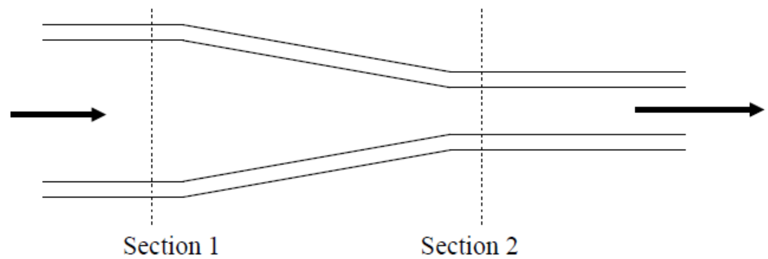
$$Q = v \cdot A$$

So if the cross-section area, A, is 1.2×10^{-3} m^2 and and the discharge, Q is 24 l/s then the velocity (v) is $v\ v = \frac{Q}{A} = \frac{24 \times 10^{-3}}{1.2 \times 10^{-3}} = 20\ m/s$

4. Basic equations

4.1 Continuity equation

We can apply the principle of continuity to pipes with cross sections which change along their length. Consider the diagram below of a pipe with a contraction:



$$Q_1 = Q_2 = Q_3 = \ldots = Q_n$$

This is the form of the continuity equation most often used. Where $Q = v.A$

A is the cross-sectional area of the flow section (pipe) and v is the mean velocity.

This equation is a very powerful tool in fluid mechanics and will be used repeatedly throughout the rest of this course.

A liquid is flowing from left to right and the pipe is narrowing in the same direction. By the continuity principle, *the mass flow rate* must be the same at each section - the mass going into the pipe is equal to the mass going out of the pipe. So we can write:

$$A_1 v_1 \rho_1 = A_2 v_2 \rho_2$$

(With the sub-scripts 1 and 2 indicating the values at the two sections)

As we are considering a liquid, usually water, which is *not* very compressible, the density changes very little so we can say $\rho_1 = \rho_2 = \rho$ This also says that the volume flow rate is constant or that Discharge at section 1 = Discharge at section 2

$$Q_1 = Q_2$$

$$A_1 v_1 = A_2 v_2$$

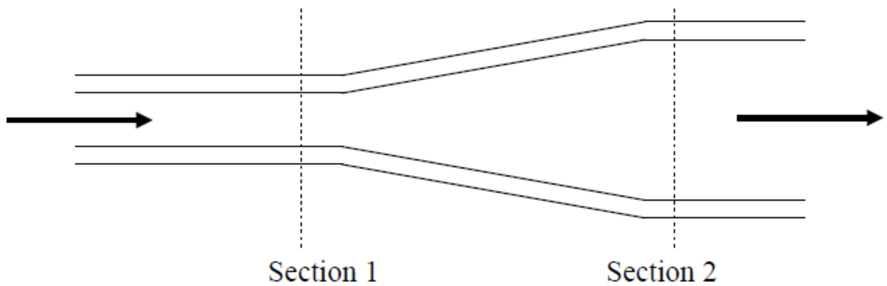
For example if the area $A_1=10*10^{-3}$ m^2 and $A_2=3*10^{-3}$ m^2 and the upstream mean velocity is v_1 is 2.1m/s ,then the downstream velocity can be calculated by

$$v_2 = \frac{A_1 v_1}{A_2} = 7\text{m/s}$$

Notice how the downstream velocity only changes from the upstream by the ratio of the two areas of the pipe. As the area of the circular pipe is a function of the diameter we can reduce the calculation further,

$$v_2 = \frac{A_1}{A_2} v_1 = \frac{\frac{\pi}{4} d_1^2}{\frac{\pi}{4} d_2^2} v_1 = \frac{d_1^2}{d_2^2} v_1 = (\frac{d_1}{d_2})^2 \, v_1$$

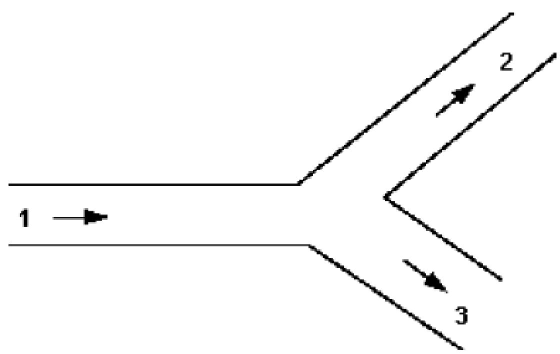
Now try this on a diffuser, a pipe which expands or diverges as in the figure below,



If the diameter at section 1 is d_1 = 30 mm and at section 2 d_2 = 40 mm and the mean velocity at section 2 is v_2 = 3m /s. The velocity entering the diffuser is given by,

$v_1 = (\frac{40}{30})^2 \times 3 = 5.3$m/s

Another example of the use of the continuity principle is to determine the velocities in pipes coming from a junction.



Total mass flow into the junction = Total mass flow out of the junction

$$\rho_1 Q_1 = \rho_2 Q_2 + \rho_3 Q_3$$

When the flow is incompressible (e.g. if it is water) $\rho_1 = \rho_2 = \rho$

$Q_1 = Q_2 + Q_3 \rightarrow A_1 v_1 = A_2 v_2 + A_3 v_3$

If pipe 1 diameter = 50mm, mean velocity 2m/s, pipe 2 diameter 40mm takes 30% of total discharge and pipe 3 diameter 60mm. What are the values of discharge and mean velocity in each pipe?

$Q_1 = A_1v_1 = \frac{\pi}{4}d_1^2 * v_1 = 0.00392 m^3/s$

$Q_2 = 0.3Q_1 = 0.001178 m^3/s$

$Q_1 = Q_2 + Q_3$, $Q_3 = Q_1 - 0.3Q_1 = 0.7Q_1 \rightarrow Q_3 = 0.00275\ m^3/s$

$Q_2 = A_2v_2 \rightarrow v_2 = 0.936 m/s$

$Q_3 = A_3v_3 \rightarrow v_3 = 0.972 m/s$

Example

As in figure the diameter at cross-section (1) is equal to (12 cm), the diameter at cross-section (2) is equal to (8 cm). If the velocity at section (1) is 1.5 m/s, calculate the velocity at section (2).

Sol.

The cross-section area at (1) is $A_1 = \frac{\pi}{4}d_1^2 = \frac{\pi}{4}(0.12)^2 = 0.0113 m^2$

$A_2 = \frac{\pi}{4}d_2^2 = \frac{\pi}{4}(0.08)^2 = 5.026*10^{-3}\ m^2$

$A_1v_1 = A_2v_2 \rightarrow v_2 = \frac{A_1}{A_2}v_1 = \frac{0.0113}{5.026*10^{-3}} *1.5 = 3.375 m/s$

4.2 Energy equation (Euler equation)

The first law of Thermodynamics states that energy can be neither created nor destroyed, but it can change from one form to another. It follows that all forms of energy are equivalent.

The flow conveyance between two points could happen because of acting of the following energy types below:

1- Potential energy (m)

Potential energy depends on the object's elevation above an arbitrary datum.

2- Flow energy (Pressure head) (m)

A fluid particle has energy due to its pressure that is greater than atmospheric pressure. $= \frac{P}{\gamma}$

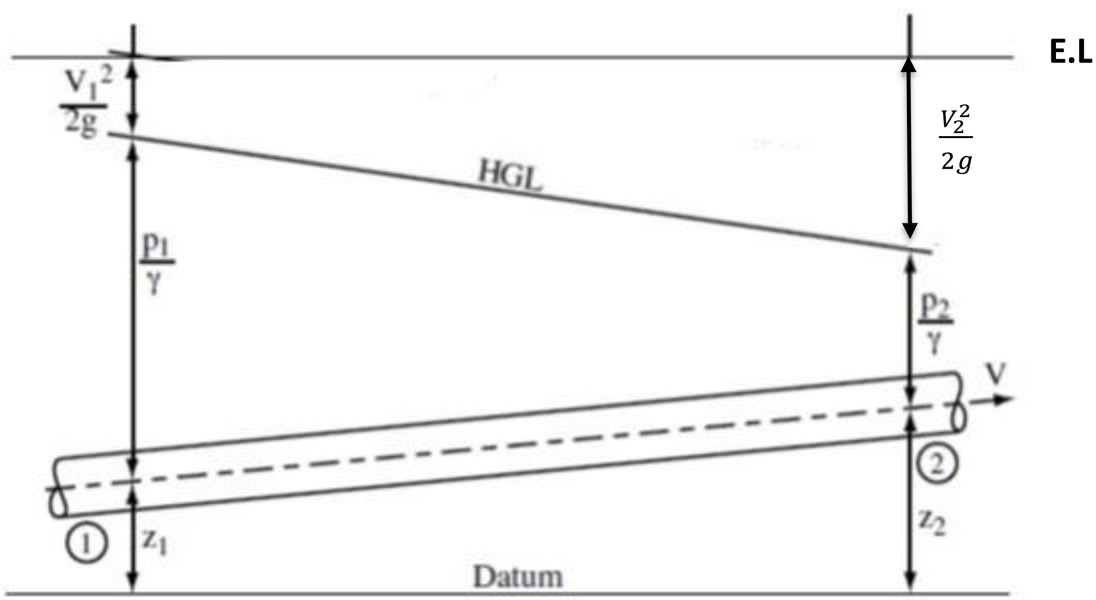
3- Kinetic energy $= \frac{V^2}{2g}$ (m)

❖ ***Bernoulli's Equation***

This equation relates the pressure, velocity and height in the steady motion of an ideal fluid. In driving Bernoulli's equation, we will assume:

1- Viscous (friction) effects are negligible (*Ideal fluid*).
2- The flow is *steady* (constant with respect to time).
3- The fluid is *incompressible.*
4- No energy is added or removed from the fluid.

If we apply the three energies above on the figure below, then we will obtain:



$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \text{Constant}$$

The Energy Grade Line, also called the Energy Line or simply EL, is a plot of the sum of the three terms in the work-energy equation, which is also the Bernoulli sum.

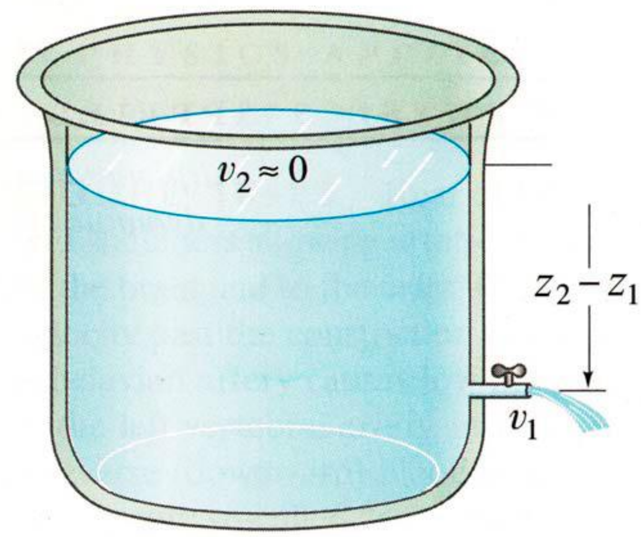
If the fluid is static, the velocities are zero, and Bernoulli's equation reduces to:

$$P_1 = P_2 + \gamma(z_2 - z_1)$$

❖ *Applications of Bernoulli's Equation*

Applying Bernoulli to Points 1,2 in the fluid:

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$



But, $P_1 = P_2$ = atmospheric pressure = 0 gage pressure.

$v_2 \approx 0 \cong$ (Still water surface in a large tank with small outlet pipe).

So,

$$0 + \frac{V_1^2}{2g} + z_1 = 0 + 0 + z_2$$

$\therefore \frac{V_1^2}{2g} = (Z_2 - Z_1)$

Or $V_1 = \sqrt{2gh}$ where h is $(Z_2 - Z_1)$

❖ *Step Method of Approaching Flow Problems*

1- Choose a convenient datum.

2- Note locations/sections where the velocity is known.

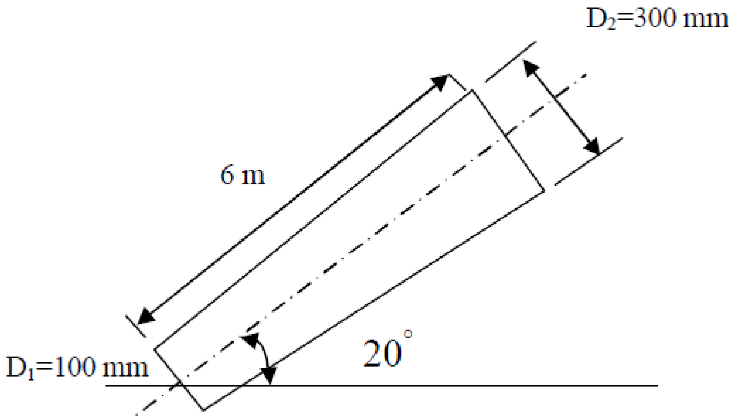
- **Reservoir** (velocity at surface = 0)
- **Continuity Equation:** $Q = AV$

3- Note locations/sections where pressure is known.

- **Reservoir** (Pressure at surface = atmospheric pressure = 0 gage pressure)
- **Jet** (pressure = 0)
- **Simple Manometers**
- **Differential Manometers**

Example:

A 6m long pipe is inclined at angle of 20 with the horizontal. The smaller section of the pipe which is at lower level is of 100mm and the larger section of pipe is of 300 mm diameter as shown in figure. If the pipe is uniformly tapering and the velocity of water at the smaller section is 1.8 m/s determine the difference of pressures between the two sections.



Sol.

$$A_1 = \frac{\pi d1^2}{4} = \pi * \frac{0.1^2}{4} = 0.00785 m^2$$

$$V_1 = 1.8 \frac{m}{s}$$

$$z_1 = 0\ m$$

$$d_2 = 0.3 m$$

$$A_2 = \frac{\pi}{4} * 0.3^2 = 0.0707 m^2$$

$$z_2 = 6 sin20 = 6 * 0.342 = 2.05 m$$

From $C.E$ $A_1V_1 = A_2V_2$

$$\therefore V_2 = \frac{A_1V_1}{A_2} = 0.00785 * \frac{1.8}{0.0707} = 0.2\ m/s$$

Applying B.E. to both sections of pipe

$$\frac{p1}{\gamma} + \frac{V1^2}{2g} + z_1 = \frac{p2}{\gamma} + \frac{V2^2}{2g} + z_2$$

$$p_1 - p_2 = \gamma\left(\frac{V_2^2 - V_1^2}{2g} + z_2\right)$$

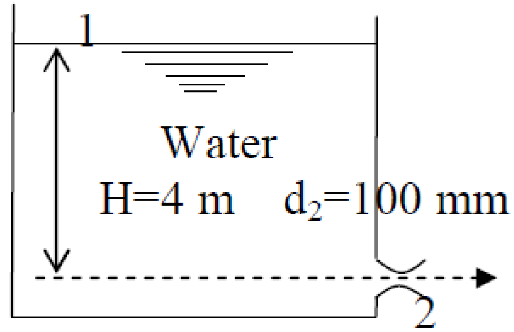
$$p_1 - p_1 = 9810\left(\frac{0.2^2 - 1.8^2}{2*9.81} + 2.05\right) = 18510 \frac{N}{m^2}$$

Example

a) Determine the velocity of efflux from the nozzle in the wall of the reservoir as in figure.

b) Find the discharge at the nozzle.

Sol.



a) $$\frac{V_1^2}{2g}+\frac{p_1}{\rho g}+z_1=\frac{V_2^2}{2g}+\frac{p_2}{\rho g}+z_2$$

With pressure datum as local atmospheric pressure, $p_1=p_2=0$ & $z_2=0$, $z_1=H$, the velocity on the surface of the reservoir is zero.

$$0+0+H=\frac{V_2^2}{2g}+0+0$$

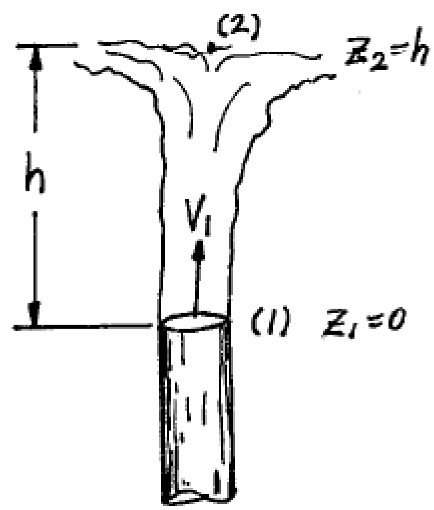
$$V_2=\sqrt{2gH}=\sqrt{2*9.81*4}=8.86\frac{m}{s}$$

b) $$Q=A_2V_2=\pi(0.05)^2(8.86)=0.07\frac{m^3}{s}=70\ \frac{L}{s}$$

Example

Water flows from a garden hose nozzle with a velocity of 15m/s. What is the maximum height that it can reach above the nozzle?

Sol.



$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$

But , p_1, p_2, v_2 are equal to zero

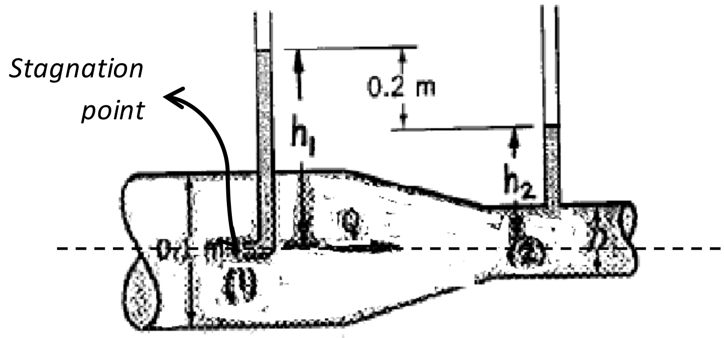
Thus

$$h=\frac{v_1^2}{2g}=\frac{15^2}{2g}=11.5\text{m}$$

Example

Water flows through the pipe contraction shown in figure. For the given 0.2m difference in manometer level, determine the flow-rate as a function of the diameter of the small pipe, D

Sol.



$z_1 = z_2$, $v_1 = 0$ (stagnation point)

Thus,

$v_2 = \sqrt{2g\frac{p_1 - p_2}{\gamma}}$, but $p_1 = \gamma h_1$ & $p_2 = \gamma h_2$

so that $p_1 - p_2 = \gamma\,(h_1 - h_2) = 0.2\gamma$

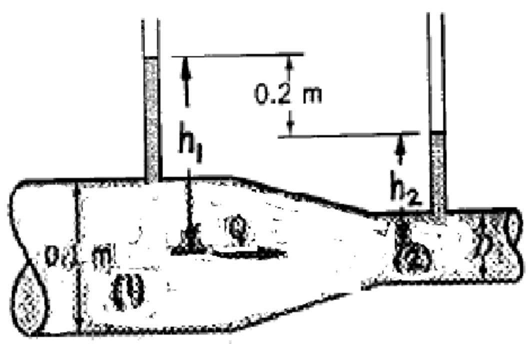
Then $v_2 = \sqrt{2g\frac{0.2\gamma}{\gamma}} = v_2 = \sqrt{2g * 0.2}$

Or $Q = A_2 v_2 = \frac{\pi}{4} D^2 v_2 = \frac{\pi}{4} D^2 \sqrt{2g * 0.2} = 1.56\, D^2\ m^3/s$ where D in m

Example

Water flows through the pipe contraction shown in figure. For the given 0.2m difference in manometer level, determine the flow-rate as a function of the diameter of the small pipe, D

Sol.



$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

With

$A_1 v_1 = A_2 v_2$ or $v_2 = \frac{\frac{\pi}{4} D_1^2}{\frac{\pi}{4} D_2^2} v_1 = (\frac{0.1}{D})^2\, v_1$

Thus, $z_1=z_2$ $\quad \frac{p_1-p_2}{\gamma} = \frac{v_2^2-v_1^2}{2g} = \frac{\{(\frac{0.1}{D})^4-1\}\,v_1^2}{2g}$

but $p_1=\gamma h_1$ & $p_2=\gamma h_2$ so that $p_1-p_2=\gamma\,(h_1-h_2) = 0.2\gamma$

Then

$$\frac{0.2\gamma}{\gamma} = \frac{\{(\frac{0.1}{D})^4-1\}\,v_1^2}{2g} \quad \text{or} \quad v_1 = \sqrt{\frac{2g*0.2}{\{(\frac{0.1}{D})^4-1\}}} \quad \text{and,}$$

$$Q = A_1v_1 = \frac{\pi}{4}(0.1)^2\sqrt{\frac{2g*0.2}{\{(\frac{0.1}{D})^4-1\}}} \quad \text{or} \quad Q = \frac{0.0156D^2}{\sqrt{(0.1)^4-D^4}} \quad \text{where D in m}$$

4.3 Conservation of Momentum (Momentum equation)

In Newtonian mechanics, the conservation of momentum is defined by Newton’s second law of motion. The 2nd law of motion states as:

- The rate of change of momentum of a body is proportional to the impressed action and takes place in the direction of the impressed action.
- If a force acts on the body, linear momentum is implied.
- If a torque (moment) acts on the body angular momentum is implied.

Newton’s 2nd Law can be written:

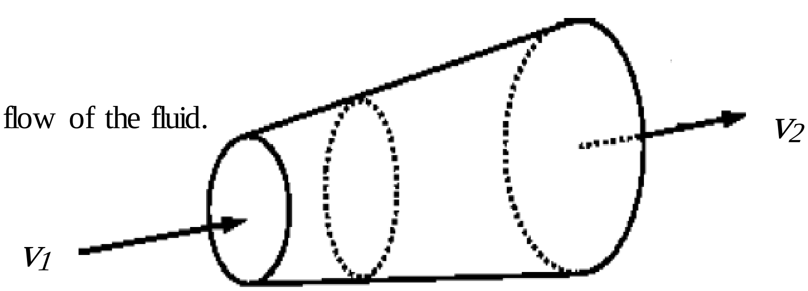
The Rate of change of momentum of a body is equal to the resultant force acting on the body, and takes place in the direction of the force.

The force exerted by the fluid using Newton’s 2nd Law is equal to the rate of change of momentum. So

Force = rate of change of momentum\

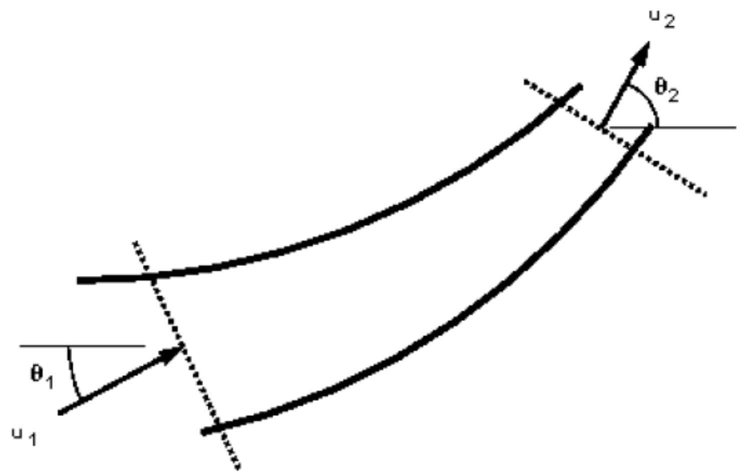
$F = \rho\, Q\,(v_{Out} - v_{In}) = \rho\, Q\,(v_2 - v_1)$

This force is acting in the direction of the flow of the fluid.



This analysis assumed that the inlet and outlet velocities were in the same direction - i.e. a one dimensional system. What happens when this is not the case?

Consider the two dimensional system in the figure below:



At the inlet the velocity vector, v_1, makes an angle, θ_1 , with the x-axis, while at the outlet v_2 make an angle θ_2. In this case we consider the forces by resolving in the directions of the coordinate axes.

The force in the x-direction

F_x = Rate of change of momentum in x – direction

= Rate of change of mass × change in velocity in x – direction

$$= \rho Q (v_{2x} - v_{1x}) = \rho Q (v_2 \cos\theta_2 - v_1 \cos\theta_1)$$

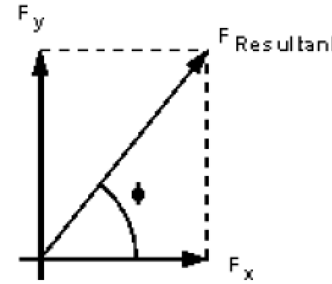
And the force in the y-direction

$$F_y = \rho Q (v_{2y} - v_{1y}) = \rho Q (v_2 \sin\theta_2 - v_1 \sin\theta_1)$$

We then find the **resultant force** by combining these vectorially:

$$F = \sqrt{F_x^2 + F_y^2}$$

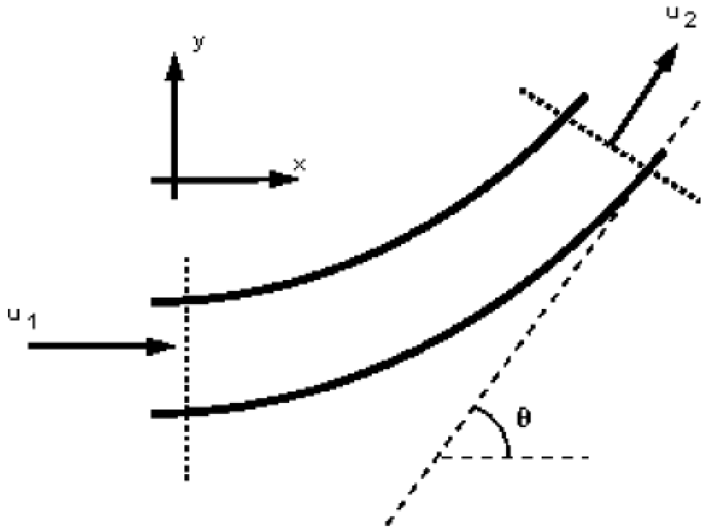
$$\Phi = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$



4.4Application of the Momentum Equation

4.4.1The forces due the flow around a pipe bend

Consider a pipe bend with a constant cross section lying in the horizontal plane and turning through an angle of (θ)



In summary we can say:

The total force ***exerted on*** the fluid = rate of change of momentum through the control volume

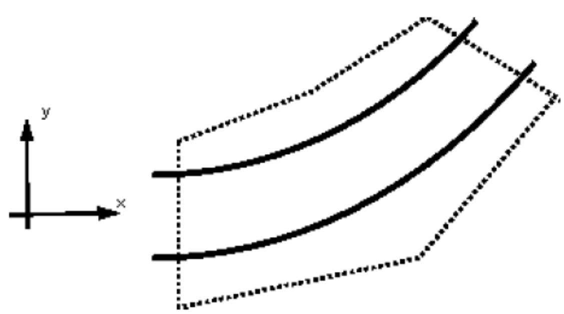
$$\mathbf{F = \rho\, Q\, (v_{out} - v_{in})}$$

Steps to analysis:

1. Draw a control volume
2. Decide on co-ordinate axis system
3. Calculate the total force
4. Calculate the pressure force
5. Calculate the body force
6. Calculate the resultant force

1. Draw a control volume

The control volume is draw in the above figure, with faces at the inlet and outlet of the bend and encompassing the pipe walls.



2. Decide on co-ordinate axis system

It is convenient to choose the co-ordinate axis so that one is pointing in the direction of the inlet velocity. In the above figure the x-axis points in the direction of the inlet velocity.

3. Calculate the total force

In the x-direction:

$F_{T_x} = \rho\, Q\, (v_{2x} - v_{1x})$

$v_{1x} = v_1$

$v_{2x} = v_2 \cos\theta$

$F_{T_x} = \rho\, Q\, (v_2 \cos\theta - v_1)$

In the y-direction:

$F_{T_y} = \rho\, Q\, (v_{2y} - v_{1y})$

$v_{1y} = v_1 \sin\theta = 0$

$v_{2y} = v_2 \sin\theta$

$F_{T_y} = \rho\, Q\, (v_2 \sin\theta)$

4. Calculate the pressure force

F_P = pressure force at 1 − pressure force at 2

$F_{P_x} = p_1 A_1 \cos 0 - p_2 A_2 \cos\theta = p_1 A_1 - p_2 A_2 \cos\theta$

$F_{P_y} = p_1 A_1 \sin 0 - p_2 A_2 \sin\theta = -p_2 A_2 \sin\theta$

5. Calculate the body force

There are no body forces in the x or y directions. The only body force is that exerted by gravity (which acts into the paper in this example - a direction we do not need to consider).

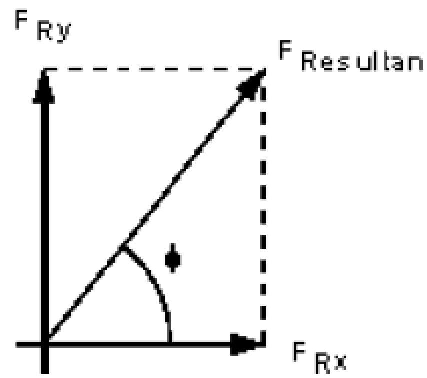
F_B = Force exerted on the fluid body (e.g. gravity)

6. Calculate the resultant force

$$F_{T_x} = F_{R_x} + F_{P_x} + F_{B_x}$$
$$F_{T_y} = F_{R_y} + F_{P_y} + F_{B_y}$$

$$F_{R_x} = F_{T_x} - F_{P_x} - 0 = \rho\, Q\,(v_2 \cos\theta - v_1) - p_1 A_1 + p_2 A_2 \cos\theta$$

$$F_{R_y} = F_{T_y} - F_{P_y} - 0 = \rho\, Q\,(v_2 \sin\theta) + p_2 A_2 \sin\theta$$



And the resultant force **on the fluid** is given by

$$R = \sqrt{{F_{R_x}}^2 + {F_{R_y}}^2}$$

And the direction of application is

$$\Phi = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right)$$

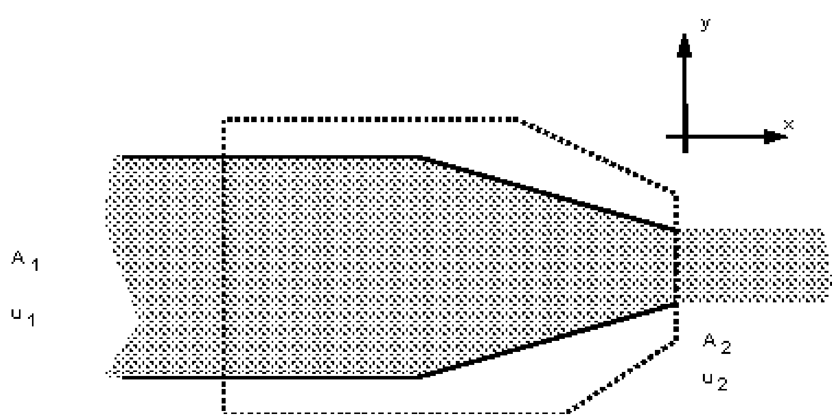
The force **on the bend** is the same magnitude but in the opposite direction.

4.4.2 Force on a pipe nozzle

Force on the nozzle at the outlet of a pipe. Because the fluid is contracted at the nozzle forces are induced in the nozzle. Anything holding the nozzle (e.g. a fireman) must be strong enough to withstand these forces.

The analysis takes the same procedure as above:

1. Draw a control volume
2. Decide on co-ordinate axis system
3. Calculate the **total** force
4. Calculate the **pressure** force
5. Calculate the **body** force
6. Calculate the **resultant** force



1 & 2 Control volume and Co-ordinate

axis are shown in the figure.

Notice how this is a one dimensional system which greatly simplifies matters.

*3. Calculate the **total** force*

$F_T = F_{T_x} = \rho\, Q\, (v_2 - v_1)$

By continuity, $Q = A_1 v_1 = A_2 v_2$, So $F_{T_x} = \rho\, Q^2 \left[\frac{1}{A_2} - \frac{1}{A_1}\right]$

*4. Calculate the **pressure** force*

$F_P = F_{P_x} =$ pressure force at 1− pressure force at 2

We use the Bernoulli equation to calculate the pressure

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

The nozzle is horizontal, $z_1 = z_2$, and the pressure outside is atmospheric, $p_2 = 0$,

And with continuity gives

$$p_1 = \frac{\rho\, Q^2}{2}\left[\frac{1}{A_2^2} - \frac{1}{A_1^2}\right]$$

*5. Calculate the **body** force*

The only body force is the weight due to gravity in the y-direction - but we need not consider this as the only forces we are considering are in the x-direction.

F_B = Force exerted on the fluid body (e.g. gravity)

*6. Calculate the **resultant** force*

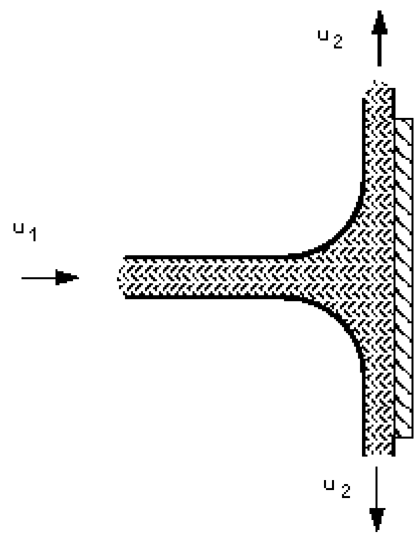
$F_{T_x} = F_{R_x} + F_{P_x} + F_{B_x}$

$F_{R_x} = F_{T_x} - F_{P_x} - 0$

$F_R = F_{R_x} = \rho\, Q^2 \left[\frac{1}{A_2} - \frac{1}{A_1}\right] - \frac{\rho\, Q^2}{2}\left[\frac{1}{A_2^2} - \frac{1}{A_1^2}\right]$

4.4.3Impact of a Jet on a Plane

We will first consider a jet hitting a flat plate (a plane) at an angle of 90°, as shown in the figure below. We want to find the reaction force of the plate i.e. the force the plate will have to apply to stay in the same position.



$F_{T_x} = \rho\, Q\, (v_{2x} - v_{1x})$

$= -\rho\, Q\, v_{1x}$

As the system is symmetrical the forces in the y-direction cancel i.e.

$F_{T_y} = 0$

Note: The pressure force is zero as the pressure at both the inlet and the outlets to the control volume are Atmospheric and the control volume is small we can ignore the body force due to the weight of gravity.

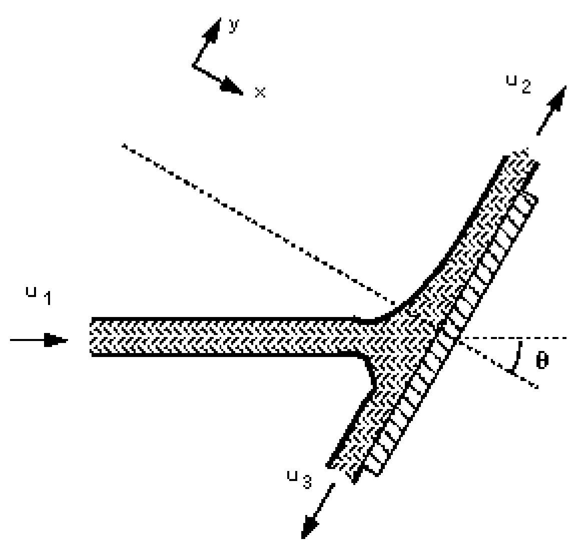
The **resultant** force is exerted **on the fluid.**

$F_R = F_{R_x} = -\rho\, Q\, v_{1x}$

The force **on the plane** is the same magnitude but in the opposite direction.

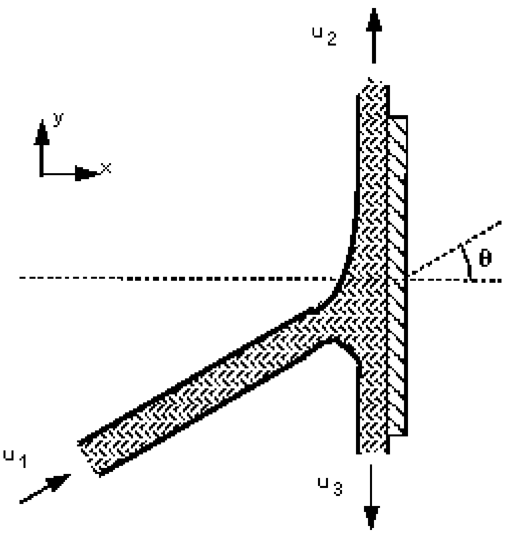
4.4.4Force due to a jet hitting an inclined plane

We have seen above the forces involved when a jet hits a plane at right angles. If the plane is tilted to an angle the analysis becomes a little more involved. This is demonstrated below.



(Note that for simplicity gravity and friction will be neglected from this analysis.)

We want to find the reaction force normal to the plate so we choose the axis system as above so that is normal to the plane. The diagram may be rotated to align it with these axes and help comprehension, as shown below



We do not know the velocities of flow in each direction. To find these we can apply Bernoulli equation

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + z_3$$

The height differences are negligible i.e. $z_1 = z_2 = z_3$ and the pressures are all atmospheric = 0. So

$$v_1 = v_2 = v_3 = v$$

By continuity

$Q_1 = Q_2 + Q_3$ i.e $A_1v_1 = A_2v_2 + A_3v_3 \rightarrow A_1 = A_2 + A_3$

$Q_1 = A_1v_1$ & $Q_2 = A_2v_2$

$Q_3 = (A_1 - A_2)\, v$

Using this we can calculate the forces in the same way as before.

1. Calculate the ***total*** force
2. Calculate the ***pressure*** force
3. Calculate the ***body*** force
4. Calculate the ***resultant*** force

*1. Calculate the **total** force in the x-direction.*

Remember that the co-ordinate system is normal to the plate.

$F_{T_x} = \rho \left[\left(Q_2 v_{2_x} + Q_3 v_{3_x} \right) - Q_1 v_{1_x} \right]$

But $v_{2_x} = v_{3_x} = 0$ as the jets are parallel to the plate with no component in the x-direction.

$v_{1_x} = v_1 \cos\theta$, So

$$F_{T_x} = -\rho\ Q_1 v_1 \cos\theta$$

*2. Calculate the **pressure** force*

All zero as the pressure is everywhere atmospheric.

3. Calculate the body force

As the control volume is small, hence the weight of fluid is small, we can ignore the body forces.

*4. Calculate the **resultant** force*

$F_{T_x} = F_{R_x} + F_{P_x} + F_{B_x}$

$F_{R_x} = F_{T_x} - 0 - 0 = -\rho\ Q_1 v_1 \cos\theta$

The force **on the plate** is the same magnitude but in the opposite direction ($\rho\ Q_1 v_1 \cos\theta$).

- We can find out how much discharge goes along in each direction on the plate. Along the plate, in the y-direction, the total force must be zero, $F_{T_y} = 0$

Also in the y-direction:

$v_{1_y} = v_1 \sin\theta \qquad v_{2_y} = v_2 \qquad v_{3_y} = -v_3$, So

$F_{T_y} = \rho \left[\left(Q_2 v_{2_y} + Q_3 v_{3_y} \right) - Q_1 v_{1_y} \right]$

$F_{T_y} = \rho \left[\left(Q_2 v_2 - Q_3 v_3 \right) - Q_1 v_1 \sin\theta \right]$

As forces parallel to the plate are zero,

$$0 = \rho A_2 v_2^2 - \rho A_3 v_3^2 - \rho A_1 v_1^2 \sin\theta$$

From above $v_1 = v_2 = v_3$

$0 = A_2 - A_3 - A_1 \sin\theta$

and from above we have $A_1 = A_2 + A_3$

$$0 = A_2 - A_3 - (A_2 + A_3) \text{ Sin } \theta$$
$$= A_2(1-\sin\theta) - A_3(1+ \text{ Sin } \theta)$$
$$A_2 = A_3\left(\frac{1+\sin\theta}{1-\sin\theta}\right)$$

$$Q_2 = Q_3\left(\frac{1+\sin\theta}{1-\sin\theta}\right)$$

As

$v_2 = v_3 = v$

$$Q_1 = Q_3\left(\frac{1+\sin\theta}{1-\sin\theta}\right) + Q_3$$
$$= Q_3\left(1 + \frac{1+\sin\theta}{1-\sin\theta}\right)$$

So we know the discharge in each direction

Example

A 600 mm diameter pipeline carries water under a head of 30 m with velocity of 3 m/s. This water main is fitted with a horizontal bend which turns the axis of the pipeline through 75°. Calculate the resultant force on the bend and its angle to the horizontal.(Gravity force = 0).

Sol.

$A_1 = A_2 = \pi \left(\frac{0.6}{2}\right)^2 = 0.283 m^2$

$v_1 = v_2 = 3$ m/s

$p_1 = \gamma h = 9800 * 30 = 294300 N/m^2$

$A_1 v_1 = A_2 v_2 = 0.283 * 3 = 0.849 m^3/sec$

$F_x = \rho Q (v_2 \cos\theta - v_1) + p_2 A_2 \cos\theta - p_1 A_1$

$= 1000 * (3*\cos 75 - 3) + 294300 * 0.283 * \cos 75 - 294300 * 0.283 = -63.618$ kN

$Fy = \rho Q (v_2 \sin\theta) + p_2 A_2 \sin\theta$

$= 1000 * (3*\sin 75) + 294300 * 0.283 * \text{sins}75 = 82.9$kN

Then $R_x = 63.618$ kN $R_y = -82.9$kN

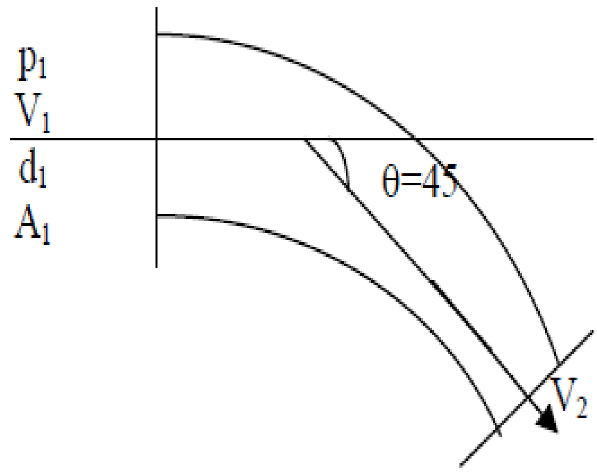
$$R = \sqrt{{F_{R_x}}^2 + {F_{R_y}}^2} = \sqrt{(63.618)^2 + (-82.9)^2} = 104.5 \text{ kN}$$

$$\Phi = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) = \tan^{-1}\left(\frac{-82.9}{63.618}\right) = -52.5°$$

Example

A pipe bend tapers from a diameter d, of (500) mm at inlet to a diameter d2 of (250mm) at outlet and turns the flow through an angle (θ) of 45º. Measurements of (p1 & p2) at inlet and outlet are 40 kN/m^2 and 23 kN/m^2. If the pipe is conveying oil which has a density ρ=850 kg/m^3. Calculate the magnitude and direction of resultant force on the bend when the oil is flowing at the rate of (0.45) m^3/s. The bend is in a horizontal plan. (Gravity force=0)

Sol.



$F_x = \rho\, Q\, (v_{2x} - v_{1x}) - (p_1 A_1 - p_2 A_2 \cos\theta)$

$$V_2 = \frac{\boldsymbol{Q}}{\boldsymbol{A_2}} = 0.45 * \frac{4}{\pi(0.25)^2} = 9.16\,\mathrm{m/s}$$

$$V_1 = \frac{Q}{A_1} = 0.45 * \frac{4}{\pi(0.5)^2} = 2.29\ \mathrm{m/s}$$

$F_x = 850 * 0.45\,((9.16 * \cos 45) - 2.29) - (40*10^3 * \frac{\pi}{4}(0.5)^2 - 23*10^3 * \frac{\pi}{4}(0.25)^2 \cos 45)$

$F_x = 7055.65\text{N}$

$F_y = \rho\, Q\, (v_{2y} - v_{1y}) + p_2 A_2 \sin\theta$

$= 850 * 0.45\,(9.16 * \sin 45 - 0) + 23*10^3 * \frac{\pi}{4}(0.25)^2 * \sin 45 = 18740.9\text{N}$

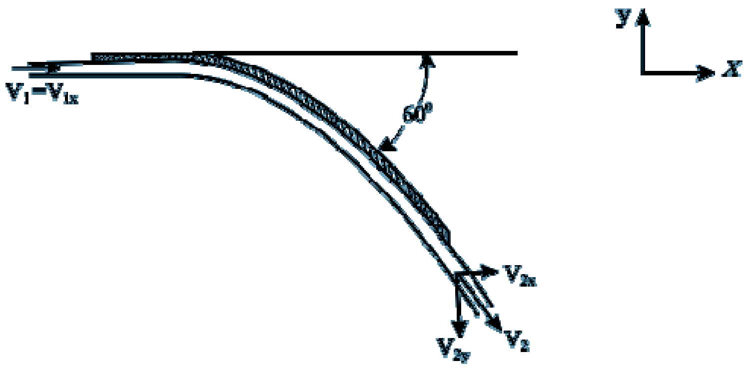
$\rightarrow R_y = -18740.9\text{N}$

$R = \sqrt{F_{R_x}{}^2 + F_{R_y}{}^2} = \sqrt{(7055.65)^2 + (-18740.9)^2} = 20643\ \text{N}$

$\Phi = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) = \tan^{-1}\left(\frac{-18740.9}{-8657}\right) = 65^{\circ}$

Example

Consider a jet that is deflected by a stationary vane, such as is given in figure. If the jet speed and diameter are 25 m/s &25 cm, respectively and jet is deflected 60º, what force is exerted by the jet?



Sol.

First solve for F_x, the x-component of force of the vane on the jet

$$F_x = (\rho QV)_{x\,out} - (\rho QV)_{x\,in}$$
$$F_x = \rho Q(V_{2x} - V_{1x})$$
$$V2x = V2\,cos60 = 25 * 0.5 = 12.5\,m/s$$
$$V_{1x} = 25\,m/s$$
$$Q = V_1 A_1 = 25 * \frac{\pi(0.25)^2}{4} = 1.227\ {m^3}/{s}$$

Therefore $F_x = (1000)\,(1.227)\,(12.5 - 25) = -15.3398$ kN

Similarly determined, the y-component of force on the jet is

$$F_y = \rho Q\left[(V_y)_{out} - (V_y)_{in}\right] = 1000 * 1.227(-21.65 - 0) = -26.5646\,kN$$

$$since\ V_{2y} = Vsin60 = 25sin60 = 21.65\,m/s$$

The resultant force is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(-15.3398)^2 + (-26.5646)^2} = 30.67kN$$

Then the force on the vane will be the reactions to the forces of the vane on the jet as

$$R_x = -F_x = +15.3398kN$$
$$R_y = -F_y = +26.5646kN$$

CHAPTER FOUR

FLOW IN CONDUITS

1. Real Fluids

The flow of real fluids exhibits viscous effect, which are they tend to "stick" to solid surfaces and have stresses within their body

You might remember from earlier in the course Newton's law of viscosity:

$$\tau \propto \frac{du}{dy}$$

This tells us that the shear stress, t, in a fluid is proportional to the velocity gradient - the rate of change of velocity across the fluid path. For a "Newtonian" fluid we can write:

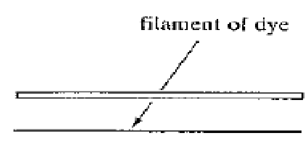
$$\tau = \mu \frac{du}{dy}$$

Where the constant of proportionality, μ, is known as the coefficient of viscosity (or simply viscosity). We saw that for some fluids - sometimes known as exotic fluids - the value of μ changes with stress or velocity gradient. We shall only deal with Newtonian fluids.

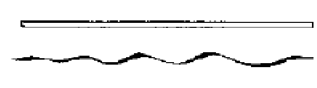
In his lecture we shall look at how the forces due to momentum changes on the fluid and viscous forces compare and what changes take place.

Laminar and turbulent flow

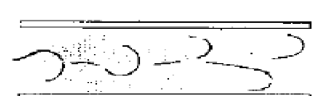
If we were to take a pipe of free flowing water and inject a dye into the middle of the stream, what would we expect to happen?



Laminar (viscous)



Transitional



Turbulent

Actually both would happen - but for different flow rates. The lower occurs when the fluid is flowing fast and the top when it is flowing slowly.

The top situation is known as **laminar** flow and the lower as **turbulent** flow.

In laminar flow the motion of the particles of fluid is very orderly with all particles moving in straight lines parallel to the pipe walls.

But what is fast or slow? And at what speed does the flow pattern change? And why might we want to know this?

$$Re = \frac{\rho v D}{\mu} \quad or \quad Re = \frac{v D}{\nu}$$

Where ρ= Mass density, v = mean velocity, D = diameter ,μ = dynamic viscosity and ν=kinematic viscosity.

This value is known as the Reynolds number, Re:

Laminar flow:	Re < 2000
Transitional flow:	2000 < Re < 4000
Turbulent flow:	Re > 4000

What are the units of this Reynolds number? We can fill in the equation with SI units:

ρ =kg/m^3 v = m/sec   D= m   μ = kg/(m.sec)

$$Re = \frac{\rho v D}{\mu} = \frac{\text{kg. m. m. s. m}}{\text{m}^3\text{. s. kg}} = 1$$

i.e. it has **no units**. A quantity that has no units is known as a non-dimensional (or dimensionless) quantity. Thus the Reynolds number, Re, is a non-dimensional number.

We can go through an example to discover at what velocity the flow in a pipe stops being laminar.

If the pipe and the fluid have the following properties:

water density	$\rho = 1000$ kg/m^3
pipe diameter	$d = 0.5$m
(dynamic) viscosity,	$\mu = 0.55\text{x}10^3$ Ns/m^2

We want to know the maximum velocity when the Re is 2000.

$$Re = \frac{\rho v D}{\mu} = 2000 \quad \rightarrow\rightarrow \quad v = \frac{\mu\, Re}{\rho D} = \frac{0.55 * 10^{-3} * 2000}{1000 * 0.5} = 0.0022 \text{ m/s}$$

In practice it very rarely occurs in a piped water system - the velocities of flow are much greater. Laminar flow does occur in situations with fluids of greater viscosity - e.g. in bearing with oil as the lubricant.

We can say that the number has a physical meaning, by doing so it helps to understand some of the reasons for the changes from laminar to turbulent flow.

$$Re = \frac{\rho v D}{\mu} = \frac{inertial\ forces}{viscous\ forces}$$

It can be interpreted that when the inertial forces dominate over the viscous forces (when the fluid is flowing faster and Re is larger) then the flow is turbulent. When the viscous forces are dominant (slow flow, low Re) they are sufficient enough to keep all the fluid particles in line, then the flow is laminar.

In summary:

Laminar flow

- $Re < 2000$
- 'low' velocity
- Dye does not mix with water
- Fluid particles move in straight lines
- Simple mathematical analysis possible
- Rare in practice in water systems.

Transitional flow

- $2000 > Re < 4000$
- 'medium' velocity
- Dye stream wavers in water - mixes slightly.

Turbulent flow

- $Re > 4000$
- 'high' velocity
- Dye mixes rapidly and completely
- Particle paths completely irregular
- Average motion is in the direction of the flow
- Cannot be seen by the naked eye
- Changes/fluctuations are very difficult to detect. Must use laser.
- Mathematical analysis very difficult - so experimental measures are used
- Most common type of flow.

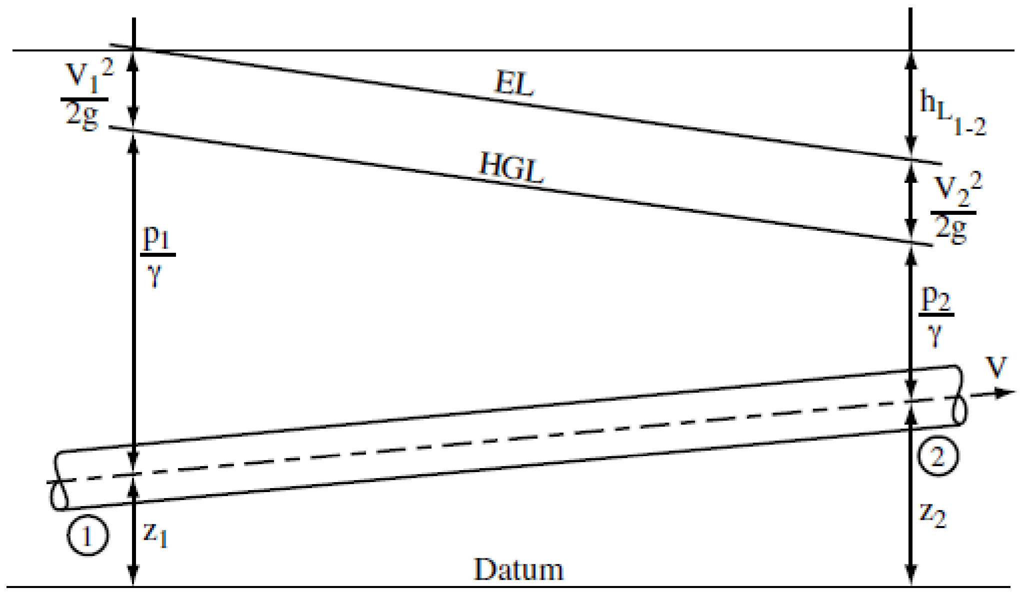
2. Modified Bernoulli's Equation

Bernoulli's equation states that the sum of pressure energy, kinetic energy and potential energy per unit mass is constant along a streamline. In most cases in closed pipes, all streamlines can be assumed to have the same energy level.

Bernoulli, in his procedure to simplify his equation didn't take into account:

1. The head loss by the fluid due to friction
2. The change in velocity distribution along any cross-section
3. He assumed that no energy is added or removed from the system, so, he didn't consider having pumps (adding energy) or turbines (extracting energy).

For those reasons, Bernoulli's Equation can be modified to take account of friction effects, h_f:



$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_f$$

Where:

h_f is the "head" loss due to friction.

The head loss (h_f) term in equation above is responsible for head loss due to fluid shear at the pipe wall, called pipe friction, The head loss due to pipe friction is always present throughout the length of the pipe.

And additional head loss caused by local disruptions of the fluid stream. The local disruptions, called local losses, are caused by valves, pipe bends, and other such fittings. Local losses may also be called minor losses if their effect, individually and/or collectively, will not contribute significantly in the determination of the flow; indeed, sometimes minor losses are expected to be inconsequential and are neglected. Or a preliminary survey of design alternatives may ignore the local or minor losses, considering them only in a later design stage. Each type of head loss will now be considered further.

DARCY-WEISBACH EQUATION

The completely general functional relation $\tau_w = F(V, D, \rho, \mu, e)$ between the wall shear stress $\boldsymbol{\tau_w}$ and the mean velocity **V**, pipe diameter **D**, fluid density **ρ**, and viscosity **μ**, and the equivalent sand-grain roughness **e** can be reduced by dimensional analysis to

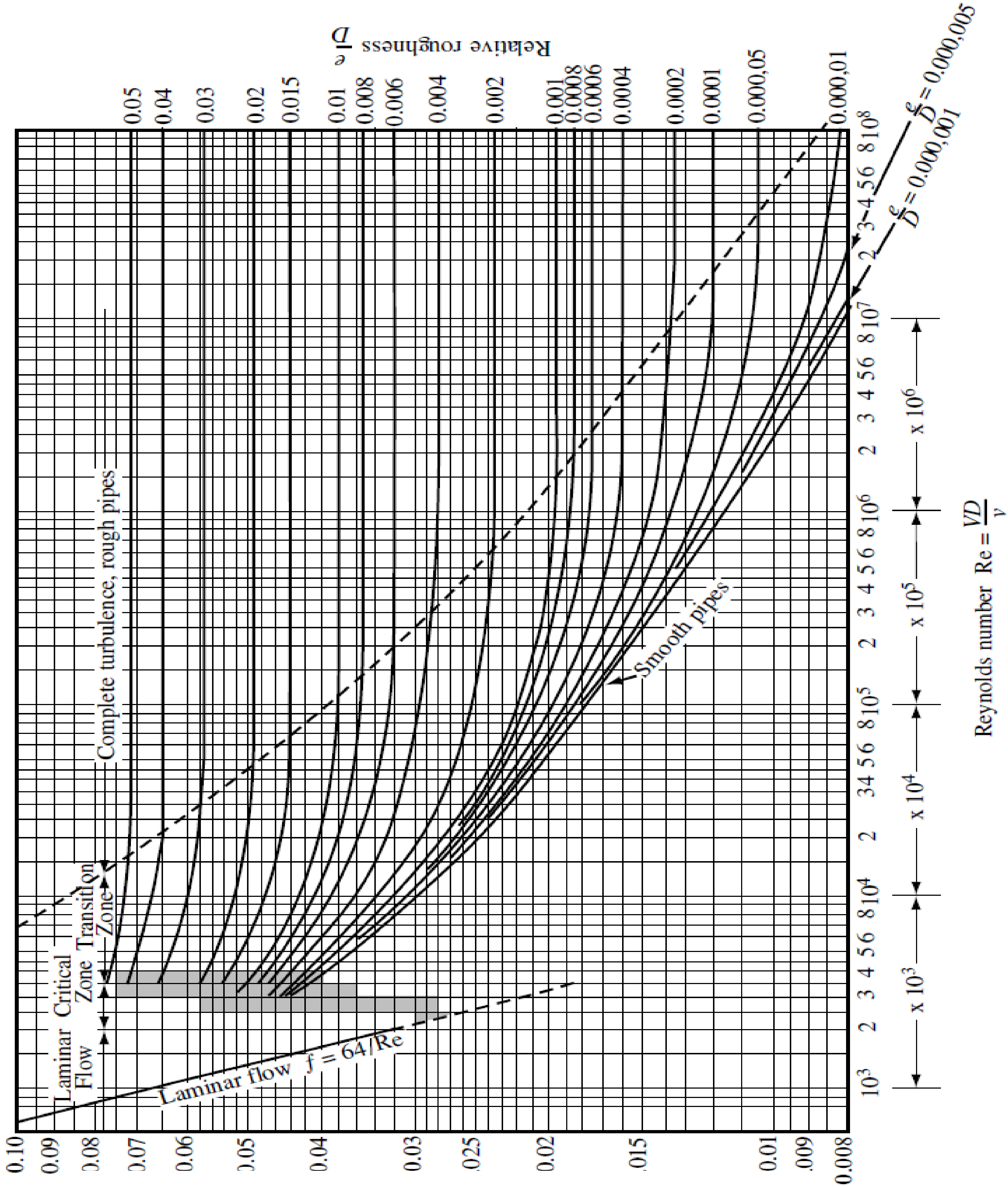
$$\frac{\tau_w}{\rho V^2} = F\left(\frac{VD\rho}{\mu}, \frac{e}{D}\right) = \frac{f}{8} \quad \& \quad \tau_w = \gamma\, h_L \frac{D}{4L}$$

$$h_f = f\frac{L}{D}\frac{v^2}{2g} = f\frac{L}{D}\frac{Q^2}{2gA^2}$$

In this equation, the friction factor (f) is a function of the pipe Reynolds number ($Re = \rho vD/\mu = vD/\nu$) and the equivalent sand-grain roughness factor$\left(\frac{e}{D}\right)$.

For each pipe material either a single value or range of $\left(\frac{e}{D}\right)$ values has been established; Table below presents common values for several materials.

Material	e, mm	e, in
Riveted Steel	0.9 - 9.0	0.035 - 0.35
Concrete	0.30 - 3.0	0.012 - 0.12
Cast Iron	0.26	0.010
Galvanized Iron	0.15	0.006
Asphalted Cast Iron	0.12	0.0048
Commercial or Welded Steel	0.045	0.0018
PVC, Drawn Tubing, Glass	0.0015	0.000 06



The Moody diagram for the Darcy-Weisbach friction factor *f*.

Because commercially available pipes of any material display some heterogeneity or unevenness in roughness, any friction factor or its empirical equivalents cannot be known with multiple-digit precision. The functional behavior of (f) is displayed fully in the Moody diagram.

In the Moody diagram, we see several zones that characterize different kinds of pipe flow. First we note that the plot is logarithmic along both axes. Below the Reynolds number Re = 2100 (some authors prefer 2300) there is only one line, which can be derived solely from the laminar, viscous flow equations without experimental input; the resulting friction factor for laminar flow is $f = 64/Re$. Because there is only one line in this region, we say all pipes are hydraulically smooth in laminar flow. Then for Reynolds numbers up to, say, 4000 is a so-called "critical" zone in which the flow changes from laminar flow to weakly turbulent flow.

EMPIRICAL EQUATIONS

Empirical head loss equations have a long and honorable history of use in pipeline problems. Their initial use preceded by decades the development of the Moody diagram, and they are still commonly used today in professional practice. Some prefer to continue to use such an equation owing simply to force of habit, while others prefer it to avoid some of the difficulties of determining the friction factor in the Darcy-Weisbach equation.

Table below summarizes the relations that describe the Darcy-Weisbach friction factor (f).

DARCY-WEISBACH FRICTION EQUATIONS

Type of Flow	Equation for f	Range
Laminar	$f = 64/Re$	$Re < 2100$
Smooth pipe	$1/\sqrt{f} = 2\log_{10}\left(Re\sqrt{f}\right) - 0.8$	$Re > 4000$ and $e/D \rightarrow 0$
Transitional Colebrook-White Eq.	$\dfrac{1}{\sqrt{f}} = 1.14 - 2\log_{10}\left(\dfrac{e}{D} + \dfrac{9.35}{Re\sqrt{f}}\right)$	$Re > 4000$
Wholly Rough	$\dfrac{1}{\sqrt{f}} = 1.14 - 2\log_{10}\left(\dfrac{e}{D}\right)$	$Re > 4000$

3. Simple Pipe Problem

Six variables enter into the problem for incompressible fluid, which are Q, L, D, h_f, v, and e. Three of them are given (L, V, and e) and three will be find. Now, the problems type can be solved as follows,

Problem	Given	To find (unknown)
I	Q, L, D, v, e	h_f
II	h_f, L, D, v, e	Q
III	Q, L, h_f, v, e	D

In each of the above problem the following are used to find the unknown quantity

i. The Darcy – Weisbach Equation.
ii. The Continuity Equation.
iii. The Moody diagram.

In place of the Moody diagram, the following explicit formula for (*f*) may be utilized with the restrictions placed on it

$$f = 0.0055\left[1 + \left(2000.\frac{\epsilon}{d} + \frac{10^6}{R_e}\right)^{\frac{1}{3}}\right] \text{Moody equation}$$

$$4 * 10^3 \leq R_e \leq 10^7 \& \frac{\epsilon}{D} \leq 0.01$$

$$f = \frac{1.325}{\left[\ln\left(\frac{\epsilon}{3.7d} + \frac{5.74}{R_e^{0.9}}\right)\right]^2} \quad 10^{-6} \leq \frac{\epsilon}{D} \leq 10^{-12} \text{ , } 5000 \leq R_e \leq 10^8$$

1% yield diff-with Darcy equation

The following formula can be used without Moody chart is

$$\frac{1}{f^{1/2}} \approx -1.8 \log\left[\frac{6.9}{Re_d} + \left(\frac{\epsilon/d}{3.7}\right)^{1.11}\right]$$

The last equation is given by Haaland which varies less than 2% from Moody chart.

3. Solution Procedures.

I. Solution for head loss (h_f).

With Q, e, and D are known

$$Re = \frac{\rho v D}{\mu} = \frac{vD}{\nu}$$

And *f* may be looked up Moody diagram or calculated from empirical equations given.

Example

Determine the head (energy) loss (friction losses) for flow of 140 l/s of oil ν=0.00001 m^2/s through 400 m of 200 mm, diameter cost–iron pipe (e=0.25m).

Sol.

$$v = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}D^2} = \frac{0.14}{\frac{\pi}{4}0.2^2} = 4.456\text{m/s}$$

$$R_e = \frac{4Q}{\pi D \upsilon} = \frac{4(0.14)}{\pi(0.2)(0.00001)} = \mathbf{89127}$$

$$Re = \frac{vD}{\nu} = \frac{4.456 \times 0.2}{0.00001} = 89127$$

The relation roughness is e/D= 0.25/200= 0.00125 from a given diagram by interpolation **f= 0.023** or by equation $f = \frac{1.325}{\left[\ln\left(\frac{e}{3.7D} + \frac{5.74}{Re^{0.9}}\right)\right]^2} = 0.0234$

$$h_f = f\frac{L}{D}\frac{v^2}{2g} = 0.023\frac{400}{0.2} \times \frac{4.456^2}{2g} = 46.58\text{m.}$$

II. Solution for discharge (Q)

v & *f* Are unknown then Darcy – Weisbach equation and moody diagram must be used simultaneously to find their values.

1. Givens: e/D

(f) Value is assumed by inspection of the Moody diagram.

2. Substitution of this trail f into the Darcy – equation produce a trial value of (v).

3. From v a trial Re is computed.

4. An improved value of *(f)* is found from moody diagram with help of Re.

5- When *(f)* has been found correct the corresponding v and Q is determined by multiplying by the area.

Example

Water at 15 ºC ($\nu = 1.13*10^{-6}$ m^2/s) flow through a 300mm diameter riveted steel pipe, e=3mm with a head loss of 6 m in 300 m. Determine the flow.

Sol.

The relative roughness is e/d = 0.003/0.3=0.01, and from diagram a trial f is taken as (0.038). By substituting into Darcy equation:

$$h_f = f\frac{L}{D}\frac{v^2}{2g} = 0.038\frac{300}{0.3}\times\frac{v^2}{2(9.81)} \quad \rightarrow\rightarrow\rightarrow \text{ v = 1.76m/s}$$

$$Re = \frac{vD}{\nu} = \frac{1.715\times 0.3}{1.13\times 10^{-6}} = 467278$$

From the Moody diagram at (Re & e/D) f=0.038

$$\text{And from Darcy} \rightarrow v = \sqrt{\frac{h_f*D*2g}{f*L}} = \sqrt{\frac{6*0.3*2*9.81}{0.038*300}} = 1.76\text{m/s}$$

$$\therefore Q = vA = \sqrt{\frac{6*0.3*2*9.81}{0.038*300}} * \frac{\pi\, 0.3^2}{4} = 0.1245\text{m}^3\text{/s}$$

III. Solution for Diameter (D)

Three unknowns in Darcy-equation f, v, D, two in the continuity equation v, D and three in the Re number equation. To element the velocity in Darcy equation & in the expression for Re, simplifies the problem as follows.

$$h_f = f\frac{L}{D}\frac{Q^2}{2g\left(\frac{D^2\pi}{4}\right)^2} \quad \text{Or} \quad D^5 = \frac{8LQ^2}{h_f g\,\pi^2}f \qquad \left(\left(\therefore D^5 = \frac{fLQ^2}{12.1h_f} \text{ in S.I units}\right)\right)$$

The solution is now affected by the following procedure:

1- Assume the value of (f).

2- Solve for (D) using $D^5 = \frac{fLQ^2}{12.1h_f}$.

3- Find Re.

4- Find the relative roughness e/D.

5- With Re and e/D, Look up new f from Moody diagram.

6- Use the new f, and repeat the procedure.

7- When the value of f does not change in the two significant steps, all equations are satisfied and the problem is solved.

Example

Determine the size of clean wrought-iron pipe (e=0.00015 ft) required to convey 8.93 cfs oil, ν=0.0001 ft^2/s , 10000 ft with a head loss of 75 ft .lb/lb.

Sol.

If *f*=0.02 (assumed value)

$$D^5 = \frac{8LQ^2}{h_f g\,\pi^2} f = \frac{8*10000*8.93^2}{75*32.2*\pi^2} * 0.02 \longrightarrow\longrightarrow \text{D= 1.35 ft.} \quad \therefore \frac{e}{D} = 0.00011$$

$$Re = \frac{vD}{\nu} = \frac{4Q}{\pi\nu}\frac{1}{D} = \frac{4*8.93}{\pi*0.0001}\frac{1}{1.35} = 81400$$

Applying $\frac{e}{D} = 0.00011$ & $Re =$81400 on Moody diagram we can get *f* value

$\therefore f = 0.0191$

In repeating the procedure, D = 1.37 ft →→ Re = 82991 →→*f* = 0.019

Therefore

D = 1.382 * 12 = 16.6 in.

4. Pumps & Turbines

If a pump is placed between the two points then the equation is modified to take account of the energy addition due to the pump.

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_P = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_f$$

h_P is the pressure head rise through the pump.

$P_P = \gamma * Q * h_P \quad (\frac{N}{m^3} * \frac{m^3}{s} * m) = \frac{N.m}{s} = \text{Watt "W"}$

Where (P_P) is the power of pump and also named water power.

The mechanical power to operate the pump must be larger; it is called the brake horsepower

$$\text{The power} = \frac{P_P}{Efficiency} = \frac{P_P}{\eta}$$

$$\frac{N.m}{s} = \frac{Joul}{s} = Watt \quad \text{(Joul is the unit of work = Newton . m)}$$

$$\text{Horse Power (HP)} = \frac{\gamma Q H \frac{ft.lb}{\sec}}{550}$$

If a turbine is placed between

HP = 745.6999 watt

the two points then the equation is further modified to take account of the energy extraction due to the turbine:

$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}+h_p-h_T=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}+h_f$$

h_T is the pressure head drop through the turbine.

$P_T = \gamma * Q * h_T \quad (\frac{N}{m^3} * \frac{m^3}{s} * m) = \frac{N.m}{s} = \text{Watt "W"}$

Where (P_T) is the power of turbine

The power $= \frac{Efficiency}{P_T} = \frac{\eta}{P_T}$

$$\eta = \frac{Power\ Output}{Power\ Input} \times 100\%$$

Pump efficiency

$$\eta P = \frac{\gamma Q h_P}{Input} \times 100\%$$

Turbine efficiency

$$\eta T = \frac{Output}{\gamma Q h_T} \times 100\%$$

Example

A pump delivers water (ν=1.007*10^{-6} m^2/s) from a tank (A) (water surface elevation=110m) to tank B (water surface elevation= 170m). The suction pipe is 45m long and 35cm in diameter the delivered pipe -is 950m long 25cm in diameter. Loss head due to friction hf_1 = 5m and hf_2 = 3m

If the piping are from

Pipe (1) = steel sheet metal (e_1= 0.05mm)

Pipe (2) = stainless – steel (e_2= 0.002mm)

Calculate the following

i) The discharge in the pipeline ii) The power delivered by the pump.

Sol.

D_1 = 35 cm = 0.35m; D_2 = 25cm = 0.25m

L_1 = 45m; L_2 = 950m

$\frac{e_1}{D_1} = \frac{0.05}{350} = 1.428 * 10^{-4}$, $\frac{e_2}{D_2} = \frac{0.002}{250} = 8 * 10^{-6}$

Assume $f_1 = 0.013$; $f_2 = 0.008$

$$h_{f_1} = f_1 \frac{L_1}{D_1}\frac{v_1^2}{2g} \quad \therefore 5 = 0.013\frac{45}{0.35} \times \frac{v_1^2}{2(9.81)} \rightarrow v_1 = 7.66m/s$$

$Re_1 = \frac{vD}{\nu} = \frac{7.66 \times 0.35}{1.007 \times 10^{-6}}$ = 2662363 = 2.66*10^6

$$h_{f_2} = f_2 \frac{L_2}{D_2}\frac{v_2^2}{2g} \quad \therefore 3 = 0.008\frac{950}{0.25} \times \frac{v_2^2}{2(9.81)} \rightarrow v_2 = 1.39m/s$$

$Re_2 = \frac{vD}{\nu} = \frac{1.39 \times 0.25}{1.007 \times 10^{-6}}$ = 2662363 = 3.45*10^5

1st. Trial

$\left(Re_1 \,\&\, \frac{e_1}{D_1}\right) \rightarrow\rightarrow f_1 = 0.0138$ $\left(Re_2 \,\&\, \frac{e_2}{D_2}\right) \rightarrow\rightarrow f_2 = 0.014$

Then,

$$\therefore h_{f_1} = 5 = 0.0138\frac{45}{0.35} \times \frac{v_1^2}{2(9.81)} \rightarrow v_1 = 7.435\frac{m}{s} \rightarrow Re_1 = 2.58 * 10^6$$

$$\therefore h_{f_2} = 3 = 0.014\frac{950}{0.25} \times \frac{v_1^2}{2(9.81)} \rightarrow v_2 = 1.051\frac{m}{s} \rightarrow Re_2 = 2.6 * 10^5$$

2nd. Trial

$\left(Re_1 \,\&\, \frac{e_1}{D_1}\right) \rightarrow\rightarrow f_1 = 0.0165$ $\left(Re_2 \,\&\, \frac{e_2}{D_2}\right) \rightarrow\rightarrow f_2 = 0.015$

$$\therefore h_{f_1} = 5 = 0.0165\frac{45}{0.35} \times \frac{v_1^2}{2(9.81)} \rightarrow v_1 = 6.8\frac{m}{s} \rightarrow Re_1 = 2.36 * 10^6$$

$$\therefore h_{f_2} = 3 = 0.015\frac{950}{0.25} \times \frac{v_1^2}{2(9.81)} \rightarrow v_2 = 1.01\frac{m}{s} \rightarrow Re_2 = 2.52 * 10^5$$

3rd. Trial

$\left(Re_1 \,\&\, \frac{e_1}{D_1}\right) \rightarrow\rightarrow f_1 = 0.0169$ $\left(Re_2 \,\&\, \frac{e_2}{D_2}\right) \rightarrow\rightarrow f_2 = 0.015$

$$\therefore h_{f_1} = 5 = 0.0169\frac{45}{0.35} \times \frac{v_1^2}{2(9.81)} \rightarrow v_1 = 6.72\frac{m}{s}$$

$$\therefore h_{f_2} = 3 = 0.015\frac{950}{0.25}\times\frac{v_1^2}{2(9.81)} \rightarrow v_2 = 1.01\frac{m}{s}$$

Q= A_2v_2 = 0.6462 m^3/s

From energy equation

$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}+h_p=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}+h_f$$

$$0 \quad +110+h_p = \quad 0 \quad +170+8 \qquad \text{Since } p_1=p_2$$

h_p = 68m

$P=\gamma Q h_p = 9810 * 0.6462 *$ 68m $= 416.8\ kW$ The power delivered by the pump.

Example

In a pipeline of diameter 350mm and length 75m, water is flowing at a velocity of 2.8 m/s. Find the head lost due to friction, using Darcy-Eq.& Moody chart, pipe material is Steel–Riveted (e=3mm) ,kinematic viscosity ν= 0.012 stoke.

Sol.

$h_f = f\frac{L}{D}\frac{v^2}{2g}$, D = 0.35m, L = 75m; v = 2.8 m/s , e/D= 0.003/0.35= 8.57*10^{-3}

$1\frac{m^2}{s} = 10^4$ stoke $\therefore$ $\nu = 0012\times10^{-4}\ \frac{m^2}{s}$

$$\text{Re} = \frac{\text{vD}}{\nu} = \frac{2.8\times0.35}{0012\times10^{-4}} = 816666 \approx 8.1\times10^5$$

$At\ \ Re\ \&\ \frac{e}{D}$ we can find f from Moody diagram f = 0.0358

$$\therefore h_f = 0.0358\frac{75}{0.35}\times\frac{(2.8)^2}{2(9.81)} = 3.0\text{ m}$$

By determine the value of f by $\frac{1}{f^{1/2}} \approx -1.8\log\left[\frac{6.9}{Re\ D}+\left(\frac{\frac{e}{D}}{3.7}\right)^{1.11}\right]$

$$\therefore \frac{1}{f^{1/2}} \approx -1.8\log\left[\frac{6.9}{8.1\times10^5\times0.35}+\left(\frac{\frac{0.003}{0.35}}{3.7}\right)^{1.11}\right] = 5.2646$$

Then f = 0.036 i.e. f difference = 0.0002

Example

Oil having absolute viscosity 0.1 Pa.s and relative density 0.85 flow through an iron pipe with diameter 305mm and length 3048 m with flow rate $44.4*10^{-3}$ m^3/s . Determine the head loss in pipe.

Sol.

$$v = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}D^2} = \frac{44.4\times10^{-3}}{\frac{\pi}{4}(0.305)^2} = 0.61 m/s \qquad Re = \frac{\rho v D}{\mu} = \frac{850\times0.61\times0.305}{0.1} = 1580$$

i.e. the flow is Laminar

$$f = \frac{64}{R_e} = \frac{64}{1580} = 0.0407$$

$$h_f = f\frac{L}{D}\frac{v^2}{2g} = 0.0407\frac{3048}{0.305}\times\frac{(0.61)^2}{2(9.81)} = 7.71m$$

5. *Minor Losses.*

A local loss is any energy loss, in addition to that of pipe friction alone, caused by some localized disruption of the flow by some flow appurtenances, such as valves, bends, and other fittings. The actual dissipation of this energy occurs over a finite but not necessarily short longitudinal section of the pipe line, but it is an accepted convention in hydraulics to lump or concentrate the entire amount of this loss at the location of the device that causes the flow disruption and loss. If a loss is sufficiently small in comparison with other energy losses and with pipe friction, it may be regarded as a minor loss. Often minor losses are neglected in preliminary studies or when they are known to be quite small, as will often happen when the pipes are very long. However, some local losses can be so large or significant that they will never be termed a minor loss, and they must be retained; one example is a valve that is only partly open.

Normally, theory alone is unable to quantify the magnitudes of the energy losses caused by these devices, so the representation of these losses depends heavily upon experimental data.

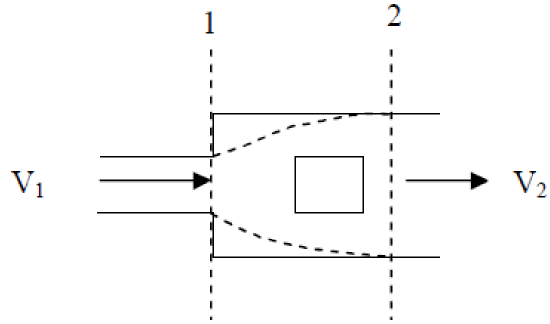
The losses which occur in pipelines because of bend, elbows, joints, valves, etc, are called minor losses h_m. the total losses in pipeline are

$$h_L = h_f + h_m$$

Local losses are usually computed from the equation: $h_m = k\frac{v^2}{2g}$

Also, others minor losses can be explain as follows,

A. Losses due to sudden expansion in pipe.

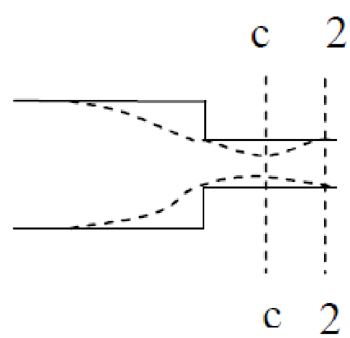


$$h_m = k\frac{v^2{}_1}{2g}$$

$$k = \left(1 - \frac{A_1}{A_2}\right)^2 = \left(1 - \frac{D_1}{D_2}\right)^2$$

If sudden expansion from pipe to a reservoir $\frac{D_1}{D_2} = 0$ Then $h_m = \frac{v^2{}_1}{2g}$

B. Head loss due to a sudden contraction in the pipe cross section.



$$h_m = k\frac{v^2{}_2}{2g}$$

$$k = \left(\frac{1}{C_C} - 1\right)^2$$

A_C is the cross – sectional area of the vena-contracts

C_C is the coefficient of contraction is defined by $C_C = \frac{A_C}{A_2}$

C. The head loss at the entrance to a pipe line.

From a reservoir the head losses is usually taken as

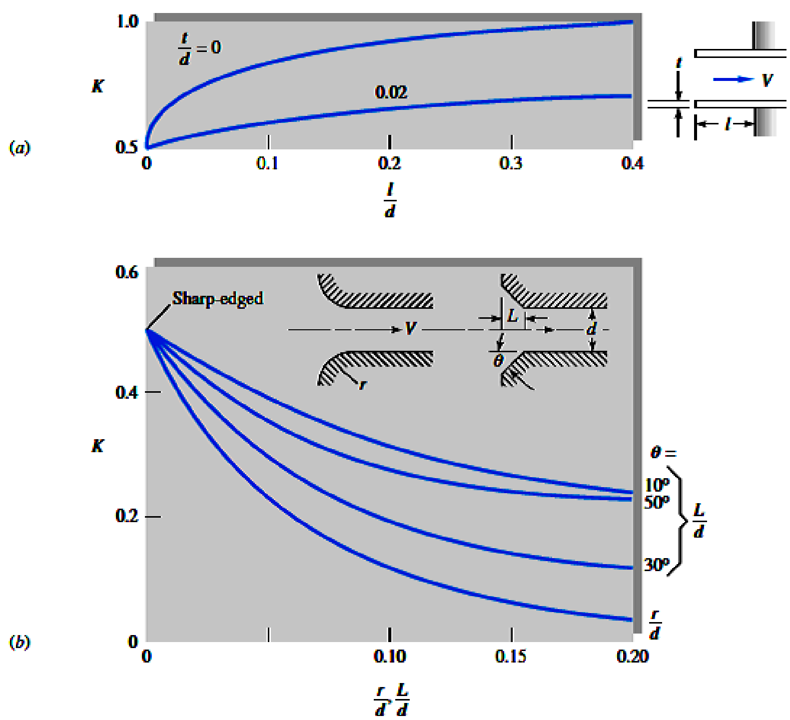
$$h_m = \frac{0.5V^2}{2g} \quad \text{is the opening is square-edged}$$

$$h_m = \frac{0.01V^2}{2g} \sim \frac{0.05V^2}{2g} \text{ if the rounded entrance}$$

Table below lists the loss coefficient K for four types of valve, three angles of elbow fitting and two tee connections. Fitting may be connected by either internal screws or flanges, hence the two are listings.

Fitting	K_L
Globe valve, fully open	10.0
Angle valve, fully open	5.0
Butterfly valve, fully open	0.4
Gate valve, fully open	0.2
3/4 open	1.0
1/2 open	5.6
1/4 open	17.0
Check valve, swing type, fully open	2.3
Check valve, lift type, fully open	12.0
Check valve, ball type, fully open	70.0
Foot valve, fully open	15.0
Elbow, 45°	0.4
Long radius elbow, 90°	0.6
Medium radius elbow, 90°	0.8
Short radius (standard) elbow , 90°	0.9
Close return bend, 180°	2.2
Pipe entrance, rounded, r/D < 0.16	0.1
Pipe entrance, square-edged	0.5
Pipe entrance, re-entrant	0.8

Entrance losses are highly dependent upon entrance geometry, but exit losses are not. Sharp edges or protrusions in the entrance cause large zones of flow separation and large losses as shown in figure.



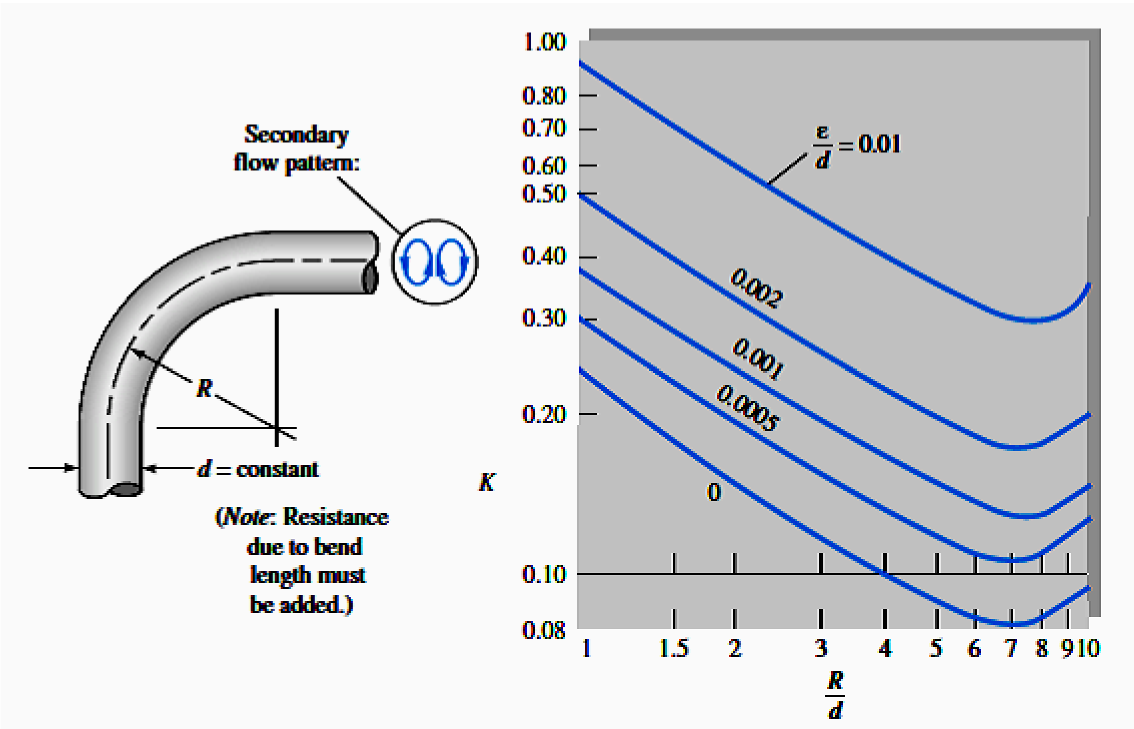
Entrance and exit loss coefficients,

(a) Reentrant inlets,

(b) Rounded and beveled inlets.

Exit losses are K=1.

As in Fig. 7.10, a bend or curve in a pipe, always induces a loss larger than the simple Moody friction loss, due to flow separation at the walls and a swirling secondary flow arising from centripetal acceleration.

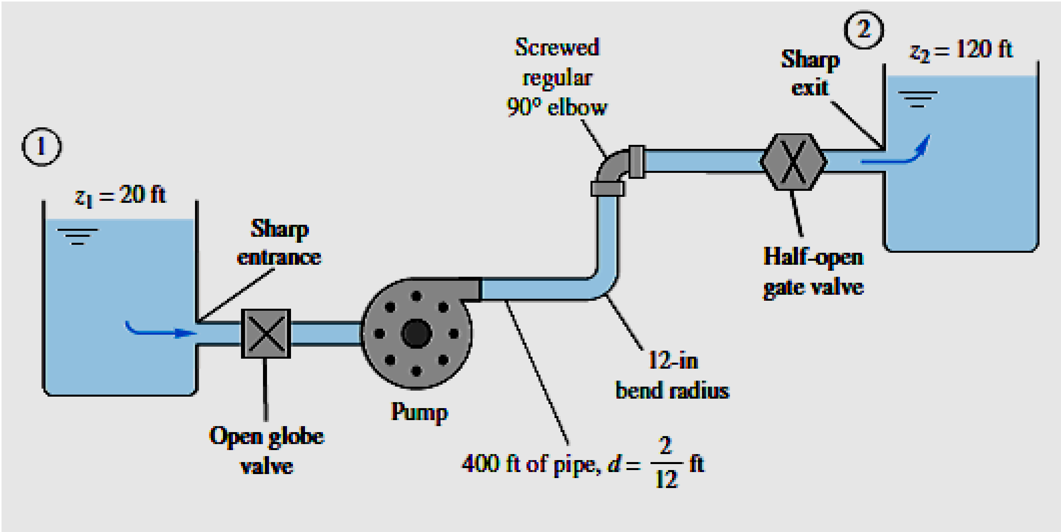


Example

Water, ρ = 1.94 slugs/ft^3, and ν= 1.1*10^{-5} ft^2/s , is pumped between two reservoir at 0.2 ft^3/s through 400ft. of 2in. diameter pipe and several minor losses, as shown in figure. The roughness ratio is e/D = 0.001. Compute the horse power required.

The minor loss coefficients are as shown .

Fitting	K
Sharp entrance	0.5
Open globe valve	6.9
12-in bend	0.15
Regular 90 elbow	0.95
Half – closed gate valve	3.8
Sharp exit	1



Sol.

Write the steady- flow energy equation between section 1 &2 the two reservoir surface:

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\left(\frac{p_2}{2g}+\frac{V_2^2}{2g}+z_2\right)+h_f+\sum h_m-h_p$$

Where h_P is the head increase across the pump & $p_1 = p_2$, $v_1 = v_2 \approx 0$,

Solve for the pump head

$$h_p = z_2 - z_1 + h_f + \sum h_m = 120 - 20 + \frac{V^2}{2g}\left(f\frac{L}{d}+\sum K\right)$$

$$V = \frac{Q}{A} = \frac{0.2}{\frac{1}{4}\pi\left(\frac{2}{12}\right)^2} = 9.17\frac{ft}{s}$$

Calculate the *Re* and pipe friction factor $\mathrm{Re} = \frac{\mathrm{vD}}{\nu} = \frac{9.17\times\frac{2}{12}}{1.1\times10^{-5}} = 139000$

$$\sum k = 13.3$$

For e/D = 0.001, from the Moody chart read $f = 0.0216$

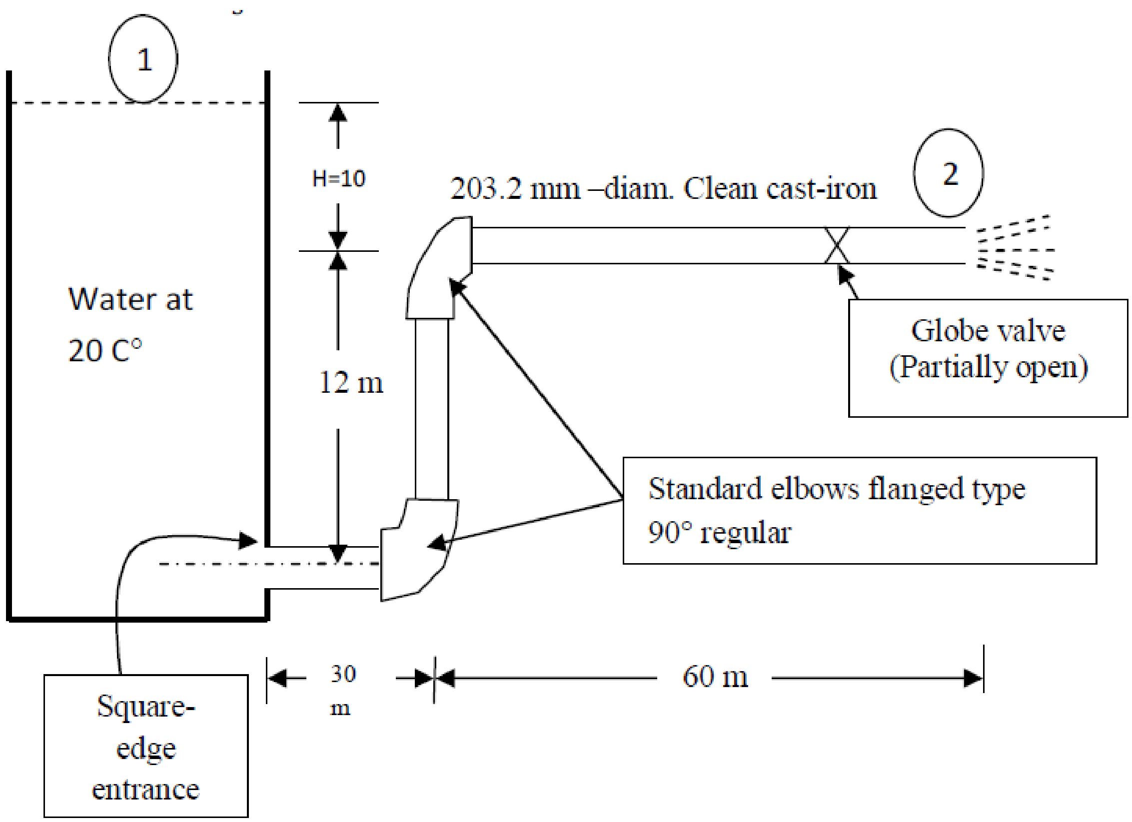
$$\therefore h_p = 100 + \frac{(9.17)^2}{2(32.2)}\left[\frac{0.0216(400)}{\left(\frac{2}{12}\right)} + 13.3\right] \qquad \therefore h_p = 184 ft\ head\ pump$$

$P_P = \rho g^* \ Q^* \ h_P = 1.94(32.2) * 0.2 * 184 = 2300 \frac{\text{ft lb}}{\text{sec}}$

1 h.p. $= 550 \frac{\text{ft lb}}{\text{sec}}$ then horsepower $= \frac{2300}{550} = 4.2$ h.p.

Example

Find the discharge through the pipeline as in below figure for H=10m,.($K_{entrance}$ = 0.5 , K_{elbow}= 0.26 , K_{valve} = 10). Roughness =0.26mm. $\nu = 1.01*10^{-6}$ m^2/s



Sol.

The energy equation applied between points 1 & 2, including all the losses, may be written

I)

$$10 + 0 + 0 = \frac{v_2^2}{2g} + 0 + 0 + 0.5\frac{v_2^2}{2g} + f\frac{102}{0.2032}\frac{v_2^2}{2g} + 2 \times 0.26\,\frac{v_2^2}{2g} + 10\,\frac{v_2^2}{2g}$$

$$\therefore 10 = \frac{v_2^2}{2g}\,(\,6.32 + 502\,f) \cdots\cdots\cdots\cdots\cdots (1)$$

e/D = 0.26/203.2 = $1.28*10^{-3}$

Assume *f = 0.0205*

Apply on eq^n (1)

$10 = \frac{v_2^2}{2g}(6.32 + 502 \times 0.0205) \rightarrow\rightarrow v_2 = 3.43 m/s$

e/D = 0.00128 $\mathrm{Re} = \frac{vD}{\nu} = \frac{3.43 \times 0.2032}{1.01 \times 10^{-6}} = 391000$

From Moody chart at (e/D = 0.00128 & Re = 391000) →→ f = 0.021

Repeating the procedure gives v_2 = 2.6m/s

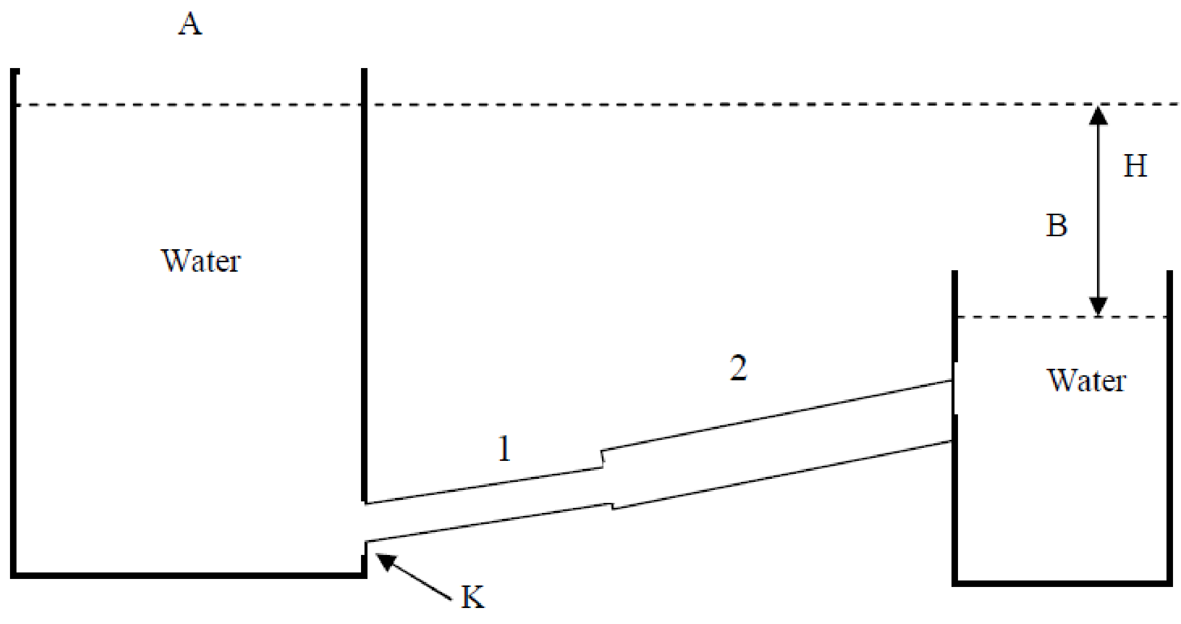
Re= 380000 & f = 0.023

The discharge is

Q= $A_2 v_2 = \frac{\pi}{4}(0.2032)^2 \times 2.6 = 84.3 l/s$

6. *Pipe in Series*

In this typical series–pipe system as in figure shown, the H (head) is required for a given Q or the Q wanted for a given H. Applying the energy equation from A to B including all losses gives:



$$H + 0 + 0 = 0 + 0 + 0 + K_e \frac{v_1^2}{2g} + f_1 \frac{L_1}{D_1}\frac{v_1^2}{2g} + \frac{(v_2 - v_1)^2}{2g} + f_2 \frac{L_2}{D_2}\frac{v_2^2}{2g} + \frac{v_2^2}{2g}$$

From continuity eqn.

$v_1 D_1^2 = v_2 D_2^2$

So,

$$H = \frac{v_1^2}{2g}\left\{K_e + f_1\frac{L_1}{D_1} + \left[1-\left(\frac{D_1}{D_2}\right)^2\right]^2 + f_2\frac{L_2}{D_2}\left(\frac{D_1}{D_2}\right)^4 + \left(\frac{D_1}{D_2}\right)^4\right\} \quad \ldots\ldots\ldots(1)$$

For known lengths and sizes of pipes this reduces to

$$H = \frac{v_1^2}{2g}(C_1 + C_2 f_1 + C_3 f_2) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Where $C_1, C_2, and\ C_3$ are known.

- Q is given then Re is computed and f may be looked up in Moody diagram then H is found by direct substitution.
- For a given H, the values of v_1, f_1, f_2 are unknown in the equation (2)

By Assuming values of f_1& f_2 (may be equaled) then v_1 is found 1st. trail and from $v_1 \rightarrow$ Re's are determined and values of f_1& f_2 look up from Moody diagram.

From these values, a better v_1 is computed from Eq. (1)since f varies so slightly with Re the trial solution converges very rapidly. The same procedures apply for more than two pieces in series.

Example

From figure in (Page 77), given : K_e = 0.5 , L_1=300m , D_1=600mm , e_1=2mm , L_2=240m , D_2=1m , e_2=0.3mm , $\nu = 3*10^{-6}$ m^2/s , and H=6m. Determine the discharge through the system.

Sol.

From energy equation:

$$H = \frac{v_1^2}{2g}\left\{K_e + f_1\frac{L_1}{D_1} + \left[1-\left(\frac{D_1}{D_2}\right)^2\right]^2 + f_2\frac{L_2}{D_2}\left(\frac{D_1}{D_2}\right)^4 + \left(\frac{D_1}{D_2}\right)^4\right\}$$

$$6 = \frac{v_1^2}{2g}\left\{0.5 + f_1\frac{300}{0.6} + \left[1-\left(\frac{0.6}{1}\right)^2\right]^2 + f_2\frac{240}{1}\left(\frac{0.6}{1}\right)^4 + \left(\frac{0.6}{1}\right)^4\right\}$$

After simplifying

$$6 = \frac{v_1^2}{2g}\ (1.0329 + 500f_1 + 31.104f_2)$$

From e_1/D_1 = 0.0033 & e_2/D_2= 0.0003, and Moody diagram values of f's are assumed for the fully turbulent range.

$f_1 = 0.026 \quad f_2 = 0.015$

By solving for V_1 with these value , $V_1 = 2.848\frac{m}{s}, V_2 = 1.025\frac{m}{s}$

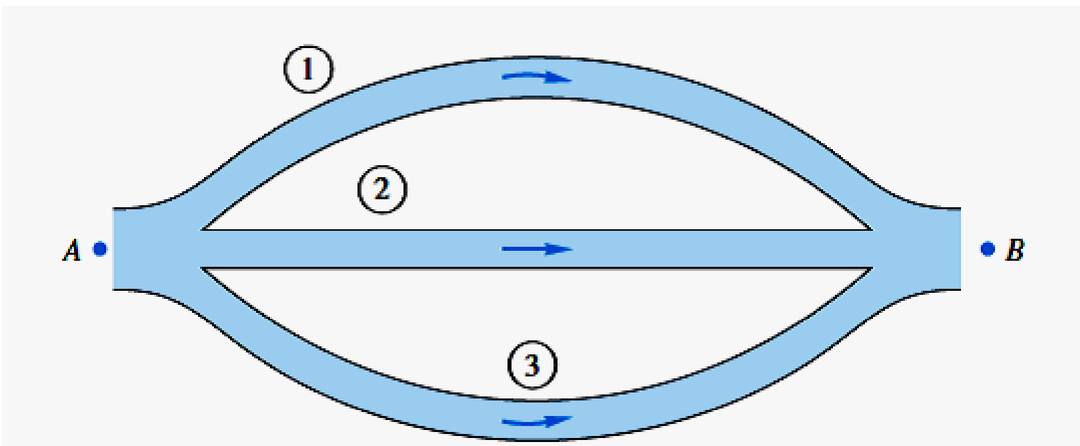
$Re_1 = \frac{2.848*0.6}{3*10^{-6}} = 569600$

$Re_2 = \frac{1.025*1.0}{3*10^{-6}} = 341667$

At these $\boldsymbol{Re's}$ and from Moody diagram, $f_1 = 0.0265, f_2 = 0.0168$, by solving again for $V_1, V_1 = 2.819\frac{m}{s}$ and $Q = 0.797\frac{m^3}{s}$

7. Pipes in Parallel.

The second type of pipe- system is the parallel flow type, in this case as shown in figure below , the head losses are same in any of the lines and the total flow is the sum of flow rate in each pipe. The minor losses are added into the lengths of each pipe as equivalent lengths. From the figure, the conditions to be satisfied are



$$h_{f_1} = h_{f_2} = h_{f_3} = \frac{p_A}{\gamma} + z_A - \left(\frac{p_B}{\gamma} + z_B\right)$$

$$Q_A = Q_1 + Q_2 + Q_3 = Q_B$$

z_A & z_B are the elevations of point A&B. Q is the discharge through the approach pipes.

Two types of problems occur

1) The elevations of HGL at A&B are known, to find the discharge.

2) Q is known, to find the distribution of flow and the head loss, size of pipe, fluid properties and roughness's are assumed to be known.

Case-1. as the simple pipe problem. Since, head loss is the drop in HGL. These discharges are added to determine the total discharge.

Case-2. The recommended procedure is as follows.

1. Assume a discharge Q' , through pipe 1.
2. Solve for h'_{f_1} using assumed discharge.
3. Using h'_{f_1} find Q'_2, Q'_3.
4. With the three discharges for a common head loss, now assume that the given Q is split up among the pipes in the same proportion as Q'_1, Q'_2, Q'_3 thus

   $$Q_1 = \frac{Q'_1}{\sum Q'} Q \;\; ; \;\; Q_2 = \frac{Q'_2}{\sum Q'} Q \;\; ; Q_3 = \frac{Q'_3}{\sum Q'} Q$$

5. Check the correctness of these discharges by computing $h_{f_1}, h_{f_2}, h_{f_3}$ for the computing Q_1, Q_2, Q_3

__Example__

In the last figure , L_1=3000ft , D_1=1ft, e_1= 0.001ft.

L_2=2000ft, D_2=8in., e_2= 0.0001ft

L_3=4000ft, D_3=16in., e_3= 0.0008ft

$$\rho = 2.00 \frac{slugs}{ft^3} \qquad \nu = 0.00003 \frac{ft^2}{s}$$

$$p_A = 80psi \, , z_A = 100ft. \, , z_B = 80ft.$$

For a total flow of 12 cfs, determine the flow through each pipe and the pressure at B.

__Sol.__

__For pipe 1__

Assume $Q'_1 = 3cfs$,, then $v'_1 = 3.82ft/s$

$$Re'_1 = \frac{3.82 \times 1}{0.00003} = 127000$$

$$\frac{e_1}{D_1} = \frac{0.001}{1} \; 0.001$$

From Moody chart f'_1= 0.022

$$\therefore h'_{f1} = 0.022 \frac{3000}{1.0} \frac{3.82^2}{64.4} = 14.97 \, ft$$

For pipe 2

$$14.97 = f_2' \frac{2000}{0.667} \frac{V_2'^2}{2g} ------ (a)$$

Assume $f'_1 = 0.020$ (Recommended fully turbulent flow)

$$then\ V_2' = 4.01 \frac{ft}{s} --\rightarrow Re_2' = \frac{4.01 * \frac{2}{3} * 1}{0.00003} = 89000$$

$$\frac{e_2}{D_2} = 0.00015$$

From Moody chart → $f_2' = 0.019$

Then from Eq. (a) $V_2' = 4.11 \frac{ft}{s} --\rightarrow Q_2' = 1.44\ cfs$

For pipe 3

$$14.97 = f_3' \frac{4000}{1.333} \frac{V_3'^2}{2g} ------ (b)$$

Assume $f_3' = 0.019\ then\ V_3' = 4.01 \frac{ft}{s} --\rightarrow Re_3' = \frac{4.01 * 1.333}{0.00003} = 178000$

$$\frac{e_3}{D_3} = 0.0006$$

From Moody chart. At $\left(Re_3' \& \frac{\epsilon_3}{d_3}\right) --\rightarrow f_3' = 0.02$ from Eq.(b).

$V_3' = 4.0 \frac{ft}{s} \& Q_3' = 5.6\ cfs$

The total discharge for the assumed conditions is

$\sum Q' = 3.0 + 1.44 + 5.6 = 10.04\ cfs$

$$Q_1 = \frac{Q_1'}{\sum Q'} . Q = \frac{3.00}{10.04}\ 12 = 3.58\ cfs$$

$$Q_2 = \frac{1.44}{10.04}\ 12 = 1.72\ cfs$$

$$Q_3 = \frac{5.6}{10.04}\ 12 = 6.7\ cfs$$

Check the values of $h_{f_1}, h_{f_2}, h_{f_3}$

$$V_1 = \frac{3.58}{\pi * \frac{1}{4}} = 4.56\frac{ft}{s}$$

Re_1 (152000)& $\frac{e_1}{D_1}$ (0.001) $\rightarrow\rightarrow\rightarrow f_1 = 0.021$ $\quad Then\ h_{f_1} = 20.4 ft.$

$$V_2 = \frac{1.72}{\frac{\pi}{9}} = 4.93\frac{ft}{s}$$

Re_2 (109200)& $\frac{e_2}{D_2}$ (0.00015) $\rightarrow\rightarrow\rightarrow f_2 = 0.019$ $\quad Then\ h_{f_2} = 21.6 ft.$

$$V_3 = \frac{6.7}{\frac{4\pi}{9}} = 4.8\frac{ft}{s}$$

Re_3 (213000)& $\frac{e_3}{D_3}$ (0.0006) $\rightarrow\rightarrow\rightarrow f_3 = 0.019$ $\quad Then\ h_{f_3} = 20.4 ft.$

is about midway between 0.018 & 0.019. to satisfy the condition $h_{f_1} = h_{f_2} = h_{f_3}$

$\therefore if\ f_2 = 0.018\ then\ h_{f2} = 20.4\ ft \quad satisfying.$

To find p_B

$$\frac{p_A}{\gamma} + z_A = \frac{p_B}{\gamma} + z_B + h_f$$

$$or\ \frac{p_B}{\gamma} = 80 * \frac{144}{64.4} + 100 - 80 - 20.4 = 178.1\ ft.$$

In which the average head loss was taken. Then

$$p_B = \frac{178.1 * 64.4}{144} = 79.6\ psi$$