



Two other common pressure units are bar and standard atmosphere :

$$1 \text{ bar} = 10^5 \text{ Pa} = 0.1 \text{ MPa} = 100 \text{ kPa}$$

$$1 \text{ atm} = 101325 \text{ Pa} = 101.325 \text{ kPa} = 1.01325 \text{ bars}$$

In the English System, the pressure unit is [pound-force per Square inch] lbf/in^2 or psi

$$1 \text{ atm} = 14.696 \text{ psi} = 760 \text{ mm Hg}$$

the actual pressure at a given position is called the absolute pressure, and it is measured relative to absolute vacuum , i.e., absolute zero pressure.

the difference between the absolute pressure and the local atmospheric pressure is called the gage pressure .

Pressures below atmospheric pressure are called Vacuum Pressure , and are measured by vacuum gage.

Absolute, gage, and vacuum pressure are all positive quantities and are related to each other by :

$$P_{\text{gage}} = P_{\text{abs}} - P_{\text{atm}}$$

(For pressure above P_{atm})

$$P_{\text{vac.}} = P_{\text{atm}} - P_{\text{abs}}$$

(" , " , below ,)



ENTROPY

The term 'entropy' which literally means transformation, was first introduced by Clausius. It is an important thermodynamics property of a working substance, which increases with the addition of heat, and decreases with its removal.

In a reversible process, over a small range of temperature, the increase or decrease of entropy, when multiplied by the absolute temperature, gives the heat absorbed or rejected by the working substance. Mathematically, heat absorbed by the working substance,

Heat transfer reversibly = Area under the curve

$$= T dS$$

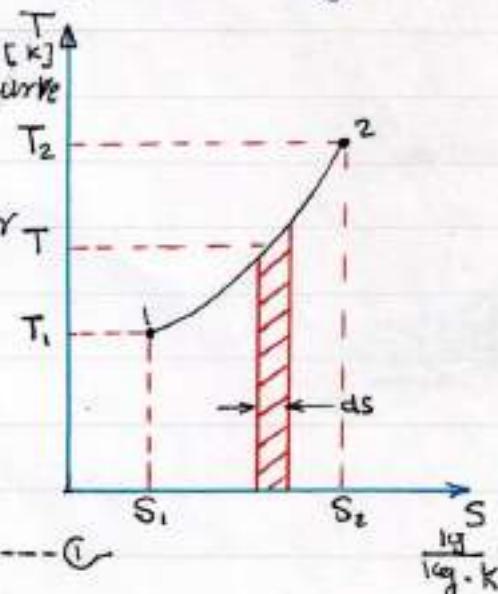
Total heat transfer $1 \rightarrow 2$ = total area under the curve

$$= \sum_{S=S_1}^{S=S_2} T dS$$

In the limit as $ds \rightarrow 0$

Heat transfer reversibly

$$\text{from } 1 \rightarrow 2 = \int_{S_1}^{S_2} T dS = Q_{\text{rev.}} \quad \text{--- (1)}$$



Differentiating eq. (1)

$$dQ_{\text{rev.}} = T \cdot dS$$

$$dS = \frac{dQ_{\text{rev.}}}{T}$$

~~~~~ \* (2)



This equation gives the relationship which must exist between  $S$ ,  $T$  and  $Q_{rev}$  in order that the area of the graph shall be heat transferred reversibly.

Now it is possible to calculate the amount of heat transferred reversibly during a non-flow process. Thus, by using equation (2) changes in ( $s$ ) can be determined. It is this function ( $s$ ) which is called **entropy**.

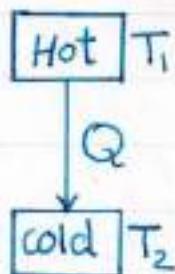
If heat is received, which makes  $Q_{rev}$  positive, then the entropy of the receiving substance has increased.

If heat is rejected, which makes  $Q_{rev}$  negative then the entropy of the rejecting substance is decreased.

Now consider a given quantity of heat energy  $Q$ . Let this quantity of heat be lost from a hot fluid at temperature  $T_1$  and let it be received by a cooler fluid at temperature  $T_2$ .

$$S = \frac{Q}{T} = \frac{Q}{T_1}$$

$$T_1 > T_2 \Rightarrow \frac{Q}{T_2} > \frac{Q}{T_1}$$



∴ the gain of entropy to the cooler fluid is greater than the loss of entropy from the hot fluid.



\* The entropy of vapours (two-phase system)

## § Liquid entropy:

$$Q = m C_p (T_2 - T_1)$$

$$Q = m C_p \Delta T \quad , \text{ for } 1\text{kg mass}$$

$$Q = C_p \Delta T \quad , \text{ for liquid}$$

$$Q = C_p \Delta T \Rightarrow$$

$$dQ = C_p dT \quad \text{---} \quad ①$$

$C_p$  = Specific heat capacity of the liquid at constant pressure.  
 = 4.187 kg/kg·K • for H<sub>2</sub>O

Dividing eq. ① throughout by T, then

$$\frac{dQ}{T} = C_p \frac{dT}{T} = ds \Rightarrow ds = C_p \frac{dT}{T}$$

$$s_2 - s_1 = \int_{T_1}^{T_2} C_p \frac{dT}{T} \Rightarrow s_2 - s_1 = C_p \ln \frac{T_2}{T_1}$$

$$\text{when } T_1 = 0^\circ C = 273K \Rightarrow s_1 = 0$$

$$s_2 - 0 = C_p \ln \frac{T_2}{273}$$

$$\therefore S = C_p \cdot \ln \frac{T}{273} \quad \text{Liquid Entropy}$$

$$\text{for H}_2\text{O}, S = 4.187 \ln \frac{T}{273}$$



**Ex:** Determine the value of the specific entropy of water at 100°C.

Sol:  $S = 4.187 \ln\left(\frac{T}{273}\right)$

$$S_f = 4.187 \ln \frac{373}{273} = 1.31 \text{ lg/kg.K}$$

from Steam tables at 100°C,  $S_f = 1.3068 \text{ lg/kg.K}$

## Entropy of evaporation:

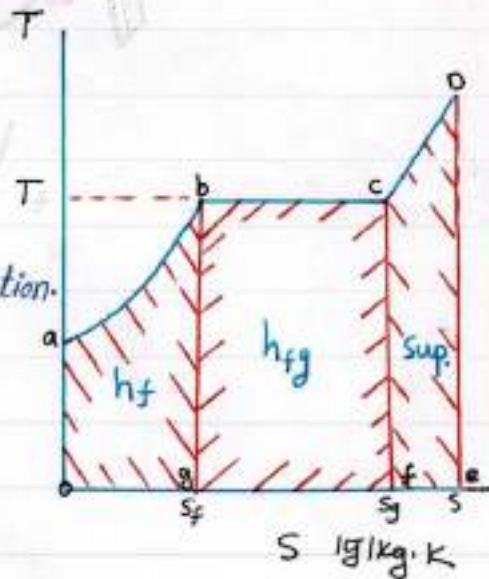
Heat transferred = Change of enthalpy

Area abgo =  $h_f$  = Specific liquid enthalpy

Area bcfg =  $h_{fg}$  = Specific enthalpy of evaporation.

Area cdef = sup = Specific superheat.

Area bcfg =  $h_{fg} = T_s(S_g - S_f)$



$S_g - S_f = \frac{h_{fg}}{T_s}$  = Specific entropy of evaporation =  $s_{fg}$   
 for wet steam

$$S = S_f + x s_{fg} \quad , \quad S_f = 4.187 \ln \frac{T_s}{273} \quad \text{for H}_2\text{O}$$

$$S = C_p \cdot \ln \frac{T_s}{273} + x \cdot \frac{h_{fg}}{T_s} \quad \text{for wet steam}$$

at  $x=1 \Rightarrow S = S_g$



**Ex:** Determine the value of the specific entropy of wet steam at a pressure of 2 MPa and 0.8 dry.

a. by calculation    b. by using values of entropy from Steam tables.

$$S = C_p \cdot \ln \frac{T_s}{273} + x \frac{h_{fg}}{T_s}$$

from Steam tables at  $p=2 \text{ MPa}$ ,  $T_s=212.42^\circ\text{C}=485.4 \text{ K}$   
 $h_{fg}=1890.74$

$$\therefore S = 4.187 \ln \frac{485.4}{273} + 0.8 * \frac{1890.74}{485.4}$$

$$= 5.525 \text{ kg/kg.K}$$

$$\begin{aligned} b. \quad S &= S_f + x S_{fg} && \text{at } 2 \text{ MPa}, S_f = 2.4473 \text{ J/g.K} \\ &= 2.4473 + 0.8 * 3.8935 && S_{fg} = 3.8935 \text{ J/g.K} \\ &= 5.562 \text{ J/g.K} \end{aligned}$$

## 8 Entropy of Superheated Vapour:

$$\text{Area cdef} = C_p v \cdot dT$$

$C_p v$  = Specific heat capacity of superheated vapour at constant pressure.

$$\begin{aligned} dQ &= C_p v \cdot dT && \div T \\ \frac{dQ}{T} &= C_p v \cdot \frac{dT}{T} \Rightarrow ds = C_p v \cdot \frac{dT}{T} \end{aligned}$$



$$s_g \int ds = C_p v \cdot \int \frac{dT}{T} \Rightarrow S - S_g = C_p v \cdot \ln \frac{T}{T_s}$$

$$S = S_g + C_p v \ln \frac{T}{T_s}$$

$$S = C_p v \cdot \ln \frac{T_s}{273} + \frac{h_{fg}}{T_s} + C_p v \ln \frac{T}{T_s}$$

**Ex:** Determine the value of Specific entropy of Steam at 1.5 MPa with a temperature of 300°C.  $C_p v = 2.0934 \text{ Jg}^{-1}\text{kg}^{-1}\text{K}$

a. by calculation      b. from Steam tables

**Sol:** a.  $S = C_p v \cdot \ln \frac{T_s}{273} + \frac{h_{fg}}{T_s} + C_p v \cdot \ln \frac{T}{T_s}$

from Steam tables at 1.5 MPa  $\rightarrow T_s = 198.32^\circ\text{C} = 471.32 \text{ K}$   
 $h_{fg} = 1947.28 \text{ Jg}^{-1}\text{kg}$

$$S = 4.187 \cdot \ln \frac{471.32}{273} + \frac{1947.28}{471.32} + 2.0934 \cdot \ln \frac{573}{471.32}$$

$$= 6.8268 \text{ Jg}^{-1}\text{kg}^{-1}\text{K}$$

b. from Superheated Steam tables at 1.5 MPa & 300°C

$$S = 6.9188 \text{ Jg}^{-1}\text{kg}^{-1}\text{K}$$

|       | 1400 kPa                                                               | (1500) | 1600 kPa |
|-------|------------------------------------------------------------------------|--------|----------|
| 300°C | 6.9533                                                                 | S      | 6.8844   |
|       | $\frac{1500 - 1400}{1600 - 1400} = \frac{S - 6.9533}{6.8844 - 6.9533}$ |        |          |

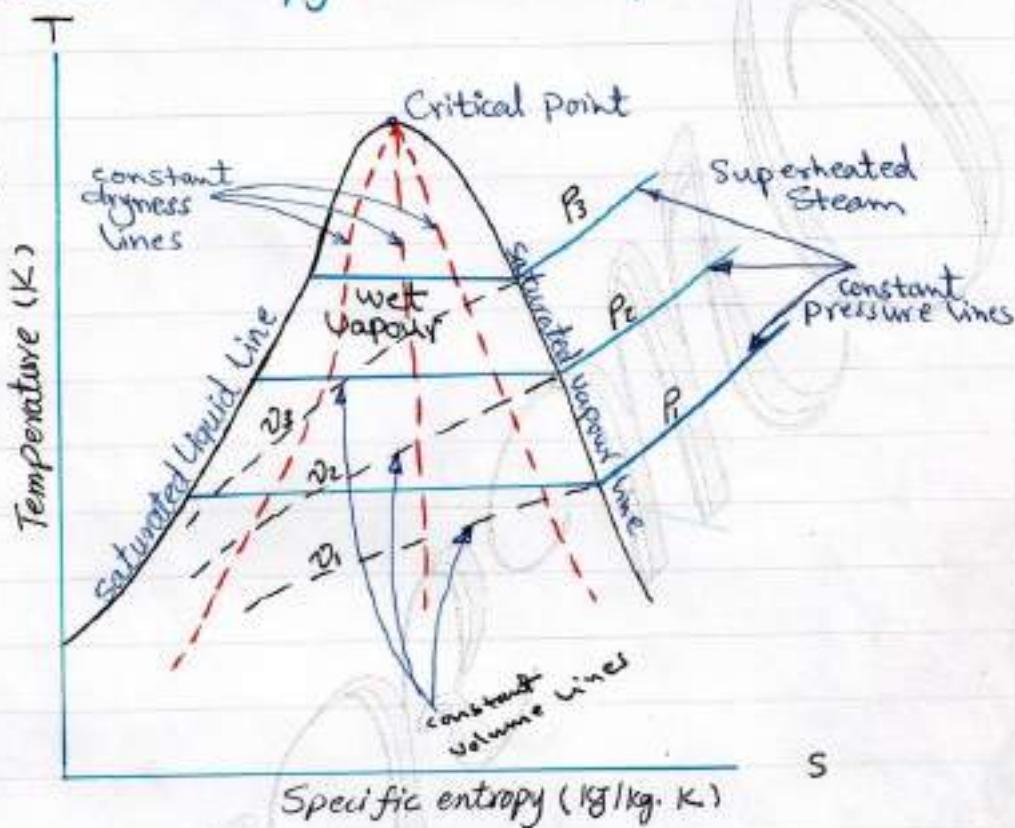


( The temperature - entropy chart for vapour )

$$P_1 < P_2 < P_3$$

$$v_1 > v_2 > v_3$$

$$S_x = S_f + x S_{fg}$$



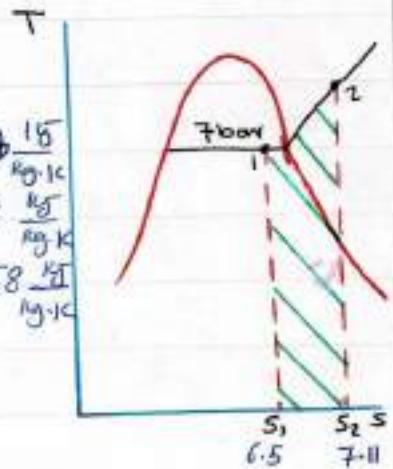
**Ex:** 1 kg of Steam at 7 bar, entropy 6.5 kg/kg.K is heated reversibly at constant pressure until the temperature is 250°C. Calculate the heat supplied and show on T-s diagram the area which represent the heat flow.

**Sol:**

at 7 bar from Steam tables ,  $S_g = 6.708 \frac{J}{kg \cdot K}$   
 $S_i < S_g \Rightarrow$  wet steam ,  $S_f = 1.9922 \frac{J}{kg \cdot K}$   
 $S_{fg} = 4.7158 \frac{J}{kg \cdot K}$

$$6.5 = 1.9922 + x \cdot 4.7158$$

$$x = 0.9558$$





$$h_1 = h_f + x \cdot h_{fg} \quad , \quad h_f = 697.2 \text{ J/g/kg} , \quad h_{fg} = 2066.3 \text{ J/g/kg}$$

$$h_1 = 697.2 + 0.9558 \cdot 2066.3 = 2672.169 \text{ J/g/kg}$$

at point ②  $P = 7 \text{ bar} , t = 250^\circ\text{C}$

from Steam tables  $t_s = 164.97^\circ\text{C}$   $\rightarrow t > t_s \Rightarrow$  Superheated

from Superheated Steam tables,  $h_2 = 2953.565 \text{ J/g/kg}$

| <u>600 kPa</u>                    | <u>700 kPa</u> | <u>800 kPa</u> |           |
|-----------------------------------|----------------|----------------|-----------|
| $h \rightarrow 250^\circ\text{C}$ | $2957.16$      | $h_2$          | $2949.97$ |
| $S \rightarrow 250^\circ\text{C}$ | $7.1816$       | $S_2$          | $7.0384$  |
| $= 281.396 \text{ J/g}$           |                |                |           |

$$Q = H_2 - H_1 = m(h_2 - h_1)$$

$$= 1(2953.565 - 2672.169)$$

$$= 281.396 \text{ J/g}$$

$$S_2 = 7.11 \text{ J/g.K}$$

**Ex:** A rigid cylinder of volume  $0.025 \text{ m}^3$  contains steam at 80 bar and  $350^\circ\text{C}$  the cylinder is cooled until the pressure is 50 bar calculate the state of the steam after cooling and amount of heat rejected by the steam. sketch the process on T-s diagram

**Sol:**

at  $P = 80 \text{ bar} , t_1 = 295.06^\circ\text{C}$   $t > t_s \Rightarrow$  Superheated

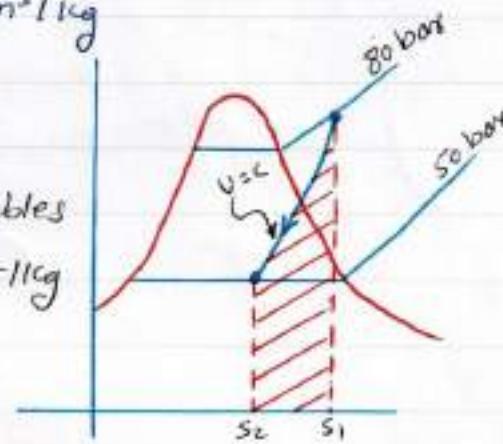
from Steam table,  $v_1 = 0.02995 \text{ m}^3/\text{kg}$

$$v_1 = \frac{V_1}{m} \Rightarrow m = \frac{0.025}{0.02995} = 0.8347 \text{ kg}$$

at  $P_1 = 80 \text{ bar} \& 350^\circ\text{C}$  from Steam tables

$$h_1 = 2987.3 \text{ J/g/kg} , \quad u_1 = 2747.67 \text{ J/g/kg}$$

$$v_1 = v_2 \quad (\text{rigid cylinder})$$





$\therefore v_2 = 0.02995 \text{ m}^3/\text{kg}$ , at  $P_2 = 50 \text{ bar}$ ,  $v_g = 0.03944 \frac{\text{m}^3}{\text{kg}}$

$\therefore v_g > v_2 \Rightarrow \text{wet Steam}$

$v_2 = x v_g \Rightarrow 0.02995 = x_2 * 0.03944 \Rightarrow x_2 = 0.759$

at 50 bar,  $u_f = 1147.78 \text{ kJ/kg}$ ,  $u_{fg} = 1449.34 \text{ kJ/kg}$

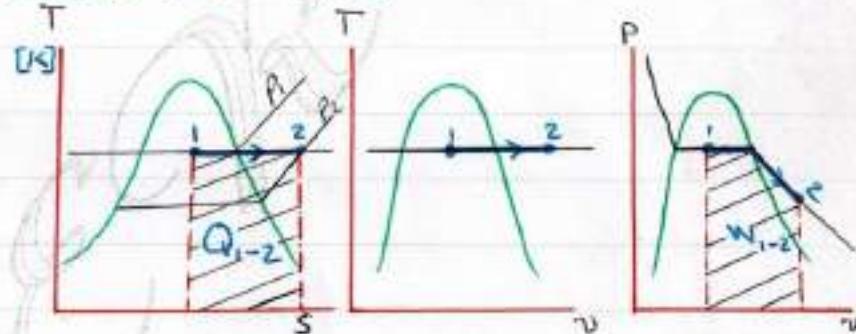
$u_2 = u_f + x u_{fg} = 1147.78 + 0.759 * 1449.34$   
 $= 2247.82 \text{ kJ/kg}$

$Q = W + \Delta U$ ,  $V = c \Rightarrow W = 0$   
 $= m(u_2 - u_1) = 0.8347(2247.82 - 2747.67)$   
 $= -417.22 \text{ kJ}$  (rejected heat)

\* Isothermal process on T-s chart:

$$Q = T(S_2 - S_1)$$

$$T_1 = T_2$$

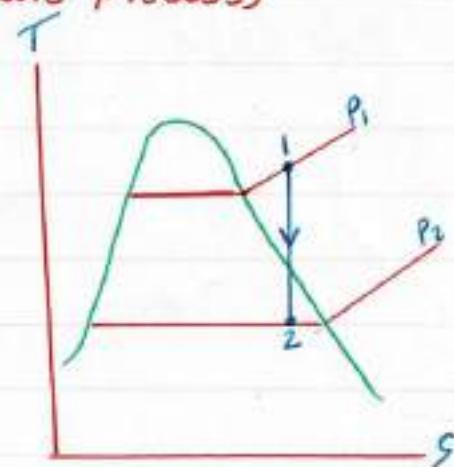


\* Isentropic process (reversible adiabatic process)

$$Q = 0, S_1 = S_2$$

$$dQ = T ds, S = \text{constant}$$

$$\therefore Q = 0 \quad \therefore dS = 0$$





## \* The Entropy of Ideal gas (single-phase system)

$$dQ = dw + dv \quad \text{for non-flow process}$$

for unit mass of gas and let its state change from  $P_1, T_1$ ,  $V_1$  to new state  $P_2, T_2, V_2$

$$dQ = dw + dv \quad \text{for unit mass}$$

$$dQ = dU + dw$$

$$dQ = C_V dT + P dV \quad \text{--- ①}$$

$$\frac{dQ}{T} = C_V \frac{dT}{T} + \frac{P}{T} dV \quad \text{--- ②}$$

$$PV = RT \Rightarrow P/T = R/V \quad \text{--- ③}$$

Sub. eq ③ in ②

$$dS = C_V \frac{dT}{T} + R \frac{dV}{V}$$

$$S_2 - S_1 = \int_{T_1}^{T_2} C_V \frac{dT}{T} + \int_{V_1}^{V_2} R \frac{dV}{V} \quad \text{--- ④}$$

$$S_2 - S_1 = C_V \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1} \quad \text{--- ⑤}$$

eq (5) represent the relation between (S, T, V).

$C_P - C_V = R$  , substituting this into eq. 5

$$S_2 - S_1 = C_V \ln \frac{T_2}{T_1} + (C_P - C_V) \ln \frac{V_2}{V_1} \quad \text{--- ⑥}$$

$$S_2 - S_1 = C_V \left( \ln \frac{T_2}{T_1} - \ln \frac{V_2}{V_1} \right) + C_P \ln \frac{V_2}{V_1} \quad \text{--- ⑦}$$



$$S_2 - S_1 = C_V \ln\left(\frac{T_2}{T_1} \cdot \frac{V_1}{V_2}\right) + C_P \ln \frac{V_2}{V_1} \quad \text{--- ⑧}$$

$$\therefore \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \Rightarrow \frac{T_2}{T_1} \cdot \frac{V_1}{V_2} = \frac{P_2}{P_1} \quad \text{--- ⑨}$$

Sub. eq. ⑨ in ⑧

$$S_2 - S_1 = C_V \ln \frac{P_2}{P_1} + C_P \ln \frac{V_2}{V_1} \quad \text{--- ⑩}$$

eq. ⑩ represent the relation between (S, P, V)

$$C_V = C_P - R, \text{ Sub. in eq ⑩}$$

$$S_2 - S_1 = C_V \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1} \quad \text{--- ⑪}$$

$$\therefore S_2 - S_1 = (C_P - R) + \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1} \quad \text{--- ⑫}$$

$$S_2 - S_1 = C_P \ln \frac{T_2}{T_1} - R (\ln \frac{T_2}{T_1} - \ln \frac{V_2}{V_1}) \quad \text{--- ⑬}$$

$$S_2 - S_1 = C_P \cdot \ln \frac{T_2}{T_1} - R \ln \left( \frac{T_2}{T_1} \cdot \frac{V_1}{V_2} \right) \quad \text{--- ⑭}$$

$$\therefore \frac{T_2}{T_1} \cdot \frac{V_1}{V_2} = \frac{P_2}{P_1}$$

$$S_2 - S_1 = C_P \cdot \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad \text{--- ⑮}$$

eq. ⑮ represent the relation between (S, T, P)



### • Isometric Process $V=C$

from eq (5) & (10)

$$S_2 - S_1 = C_V \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1} \quad \dots \dots (5)$$

$$V_2 = V_1 \Rightarrow V_2/V_1 = 1 \Rightarrow \ln \frac{V_2}{V_1} = \ln 1 = 0$$

$$\therefore S_2 - S_1 = C_V \ln \frac{T_2}{T_1}$$

and  $S_2 - S_1 = C_V \ln \frac{P_2}{P_1} + C_P \ln \frac{V_2}{V_1} \quad \dots \dots (10)$

$$V_2 = V_1 \Rightarrow \ln V_2/V_1 = 0$$

$$\therefore S_2 - S_1 = C_V \ln \frac{P_2}{P_1}$$

$$S_2 - S_1 = C_V \ln \frac{T_2}{T_1} = C_V \ln \frac{P_2}{P_1} \quad \text{for constant volume}$$

### • Isobaric process $P=C$

from eq (10) & (14)

$$S_2 - S_1 = C_V \ln \frac{P_2}{P_1} + C_P \ln \frac{V_2}{V_1} \quad \dots \dots (10)$$

$$S_2 - S_1 = C_P \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad \dots \dots (14)$$

But  $P_1 = P_2 \Rightarrow \ln (P_2/P_1) = 0$ , Sub. in eq (10) & (14)

$$S_2 - S_1 = C_P \ln \frac{V_2}{V_1} = C_P \ln \frac{T_2}{T_1} \quad \text{for constant pressure}$$



## Isothermal process $T=C$

from eqn 5 & 14

$$S_2 - S_1 = C_V \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1} \quad \text{--- (3), } T_1 = T_2 \Rightarrow \ln \frac{T_2}{T_1} = 0$$

$$S_2 - S_1 = R \ln \frac{V_2}{V_1}$$

and

$$S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad \text{--- (4)} \quad \ln \frac{T_2}{T_1} = 0$$

$$S_2 - S_1 = -R \ln \frac{P_2}{P_1} = R \ln \frac{P_1}{P_2}$$

$$S_2 - S_1 = R \ln \frac{V_2}{V_1} = R \ln \frac{P_1}{P_2}$$

for constant temperature

## Polytropic Process $PV^n = C$

Heat transferred =  $\frac{\gamma-n}{\gamma-1} \cdot \text{Work done}$

$$dQ = \frac{\gamma-n}{\gamma-1} P dV \quad \left. \right\} \div T$$

$$\frac{dQ}{T} = ds = \frac{\gamma-n}{\gamma-1} \frac{P dV}{T}$$

$$PV = RT \Rightarrow P/T = R/V$$

$$ds = \frac{\gamma-n}{\gamma-1} \cdot R \cdot \frac{dV}{V}$$

$$S_2 - S_1 = \frac{\gamma-n}{\gamma-1} R \int_{V_1}^{V_2} \frac{dV}{V}$$



$$S_2 - S_1 = \frac{\gamma-n}{\gamma-1} R \ln \frac{V_2}{V_1} \quad , \quad R = C_V(\gamma-1)$$

$$S_2 - S_1 = \frac{\gamma-n}{\gamma-1} \cdot C_V(\gamma-1) \ln \frac{V_2}{V_1}$$

$$S_2 - S_1 = C_V(\gamma-n) \ln \frac{V_2}{V_1} \quad , \quad \frac{V_2}{V_1} = \left( \frac{T_1}{T_2} \right)^{\frac{1}{n-1}} = \left( \frac{P_1}{P_2} \right)^{\frac{1}{n}}$$

$$S_2 - S_1 = C_V(\gamma-n) \ln \left( \frac{T_1}{T_2} \right)^{\frac{1}{n-1}}$$

$$S_2 - S_1 = C_V \frac{\gamma-n}{n-1} \cdot \ln \frac{T_1}{T_2}$$

$$S_2 - S_1 = C_V(\gamma-n) \ln \left( \frac{P_1}{P_2} \right)^{\frac{1}{n}}$$

$$S_2 - S_1 = C_V \frac{\gamma-n}{n} \ln \frac{P_1}{P_2}$$

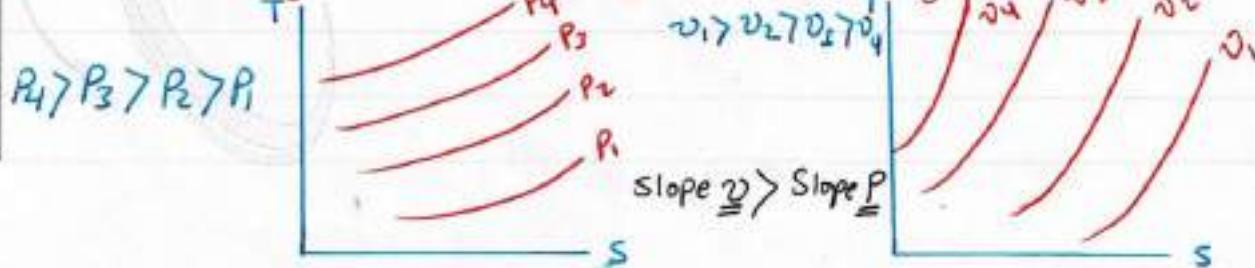
and

$$S_2 - S_1 = C_V \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1} = C_P \ln \frac{V_2}{V_1} + C_V \ln \frac{P_2}{P_1} = C_P \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

- Isentropic process  $S=C$

$$S_2 - S_1 = 0$$

\* The entropy chart for a gas (perfect gas)





**Ex:** Air at 15°C and 1.05 bar occupies 0.02m<sup>3</sup>. The air is heated at constant volume until the pressure is 4.2 bars, and then cooled at constant pressure back to the original temperature. Calculate the heat flow to or from the air and the net entropy change. Sketch the processes on T-s diagram.

**Sol:**

$$R = 0.287 \text{ kg/kg.K}$$

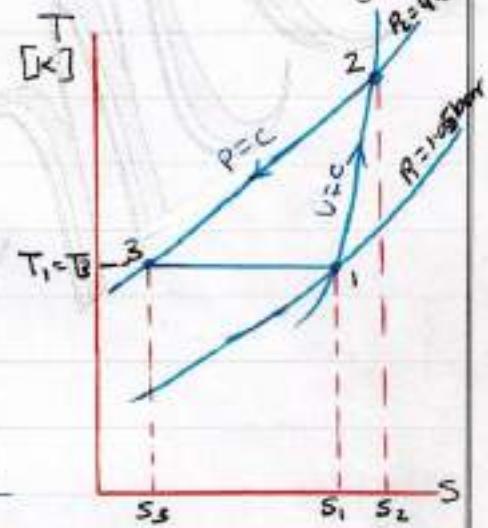
$$C_v = 0.718 \text{ kg/kg.K} \quad C_p = 1.005 \text{ kg/kg.K}$$

$$P_1 V_1 = m R T_1 \quad T_1 = 15 + 273 = 288 \text{ K}$$

$$1.05 \text{ bar} * 0.02 = m * 0.287 * 288$$

$$m = 0.0254 \text{ kg}$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \Rightarrow T_2 = \frac{P_2 \cdot T_1}{P_1} = \frac{4.2 * 288}{1.05} = 1152 \text{ K}$$



$$Q_{1-2} = m C_v (T_2 - T_1) = 0.0254 * 0.718 * (1152 - 288) \\ = 15.75 \text{ kJ}$$

$$Q_{2-3} = m C_p (T_3 - T_2) = 0.0254 * 1.005 * (288 - 1152) \\ = -22.05 \text{ kJ}$$

$$Q_{1-3} = Q_{1-2} + Q_{2-3} = 15.75 + (-22.05) \\ = -6.3 \text{ kJ}$$

$$\text{for } V=C \quad S_2 - S_1 = C_v \ln \frac{T_2}{T_1} \quad \text{kg/kg.K}$$

$$S_2 - S_1 = m C_v \ln \frac{T_2}{T_1} \quad \text{kg/K}$$

$$S_2 - S_1 = 0.0254 * 0.718 * \ln \frac{1152}{288} = 0.0253 \text{ kg/K}$$



for  $P=c$  from 2 → 3

$$S_3 - S_2 = m C_p \ln \frac{T_3}{T_2} = 0.0254 * 1.005 * \ln \frac{288}{1152} = -0.0354 \text{ Jg/K}$$

$$\Delta S_{\text{net}} = \Delta S_{1-2} + \Delta S_{2-3} \\ = 0.0253 - 0.0354 \\ = -0.0101 \text{ Jg/K}$$

**Ex:** 0.03 m<sup>3</sup> of nitrogen contained in a cylinder behind a piston is initially at 1.05 bar and 15°C. The gas is compressed isothermally and reversibly until the pressure is 4.2 bar. Calculate the change of entropy, the heat flow, the work done. Sketch the process on P-V & T-s diagram.

**Sol:**  $R = R_0/M = 8.314/28 = 0.297 \text{ Jg/K}$

$P_1 V_1 = m R T_1 \quad \Rightarrow T_1 = 15 + 273 = 288 \text{ K}$

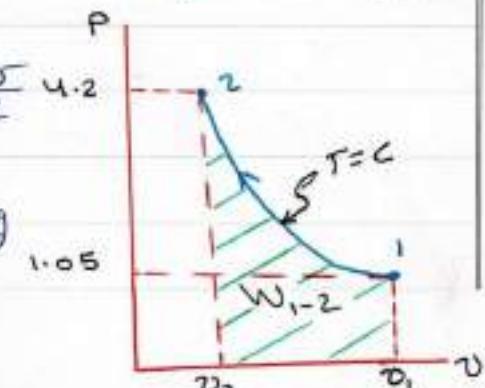
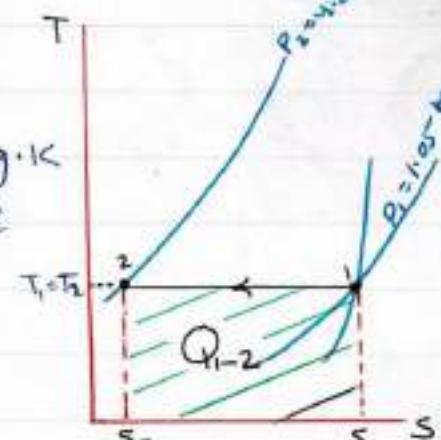
$1.05 * 10^2 * 0.03 = m * 0.297 * 288$

$m = 0.0368 \text{ kg}$

$S_2 - S_1 = C_p \ln \frac{T_2}{T_1} + m R \ln \frac{P_1}{P_2}$

$\therefore S_2 - S_1 = m R \ln \frac{P_1}{P_2} = 0.0368 * 0.297 * \ln \frac{1.05}{4.2} = -0.01516 \text{ Jg/K}$

$Q_{1-2}$  = area under the line representing the process 1 → 2





$Q = T_1(S_2 - S_1) = 288 * -0.01516 = -4.37 \text{ kJ}$

$Q = W + \Delta U \Rightarrow T = C \Rightarrow \Delta U = 0$

$\therefore Q = W$

$\Rightarrow W = -4.37 \text{ kJ}$

### Entropy & Irreversibility

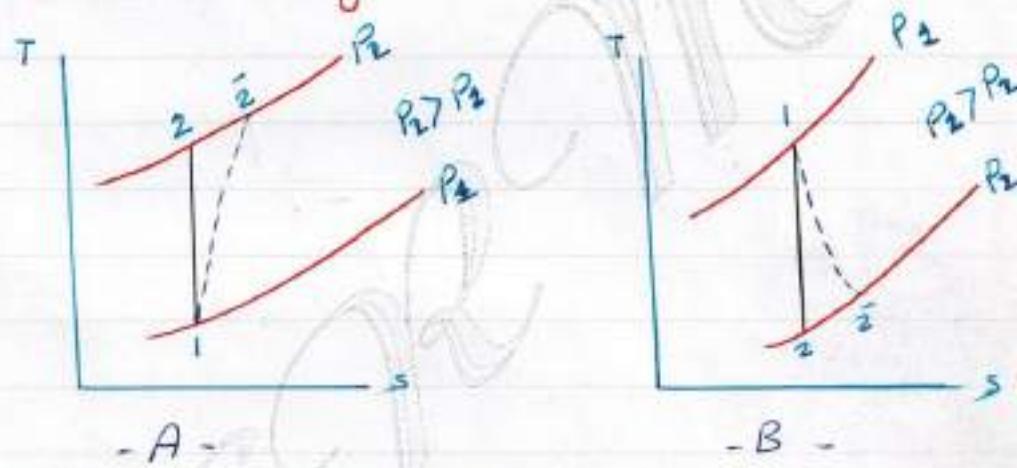


fig. A represent the compression of fluid.

fig. B represent the expansion of fluid.

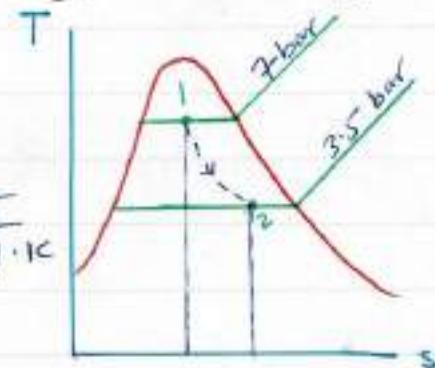
**Ex:** Steam at 7 bar, dryness fraction 0.96, is throttled down to 3.5 bar, calculate the change of entropy per kg of steam.

**Sol:**

$$\text{at 7 bar, } S_1 = S_{fr} + S_{fg}$$

$$\therefore S_1 = 1.99 + 0.96 * 4.717 = 6.522 \frac{\log \frac{T_2}{T_1}}{\log \frac{P_2}{P_1}}$$

Throttling  $\Rightarrow h_1 = h_2$





$$h_1 = h_f + x h_{fg} = 697 + 0.96 \times 2067 \\ = 2682 \text{ kJ/kg}$$

$\therefore h_2 = 2682 \text{ kJ/kg}$ , from Steam tables at  $P=3.5 \text{ bar}$

$$h_g = 2732 \text{ kJ/kg}, h_f = 584 \text{ kJ/kg}$$

$h_f < h_2 < h_g \Rightarrow \text{wet Steam}$

$$h_2 = h_f + x_2 h_{fg} \Rightarrow 2682 = 584 + x_2 \cdot 2148 \Rightarrow x_2 = 0.977$$

$$S_2 = S_f + x_2 S_{fg}, \text{ at } P=3.5 \text{ bar}$$

$$S_2 = 1.727 + 0.977 \times 5.214 \\ = 6.817 \text{ kJ/kg.K}$$

**Ex:** In an air turbine the air expands from 6.8 bar and  $430^\circ\text{C}$  to 1.013 bar and  $150^\circ\text{C}$ . The heat loss from the turbine can be assumed to be negligible. Show that the process is irreversible, and calculate the change of entropy.  $\gamma=1.4$

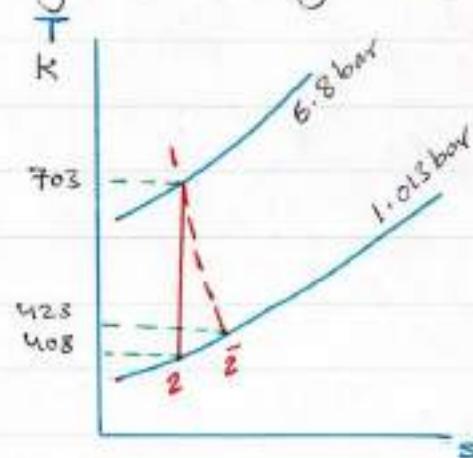
**Sol:**

since the process is with negligible heat loss.

$\therefore$  the process is adiabatic.

If it is adiabatic & reversible,

$\therefore$  it is isentropic



$$T_1 = 430 + 273 = 703 \text{ K}$$

$$\frac{T_1}{T_2} = \left( \frac{P_1}{P_2} \right)^{\frac{\gamma-1}{\gamma}} \Rightarrow \frac{703}{T_2} = \left( \frac{6.8}{1.013} \right)^{\frac{0.4}{1.4}} \Rightarrow T_2 = 408 \text{ K} = 135^\circ\text{C}$$



the process is irreversible because  $T_2 < T_{\bar{2}}$   
(If  $T_2 = T_{\bar{2}} \Rightarrow$  reversible process)

$$S_1 - S_{\bar{2}} = S_{\bar{2}} - S_{\bar{1}} \quad , \quad S_1 = S_{\bar{2}} \quad (\text{reversible adiabatic})$$

the process  $2 \rightarrow \bar{2} \rightarrow P=c$

$$\therefore S_{\bar{2}} - S_{\bar{1}} = C_p \ln \frac{T_{\bar{2}}}{T_2} = 1.005 \ln \frac{423}{408} = 0.0355 \text{ J/g.K}$$

If  $\Delta S_{\text{total}} > 0$  irreversible process  
 $= 0$  reversible process } for adiabatic process  
 $< 0$  impossible process

H.W:

1. Calculate the change of entropy of 1kg of air expanding polytropically in a cylinder behind a piston from 6.3 bar and 550°C to 1.05 bar. The index of expansion is 1.3.

2. 0.05 kg of carbon dioxide ( $M=44$ ) is compressed from 1 bar & 15°C until the pressure is 8.3 bar and the volume is then 0.004m<sup>3</sup>. Calculate the change of entropy.

Take:  $C_p = 0.88 \frac{\text{kJ}}{\text{kg.K}}$



## (Sheet No. 6)

**Q1:**

A rigid cylinder containing  $0.006\text{ m}^3$  of nitrogen at  $1.04\text{ bar}$ ,  $15^\circ\text{C}$ , is heated reversibly until the temperature is  $90^\circ\text{C}$ . Calculate the change of entropy and the heat supplied. Sketch the process on T-s diagram. Take  $\gamma = 1.4$ , assume nitrogen is a perfect gas.

$$\text{Ans. } [0.00125 \frac{\text{kJ}}{\text{K}}, 0.407 \text{ kJ}]$$

**Q2:**

$1\text{ m}^3$  of air is heated reversibly at constant pressure from  $15^\circ\text{C}$  to  $300^\circ\text{C}$ , and then cooled reversibly at constant volume back to the initial temperature, the initial pressure is  $1.03\text{ bar}$ . Calculate the net heat flow and the overall change of entropy. Sketch the process on T-s diagram.

$$\text{Ans. } [101.5 \text{ kJ}, 0.246 \text{ kJ/K}]$$

**Q3:**

$1\text{ kg}$  of air is allowed to expand reversibly in a cylinder behind a piston in such away that the temperature remains constant at  $260^\circ\text{C}$  while the volume is doubled. The piston is then moved in, and heat is rejected by the air reversibly at constant pressure until the volume is the same as it was initially. Calculate the net heat flow and the overall change of entropy. Sketch the process on T-s diagram.

$$\text{Ans. } [-161.9 \frac{\text{kJ}}{\text{kg}}, -0.497 \frac{\text{kJ}}{\text{kg.K}}]$$

**Q4:**  $1\text{ kg}$  of air at  $1.02\text{ bar}, 20^\circ\text{C}$  undergoes a process in which the pressure is raised to  $6.12\text{ bar}$ , and the volume becomes  $0.25\text{ m}^3$ . Calculate the change of entropy.  $\text{Ans. } [0.087 \text{ kJ/kg.K}]$



**Q5:** Steam expands reversibly in a cylinder behind a piston from 6 bar dry saturated to a pressure of 65 kpa. Assuming that the cylinder is perfectly thermally insulated. Calculate the work done during the expansion per kg of steam. **Ans. [323.81 kJ/kg]**

**Q6:** In a centrifugal compressor the air is compressed through a pressure ratio of 4 to 1, and the temperature of the air increases by a factor of 1.65. Show that the process is irreversible and calculate the change of entropy per kg of air. Assume the process is adiabatic. **Ans. [0.105 10^3 J/kg.K]**

**Q7:** A turbine is supplied with steam at 40bar, 400°C, which expands through the turbine in steady flow to an exit pressure of 0.2bar and dryness fraction of 0.93. The inlet velocity is negligible but the steam leaves at high velocity through a duct of 0.14m<sup>2</sup> cross-sectional area. If the mass flow rate is 3kg/s, and the mechanical efficiency is 90%. Calculate the horse power output of the turbine. Show that the process is irreversible and calculate the change of entropy, negligible the heat losses from the turbine.

**Ans. [201.8 kW > 0.643 10^3 J/kg.K]**