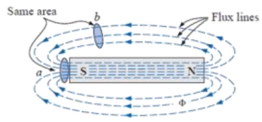
Magnetic Circuits

INTRODUCTION

Magnetism plays an integral part in almost every electrical device used today in industry, research, or the home. Generators, motors, transformers, circuit breakers, televisions, computers, tape recorders, and telephones all employ magnetic effects to perform a variety of important tasks.

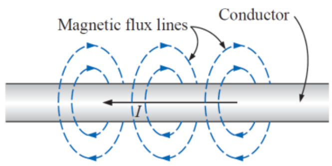
MAGNETIC FIELDS

In the region surrounding a permanent magnet there exists a magnetic field, which can be represented by **magnetic flux lines** similar to electric flux lines. Magnetic flux lines, however, do not have origins or terminating points as do electric flux lines but exist in continuous loops, as shown in Figure below.
The symbol for magnetic flux is the Greek letter Φ (phi).

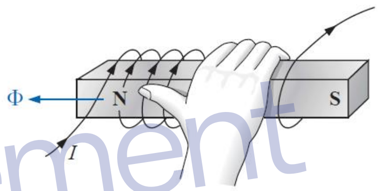


The magnetic flux lines radiate from the north pole to the south pole, returning to the north pole through the metallic bar.

A magnetic field is present around every wire that carries an electric current. The direction of the magnetic flux lines can be found simply by placing the thumb of the right hand in the direction of conventional current flow and noting the direction of the fingers. (This method is commonly called the right-hand rule.)



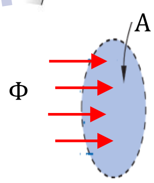
In the SI system of units, magnetic flux is measured in *webers*. The number of flux lines per unit area is called the **flux density,** is denoted by the capital letter *B*, and is measured in *teslas*. Its magnitude is determined by the following equation:



$$B = \frac{\Phi}{A}$$

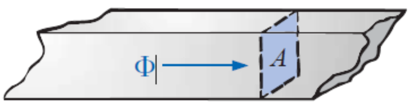
B = teslas (T)
Φ = webers (Wb)
A = square meters (m^2)

where Φ is the number of flux lines passing through the area *A*.



Example
Determine the flux density

$$B = \frac{\Phi}{A} = \frac{6\times10^{-5}}{1.2\times10^{-3}} = 5\times10^{-2}T$$



$\Phi = 6 \times 10^{-5}$ Wb
$A = 1.2 \times 10^{-3}\ m^2$

PERMEABILITY

If cores of different materials with the same physical dimensions are used in the electromagnet described in Section 11.2, the strength of the magnet will vary in accordance with the core used. This variation in strength is due to the greater or lesser number of flux lines passing through the core. Materials in which flux lines can readily be set up are said to be *magnetic* and to have *high permeability*. The **permeability** (**μ**) of a material, therefore, is a measure of the ease with which magnetic flux lines can be established in the material. It is similar in many respects to conductivity in electric circuits. The permeability of free space μ_0 (vacuum) is

$$\mu_0 = 4\pi\times10^{-7}\frac{W}{A.m}$$

The ratio of the permeability of a material to that of free space is called its **relative permeability;** that is,

$$\mu_r = \frac{\mu}{\mu_0}$$

RELUCTANCE

The resistance of a material to the flow of charge (current) is determined for electric circuits by the equation

$$R = \rho\frac{l}{A}\ \Omega$$

The reluctance of a material to the setting up of magnetic flux lines in the material is determined by the following equation:

$$\mathscr{R} = \frac{l}{\mu A} \qquad At/Wb$$

Where $\mathscr{R}$ is the reluctance, l is the length of the magnetic path, and A is the cross-sectional area.

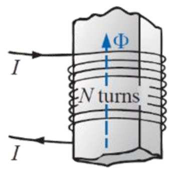
OHM'S LAW FOR MAGNETIC CIRCUITS

For magnetic circuits, the effect desired is the flux Φ. The cause is the **magnetomotive force (mmf)** $\mathscr{F}$ which is the external force (or "pressure") required to set up the magnetic flux lines within the magnetic material. The opposition to the setting up of the flux Φ is the reluctance $\mathscr{R}$.
Substituting, we have

$$\Phi = \frac{\mathscr{F}}{\mathscr{R}}$$

The magnetomotive force $\mathscr{F}$ is proportional to the product of the number of turns around the core (in which the flux is to be established) and the current through the turns of wire.

$$\mathscr{F} = NI \text{ (ampere-turns, At)}$$



MAGNETIZING FORCE

The magnetomotive force per unit length is called the **magnetizing force** (H).

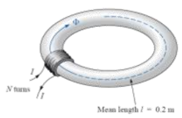
$$H = \frac{\mathscr{F}}{l} \quad \text{(At/m)}$$

Substituting for the magnetomotive force will result in

$$H = \frac{NI}{l} \quad \text{(At/m)}$$

Example:
Determine the magnetizing force for the following figure if N=20 and I=2A.
Sol

$$H = \frac{NI}{l} = \frac{40}{0.2} = 200 \quad \text{(At/m)}$$



The flux density and the magnetizing force are related by the following equation:

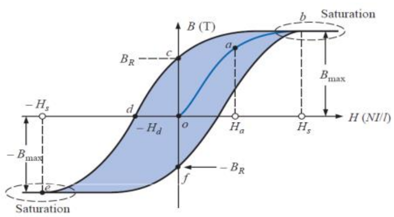
$$B = \mu H$$

HYSTERESIS

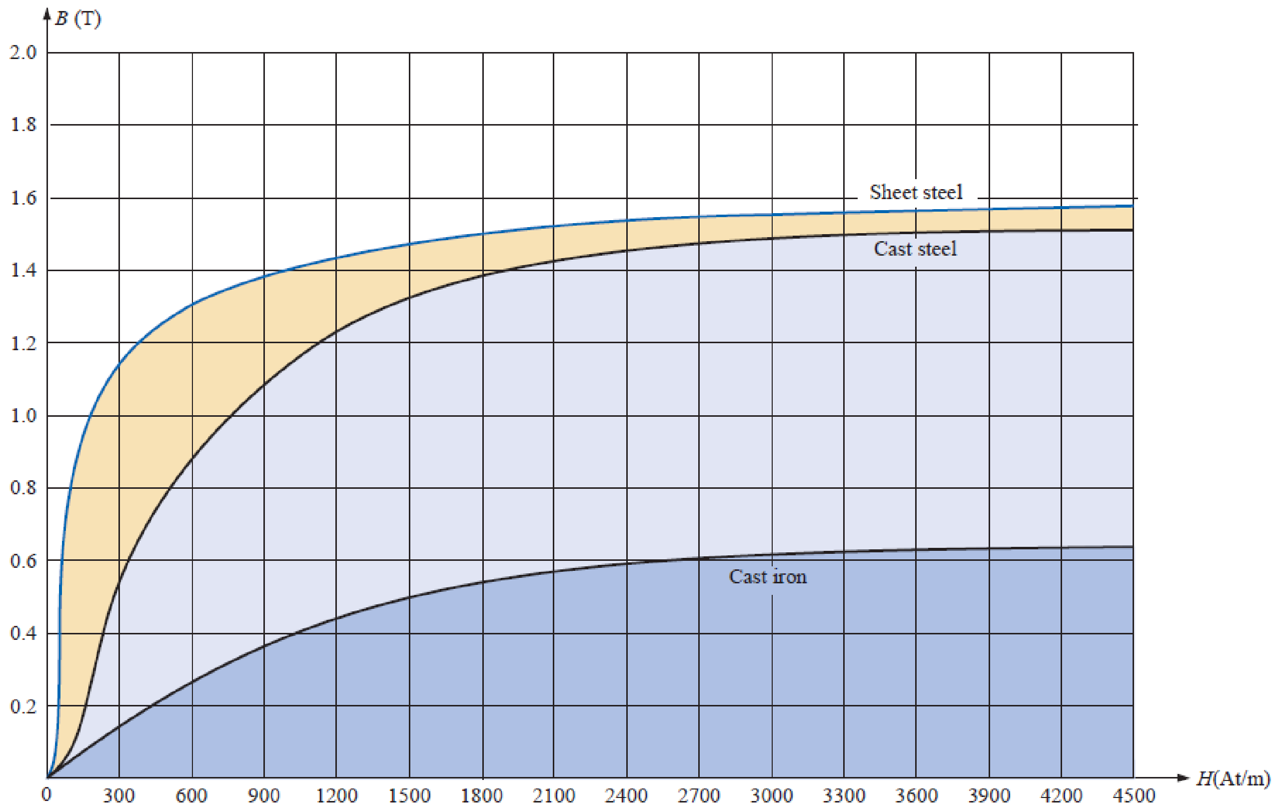
A curve of the flux density B versus the magnetizing force H of a material is of particular importance to the engineer. Curves of this type can usually be found in manuals, descriptive pamphlets, and brochures published by manufacturers of magnetic materials. A typical B-H curve for a ferromagnetic material such as steel can be derived using the following setups.
The core is initially unmagnetized and the current I =0. If the current I is increased to some value above zero, the magnetizing force H will increase to a value determined by

$$H\uparrow = \frac{NI\uparrow}{l}$$

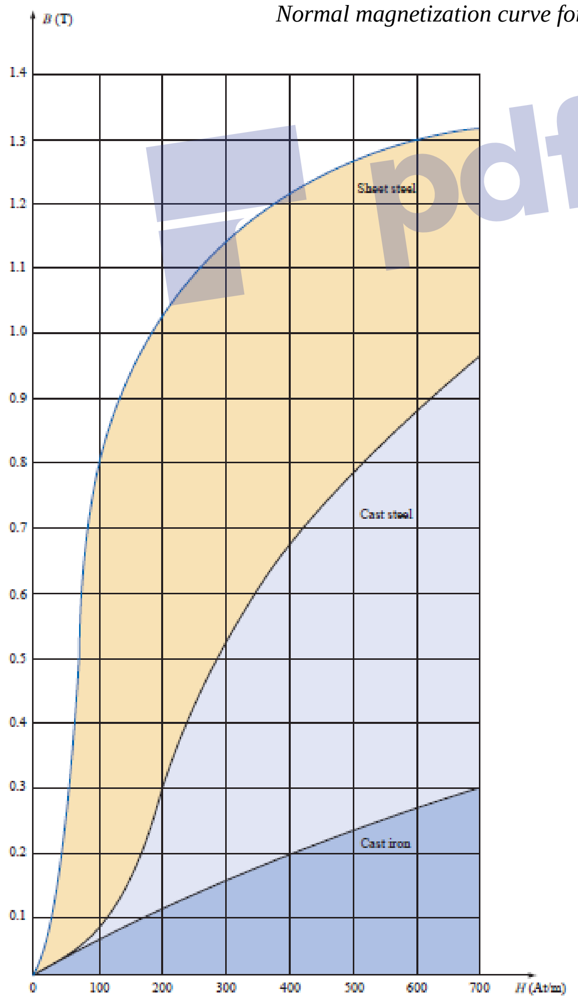
The flux Φ and the flux density B will also increase with the current I (or H).



Hysteresis curve.



Normal magnetization curve for three ferromagnetic materials



low magnetizing force region for three ferromagnetic materials

AMPÈRE'S CIRCUITAL LAW

	Electric Circuits	Magnetic Circuits
Cause	E	$\mathscr{F}$
Effect	I	Φ
Opposition	R	$\mathscr{R}$

If we apply the "cause" analogy to Kirchhoff's voltage law $\sum_{\circlearrowright} V = 0$, we obtain the following:

$$\sum_{\circlearrowright} \mathscr{F} = 0$$

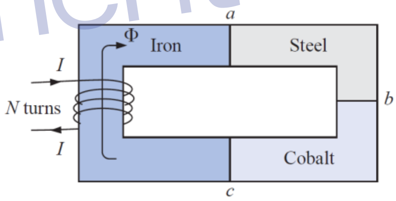
which, in words, states that the algebraic sum of the rises and drops of the mmf around a closed loop of a magnetic circuit is equal to zero; that is, the sum of the rises in mmf equals the sum of the drops in mmf around a closed loop.
This equation referred to as **Ampère's circuital law.** When it is applied to magnetic circuits, sources of mmf are expressed as

$$\mathscr{F} = NI \text{ (ampere-turns, At)}$$

And

$$\mathscr{F} = Hl \text{ (ampere-turns, At)}$$

Example:
Consider the magnetic circuit appearing in Figure below constructed of three different ferromagnetic materials.

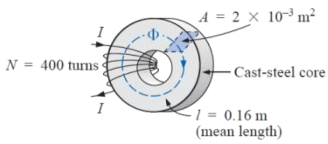


***Solutions*.**
Applying Ampère's circuital law, we have

$$\sum_{\circlearrowright} \mathscr{F} = 0$$

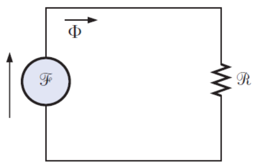
$$NI - H_{ab}l_{ab} + H_{bc}l_{bc} + H_{ca}l_{ca} = 0$$
$$NI = H_{ab}l_{ab} + H_{bc}l_{bc} + H_{ca}l_{ca}$$

Example
For the series magnetic circuit:
a. Find the value of I required to develop a magnetic flux of $\Phi = 4 \times 10^{-4}$ Wb.
b. Determine μ and μ_r for the material under these conditions.



Solutions:

$B = \frac{\Phi}{A} = \frac{4\times10^{-4}}{2\times10^{-3}} = 0.2$ T
Using the B-H curves, we can determine the magnetizing force H:
H (cast steel) = 170 At/m
Applying Ampère's circuital law yields
$NI = Hl$
$I = \frac{Hl}{N} = \frac{170\times0.16}{400} = 68\ mA$



b. The permeability of the material can be found as

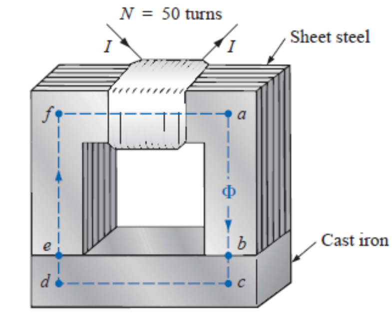
$$\mu = \frac{B}{H} = \frac{0.2}{170} = 1.176 \times 10^{-3} \text{ Wb/A.m}$$

and the relative permeability is

$$\mu_r = \frac{\mu}{\mu_0} = \frac{1.176 \times 10^{-3}}{4\pi \times 10^{-7}} = 935.83$$

Example:

The electromagnet of Figure below has picked up a section of cast iron. Determine the current I required to establish the indicated flux in the core, if $l_{ab} = l_{cd} = l_{ef} = l_{fa} = 101.6 \times 10^{-3}\ m$, $l_{bc} = l_{de} = 12.7 \times 10^{-3}\ m$, $\Phi = 3.5 \times 10^{-4}\ T$ and $A = 6.452 \times 10^{-4}\ m^2$



Solution:

The flux density for each section is

$B = \frac{\Phi}{A} = \frac{3.5\times10^{-4}}{6.452\times10^{-4}} = 0.542\ T$

and the magnetizing force is

H (sheet steel) = 70 At/m

H (cast iron) = 1600 At/m

Determining Hl for each section yields

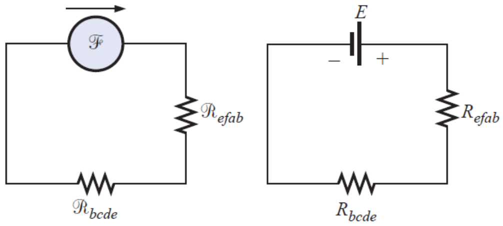
$l_{efab} = l_{ef} + l_{fa} + l_{ab} = 3 \times 101.6 \times 10^{-3} = 304.8 \times 10^{-3}\ m$

$l_{bcde} = l_{bc} + l_{cd} + l_{de} = 101.6 \times 10^{-3} + 2 \times 12.7 \times 10^{-3} = 127 \times 10^{-3}\ m$

$H_{efab} l_{efab} = 70 \times 304.8 \times 10^{-3} = 21.34\ At$

$H_{bcde} l_{bcde} = 1600 \times 127 \times 10^{-3} = 203.2\ At$

The magnetic circuit equivalent and the electric circuit analogy for the system



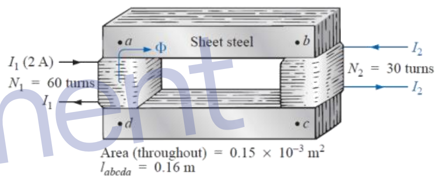
Applying Ampère’s circuital law,

$NI = H_{efab} l_{efab} + H_{bcde} l_{bcde} = 21.34 + 203.2 = 224.54$

$I = \frac{224.54}{50} = 4.49\ A$

Example:

Determine the secondary current I_2 for the transformer of Figure below if the resultant clockwise flux in the core is $1.5 \times 10^{-5}\ Wb$



Solution:

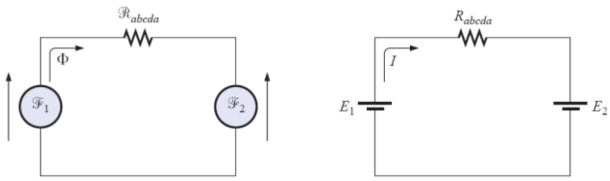
The flux density for each section is

$B = \frac{\Phi}{A} = \frac{1.5\times10^{-5}}{0.15\times10^{-3}} = 0.1\ T$

and the magnetizing force is

H (sheet steel) = 20 At/m

Applying Ampère’s circuital law,



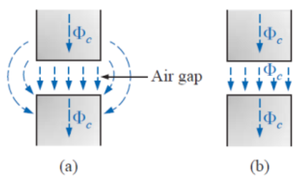
$N_1 I_1 - N_2 I_2 = H l_{abcda}$

$60 \times 2 - 30 I_2 = 20 \times 0.16$

$I_2 = \frac{120-3.2}{30} = 3.89\ A$

AIR GAPS

The spreading of the flux lines outside the common area of the core for the air gap in Fig. a is known as *fringing*. For our purposes, we shall neglect this effect and assume the flux distribution to be as in Fig. b.



The flux density of the air gap in is given by

$B_g = \frac{\Phi_g}{A_g}$

where, for our purposes,

$\Phi_g = \Phi_{core}$

And

$A_g = A_{core}$

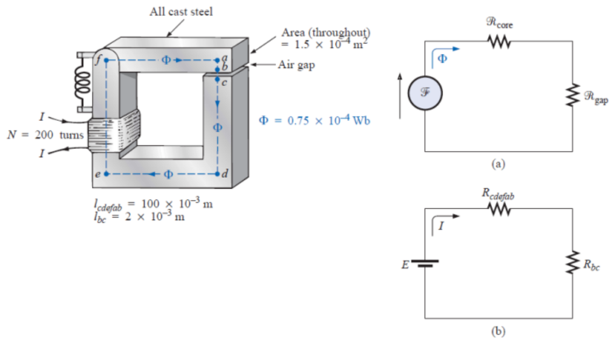
For most practical applications, the permeability of air is taken to be equal to that of free space. The magnetizing force of the air gap is then determined by

$H_g = \frac{B_g}{\mu_0}$

and the mmf drop across the air gap is equal to $H_g l_g$. An equation for H_g is as follows:

$$H_g = \frac{B_g}{\mu_0} = \frac{B_g}{4\pi \times 10^{-7}} = 7.96 \times 10^5 B_g \text{ (At/m)}$$

Example:
Find the value of I required to establish a magnetic flux of $\Phi = 0.75 \times 10^{-4}$ Wb in the series magnetic circuit of following Figure.



Solution:

The flux density for each section is

$$B = \frac{\Phi}{A} = \frac{7.5\times10^{-4}}{1.5\times10^{-4}} = 0.5\ T$$

From the *B-H* curves,

H (cast steel) = 280 At/m

$$H_g = 7.96 \times 10^5 B_g = 7.96 \times 10^5 \times 0.5 = 3.98 \times 10^5$$

Applying Ampère's circuital law,

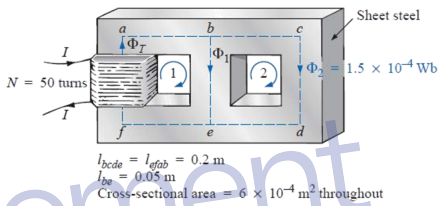
$$NI - Hl_{abcda} - H_g l_g = 0$$

$$I = \frac{280\times100\times10^{-3} + H_g l_g}{N} = \frac{280\times100\times10^{-3} + 3.98\times10^5\times10^{-3}}{200} = 4.12\ A$$

SERIES-PARALLEL MAGNETIC CIRCUITS

EXAMPLE

Determine the current I required to establish a flux of 1.5×10^{-4} Wb in the section of the core



Solution:

The equivalent magnetic circuit and the electric circuit analogy.

We have

The flux density for each section is

$$B_2 = \frac{\Phi_2}{A} = \frac{1.5\times10^{-4}}{6\times10^{-4}} = 0.25\ T$$

From the *B-H* curves,

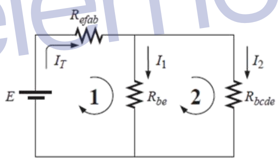
H_{bcde} (sheet steel) = 40 At/m

Applying Ampère's circuital law around loop 2

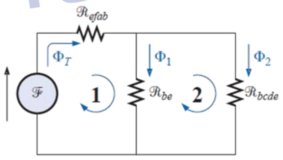
$$\Sigma_{\circlearrowright} \mathcal{F} = 0$$

$$H_{be} l_{be} - H_{bcde} l_{bcde} = 0$$

$$H_{be} = \frac{H_{bcde} l_{bcde}}{l_{be}} = \frac{40\times0.2}{0.05} = 160\ At/m$$



Electric circuit analogy for the series parallel system



Magnetic circuit eauivalent

From the *B-H* curves,

$$B_1 = 0.97\ T$$

$$\Phi_1 = B_1 A = 0.97 \times 6 \times 10^{-4} = 5.82 \times 10^{-4}\ Wb$$

The total flux density can be expressed as

$$\Phi_T = \Phi_1 + \Phi_2 = 5.82 \times 10^{-4} + 1.5 \times 10^{-4} = 7.32 \times 10^{-4}\ Wb$$

$$B_T = \frac{\Phi_T}{A} = \frac{7.32\times10^{-4}}{6\times10^{-4}} = 1.22\ T$$

From the *B-H* curves,

H_{efab} (sheet steel) = 400 At/m

Applying Ampère's circuital law around loop 1

$$\Sigma_{\circlearrowright} \mathcal{F} = 0$$

$$NI - H_{be} l_{be} - H_{efab} l_{efab} = 0$$

$$I = \frac{160\times0.05 + 200\times0.2}{50} = 1.78\ A$$

To demonstrate that m is sensitive to the magnetizing force H, the permeability of each section is determined as follows.

For section *bcde*,

$$\mu = \frac{B}{H} = \frac{0.25}{40} = 6.25 \times 10^{-3}$$

$$\mu_r = \frac{\mu}{\mu_0} = \frac{6.25\times10^{-3}}{4\pi \times 10^{-7}} = 4972.2$$

For section *be*,

$$\mu = \frac{B}{H} = \frac{0.97}{160} = 6.06 \times 10^{-3}$$

$\mu_r = \frac{\mu}{\mu_0} = \frac{6.06\times10^{-3}}{4\pi \times 10^{-7}} = 4821$

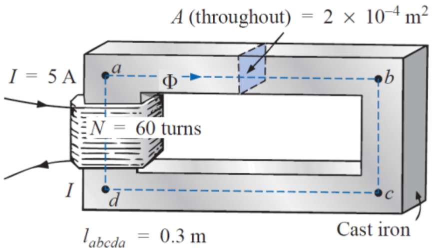
For section *be*,

$\mu = \frac{B}{H} = \frac{1.22}{40} = 3.05 \times 10^{-3}$

$\mu_r = \frac{\mu}{\mu_0} = \frac{3.05\times10^{-3}}{4\pi \times 10^{-7}} = 2426.41$

Example:

Calculate the magnetic flux Φ for the magnetic circuit shown below:



Solution:

By Ampère's circuital law,

$\sum_{\circlearrowright} \mathcal{F} = 0$

$NI - H_{abcda} l_{abcda} = 0$

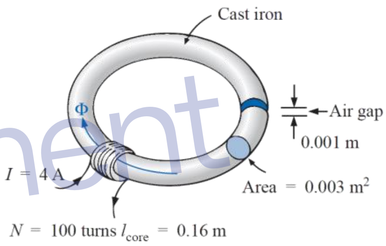
$H_{abcda} = \frac{NI}{l_{abcda}} = \frac{5 \times 60}{0.3} = 1000 \text{ At/m}$

B (cast iron from Figure)=0.39 T

$\Phi = \text{BA} = 0.39 \times 2 \times 10^{-4} = 0.78 \times 10^{-4}\ Wb$

Example:

Find the magnetic flux Φ for the series magnetic circuit of Figure below for the specified impressed mmf.



Solution:

Assuming that the total impressed mmf *NI* is across the air gap,

$NI - H_g l_g$

$H_g = \frac{NI}{l_g} = \frac{4 \times 100}{0.001} = 4 \times 10^5\ At/m$

$B_g = \mu_0 H_g = 4\pi \times 10^{-7} \times 4 \times 10^5 = 0.503\ T$

$\Phi_g = \Phi_{core} = B_g A = 0.503 \times 0.003 = 1.51 \times 10^{-3}\ Wb$

$H_{core}\ (cast\ iron\ from\ B - H\ curve) = 1500\ At/m$