

THERMODYNAMICS

6

CHAPTER SIX

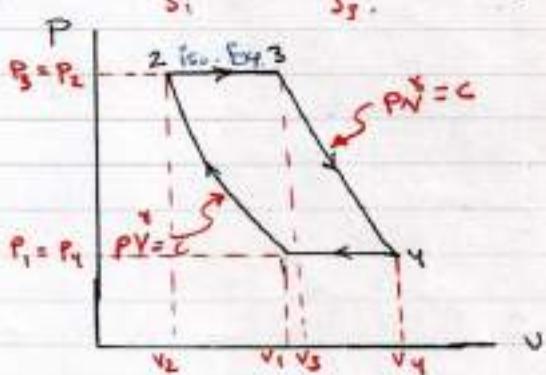
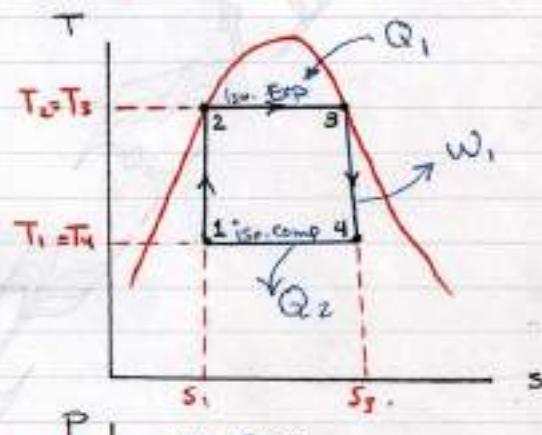
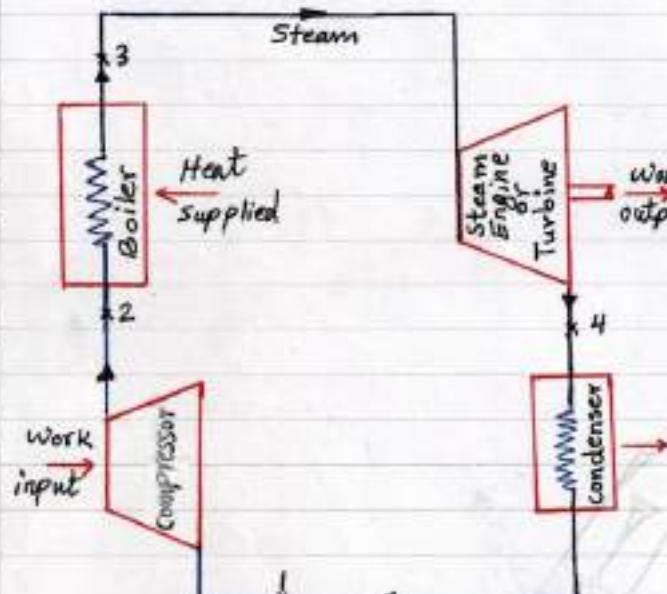
The Steam Cycles



The Steam Cycles

In a vapour cycle, all the theory of air cycles remains the same, except the working substance, which is steam. The steam may be in any form, wet, dry saturated or superheated.

* Carnot Cycle with steam as working substance :



Process 1 → 2

Isentropic compression of wet steam in compressor the pressure and temperature rises, and no heat is absorbed or rejected during this process. $Q=0$

Process 2 → 3

The saturated water at point 2, is isothermally converted into dry saturated steam in a boiler, and the heat is absorbed at constant temperature.

$q_{2 \rightarrow 3} = \text{change in entropy} \times \text{Absolute temperature}$

$$q_{2 \rightarrow 3} = (S_3 - S_2) \cdot T_2 = (S_3 - S_2) \cdot T_3$$



Process 3 → 4

The dry steam now expands isentropically in a steam engine or turbine. the pressure and temperature falls from P_3 to P_4 and T_3 to T_4 respectively, and no heat is supplied or rejected during this process.

Process 4 → 1

The wet steam is now isothermally condensed in condenser and the heat is rejected at constant temperature and pressure.

$$q_{4 \rightarrow 1} = (S_4 - S_1) \cdot T_1 = (S_4 - S_1) \cdot T_4$$

$$\sum Q = \sum W$$

∴ Work done during the cycle = Heat absorbed - Heat rejected

$$W.D = Q_1 - Q_2 \quad , \text{ for } 1 \text{ kg}$$

$$W.D = q_{2 \rightarrow 3} - q_{4 \rightarrow 1}$$

$$W.D = (S_3 - S_2)T_2 - (S_4 - S_1)T_1$$

$$= (S_3 - S_2)(T_2 - T_1) \quad , \quad S_3 = S_4 \Rightarrow S_2 = S_1$$

or

$$W.D = (S_4 - S_1)(T_2 - T_1) \quad , \quad T_2 = T_3 \Rightarrow T_1 = T_4$$

or

$$W.D = (S_4 - S_1)(T_3 - T_4)$$

or

$$W.D = (S_3 - S_2)(T_3 - T_4)$$

$$\gamma = \frac{\text{Work done}}{\text{heat added}} = \frac{(S_3 - S_2)(T_2 - T_1)}{(S_3 - S_2)T_2} = \frac{T_2 - T_1}{T_2}$$

$$\gamma = 1 - \frac{T_1}{T_2}$$



Notes:

1. Since the heat absorbed is at the highest temperature and rejected at the lowest temperature, the Carnot cycle would give a maximum possible efficiency.
2. It may be noted that it is impossible to make a steam engine working on Carnot cycle, the isothermal expansion $2 \rightarrow 3$ and the isothermal compression $4 \rightarrow 1$ will have to be carried out extremely slow to ensure that the steam is always at constant temperature. But the isentropic expansion $3 \rightarrow 4$ and isentropic compression $1 \rightarrow 2$ should be carried out as quickly as possible in order to approach ideal isentropic condition. We know that sudden changes in the speed of an engine are not possible in actual practice, therefore it is impossible to realise Carnot's engine in actual practice.
3. At State 1 the steam is wet at T_1 , it is difficult to stop condensation at the point 1 and then compress it just to state 2. It is more convenient to allow the condensation process to proceed to completion.
4. The working fluid is water at the new state, and this can be conveniently pumped to boiler pressure, the pump has much smaller dimensions than it would have if it had to pump a wet vapour, the compression process is carried out more efficiently, and the equipment required is simpler and less expensive.

Ex: A power plant is supplied with dry saturated steam at a pressure of 16 bar and exhausts at 0.2 bar. find the efficiency of the Carnot Cycle.

Sol: from Steam tables at 16 bar, $T_s = 201.4 + 273 = 474.4 K$
 $= T_2$

from Steam tables, at 0.2 bar, $T_s = 60.1 + 273 = 333.1 K = T_1$

$$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \frac{333.1}{474.4} = 0.298 \text{ or } 29.8\%$$



Ex: In a carnot cycle, heat is supplied at 350°C and is rejected at 25°C . The working fluid is water, which while receiving heat, evaporates from liquid at 350°C to steam at 350°C . From Steam tables the entropy change for this process is 1.438 kJ/kg.K .

If the cycle operates on a stationary mass of 1 kg of water, find the heat supplied, work done and heat rejected per cycle. What is the pressure of water during heat reception.

$$\begin{aligned}\text{Sol: the heat supplied} &= (S_3 - S_2) T_2 \\ &= 1.438 * 623 \\ &= 895.87 \text{ J/g/Kg}\end{aligned}\quad \begin{aligned}T_2 &= 350 + 273 \\ &= 623 \text{ K}\end{aligned}$$

$$\begin{aligned}\text{the work done} &= (S_3 - S_2)(T_2 - T_1) \\ &= 1.438 * (623 - 298) \\ &= 467.35 \text{ J/g/Kg}\end{aligned}\quad \begin{aligned}T_1 &= 25 + 273 \\ &= 298 \text{ K}\end{aligned}$$

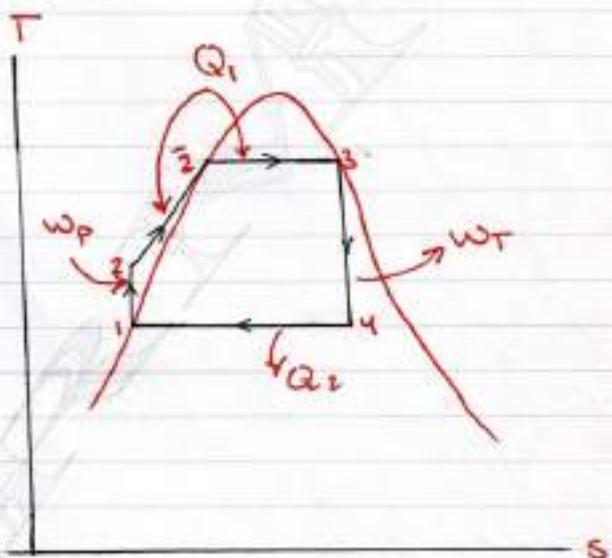
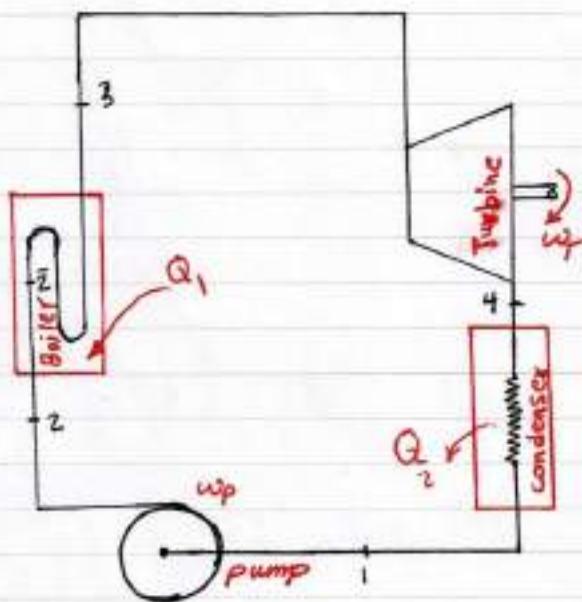
$$\begin{aligned}\text{the heat rejected} &= (S_4 - S_1) \cdot T_1 \quad , \quad S_4 - S_1 = S_3 - S_2 \\ &= 1.438 * 298 \\ &= 428.52 \text{ J/g/Kg}\end{aligned}$$

from Steam tables at $T_s = 350^{\circ}\text{C}$, $P = 165.35 \text{ bar}$



* Rankine Cycle :

The Rankine cycle is an ideal cycle for comparing the performance of Steam plants. It is modified form of Carnot Cycle, in which the condensation process (4→1) is continued until the steam is condensed into water.



- By the non-flow Energy Equation ($\Delta Z \approx 0$, $\Delta C \approx 0$)

1. Boiler $2 \rightarrow 3$ (Heat added, Q_1)

$$h_2 + q = h_3 + w, \quad w = 0 \quad \text{in the boiler}$$

$$\therefore q_{2 \rightarrow 3} = h_3 - h_2 \Rightarrow Q_{2 \rightarrow 3} = m(h_3 - h_2)$$

2. Turbine $3 \rightarrow 4$ (work output, W_T)

$$h_3 + q = h_4 + w, \quad q = 0, \text{ isentropic Exp.}$$

$$w_{3 \rightarrow 4} = h_3 - h_4 \Rightarrow$$

$$W_{3 \rightarrow 4} = m(h_3 - h_4)$$



3. Condenser $4 \rightarrow 1$ (Heat rejected, Q_2)

$$h_4 + q = h_1 + w \quad , \quad w=0$$

$$q_{4 \rightarrow 1} = h_1 - h_4 \quad , \text{ for positive value} \quad , \quad Q_{4 \rightarrow 1} = m(h_4 - h_1)$$

4. Pump $1 \rightarrow 2$ (work input, W_p)

$$h_1 + q = h_2 + w \quad , \quad q=0$$

$$w_{1 \rightarrow 2} = h_2 - h_1 \quad , \text{ for positive value} \quad , \quad W_{1 \rightarrow 2} = m(h_2 - h_1)$$

$$\text{or} \quad W_{\text{pump}} = v_f(P_2 - P_1)$$

$$\text{Net work done} = W_{3 \rightarrow 4} - W_{1 \rightarrow 2}$$

$$= W_T - W_c$$

$$W.D = (h_3 - h_4) - (h_2 - h_1)$$

If the feed pump work is neglected, $W_p = 0$

$$W.D = h_3 - h_4$$

$$\text{Heat added} = q_1 = h_3 - h_1$$

$$\text{Rankine efficiency, } \eta_R = \frac{W.D}{Q_{\text{add}}} = \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)}$$

$$\text{If } w_p = 0 \quad \eta_R = h_3 - h_4 / h_3 - h_1$$

* **Efficiency ratio:** It is also known as relative efficiency. It is defined as the ratio of thermal efficiency (or actual cycle efficiency) to Rankine efficiency (or ideal cycle efficiency). Mathematically,

$$\text{Efficiency ratio} = \frac{\text{Thermal efficiency}}{\text{Rankine efficiency}}$$



* **Work ratio :** It is defined as the ratio of net work output to the gross (engine or turbine) output.

$$\text{Work ratio} = \frac{\text{Net work output}}{\text{Gross output}} = \frac{\text{Turbine work - Compressor Work}}{\text{Turbine work}}$$

It may be noted that the Carnot cycle, despite of its high ideal thermal efficiency, has low work ratio. It is one of the reasons that Carnot cycle is not attempted. The higher value of work ratio also means a smaller size of the plant.

* **Specific Steam Consumption (S.S.C.) :**

It is also known as steam rate or specific rate of flow of steam. It is defined as the mass of steam that must be supplied to a steam engine or turbine in order to develop a unit amount of work or power output. The amount of work or power output is usually expressed in kilowatt hour (kWh).

$$S.S.C. = \frac{1 \text{ kWh}}{W} = \frac{3600}{W} = \frac{3600}{h_3 - h_4} \quad \text{kg/1kw.h} \rightarrow W_p = 0$$

W = net work done

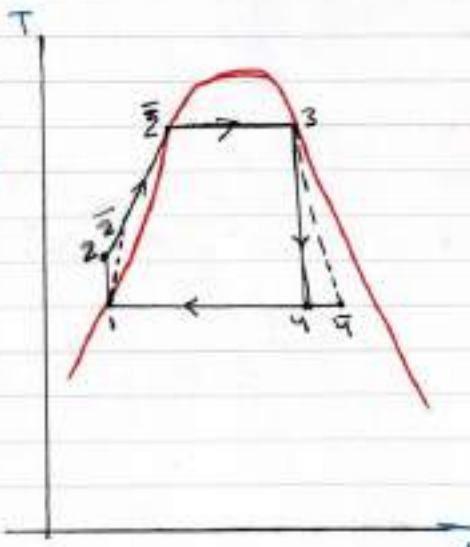
* **Isentropic efficiency:**

a. **For expansion process: (Turbine, 3 → 4)**

$$\gamma_{ise.} = \frac{\text{actual W.D.}}{\text{Isentropic W.D.}} = \frac{h_3 - h_4}{h_3 - h_{4'}}$$

b. **For compression process: (Pump)**

$$\gamma_{ise.} = \frac{\text{Isentropic W.D.}}{\text{actual W.D.}} = \frac{h_2 - h_1}{h_{2'} - h_1}$$





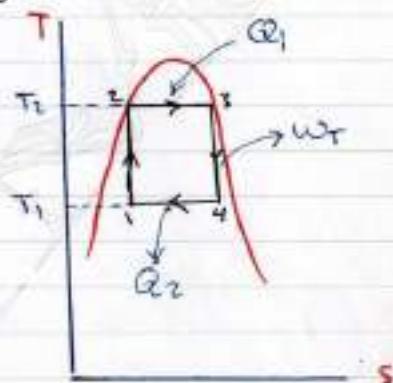
Ex: A steam power plant operates between a boiler pressure of 42 bar and a condenser pressure of 0.035 bar. Calculate for these limits the cycle efficiency, the work ratio, and the specific steam consumption:

- for a Carnot cycle using wet steam.
- for a Rankine cycle with dry saturated steam at entry to the turbine.
- for the Rankine cycle of (b) when the expansion process has an isentropic efficiency of 80%.

Sol: from Steam tables

at $P = 42 \text{ bar}$, $T_s = 253.2^\circ\text{C}$

at $P = 0.035 \text{ bar}$, $T_b = 26.7^\circ\text{C}$



a

$$\eta_{\text{Carnot}} = 1 - \frac{T_{\text{min}}}{T_{\text{max}}} = 1 - \frac{26.7 + 273}{253.2 + 273} \\ = 0.432 \text{ or } 43.2\%$$

$$Q_1 = h_3 - h_2 = h_g - h_f \quad , \text{ at } P = 42 \text{ bar} \\ = 2800 - 1105 \\ = 1695 \text{ J/g/kg}$$

$$\text{Net Work done} = \eta_{\text{Carnot}} * Q_1 \\ = 0.432 * 1695 \\ = 732.3 \text{ J/kg}$$

$$W_T = h_3 - h_4 \quad , \quad S_3 = S_4 = S_g \quad \text{at } P = 42 \text{ bar} \\ = 6.05 \text{ J/g/kg}$$

$$S_4 = S_f + x_4 S_{fg} \quad , \text{ at } P = 0.035 \text{ bar}$$

$$6.05 = 0.391 + x_4 * 8.13 \Rightarrow x_4 = 0.696$$

$$h_4 = h_f + x_4 h_{fg} \quad , \text{ at } P = 0.035 \text{ bar} \\ = 112 + 0.696 * 2438 = 1808 \text{ J/g/kg}$$

$$W_T = h_3 - h_4 = 2800 - 1808 = 992 \text{ J/g/kg}$$

$$\text{Work ratio} = \frac{\text{net work}}{\text{gross work}} = \frac{732.3}{992} = 0.738$$

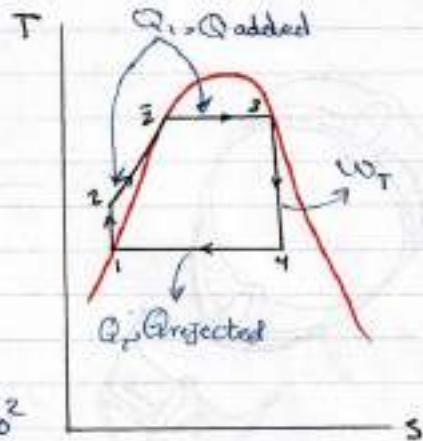
$$\text{S.S.C.} = \frac{3600}{W_T} = \frac{3600}{732.3} = 4.92 \text{ kg/kw.h}$$



b. $h_3 = 2800 \text{ kJ/kg}$, $h_4 = 1808 \text{ kJ/kg}$
 $h_2 = 116.1 \text{ kJ/kg}$, $h_1 = 111.9 \text{ kJ/kg}$
 $v_f = v_i = 0.001 \text{ m}^3/\text{kg}$

$$\gamma_R = \frac{W.D}{Q_1} = \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)}$$

$$\begin{aligned} \text{Pump work} &= h_2 - h_1 = v_i(P_2 - P_1) \\ &= 0.001(42 - 0.035) \times 10^2 \\ &= 4.2 \text{ kJ/kg} \end{aligned}$$



$$h_2 = h_1 + \text{Pump work} = 111.9 + 4.2 = 116.1 \text{ kJ/kg}$$

$$w_T = h_3 - h_4 = 2800 - 1808 = 992 \text{ kJ/kg}$$

$$w_P = 4.2 \text{ kJ/kg}$$

$$q_1 = h_3 - h_2 = 2800 - 116.1 = 2684 \text{ kJ/kg}$$

$$\text{work ratio} = \frac{w_{net}}{\text{gross work}} = \frac{992 - 4.2}{992} = 0.995$$

$$\text{S.S.C.} = \frac{3600}{w_{net}} = \frac{3600}{992 - 4.2} = 3.64 \text{ kg/kW.h}$$

$$\gamma_R = \frac{W.D}{Q_1} = \frac{992 - 4.2}{2684} = 0.368 \text{ or } 36.8\%$$

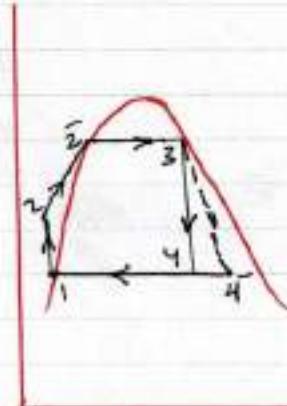
c.

$$\text{Mise.} = \frac{h_3 - h_4}{h_3 - h_1} = 0.8 \Rightarrow 0.8(2800 - 1808) = 2800 - h_4 \\ \text{so } h_4 = 2006.4 \text{ kJ/kg}$$

$$\eta_R = \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)} = \frac{(2800 - 2006.4) - 4.2}{2800 - 116.1} = 0.294 \text{ or } 29.4\%$$

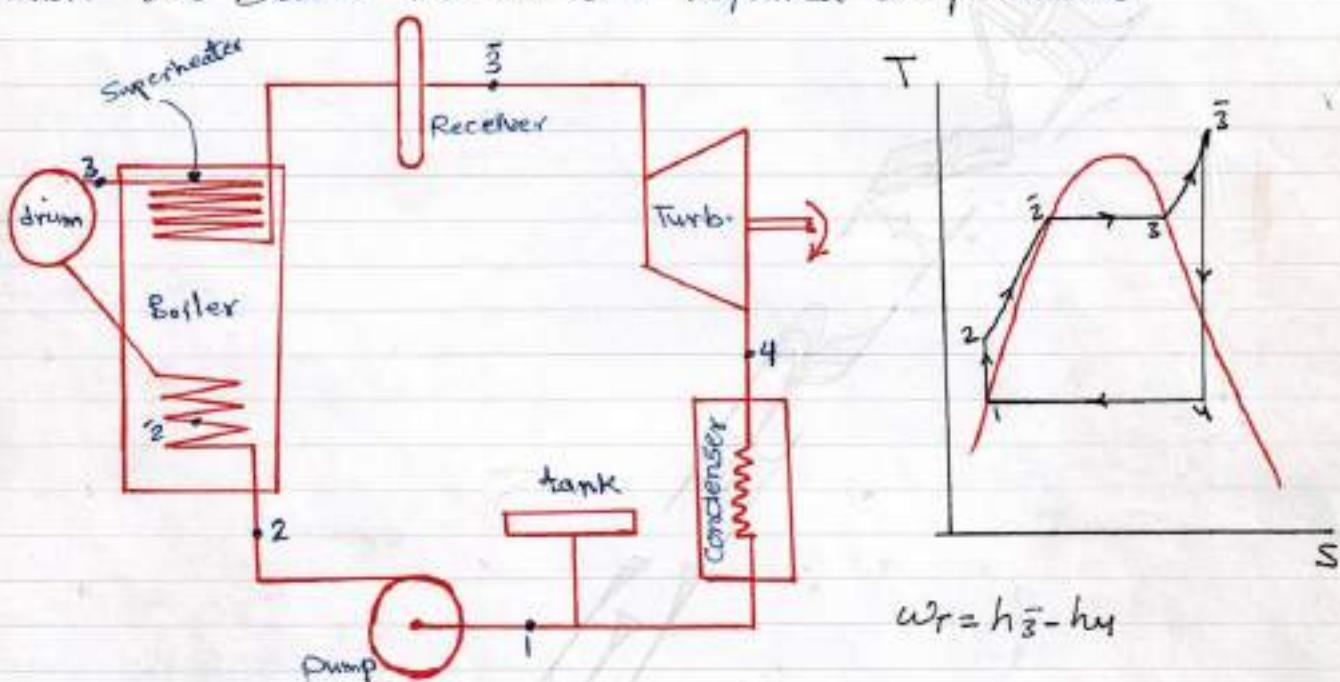
$$W.R = \frac{(2800 - 2006.4) - 4.2}{2800 - 2006.4} = 0.9947$$

$$\text{S.S.C.} = \frac{3600}{793.6 - 4.2} = 4.56 \text{ kg/kW.h}$$



* Rankine Cycle with Superheated Steam:

The average temperature at which heat is supplied in the boiler can be increased by superheating the steam. Usually the dry saturated steam from the boiler drum is passed through a second bank of smaller bore tubes within the boiler. This bank is situated such that it is heated by hot gases from the furnace until the steam reaches the required temperature.



Assuming $W_p = 0$ (always)

$$Q_i = Q_{add} = h_3 - h_1 \quad , \quad W.D = W_T = h_3 - h_4$$

$$\gamma_h = \frac{W.D}{Q_{add.}} = \frac{h_3 - h_4}{h_3 - h_1}$$

Ex: A Steam turbine operating on the Rankine cycle received steam from the boiler at 3.5 MPa and 350°C and exhaust to the condenser at 10 kPa. The condensate is then returned to the boiler by the feed pump. Calculate, neglecting all losses:

- the energy supplied in the boiler per kg of steam,
- the dryness fraction of steam entering the condenser,
- the Rankine efficiency.



Sol: At 10 kPa, $h_1 = h_f = 192.1 \text{ kJ/kg}$
at 3.5 MPa & 350°C, $h_3 = 3104.1 \text{ kJ/kg}$, $S_3 = 6.6579 \text{ kJ/kg.K}$
 $S_4 = S_f + x_4 S_{fg}$, at 10 kPa
 $S_3 = S_4 \Rightarrow 6.657 = 0.6493 + x_4 \cdot 7.5 \Rightarrow x_4 = 0.8$

$$h_4 = h_f + x_4 h_{fg} = 191.8 + 0.8 \times 2392.8 \quad , \text{ at } 10 \text{ kPa} \\ = 2106 \text{ kJ/kg}$$

$$\begin{aligned} \text{Energy Supplied} &= h_3 - h_1 \\ &= 3104 - 192 \\ &= 2912.2 \text{ kJ/kg} \end{aligned}$$

$$\eta_R = \frac{h_3 - h_4}{h_3 - h_1} = \frac{3104 - 2106}{3104 - 191.8} = 0.3427 \text{ or } 34.27\%$$

Ex: Compare the Rankine cycle performance of example in page (211) with that obtained when the steam is superheated to 500°C, neglecting feed water work.

Sol: from Steam tables at 42 bar & 500°C

$$h_1 = 3442.6 \text{ kJ/kg} \rightarrow S_p = 7.066 \text{ kJ/kg.K} \\ = S_2$$

$$S_2 = S_f + x_2 S_{fg}$$

$$7.066 = 0.391 + x_2 \cdot 8.18 \Rightarrow x_2 = 0.821$$

$$h_2 = h_f + x_2 h_{fg} = 112 + 0.821 \times 2438 \\ = 2113 \text{ kJ/kg}$$

$$h_3 = h_f = 112 \text{ kJ/kg}$$

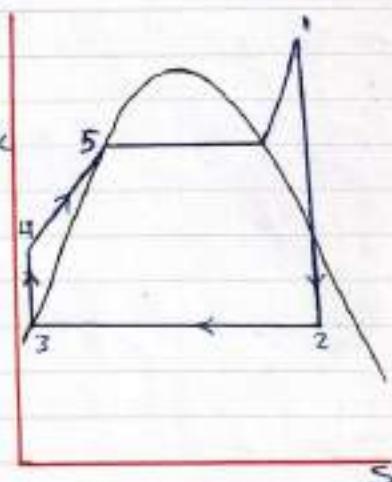
$$w_T = h_1 - h_2 = 3442.6 - 2113 = 1329.6 \text{ kJ/kg}$$

$$Q_1 = \text{heat Supplied} = h_1 - h_3 = 3442.6 - 112 = 3330.6 \text{ kJ/kg}$$

$$\eta_R = \frac{h_1 - h_2}{h_1 - h_3} = \frac{1329.6}{3330.6} \rightarrow 0.399 \text{ or } 39.9\%$$

$$S.S.C = \frac{3600}{h_1 - h_2} = \frac{3600}{1329.6} = 2.71 \text{ kg/s.h}$$

* the thermal efficiency has increased due to superheating and the improvement in S.S.C is even more marked



* Reheat Steam Cycle :

It is desirable to increase the average temperature at which heat is supplied to the steam, and also to keep the steam as dry as possible in the lower pressure stages of the turbine. The wetness at exhaust should be not greater than 10%. The high boiler pressures are required for high efficiency, but that expansion in one stage can result in exhaust steam which is wet. This is a condition which is improved by superheating the steam. The exhaust steam condition can be improved most effectively by reheating the steam, the expansion being carried out in two stages.

$$\text{Heat Supplied} = Q_{234} + Q_{56}$$

$$w_p \approx 0$$

$$Q_{234} = Q_{134} = h_u - h_i$$

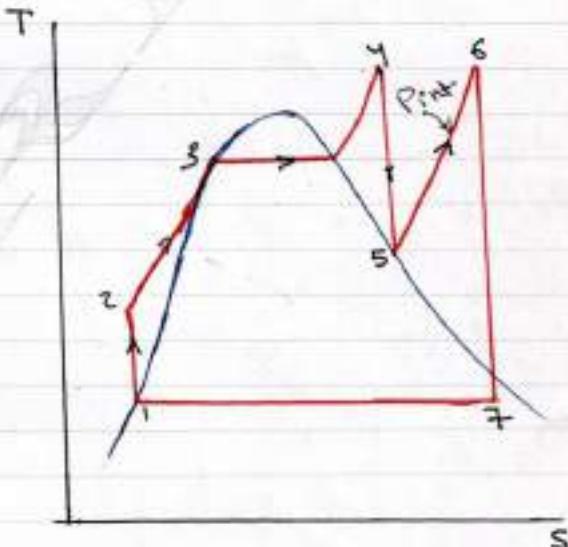
$$Q_{56} = h_6 - h_5$$

$$\text{Work output} = W_{45} + W_{67}$$

$$W_{45} = h_4 - h_5$$

$$W_{67} = h_6 - h_7$$

$$\eta_t = \frac{(h_u - h_5) + (h_6 - h_7)}{(h_u - h_i) + (h_6 - h_5)}$$



Ex: Calculate new cycle efficiency if reheat is included in the plant of above example. The steam condition at inlet to the turbine are 42 bar and 500°C and condenser pressure is 0.035 bar as before. Assume that the steam is just dry saturated on leaving the first turbine, and reheated to the initial temperature. Neglect feed water work.

$$\text{Sol: } h_i = 3442.6 \text{ J/g/kg}, s_i = 7.066 \text{ J/g.K}$$

$$s_1 = s_2 = s_g \Rightarrow P_2 \approx 2.4 \text{ bar} \\ = P_{\text{intermediate}}$$



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for $P_{int} = 2.4 \text{ bar}$

$$h_2 = h_g = 2715 \text{ kJ/kg}$$

$h_6 = 3487 \text{ kJ/kg}$,
 (at $P=2.4 \text{ bar}$ & 500°C)

$$S_6 = S_7$$

$$h_7 = 2535 \text{ kJ/kg}$$

$$h_3 = h_f \text{ at } P_{cond.} = 0.035 \text{ bar} \\ = 112 \text{ kJ/kg}$$

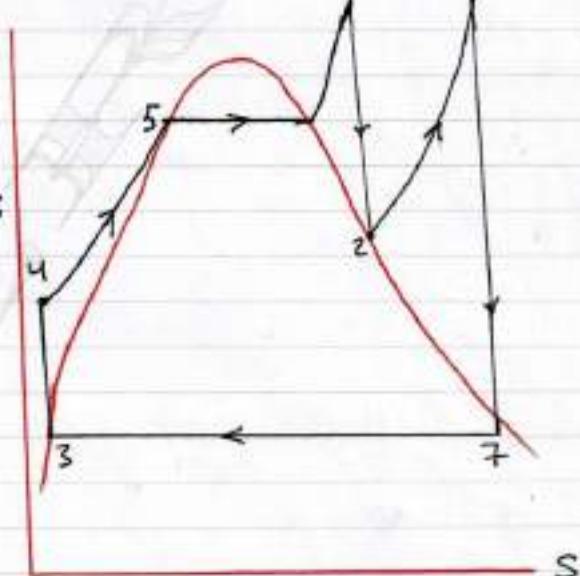
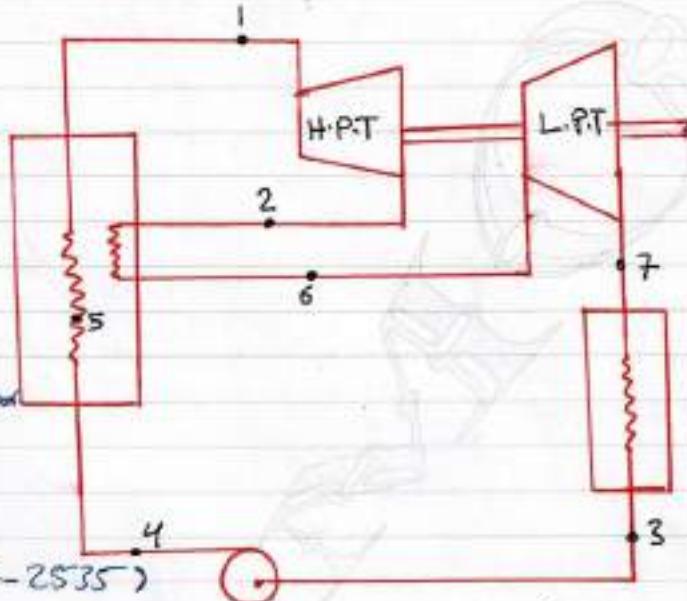
$$W_{net} = (h_1 - h_2) + (h_6 - h_7) \\ = (3442.6 - 2715) + (3487 - 2535) \\ = 1688 \text{ kJ/kg}$$

$$\text{Heat supplied} = (h_1 - h_3) + (h_6 - h_e) \\ = 3104 \text{ kJ/kg}$$

$$\eta_{th} = \frac{W_{net}}{Q_{sup}} = \frac{1682}{3104} = 0.41 \text{ or } 41\%$$

$$S.S.C. = \frac{3600}{1682} = 2.14 \text{ kg/kW.h}$$

• η_{th} & S.S.C. improved.



Sheet No. 8

Q1: A. Steam is supplied dry saturated at 40 bar to a turbine and the condenser pressure is 0.035. If the plant operates on the Rankine Cycle, calculate per kg of steam: a. the work output neglecting feed pump work, b. the work required for feed pump c. the heat transferred to the condenser cooling water, and the amount of cooling water required through the condenser if the temperature rise of water is assumed to be 5.5 K, d. the heat supplied e. the Rankine efficiency
Ans. [982.41kg, 4KJ, 1706.61kg, 74.11kg, 26851kg, 36.6%]

B. for the same steam conditions calculate the efficiency for a Carnot cycle operating with wet steam.

Ans. [43%]

Q2: Repeat problem (1-A) for a steam supply condition of 40 bar and 350°C and the same condenser pressure.

Ans. [1125KJ, 41kg, 18571kg, 80.51kg, 2978KJ, 37.8%]

Q3: Steam is supplied to a two-stage turbine at 40 bar and 350°C. It expands in the first turbine until it is just dry saturated, then it is reheated to 350°C and expanded through the second-stage turbine, the condenser pressure is 0.035 bar. Calculate the work output and the heat supplied per kg of steam for the plant, assuming ideal processes and neglecting the feed pump term, and calculate the cycle efficiency.

Ans. [12901kg, 33621kg, 38.4%]

Q4: If the expansion processes in the turbine of (Q3) have isentropic efficiencies of 84% and 78%, respectively in the 1st and 2nd stages, calculate the work output and the heat supplied per kg of steam, the thermal efficiency and the specific steam consumption.

1026kg → 33111kg → 31.1%, → 3.51 kg/kWh