University of Anbar

College of Engineering

Mechanical Engineering Dept.

Advanced Heat Transfer/ I
Conduction and Radiation

Handout Lectures for MSc. / Power
Chapter Two
One-dimensional, Steady-State Condition

Course Tutor

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Chapter Two

One-dimensional, Steady-State Condition

2.1 Introduction

Several different physical shapes may fall in the category of one-dimensional systems. Cylindrical and spherical systems are one-dimensional when the temperature in the body is a function only of radial distance and is independent of azimuth angle of axial distance. In some two-dimensional problems the effect of a second – space coordinate may be so small as to justify its neglect, and the multidimensional heat flow problem may be approximated with a one-dimensional analysis. In these cases, the differential equations are simplified, and we are led to a much easier solution as a result of this simplification.

2.2 General Formulation

Consider a long, hollow cylinder or a thick-walled, closed shell of constant wall thickness whose cross section is shown in Figure 2.1. This cylinder or shell contains a fluid at temperature T_i, and is surrounded by an ambient at temperature T_o. Let us suppose that $T_i > T_o$. The inside and outside heat transfer coefficients are h_i and h_o, respectively. We wish to know the temperature distribution of and the heat transfer through this cylinder or shell.

s = space coordinate

$A(s)$ = conduction heat transfer area

$$q_s = q_{s+ds} = q_s + \frac{\partial}{\partial s}(q_s)ds$$

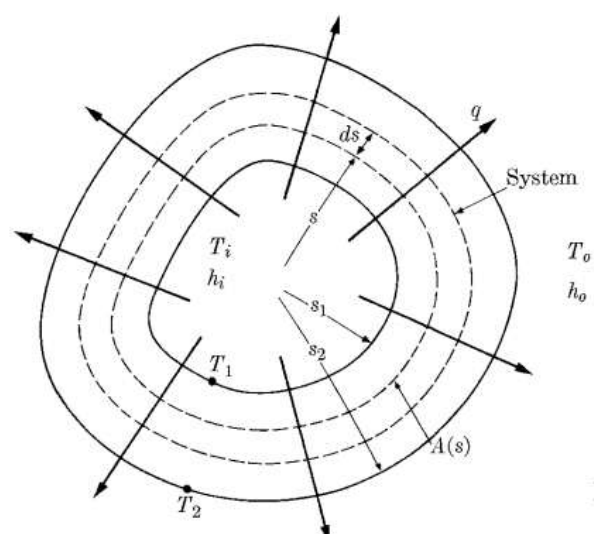


Figure 2.1: a long, hollow cylinder or a thick-walled, closed shell of constant wall thickness.

The method of solution employed here is convenient for one-dimensional problems in which q = *const*. at every cross section.

Assumptions

- ✓ Steady-State Condition
- ✓ One-dimensional problem (length >> thickness)
- ✓ Uniform thermal conductivity (k)
- ✓ Convection heat transfer coefficients (h_i and h_o) are uniform over the whole area.

$$q_s = q_{s+ds} = q_s + \frac{\partial}{\partial s}(q_s)ds \qquad (2\text{-}1)$$

$$q_s = -kA_s \frac{dT}{ds} \qquad (2\text{-}2)$$

$$\frac{d}{ds}\left(-kA_s \frac{dT}{ds}\right) = 0 \qquad (2\text{-}3)$$

From assumption, k = c

$$\frac{d}{ds}\left(A_s \frac{dT}{ds}\right) = 0 \qquad (2\text{-}4)$$

The boundary conditions from Figure 2.1,

Boundary condition (1) $\quad k \frac{dT_{(s1)}}{ds} = h_i[T_{(s1)} - T_i]$

Boundary condition (2) $\quad -k \frac{dT_{(s2)}}{ds} = h_o[T_{(s2)} - T_o]$

By integrating Eq. (2.4) twice, we can get

$$A_s \frac{dT}{ds} = C \quad \rightarrow \quad dT = C \frac{ds}{A_s} \quad \rightarrow \quad \int dT = C \int \frac{ds}{A_s} \quad \rightarrow \quad T = C \int \frac{ds}{A_s} + D$$

Substituting the solution of Eq. (2.4) into both boundary conditions,

$$\frac{kC}{A_{s1}} = h_i[C \int^{s1} \frac{ds}{A_s} + D - T_i] \qquad (2\text{-}5)$$

$$-\frac{kC}{A_{s2}} = h_o[C \int^{s2} \frac{ds}{A_s} + D - T_o] \qquad (2\text{-}6)$$

Thus, after finding C and D from Eqs (2.5 and 2.6) will be,

$$\frac{T - T_o}{T_i - T_o} = U_o[\frac{A_{s2}}{k} \int_s^{s2} \frac{ds}{A_s} + \frac{1}{h_o}] \qquad (2\text{-}7)$$

Where U_o is overall heat transfer coefficient based on A_{s2}

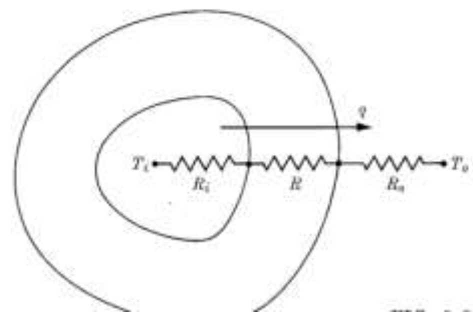
Therefore, $q = \frac{T_i - T_o}{R_i + R + R_o}$ (2-8)

It is sometimes convenient to simplify Eq. (2.8) by writing it in terms of the so-called *over-all coefficient of heat transfer U*, which is defined according to

$q = UA\,(T_i - T_o)$

$\frac{1}{UA} = R_i + R + R_o$

Where, $R_i = \frac{1}{h_i A_{s1}}$, $R_o = \frac{1}{h_o A_{s2}}$, $R = \frac{1}{k}\int_{s1}^{s2}\frac{ds}{A_s}$



Since U depends on A, the statement of U is ambiguous until an area is chosen. Noting that

$UA = U_i A_{s1} = U_o A_{s2}$ (2-9)

where U_i and U_o denote the *over-all heat transfer coefficients* based on the inner and outer surface areas, respectively, we may write the outside coefficient U_o, for example, as

$\frac{1}{U_o} = \frac{A_{s2}/A_{s1}}{h_i} + \frac{A_{s2}}{k}\int_{s1}^{s2}\frac{ds}{A_s} + \frac{1}{h_o}$ (2-10)

For convenience, we shall apply the procedure of the foregoing problem to three important cases, the *Cartesian*, *Cylindrical*, and *Spherical* geometries. The *over-all heat transfer coefficients* based on the outer surface area and the temperature distributions of these geometries are:

Cartesian: $\frac{1}{U_o} = \frac{1}{U} = \frac{1}{h_i} + \frac{L}{k} + \frac{1}{h_o}$ (2-11)

Cylindrical: $\frac{1}{U_o} = \frac{(R_2/R_1)}{h_i} + \frac{R_2}{k}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{h_o}$ (2-12)

Spherical: $\frac{1}{U_o} = \frac{(R_2/R_1)^2}{h_i} + \frac{R_2}{k}\left(\frac{R_2}{R_1} - 1\right) + \frac{1}{h_o}$ (2-13)

Cartesian: $$\frac{T-T_o}{T_i-T_o} = U_o\left(\frac{x_2-x}{L}+\frac{1}{h_o}\right) \tag{2-14}$$

Cylindrical: $$\frac{T-T_o}{T_i-T_o} = U_o\left[\frac{R_2}{k}\ln\left(\frac{R_2}{r}\right)+\frac{1}{h_o}\right] \tag{2-15}$$

Spherical: $$\frac{T-T_o}{T_i-T_o} = U_o\left[\frac{R_2}{k}\left(\frac{R_2}{r}-1\right)+\frac{1}{h_o}\right] \tag{2-16}$$

Homework:

Derive the following Equations,

Equation 2.7, Equation 2.8, Equation 2.12, Equation 2.3, Equation 2.15 and Equation 2.16.

2.3 Composite Structures

Assume that the hollow cylinder or the thick-walled, closed shell of Figure 2-1 is composed of N layers of materials having different thicknesses and thermal conductivities (Figure 2.2). The contact resistance between the layers is negligible. We wish to find the heat transfer from the inner fluid to the surrounding ambient, and the temperature distribution of the structure. Extending the analogy between the diffusion of heat and electric current to the present case, we readily obtain

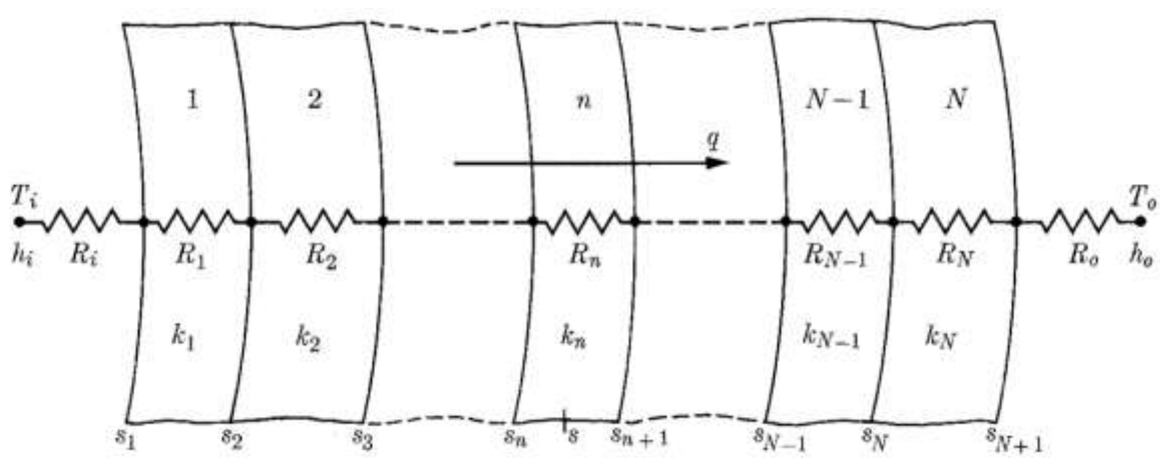


Figure 2.2: The hollow cylinder or the thick-walled with composite wall of N layers.

$$\frac{1}{UA} = R_i + \sum_{n=1}^{N} R_n + R_o \tag{2-17}$$

The explicit form of U based on the outside surface is

$$\frac{1}{U_o} = \frac{A_{S_{N+1}}/A_{S_1}}{h_i} + A_{S_{N+1}} \sum_{n=1}^{N} \frac{1}{k_n} \int_{S_n}^{S_{n+1}} \frac{ds}{A_s} + \frac{1}{h_o} \tag{2-18}$$

Equation (2.18) reduces to Eq. (2.10) for $N = 1$.

To obtain the temperature distribution in the structure we first express q in terms of the temperature difference ($T - T_o$) and the corresponding resistances (from the series R_n, R_{n+1}, R_{n+2}, ... ,R_N). The result is

$$q = \frac{T - T_o}{\frac{1}{k_n}\int_{s}^{s_{n+1}}(\frac{ds}{A_s}) + \sum_{m=n+1}^{N}(\frac{1}{k_m})\int_{s_m}^{s_{m+1}}(\frac{ds}{A_s}) + \frac{1}{h_o A_{s_{N+1}}}} \tag{2-19}$$

Where T denotes the temperature of the location s (see Figure 2.2). Then eliminating q between Equation 2.19 and [$q = UA\,(T_i - T_o)$], we find that the desired temperature distribution in terms of U_o is

$$\frac{T - T_o}{T_i - T_o} = U_o[\frac{A_{s_{N+1}}}{k_n}\int_{s}^{s_{n+1}}(\frac{ds}{A_s}) + A_{s_{N+1}} \sum_{m=n+1}^{N}\left(\frac{1}{k_m}\right)\int_{s_m}^{s_{m+1}}\left(\frac{ds}{A_s}\right) + \frac{1}{h_o}] \tag{2-20}$$

Equation (2.20) reduces to Eq. (2.7) for $N = 1$.

The Cartesian, cylindrical, and spherical forms of Equations (2.18) and (2.20) are listed below:

Cartesian: $$\frac{1}{U_o} = \frac{1}{U} = \frac{1}{h_i} + \sum_{n=1}^{N} \frac{L_n}{k_n} + \frac{1}{h_o} \tag{2-21}$$

Cylindrical: $$\frac{1}{U_o} = \frac{(R_{N+1}/R_1)}{h_i} + R_{N+1} \sum_{n=1}^{N}[\frac{1}{k_n} \ln\left(\frac{R_{n+1}}{R_n}\right)] + \frac{1}{h_o} \tag{2-22}$$

Spherical: $$\frac{1}{U_o} = \frac{(R_{N+1}/R_1)^2}{h_i} + R_{N+1}^2 \sum_{n=1}^{N}[\frac{1}{k_n}(\frac{1}{R_n} - \frac{1}{R_{n+1}})] + \frac{1}{h_o} \tag{2-23}$$

Cartesian: $$\frac{T - T_o}{T_i - T_o} = U_o\left(\frac{x_{n+1} - x}{k_n} + \sum_{m=n+1}^{N}(\frac{x_{m+1} - x_m}{k_m}) + \frac{1}{h_o}\right) \tag{2-24}$$

Cylindrical: $$\frac{T - T_o}{T_i - T_o} = U_o\left[\frac{R_{N+1}}{k_n}\ln\left(\frac{R_{n+1}}{r}\right) + R_{N+1}\sum_{m=n+1}^{N}\frac{1}{k_m}\ln(\frac{R_{m+1}}{R_m}) + \frac{1}{h_o}\right] \tag{2-25}$$

Spherical: $\frac{T-T_o}{T_i-T_o} = U_o \left[\frac{R_{N+1}}{k_n}\left(\frac{R_{N+1}}{r} - \frac{R_{N+1}}{R_{n+1}}\right) + R_{N+1}^2 \sum_{m=n+1}^{N} \frac{1}{k_m}\left(\frac{1}{R_m} - \frac{1}{R_{m+1}}\right) + \frac{1}{h_o}\right]$

(2-26)

In practice, the combination of series- and parallel-connected structures is also important, especially in Cartesian geometry.

In this section a number of physical and mathematical facts are demonstrated in terms of examples selected from Cartesian, Cylindrical, and Spherical geometries.

Examples:

<u>Q1:</u> Consider a wall composed of hollow concrete blocks, such as is used in building construction (Fig. 2.3). Actually, the heat transfer through this type of wall is not one-dimensional. However, a one-dimensional analysis gives satisfactory results for practical problems.

<u>Solution:</u>

By employing the electrical analogy, we readily obtain

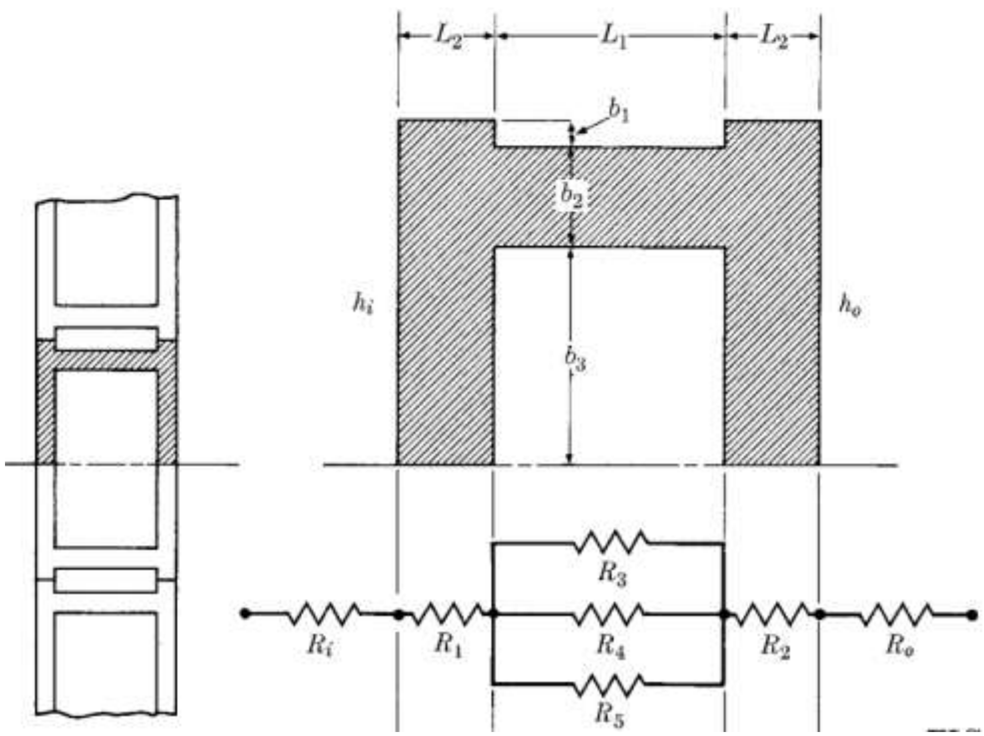


Figure 2.3: A wall composed of hollow concrete blocks.

$\frac{1}{UA} = R_i + \sum_{n=1}^{N} R_n + R_o = R_i + R_1 + \frac{1}{1/R_3+1/R_4+1/R_5} + R_2 + R_o$

Thus we have, per unit width of the wall perpendicular to the cross section shown in Figure 2.3,

$$\frac{1}{h_i(b_1+b_2+b_3)} + \frac{L_2}{k_2(b_1+b_2+b_3)} + \frac{1}{k_1b_1/L_1 + k_2b_2/L_1 + k_1b_3/L_1} + \frac{L_2}{k_2(b_1+b_2+b_3)} + \frac{1}{h_o(b_1+b_2+b_3)},$$

and hence per unit area of the wall

$$\frac{1}{U} = \frac{1}{h_i} + \frac{L_2}{k_2} + \frac{L_1}{\epsilon_1 k_1 + \epsilon_2 k_2} + \frac{L_2}{k_2} + \frac{1}{h_o},$$

where

$$\epsilon_1 = (b_1 + b_3)/(b_1 + b_2 + b_3)$$

and

$$\epsilon_2 = b_2/(b_1 + b_2 + b_3)$$

Q2: The fuel element of a pool reactor is composed of fiat plates of thickness $2L_1$ and cladding material of thickness (L_2 - L_1) bonded to the surfaces of these plates as shown in Figure 2.4. Uniform (nuclear) internal energy $\dot{g}$ is assumed to be generated in the plates only. The heat transfer coefficient is h, the temperature of the coolant T_∞. We need to know the temperature distribution of the fuel element.

Solution:

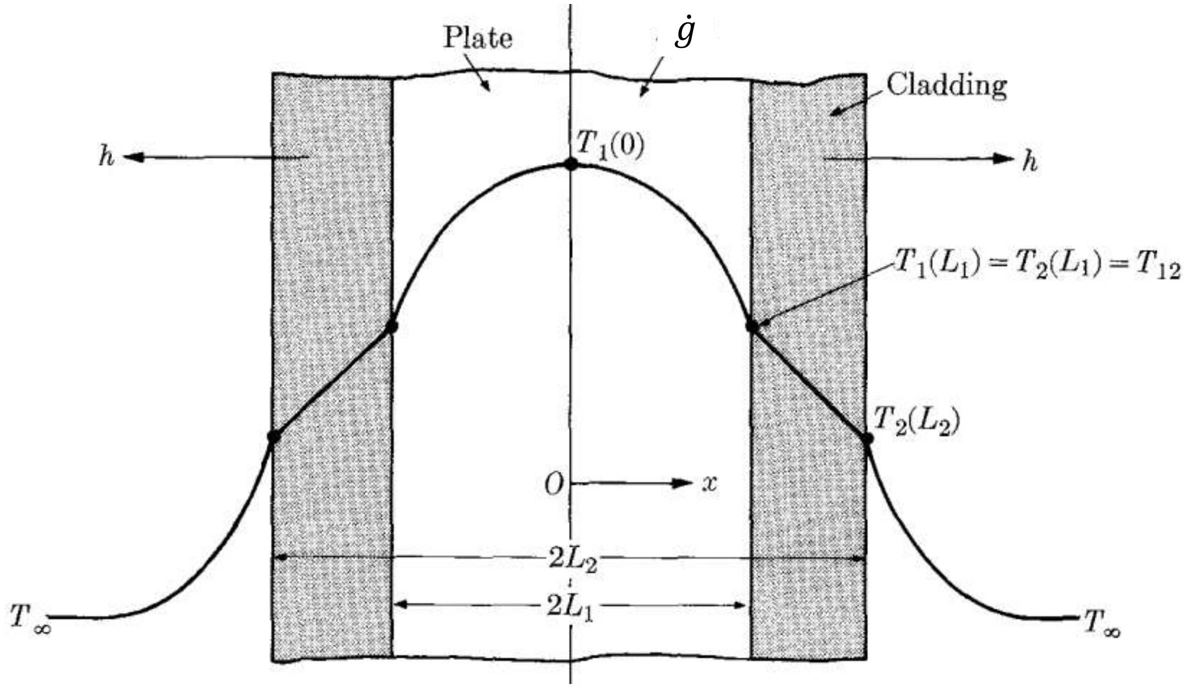
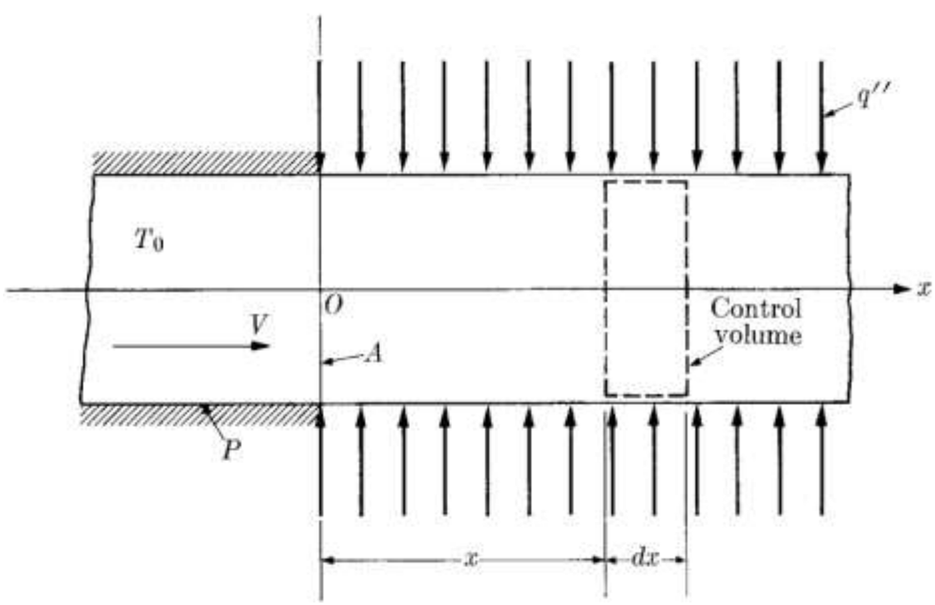


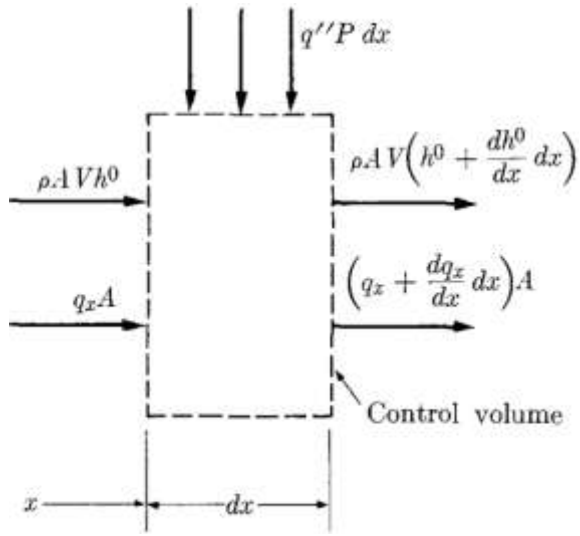
Figure 2.4: The fuel element of a pool reactor.

<u>Q3:</u> A constant-property inviscous liquid having the far upstream temperature T_o and velocity V flows steadily through an infinitely long tube of cross-sectional area A and periphery P. The wall thickness of the tube is negligible. The downstream half of the tube is subjected to the constant heat flux q''; the upstream half is insulated (see Figure 2.5). We wish to know the axial temperature distribution of the liquid, based on a radially lumped analysis.

<u>Solution:</u>



Consider the radially lumped, axially differential control volume shown in Figure 2.5. Let us apply the general laws to this control volume as follows.



Q4: An electric wire of radius R is uniformly insulated with plastic to produce an outer radius R_o (see Figure 2.6). The electrical resistance and thermal conductivity of this wire are ρ_e (*ohms* × *length*) and k_w, respectively. The thermal conductivity of the insulation is k, the heat transfer coefficient h, and the ambient temperature T_∞. We wish to determine the maximum current that this wire can carry without heating the plastic above its allowable operating temperature T_{max}.

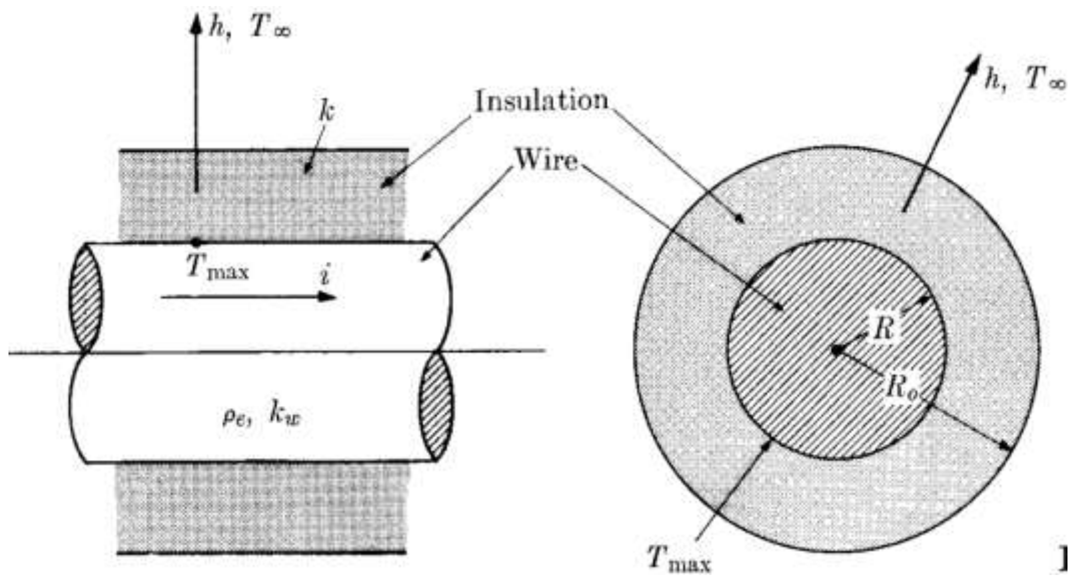


Figure 2.6: An electric wire.

Q5: The fuel element of a reactor consists of a sphere of fissionable material with radius R, surrounded by a spherical shell of cladding with outer radius R_o (see Figure 2.7). The temperature of the coolant is T_∞, and the heat transfer coefficient is h. The nuclear internal energy generated in the sphere can be approximated by a parabola as: $\dot{g}_{(r)} = \dot{g}_o[1 - \left(\frac{r}{R}\right)^2]$, where $\dot{g}_o$ is the nuclear energy generation at the center of the sphere. We wish to know the temperature distribution in the fuel element.

Solution:

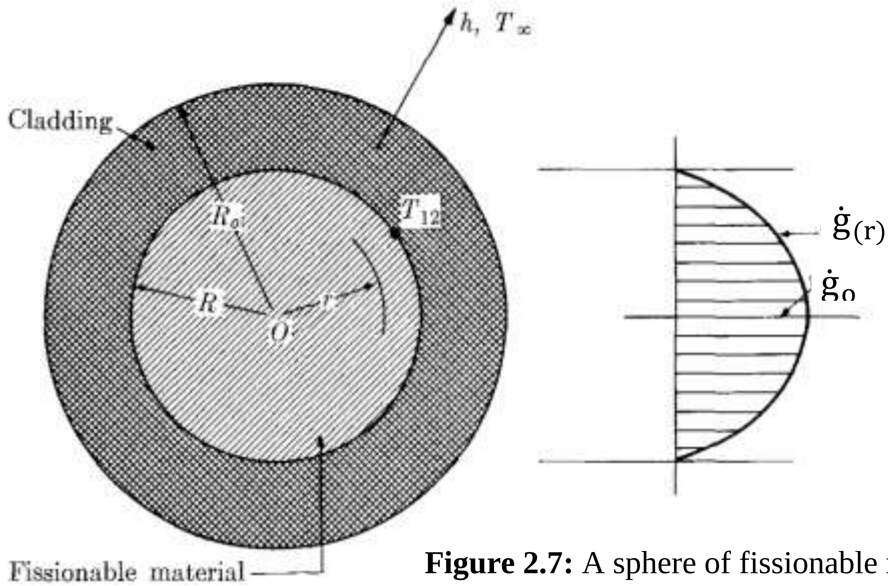


Figure 2.7: A sphere of fissionable material.

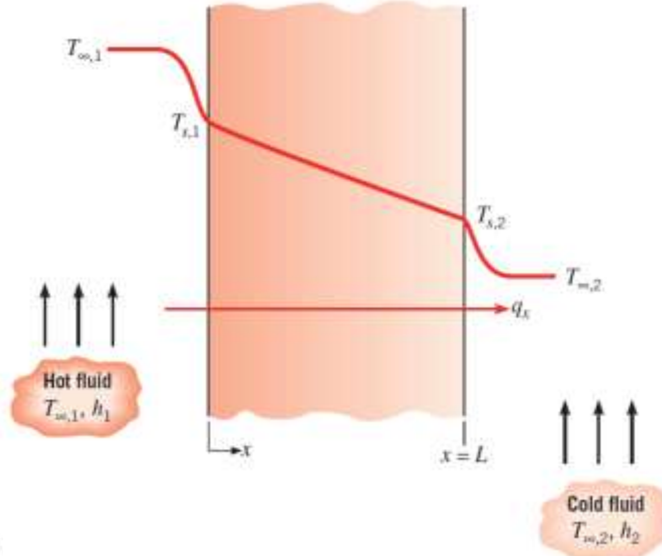
Q6: Derive an expression for the temperature distribution and the heat flow through the walls of the following three geometries for the following four cases (each of cases (**i** & **ii**) should be worked with both cases (**iii** & **iv**)):

Case (i): without heat generation, **Case (ii):** with heat generation ($\dot{g}$)

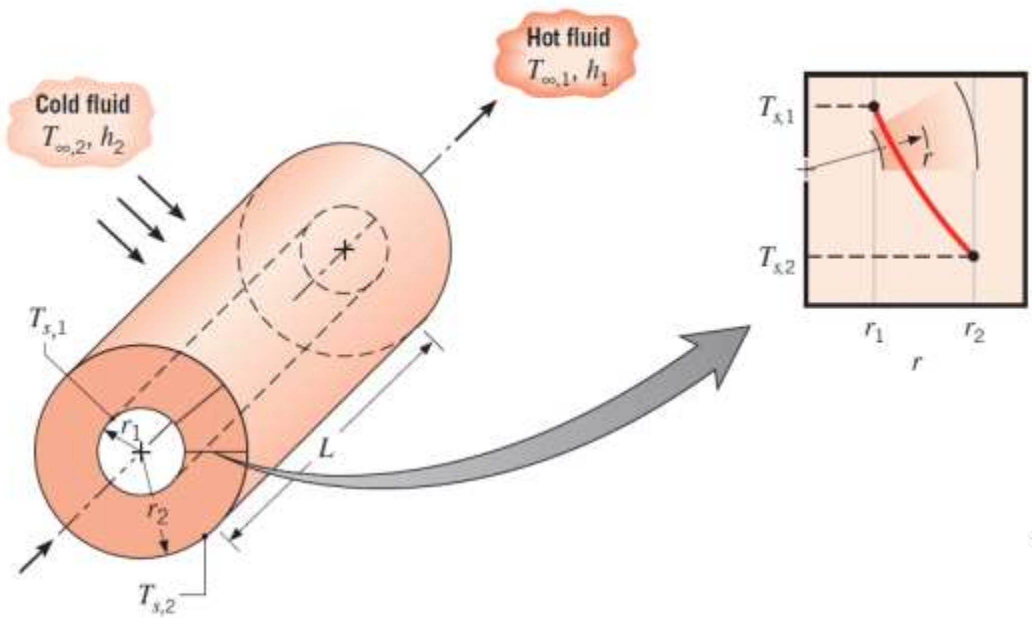
Case (iii): constant thermal conductivity (k).

Case (iv): variable thermal conductivity ($k = k_o(1 + \beta\ T)$), where (k_o) is known conductivity at a reference temperature (T_o) and (β) is the coefficient of thermal conductivity.

Geometry (I) In finite plate wall



Geometry (II) Hollow long cylinder



Geometry (III) Hollow sphere