

THERMODYNAMICS

2

CHAPTER TWO

IDEAL GAS



## (IDEAL GAS)

(Perfect gas)

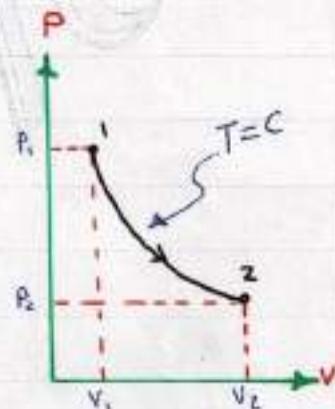
### - Boyle's Law:

It is defined as: when a change of state of any gas in which the mass and the temperature remain constant, the volume varies inversely as the pressure.

$$P \propto \frac{1}{V} \Rightarrow P = C \cdot \frac{1}{V}$$

$$\therefore PV = C \quad \text{Boyle's Law}$$

$$\therefore P_1 V_1 = P_2 V_2$$



### Ex:

A gas whose original pressure and volume were  $300 \text{ kNm}^{-2}$  and  $0.14 \text{ m}^3$  is expanded until its new pressure is  $60 \text{ kNm}^{-2}$  while its temperature remains constant. What is its new volume?

### Sol:

$$PV_1 = P_2 V_2 \quad \text{or} \quad V_2 = V_1 \cdot \frac{P_1}{P_2}$$

$$V_2 = 0.14 \cdot \frac{300}{60}$$

$$= 0.7 \text{ m}^3$$



## - Charles's Law :-

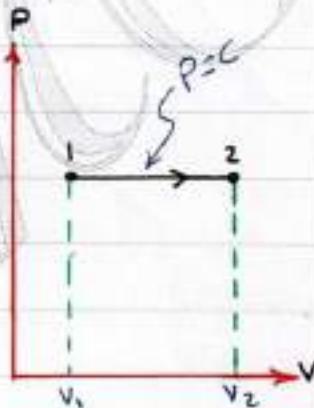
It is defined as: When the change of state of any gas in which the mass and pressure remain constant, the volume varies in proportion with the absolute temperature.

$$V \propto T \Rightarrow V = CT$$

$$\therefore \frac{V}{T} = C$$

Charles's Law

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$



**Ex:-** A quantity of gas whose original volume and temperature are  $0.2\text{ m}^3$  and  $303^\circ\text{C}$ , respectively, is cooled at constant pressure until its volume becomes  $0.1\text{ m}^3$ . What will be the final temperature of the gas?

**SOL:-**

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow T_2 = T_1 \cdot \frac{V_2}{V_1}$$

$$T_1 = 303 + 273 = 576\text{ K}$$

$$T_2 = 576 \times \frac{0.1}{0.2} = 288\text{ K}$$

$$\therefore t_2 = 288 - 273 = 15^\circ\text{C}$$



## (The characteristic equation of a perfect gas)

the process  $1 \rightarrow A$

$PV = C$  (Boyle's ( $T = \text{const.}$ ))

$$P_1 V_1 = P_A V_A \rightarrow T_1 = T_A$$

$$V_A = \frac{P_1}{P_A} \cdot V_1 \quad \dots \dots \quad (1)$$

$$V = CT \quad (\text{charles}) \quad (P = \text{const.})$$

$$\frac{V_A}{T_A} = \frac{V_2}{T_2} = \frac{V}{T} = C$$

$$V_A = \frac{T_A}{T_2} \cdot V_2 \quad \dots \dots \quad (2)$$

from Boyle's Law ( $T_A = T_1$ ) Sub in eq.(2)

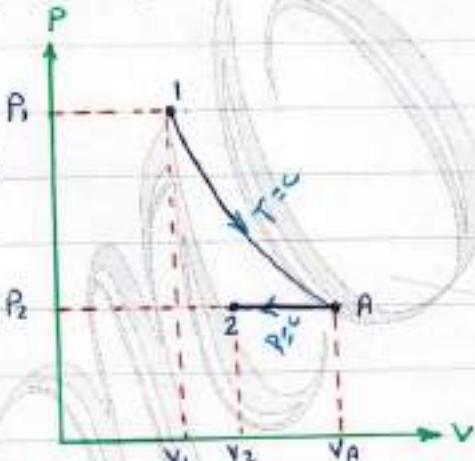
$$V_A = \frac{T_1}{T_2} \cdot V_2 \quad \dots \dots \quad (3)$$

from eq's (1) & (3):

$$V_A = \frac{P_1}{P_A} \cdot V_1 = \frac{T_1}{T_2} \cdot V_2 \rightarrow \text{from charles law}$$

$$P_A = P_2$$

$$\frac{P_1}{P_2} \cdot V_1 = \frac{T_1}{T_2} \cdot V_2 = C$$





$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = R \rightarrow R = \text{gas constant}$$

$$P_1 V_1 = R T_1 \quad \text{for } 1 \text{ kg}$$

$$P_1 V_1 = m R T_1 \quad \text{for } m \text{ kg}$$

$$\therefore m = n \cdot M$$

Whereas  
 $m$  = mass kg

$n$  = number of moles mol

$M$  = molecular weight

$$P V = m R T \quad (\text{In general})$$

$$P V = n M R T$$

$$M R = \frac{P V}{n T} \quad (P \neq T \text{ are constant})$$

$$M R = C \cdot \frac{V}{n} \quad , \left( \frac{V}{n} = C \right) \quad (\text{Avogadro's theorem})$$

$$\therefore M R = R_0$$

$\rightarrow R_0$  = Universal gas Constant

**Avogadro's theorem:** volume of one mole of any gas is same for all gases at any temperature and pressure.



$$\therefore R_0 = \frac{PV}{nT}$$

O <sub>2</sub>	32
N <sub>2</sub>	28
CO <sub>2</sub>	44

∴ Volume of 1 mole at 0°C and P = P<sub>atm</sub> = 1.0132 bar equal to 22.7 m<sup>3</sup>.

$$\therefore R_0 = \frac{1.0132 \times 10^5 \times 22.7}{1 \times (0 + 273)} = 8.314 \text{ J/g·K}$$

**Ex:** for O<sub>2</sub>: R = R<sub>0</sub>/M = 8.314/32 = 0.259

**Ex:** A volume of 3.6 m<sup>3</sup> of O<sub>2</sub> initially at 220°C and pressure 400 kPa is compressed reversibly at constant temperature to a final volume of 0.06 m<sup>3</sup>. Calculate the mass, the final pressure, the increase in internal energy and the work done.

**Sol:**

$$PV = mRT \quad , \text{ for O}_2 \quad R = R_0/M = 0.26 \text{ J/kg·K}$$

$$m = 400 \times 10^3 \times 3.6 / 0.26 \times 10^3 \times (220 + 273) = \frac{400 \times 10^3 \times 3.6}{0.26 \times 10^3 \times 493} = 11.23 \text{ kg}$$

$$P_1 V_1 = P_2 V_2 \Rightarrow P_2 = (3.6 / 0.06) * 400 = 24 \text{ MPa}$$

$$Q = w + \Delta U \quad , \quad \Delta U = 0 \quad (\text{since } T = C)$$

$$w = \int P dV = C \int \frac{dV}{V} = P_1 V_1 \int \frac{dV}{V} = P_1 V_1 \ln \frac{V_2}{V_1}$$

$$w = 400 \times 10^3 \times 3.6 \times \ln \frac{0.06}{3.6} = Q = -5.896 \text{ MJ}$$



## - Joule's Law :

It is defined as:

the internal energy of a gas is a function of temperature only and is independent of changes in pressure and volume.

$$\therefore \text{If } T=c \Rightarrow \Delta U=0$$

## - The specific heat Capacities of a gas:

### a. the constant volume heating of a gas:

let a mass of gas  $m$  be heated at constant volume such that its temperature rises from  $T_1$  to  $T_2$  and its pressure rises from  $P_1$  to  $P_2$ . Then

$$\text{Heat received by the gas} = m C_V (T_2 - T_1)$$

$$\text{from NFEE , } Q = \Delta U + W$$

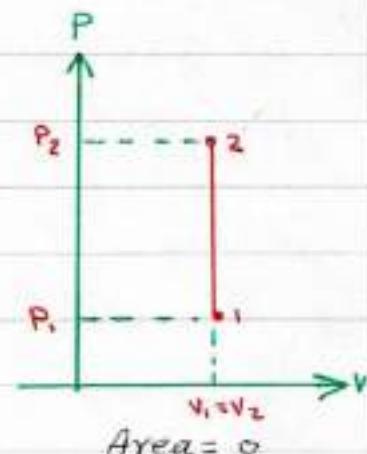
$$\therefore V=c \Rightarrow W=0$$

$$Q = \Delta U ,$$

$$\text{or } m C_V (T_2 - T_1) = U_2 - U_1$$

from the perfect gas equation  
 (characteristic equation)

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \Rightarrow P_2 = P_1 \frac{T_2}{T_1}$$





**Sax:** 2 kg of gas, occupying 0.7 m<sup>3</sup>, had an original temperature of 15°C. It was then heated at constant volume until its temperature became 135°C. How much heat was transferred to the gas and what was its final pressure?

Take,  $C_V = 0.72 \text{ kJ/kg K}$  and  $R = 0.29 \text{ kJ/kg K}$

$$\begin{aligned}\text{Sol:} \quad \text{Heat transferred at } (V=c) &= m C_V (T_2 - T_1) \\ &= 2 \times 0.72 \times (135 - 15) \\ &= 2 \times 0.72 \times 120 \\ &= 172.8 \text{ kJ}\end{aligned}$$

$$\text{Now } P_1 V_1 = m R T_1$$

$$\therefore P_1 = \frac{m R T_1}{V_1} = \frac{2 \times 0.29 \times 288}{0.7} = \frac{167.04}{0.7} = 238.6 \text{ kPa}$$

Since the volume remains constant, then

$$\begin{aligned}\frac{P_1}{T_1} &= \frac{P_2}{T_2} \quad \therefore P_2 = P_1 \frac{T_2}{T_1} \\ &= 238.6 \times \frac{408}{288} \\ &= 338.1 \text{ kN/m}^2\end{aligned}$$

$$\text{Or } P_2 V_2 = m R T_2$$

$$\begin{aligned}P_2 &= m R T_2 / V_2 = 2 \times 0.29 \times (135 + 273) / 0.7 \\ &= 338.1 \text{ kPa}\end{aligned}$$



b. the constant pressure heating of a gas :

Let a mass of gas ( $m$ ) be heated at constant pressure such that its temperature rises from  $T_1$  to  $T_2$  and its volume increases from  $V_1$  to  $V_2$ . Then

$$\text{Heat received by the gas} = m C_p (T_2 - T_1)$$

$$Q = \Delta U + W$$

$$\begin{aligned} m C_p (T_2 - T_1) &= (U_2 - U_1) + P(V_2 - V_1) \\ &= (U_2 + PV_2) - (U_1 + PV_1) \\ &= H_2 - H_1 \end{aligned}$$

Ans.  
See

or

$$U_2 - U_1 = m C_p (T_2 - T_1) - P(V_2 - V_1)$$

when  $PV = mRT$  then

$$\begin{aligned} U_2 - U_1 &= m C_p (T_2 - T_1) - m R (T_2 - T_1) \\ &= m (T_2 - T_1) (C_p - R) \end{aligned}$$

from the characteristic equation of a perfect gas :

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}, P_1 = P_2$$

$$V_2 = V_1 \cdot \frac{T_2}{T_1}$$

Note :  $H = m C_p T$  ,  $h = C_p T$  ,  $\Delta h = C_p \Delta T$   
 $U = m C_v T$  ,  $u = C_v T$  ,  $\Delta u = C_v \Delta T$



**Ex:** A gas whose pressure, volume and temperature are  $275 \text{ kN/m}^2$ ,  $0.09 \text{ m}^3$  and  $185^\circ\text{C}$ , respectively, has its state changed at constant pressure until its temperature becomes  $15^\circ\text{C}$ . How much heat is transferred from the gas and how much work is done on the gas during the process?

Take:  $R = 0.29 \text{ kJ/kg}\cdot\text{K}$  ,  $C_p = 1.005 \text{ kJ/kg}\cdot\text{K}$ .

**Sol:**  $P_1 V_1 = m R T_1$

$$\therefore m = P_1 V_1 / R T_1 = 275 \times 10^3 \times 0.09 / 0.29 \times 10^3 \times 458 \\ = 0.186 \text{ kg}$$

$$\text{Heat transferred} = m C_p (T_2 - T_1)$$

$$= 0.186 \times 1.005 \times (288 - 458)$$

=  $-31.78 \text{ kJ}$  (the heat has been extracted from the gas)

$\therefore P = \text{constant}$  (given)

$$\therefore \frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow V_2 = V_1 \times \frac{T_2}{T_1} = 0.09 \times \frac{288}{458} \\ = 0.0566 \text{ m}^3$$

$$\text{Work done} = P(V_2 - V_1)$$

$$= 275 \times (0.0566 - 0.09)$$

$$= -9.19 \text{ kJ}$$



## Relation Between Specific Heat ( $C_p$ & $C_v$ ):-

from the non-flow energy equation NFE

$$Q = W + \Delta U \quad , \quad \Delta U = m C_v (T_2 - T_1)$$

$$W = \int P \cdot dV = P_2 V_2 - P_1 V_1$$

Ideal Gas Law  $PV = mRT$

$$W = mR(T_2 - T_1)$$

$$\therefore Q = mR(T_2 - T_1) + mC_v(T_2 - T_1)$$

$$= m(R + C_v)(T_2 - T_1)$$

$$\therefore Q = \Delta H = mC_p(T_2 - T_1)$$

$$\therefore mC_p(T_2 - T_1) = m(R + C_v)(T_2 - T_1)$$

$$\therefore C_p = R + C_v \Rightarrow C_p - C_v = R$$

$$\gamma = \frac{C_p}{C_v}$$

$\gamma$  = the ratio of the specific heat.

$$\frac{C_p}{C_v} - \frac{C_v}{C_v} = \frac{R}{C_v}$$

$$\gamma - 1 = R/C_v \Rightarrow C_v = \frac{R}{\gamma - 1}$$

$$C_p = \gamma * C_v \Rightarrow$$

$$C_p = \frac{\gamma R}{\gamma - 1}$$



**Ex:** A perfect gas have specific heat as  $C_p = 0.846 \frac{kg}{kg \cdot K}$   
 $C_v = 0.657 \frac{kg}{kg \cdot K}$ . find the gas constant and molecular weight of gas.

**Sol:**  $R = C_p - C_v = 0.846 - 0.657 = 0.189 \frac{kg}{kg \cdot K}$   
 $= 189 \frac{J}{kg \cdot K}$

$$M = R_0 / R = 8314 / 189 = 44$$

**Ex:** A perfect gas has molecular weight of 26 and a value of  $\gamma = 1.26$ . Calculate the heat rejected per 1 kg when :

a. the gas is contained in a rigid vessel at 3 bar and 315 °C and cooled until the pressure falls to 1.5 bar.

b. the gas enters pipe at 280 °C and flow steady in the end of the pipe where T = 250 °C. Neglect any change in velocity of the gas.

**Sol:** a.  $Q = mV + \Delta U$ , a rigid vessel  $\Rightarrow V = C \Rightarrow dV = 0$   
 $w = S_p \cdot dV = 0$

$$\therefore Q = \Delta U = mC_v(T_2 - T_1) \text{ , for } V = C \text{ only}$$

$$M = 26, \gamma = 1.26$$

$$R = R_0 / M = 8314 / 26 = 0.3198 \frac{kg}{kg \cdot K}$$

$$C_v = \frac{R}{\gamma - 1} = \frac{0.3198}{1.26 - 1} = 1.229 \frac{kg}{kg \cdot K}$$



$$Q = mC_V(T_2 - T_1) = 1 * 1.229 * (T_2 - T_1)$$

since  $V=C \Rightarrow U_1 = U_2$

from the characteristic equation

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\therefore T_2 = \frac{P_2}{P_1} \cdot T_1 = \frac{1.5}{3} (315 + 273) = 294 K$$

$$Q = 1 * 1.229 * (294 - 588)$$

$$= -361 \text{ kJ} \quad (\text{rejected})$$

$$= 361 \text{ kJ} \quad (\text{rejected})$$

### b. Steady Flow Energy Equation SFEE

$$gZ_1 + P_1 V_1 + U_1 + \frac{1}{2} C_1^2 + q = gZ_2 + P_2 V_2 + U_2 + \frac{1}{2} C_2^2 + W$$

$$\Delta Z \approx 0, \Delta C \approx 0, W \approx 0$$

$$\therefore h_1 + q = h_2 \Rightarrow q = h_2 - h_1$$

$$Q = m(h_2 - h_1) = m(C_p(T_2 - T_1))$$

$$C_p = \gamma \cdot C_V = 1.26 * 1.229 = 1.548 \text{ kJ/kg.K}$$

$$\therefore Q = 1 * 1.548 * (250 - 280)$$

$$= -46.44 \text{ kJ} \quad (\text{rejected})$$

$$= 46.44 \text{ kJ} \quad (\text{rejected})$$



**Ex:** Five kilogram of oxygen are heated from 250 to 400K at constant pressure. Calculate the change of enthalpy, change in internal energy, heat transferred and work done.  $\gamma = 1.4$

**Sol:**

a non-flow energy equation

$$Q = W + \Delta U$$

$$W = \int p dV = p(V_2 - V_1) , \quad p = c \text{ (given)}$$

$$Q = (P_2 V_2 - P_1 V_1) + (U_2 - U_1)$$

$$Q = (P_2 V_2 + U_2) - (P_1 V_1 + U_1)$$

$$= H_2 - H_1 = mC_p(T_2 - T_1) \quad \text{for } p=c \text{ only}$$

for O<sub>2</sub>:

$$N = 32 , \gamma = 1.4$$

$$R = R_0/N = 8.314/32 = 0.2598 \text{ Jg/Kg} \cdot \text{K}$$

$$C_p = \frac{\gamma R}{\gamma - 1} = \frac{1.4 \times 0.2598}{1.4 - 1} = 0.9093 \text{ Jg/Kg} \cdot \text{K}$$

$$\Delta H = 5 * 0.9093 * (400 - 250) = 692 \text{ Jg}$$

$$\Delta U = m C_V (T_2 - T_1)$$

$$C_V = C_p - R = 0.9093 - 0.2598 = 0.6495 \text{ Jg/Kg} \cdot \text{K}$$

$$\Delta U = 5 * 0.6495 * (400 - 250) =$$

$$Q = W + \Delta U , \text{ at } p=c \Rightarrow Q = \Delta H$$

$$\therefore W = Q - \Delta U$$

$$= 682 - 487 = 195 \text{ Jg} \quad (\text{work done by the system})$$



**Ex:** An Oxygen cylinder has a capacity of 300L, and contains O<sub>2</sub> at 3.1 MPa and 18°C. The valve is open and some gas is used. If the pressure and temperature of the oxygen fall to 1.7 MPa and 15°C respectively. Find the mass of oxygen used. If after the valve is closed the Oxygen remains in the cylinder gradually attains its initial temperature of 18°C. Find the amount of heat transfer through the cylinder wall. Oxygen density at 0°C and 0.1013 MPa is 1.429  $\frac{\text{kg}}{\text{m}^3}$  and  $\gamma = 1.4$ .

**Sol:**  $P_1 V_1 = m R T_1 \Rightarrow m_1 = \frac{P_1 V_1}{R T_1}$

$$R(O_2) = 8.314 / 32 = 0.26 \text{ kg/kg.K}$$

$$\text{Initial mass } m_1 = \frac{3.1 \times 10^3 \times 300 \times 10^{-3}}{0.26 \times (18+273)} = 12.29 \text{ kg}$$

$$\text{final mass } m_2 = \frac{1.7 \times 10^3 \times 300 \times 10^{-3}}{0.26 \times (15+273)} = 6.8 \text{ kg}$$

$$\text{mass used} = m_1 - m_2 = 12.29 - 6.8 = 5.49 \text{ kg}$$

$$Q = W + \Delta U \quad , \quad V = C \quad \Rightarrow \quad W = 0$$

$$Q = m C_V (T_2 - T_1)$$

$$C_V = R / \gamma - 1 = 0.26 / 1.4 - 1 = 0.65 \text{ kg/kg.K}$$

$$Q = 6.8 \times 0.65 \times (18 - 15) = 13.26 \text{ Jg} \\ (\text{heat gain})$$



## (Sheet No. 2)

1000

**Q1:** The molecular weight of Carbon dioxide  $\text{CO}_2$  is 44. In an experiment the value of  $\gamma$  was found to be 1.3. Assuming that  $\text{CO}_2$  is a perfect gas. Calculate the gas constant  $R$  and the Specific heats at constant  $P \& V$ . **Ans.** [0.189, 0.63 and 0.819  $\text{kJ/kg}\cdot\text{K}$ ]

**Q2:** Calculate the internal energy and enthalpy of 1kg of air occupying  $0.05 \text{ m}^3$  at 20bar. If the internal energy is increased by 120  $\text{kJ/kg}$  as the air compressed to 50 bar. Calculate the new volume occupied by 1kg of air. **Ans.** [250.1 - 350.1  $\text{kg}\text{N}_\text{a}$  =  $0.0296 \text{ m}^3$ ]

$$R_{\text{air}} = 0.287$$

**Q3:** When a certain perfect gas is heated at constant pressure from  $15^\circ\text{C}$  to  $95^\circ\text{C}$ , the heat required is  $1130 \text{ kJ/kg}$ . While when the same gas is heated at constant volume between the same temperature limits above, the heat required is  $808 \text{ kJ/kg}$ . Calculate  $C_p, C_v, \gamma, R$  and  $M$  of the gas.

$$\text{Ans.} [14.2, 10.1 \text{ kg}\text{N}_\text{a}\cdot\text{K}^{-1}, 1.405, 4.1 \text{ kg}\text{N}_\text{a}\cdot\text{K}^{-1}, 2.028]$$

**Q4:** In an air compressor the pressure at inlet and outlet are 1bar & 5bar respectively. The temperature of the air at inlet is  $15^\circ\text{C}$  and the volume at



beginning of compression is three times that at the end of compression. Calculate the temperature of air at outlet and the increase in internal energy.

Ans. [207°C, 138 kJ/kg]

**Q5:** The exhaust gas leaving an oil engine is passed through a heat exchanger which consists of tubes surrounding by water. The gas enters at 327°C and leaves at 193°C. The water enters the exchanger at 17°C and leaves at 56°C. If the mass flow rate of water is 2.54 kg/min. Determine the mass flow rate of gas. Take:  $C_p = 1.04 \text{ kJ/kg}\cdot\text{K}$  -  $C_{pw} = 4.186 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

Ans. [2.98 kg/min]

**Q6:** A working fluid enters a steady-flow system with velocity 30 m/s and leaves with velocity 140 m/s. The mass flow rate is 9 kg/s. The properties of the fluid are at entry, 13.8 bar, 0.122 m<sup>3</sup>/kg, internal energy 422 kJ/kg, and exit properties are 1.035 bar, 0.805 m<sup>3</sup>/kg, internal energy 208  $\frac{\text{kJ}}{\text{kg}}$ . The heat transfer from the system is 4.22 kJ/kg. Determine the work transfer in kW from the system.

Ans. [2565 kW]