University of Anbar
College of Engineering
Mechanical Engineering Dept.

Advanced Heat Transfer/ I
Conduction and Radiation

Handout Lectures for MSc. / Power
Chapter Six
Radiation Heat Transfer

Course Tutor

Assist. Prof. Dr. Waleed M. Abed

- J. P. Holman, "*Heat Transfer*", McGraw-Hill Book Company, 6th Edition, 2006.
- T. L. Bergman, A. Lavine, F. Incropera, D. Dewitt, "*Fundamentals of Heat and Mass Transfer*", John Wiley & Sons, Inc., 7th Edition, 2007.
- Vedat S. Arpaci, "*Conduction Heat Transfer*", Addison-Wesley, 1st Edition, 1966.
- P. J. Schneider, "*Conduction Teat Transfer*", Addison-Wesley, 1955.
- D. Q. Kern, A. D. Kraus, "*Extended surface heat transfer*", McGraw-Hill Book Company, 1972.
- G. E. Myers, "*Analytical Methods in Conduction Heat Transfer*", McGraw-Hill Book Company, 1971.
- J. H. Lienhard IV, J. H. Lienhard V, "*A Heat Transfer Textbook*", 4th Edition, Cambridge, MA : J.H. Lienhard V, 2000.

Chapter Six
Radiation Heat Transfer

6.1 Introduction

Heat transfer by conduction and convection requires the presence of a temperature gradient in some form of matter. In contrast, heat transfer by thermal radiation requires no matter. It is an extremely important process, and in the physical sense it is perhaps the most interesting of the heat transfer modes. It is relevant to many industrial heating, cooling, and drying processes, as well as to energy conversion methods that involve fossil fuel combustion and solar radiation.

6.2 Processes and Properties of Radiation

6.2.1 Fundamental Concepts

Consider a solid that is initially at a higher temperature T_s than that of its surroundings T_{sur}, but around which there exists a vacuum (see Figure 6.1). The presence of the vacuum precludes energy loss from the surface of the solid by conduction or convection. However, our intuition tells us that the solid will cool and eventually achieve thermal equilibrium with its surroundings. This cooling is associated with a reduction in the internal energy stored by the solid and is a direct consequence of the *emission* of thermal radiation from the surface. In turn, the surface will intercept and absorb radiation originating from the surroundings. However, if $T_s > T_{sur}$ the net heat transfer rate by radiation $q_{rad,net}$ is from the surface, and the surface will cool until T_s reaches T_{sur}.

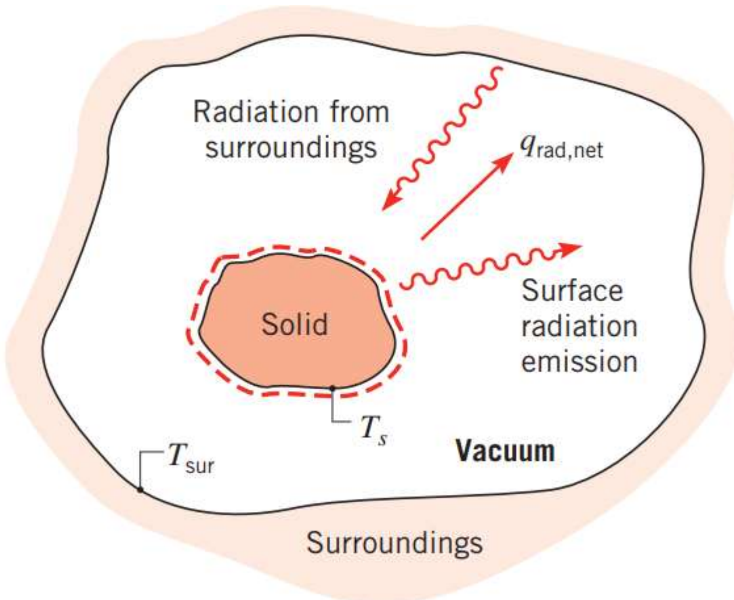


Figure 6.1: Radiation cooling of a hot solid.

We associate thermal radiation with the rate at which energy is emitted by matter as a result of its finite temperature. At this moment thermal radiation is being emitted by all the matter that surrounds you: by the furniture and walls of the room, if you are indoors, or by the ground, buildings, and the atmosphere and sun if you are outdoors. The mechanism of emission is related to energy released as a result of oscillations or transitions of the many electrons that constitute matter. These oscillations are, in turn, sustained by the internal energy, and therefore the temperature, of the matter. Hence we associate the emission of thermal radiation with thermally excited conditions within the matter.

We know that radiation originates due to *emission* by matter and that its subsequent transport does not require the presence of any matter. But *what is the nature of this transport*? One theory views radiation as the propagation of a collection of particles termed *photons* or *quanta*. Alternatively, radiation may be viewed as the propagation of *electromagnetic waves*. In any case we wish to attribute to radiation the standard wave properties of *frequency* ν and *wavelength* λ. For radiation propagating in a particular medium, the two properties are related by

$$\lambda = \frac{c}{\nu} \quad (6\text{-}1)$$

where c is the speed of light in the medium. For propagation in a vacuum, c_o = 2.998×10^8 m/s. The unit of wavelength is commonly the micrometer (μm), where 1 μm 10^{-6} m.

The complete electromagnetic spectrum is delineated in Figure 6.2. The short wavelength *gamma* rays, *X* rays, and ultraviolet (*UV*) radiation are primarily of interest to the high-energy physicist and the nuclear engineer, while the long wavelength microwaves and radio waves ($\lambda > 10^5$ μm) are of concern to the electrical engineer. It is the intermediate portion of the spectrum, which extends from approximately 0.1 to 100 μm and includes a portion of the *UV* and all of the visible and infrared (*IR*), that is termed *thermal radiation* because it is both caused by and affects the thermal state or temperature of matter. For this reason, *thermal radiation* is pertinent to heat transfer.

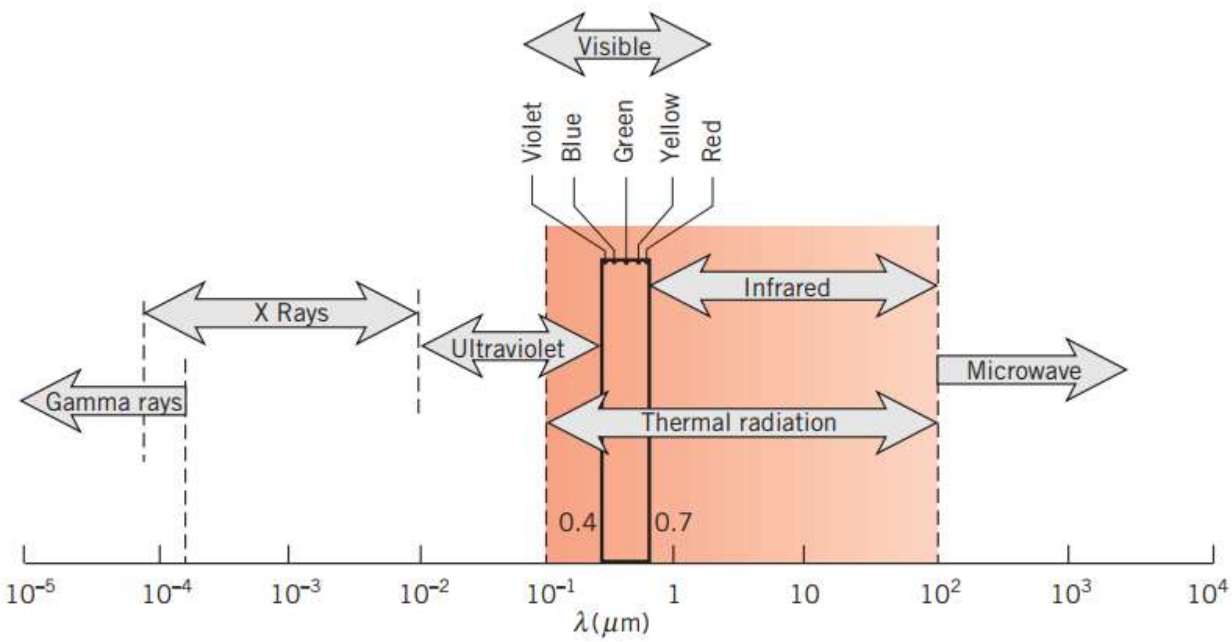


Figure 6.2: Spectrum of electromagnetic radiation.

6.2.2 Radiation Heat Fluxes

Various types of heat fluxes are pertinent to the analysis of radiation heat transfer. Table 6.1 lists four distinct radiation fluxes that can be defined at a surface. The *emissive power*, E (W/m^2), is the rate at which radiation is emitted from a surface per unit surface area, over all wavelengths and in all directions. This *emissive power* was related to the behavior of a *blackbody* through the relation $E = \varepsilon\sigma T_s^4$, where ε is a surface property known as the *emissivity*.

Table 6.1: Radiative fluxes (over all wavelengths and in all directions).

Flux (W/m^2)	Description	Comment
Emissive power, E	Rate at which radiation is emitted from a surface per unit area	$E = \varepsilon\sigma T_s^4$
Irradiation, G	Rate at which radiation is incident upon a surface per unit area	Irradiation can be reflected, absorbed, or transmitted
Radiosity, J	Rate at which radiation leaves a surface per unit area	For an opaque surface $J = E + \rho G$
Net radiative flux, $q''_{rad} = J - G$	Net rate of radiation leaving a surface per unit area	For an opaque surface $q''_{rad} = \varepsilon\sigma T_s^4 - \alpha G$

Radiation from the surroundings, which may consist of multiple surfaces at various temperatures, is incident upon the surface. The surface might also be irradiated by the sun or by a laser. In any case, we define the *irradiation*, G (W/m^2), as the rate at which radiation is incident upon the surface per unit surface area, over all wavelengths and from all directions. The two remaining heat fluxes of Table 6.1 are readily described once we consider the fate of the irradiation arriving at the surface. When radiation is incident upon a semitransparent medium, portions of the irradiation may be *reflected*, *absorbed*, and *transmitted* illustrated in Figure 6.3.a. Transmission, τ, refers to radiation passing through the medium, as occurs when a layer of water or a glass plate is irradiated by the sun or artificial lighting.

Absorption occurs when radiation interacts with the medium, causing an increase in the internal thermal energy of the medium. Reflection is the process of incident radiation being redirected away from the surface, with no effect on the medium. We define reflectivity, ρ, as the fraction of the irradiation that is reflected, absorptivity, α, as the fraction of the irradiation that is absorbed, and transmissivity, τ, as the fraction of the irradiation that is transmitted. Because all of the *irradiation* must be *reflected*, *absorbed*, or *transmitted*, it follows that

$$\rho + \alpha + \tau = 1 \qquad (6\text{-}2)$$

A medium that experiences no transmission ($\tau = 0$) is opaque, in which case

$$\rho + \alpha = 1 \qquad (6\text{-}3)$$

With this understanding of the partitioning of the irradiation into reflected, absorbed, and transmitted components, two additional and useful radiation fluxes can be defined. The *radiosity*, J (W/m^2), of a surface accounts for all the radiant energy leaving the surface. For an opaque surface, it includes emission and the reflected portion of the irradiation, as illustrated in Figure 6.3.b. It is therefore expressed as

$$J = E + G_{ref} = E + \rho G \qquad (6\text{-}4)$$

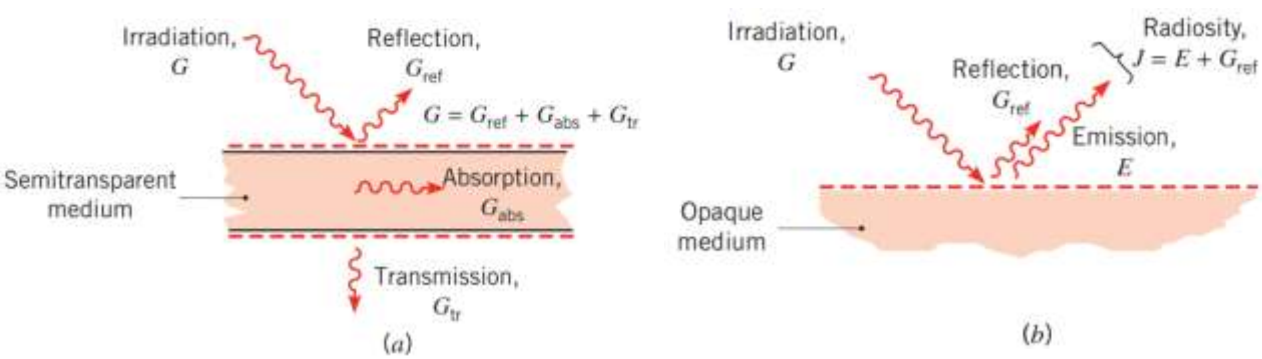


Figure 6.3: Radiation at a surface. (**a**) Reflection, absorption, and transmission of irradiation for a semitransparent medium. (**b**) The radiosity for an opaque medium.

Radiosity can also be defined at a surface of a semitransparent medium. In that case, the radiosity leaving the top surface of Figure 6.3.a (not shown) would include radiation transmitted through the medium from below.

Finally, the *net radiative flux from a surface*, (W/m^2), is the difference between the outgoing and incoming radiation

$$q''_{rad} = J - G \quad (6\text{-}5)$$

Combining Equations 6.5, 6.4, and 6.3, the *net flux* for an opaque surface is

$$q''_{rad} = E + \rho G - G = \varepsilon\sigma T_s^4 - \alpha G \quad (6\text{-}6)$$

A similar expression may be written for a semitransparent surface involving the transmissivity. Because it affects the temperature distribution within the system, the *net radiative flux* (or net radiation heat transfer rate, $q'' = q''_{rad}\, A$), is an important quantity in heat transfer analysis. As will become evident, the quantities *E*, *G*, and *J* are typically used to determine q''_{rad}, but they are also intrinsically important in applications involving *radiation detection* and *temperature measurement*.

6.2.3 Blackbody Radiation

To evaluate the *emissive power*, *irradiation*, *radiosity*, or *net radiative heat flux* of a real opaque surface, to do so, it is useful to first introduce the concept of a *blackbody*.

- ✓ A *blackbody* absorbs all incident radiation, regardless of wavelength and direction.
- ✓ For a prescribed temperature and wavelength, no surface can emit more energy than a blackbody.
- ✓ Although the radiation emitted by a blackbody is a function of wavelength and temperature, it is independent of direction. That is, the blackbody is a diffuse emitter.

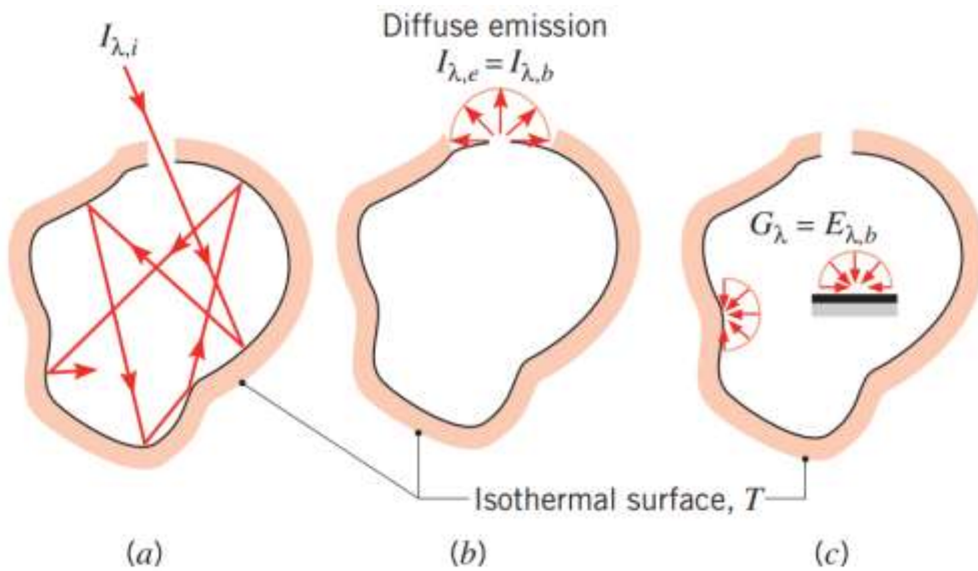


Figure 6.4: Characteristics of an isothermal blackbody cavity. (**a**) Complete absorption. (**b**) Diffuse emission from an aperture. (**c**) Diffuse irradiation of interior surfaces.

As the perfect absorber and emitter, the *blackbody* serves as a standard against which the *radiative properties* of actual surfaces may be compared.

Although closely approximated by some surfaces, it is important to note that no surface has precisely the properties of a blackbody. The closest approximation is achieved by a cavity whose inner surface is at a uniform temperature. If radiation enters the cavity through a small aperture (Figure 6.4.a), it is likely to experience many reflections before reemergence. Since some radiation is absorbed by the inner surface upon each reflection, it is eventually almost entirely absorbed by the cavity, and blackbody behavior is approximated. From thermodynamic principles it may then be argued that radiation leaving the aperture depends only on the surface temperature and corresponds to blackbody emission (Figure 6.4.b). Since blackbody emission is diffuse, the spectral intensity $I_{\lambda,b}$ of radiation leaving the cavity is independent of direction. Moreover, since the radiation field in the cavity, which is the cumulative effect of emission and reflection from the cavity surface, must be of the same form as the radiation emerging from the aperture, it also follows that a blackbody radiation field exists within the cavity. Accordingly, any small surface in the cavity (Figure 6.4.c) experiences irradiation for which $G_\lambda = E_{\lambda,b}(\lambda, T)$. This surface is *diffusely irradiated*, regardless of its orientation. *Blackbody radiation exists within the cavity irrespective of whether the cavity surface is highly reflecting or absorbing*.

6.3 Radiation Exchange between Surfaces

6.3.1 The View Factor

A. The View Factor Integral

The view factor F_{ij} is defined as the *fraction of the radiation leaving surface i that is intercepted by surface j*. To develop a general expression for F_{ij}, we consider the arbitrarily oriented surfaces A_i and A_j of Figure 6.5. Elemental areas on each surface, dA_i and dA_j, are connected by a line of length R, which forms the polar angles θ_i and θ_j, respectively, with the surface normals $\mathbf{n}_i$ and $\mathbf{n}_j$. The values of R, θ_i, and θ_j vary with the position of the elemental areas on A_i and A_j.

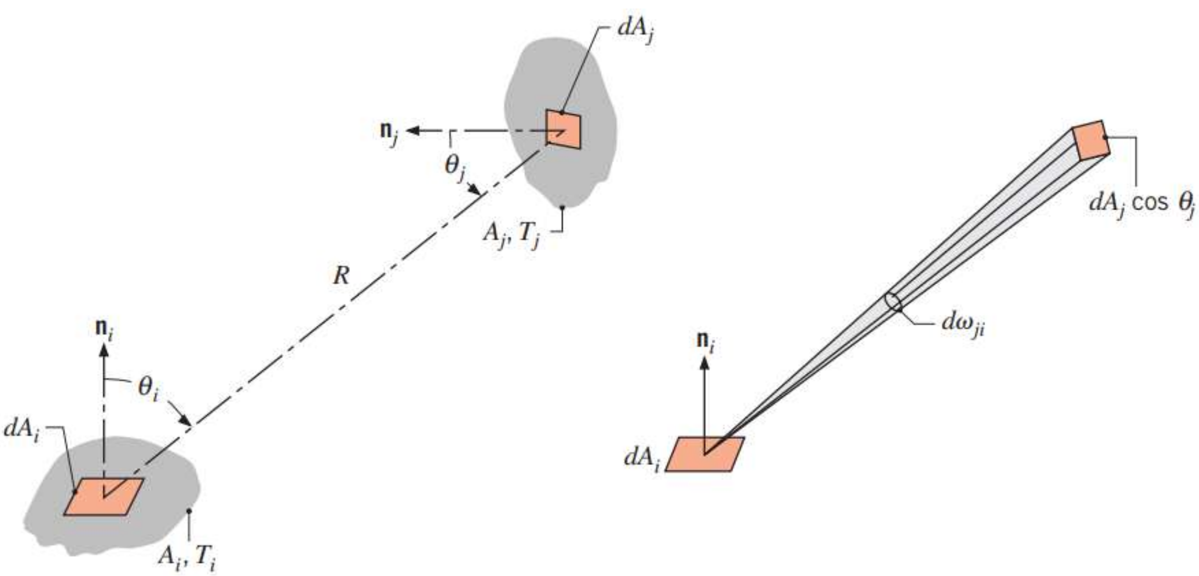


Figure 6.5: View factor associated with radiation exchange between elemental surfaces of area dA_i and dA_j.

From the definition of the *radiation intensity*, the rate at which radiation *leaves* dA_i and is *intercepted* by dA_j may be expressed as

$$dq_{i\rightarrow j} = I_{e+r,i} \cos\theta_i \, dA_i d\omega_{j-i} \quad (6\text{-}7)$$

where $I_{e+r,i}$ is the intensity of radiation leaving surface *i* by emission and reflection and $d\omega_{j-i}$ is the solid angle subtended by dA_j when viewed from dA_i. With $d\omega_{j-i} = (\cos\theta_j \, dA_j)/R^2$, it follows that

$$dq_{i\rightarrow j} = I_{e+r,i} \frac{\cos\theta_i \cos\theta_j}{R^2} dA_i dA_j \quad (6\text{-}8)$$

Assuming that surface *i* emits and reflects diffusely, we then obtain

$$dq_{i\rightarrow j} = J_i \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i dA_j \quad (6\text{-}9)$$

The total rate at which radiation leaves surface *i* and is intercepted by *j* may then be obtained by integrating over the two surfaces. That is,

$$dq_{i\rightarrow j} = J_i \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i dA_j \quad (6\text{-}10)$$

where it is assumed that the *radiosity* J_i is uniform over the surface A_i. From the definition of the view factor as the fraction of the radiation that leaves A_i and is intercepted by A_j,

$$F_{ij} = \frac{q_{i\rightarrow j}}{A_i J_i} \quad (6\text{-}11)$$

it follows that

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i dA_j \quad (6\text{-}12)$$

Similarly, the view factor F_{ji} is defined as the fraction of the radiation that leaves A_j and is intercepted by A_i. The same development then yields

$$F_{ji} = \frac{1}{A_j} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i dA_j \quad (6\text{-}13)$$

Either Equation 6.12 or 6.13 may be used to determine the *view factor* associated with any two surfaces that are *diffuse emitters and reflectors* and have *uniform radiosity*.

B. Relations between Shape Factors

i. Reciprocity relation

An important view factor relation is suggested by Equations 6.12 and 6.13. In particular, equating the integrals appearing in these equations, it follows that

$$A_i F_{ij} = A_j F_{ji} \tag{6-14}$$

This expression, termed the *reciprocity relation*, is useful in determining one view factor from knowledge of the other.

ii. Summation rule

Another important view factor relation pertains to the surfaces of an enclosure (Figure 6.6). From the definition of the view factor, the *summation rule*

$$\sum_{j=1}^{N} F_{ij} = 1 \tag{6-15}$$

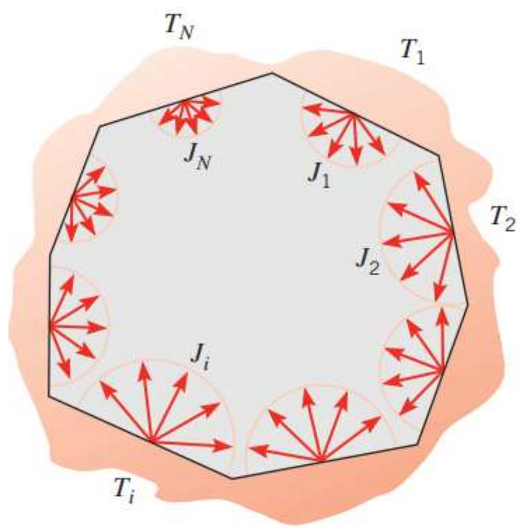
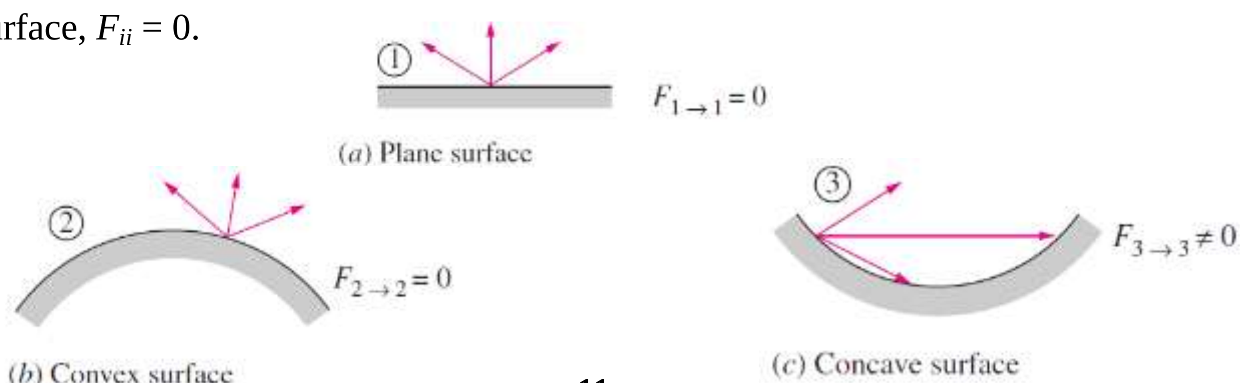


Figure 6.6: Radiation exchange in an enclosure.

may be applied to each of the *N* surfaces in the enclosure. This rule follows from the conservation requirement that all radiation leaving surface *i* must be intercepted by the enclosure surfaces. The term F_{ii} appearing in this summation represents the fraction of the radiation that leaves surface *i* and is directly intercepted by *i*. If the surface is concave, it sees itself and F_{ii} is *nonzero*. However, for a plane or convex surface, F_{ii} = 0.



A total of N view factors may be obtained from the N equations associated with application of the *summation rule*, Equation 6.15, to each of the surfaces in the enclosure. In addition, $N(N-1)/2$ view factors may be obtained from the $N(N-1)/2$ applications of the reciprocity relation, Equation 6.14, which are possible for the enclosure. Accordingly, only $[N^2-N-N(N-1)/2]=N(N-1)/2$ view factors need be determined directly. *For example*, in a three-surface enclosure this requirement corresponds to only 3(3-1)/2= 3 view factors. The remaining *six* view factors may be obtained by solving the *six* equations that result from use of Equations 6.14 and 6.15.

To illustrate the foregoing procedure, consider a simple, two-surface enclosure involving the spherical surfaces of Figure 6.7. Although the enclosure is characterized by N_2=4 view factors (F_{11}, F_{12}, F_{21}, F_{22}), only $N(N-1)/2=1$ view factor need be determined directly. In this case such a determination may be made by *inspection*. In particular, since all radiation leaving the inner surface must reach the outer surface, it follows that $F_{12}=1$. The same may not be said of radiation leaving the outer surface, since this surface sees itself. However, from the *reciprocity relation*, Equation 6.14, we obtain

$$F_{21} = \left(\frac{A_1}{A_2}\right) F_{12} = \left(\frac{A_1}{A_2}\right) \quad (6\text{-}16)$$

From the summation rule, we also obtain

$$F_{11} + F_{12} = 1$$

in which case F_{11}= 0, and i

$$F_{21} + F_{22} = 1$$

in which case

$$F_{22} = 1 - \left(\frac{A_1}{A_2}\right)$$

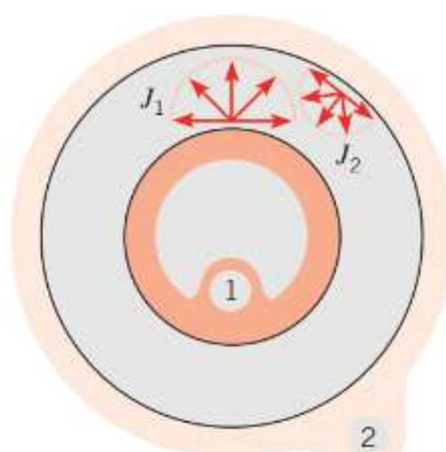
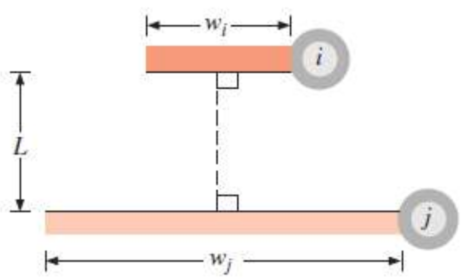
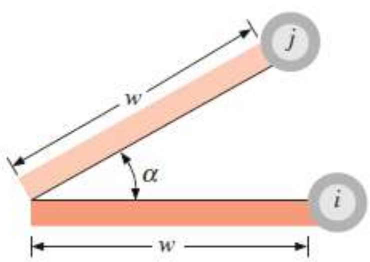
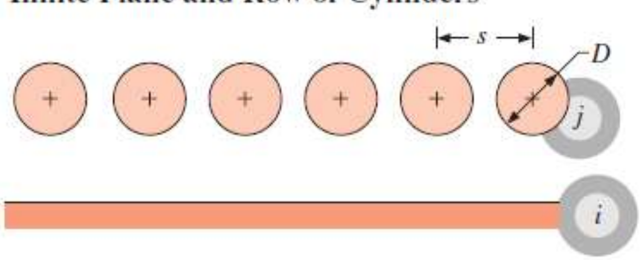


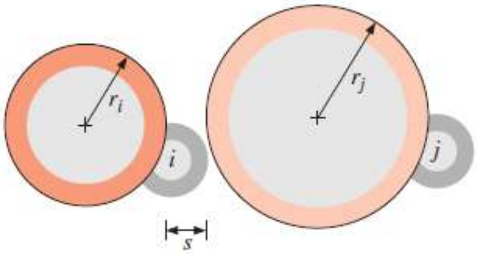
Figure 6.7: View factors for the enclosure formed by two spheres.

The view factors for several common geometries are presented in Tables 6.1 and 6.2. The configurations of Table 6.1 are assumed to be infinitely long (in a direction perpendicular to the page) and are hence *two-dimensional*. The configurations of Table 6.2 are three-dimensional.

Table 6.1: View Factors for *Two-Dimensional* Geometries.

Geometry	Relation
Parallel Plates with Midlines Connected by Perpendicular 	$F_{ij} = \dfrac{[(W_i + W_j)^2 + 4]^{1/2} - [(W_j - W_i)^2 + 4]^{1/2}}{2W_i}$ $W_i = w_i/L,\ W_j = w_j/L$
Inclined Parallel Plates of Equal Width and a Common Edge 	$F_{ij} = 1 - \sin\left(\dfrac{\alpha}{2}\right)$
Innite Plane and Row of Cylinders 	$F_{ij} = 1 - \left[1 - \left(\dfrac{D}{s}\right)^2\right]^{1/2} + \left(\dfrac{D}{s}\right)\tan^{-1}\left[\left(\dfrac{s^2 - D^2}{D^2}\right)^{1/2}\right]$

Parallel Cylinders of Different Radii

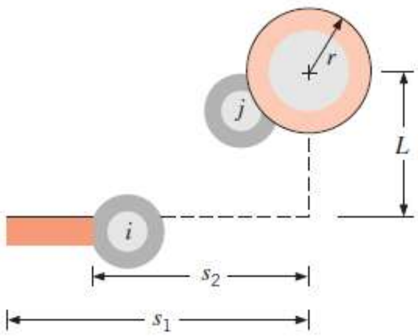


$$F_{ij} = \frac{1}{2\pi}\left\{\pi + [C^2 - (R+1)^2]^{1/2} - [C^2 - (R-1)^2]^{1/2} + (R-1)\cos^{-1}\left[\left(\frac{R}{C}\right) - \left(\frac{1}{C}\right)\right] - (R+1)\cos^{-1}\left[\left(\frac{R}{C}\right) + \left(\frac{1}{C}\right)\right]\right\}$$

$R = r_j/r_i,\ S = s/r_i$

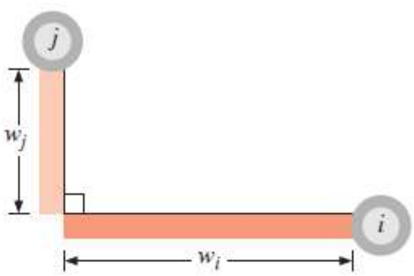
$C = 1 + R + S$

Cylinder and Parallel Rectangle



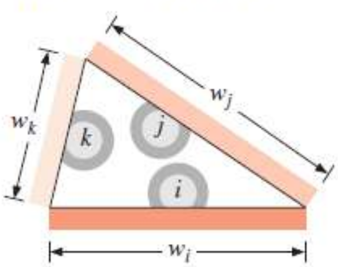
$$F_{ij} = \frac{r}{s_1 - s_2}\left[\tan^{-1}\frac{s_1}{L} - \tan^{-1}\frac{s_2}{L}\right]$$

Perpendicular Plates with a Common Edge



$$F_{ij} = \frac{1 + (w_j/w_i) - [1 + (w_j/w_i)^2]^{1/2}}{2}$$

Three-Sided Enclosure

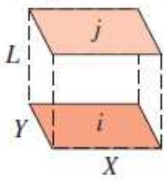


$$F_{ij} = \frac{w_i + w_j - w_k}{2w_i}$$

Geometry | **Relation**

Aligned Parallel Rectangles

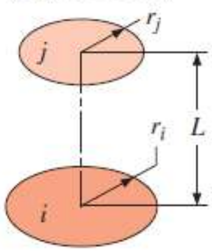
(Figure 13.4)



$$\overline{X} = X/L,\ \overline{Y} = Y/L$$

$$F_{ij} = \frac{2}{\pi \overline{X}\,\overline{Y}} \left\{ \ln \left[\frac{(1+\overline{X}^2)(1+\overline{Y}^2)}{1+\overline{X}^2+\overline{Y}^2} \right]^{1/2} + \overline{X}(1+\overline{Y}^2)^{1/2} \tan^{-1} \frac{\overline{X}}{(1+\overline{Y}^2)^{1/2}} + \overline{Y}(1+\overline{X}^2)^{1/2} \tan^{-1} \frac{\overline{Y}}{(1+\overline{X}^2)^{1/2}} - \overline{X}\tan^{-1}\overline{X} - \overline{Y}\tan^{-1}\overline{Y} \right\}$$

Coaxial Parallel Disks
(Figure 13.5)

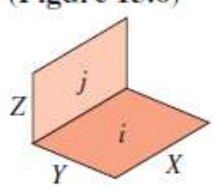


$$R_i = r_i/L,\ R_j = r_j/L$$

$$S = 1 + \frac{1+R_j^2}{R_i^2}$$

$$F_{ij} = \frac{1}{2}\{S - [S^2 - 4(r_j/r_i)^2]^{1/2}\}$$

Perpendicular Rectangles with a Common Edge
(Figure 13.6)



$$H = Z/X,\ W = Y/X$$

$$F_{ij} = \frac{1}{\pi W}\left(W \tan^{-1}\frac{1}{W} + H \tan^{-1}\frac{1}{H} - (H^2+W^2)^{1/2}\tan^{-1}\frac{1}{(H^2+W^2)^{1/2}} + \frac{1}{4}\ln\left\{ \frac{(1+W^2)(1+H^2)}{1+W^2+H^2} \left[\frac{W^2(1+W^2+H^2)}{(1+W^2)(W^2+H^2)} \right]^{W^2} \times \left[\frac{H^2(1+H^2+W^2)}{(1+H^2)(H^2+W^2)} \right]^{H^2} \right\} \right)$$

The view factors for several common geometries are presented in Figures 6.8 through 6.11.

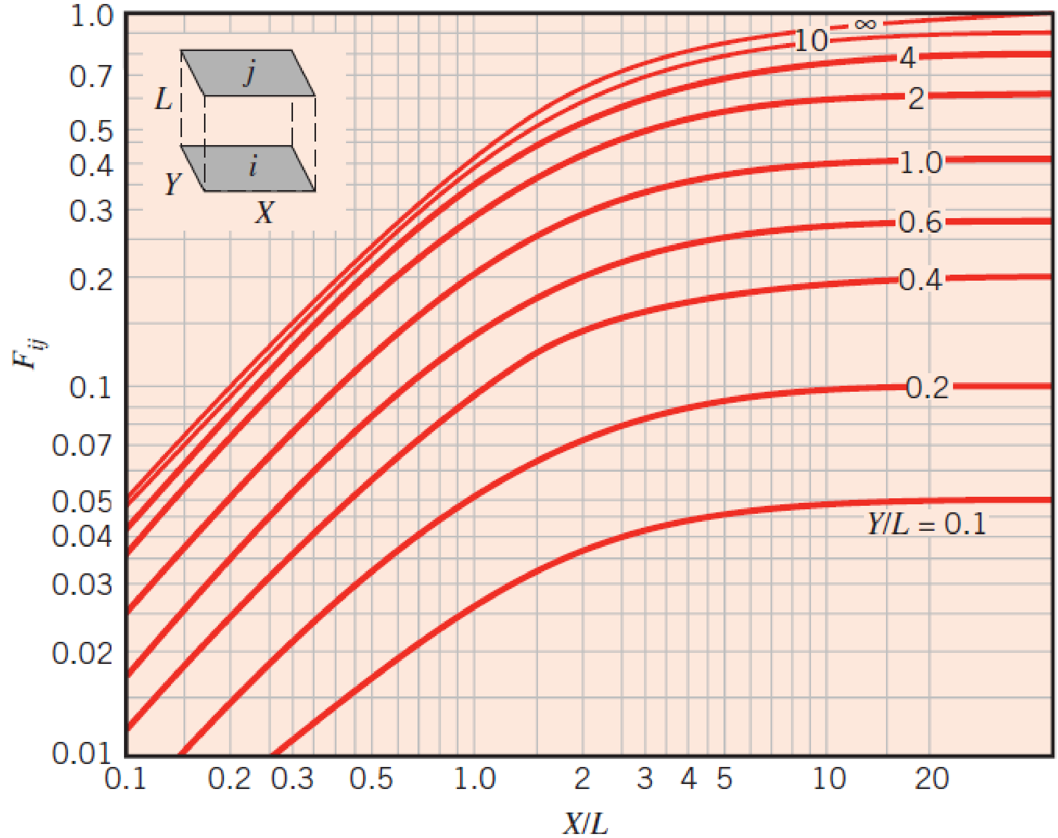


Figure 6.8: View factor for aligned parallel rectangles.

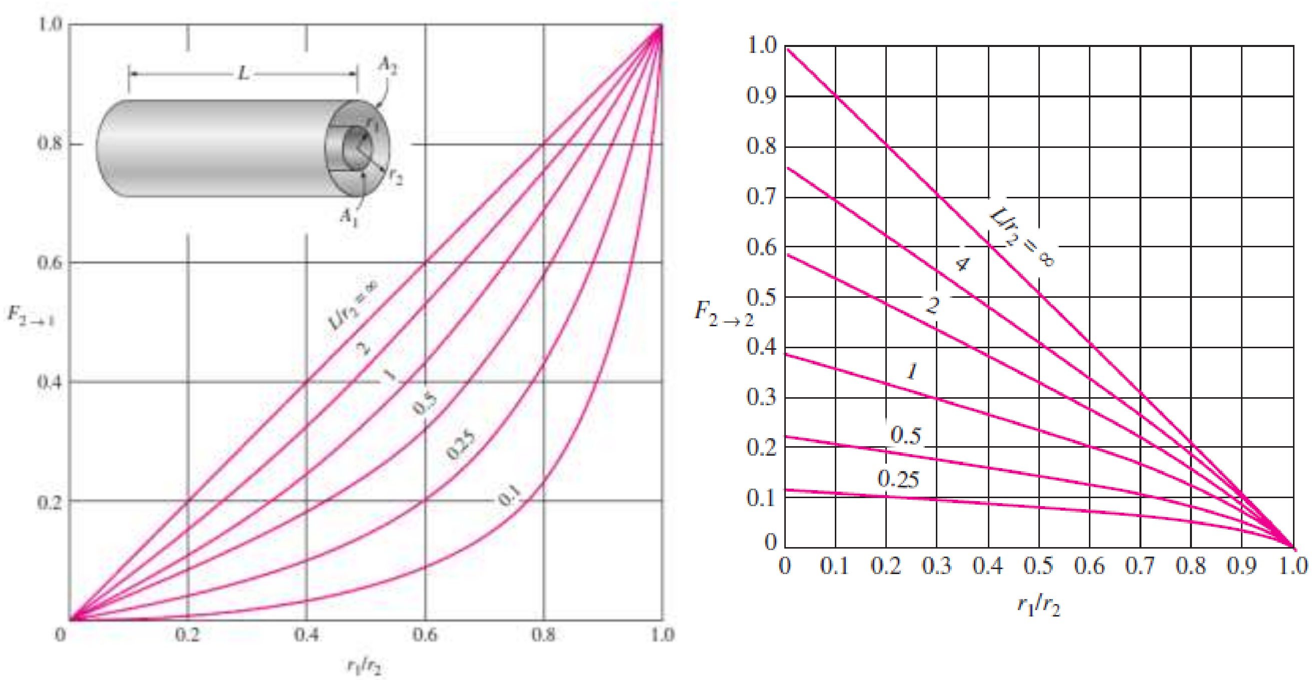


Figure 6.9: View factors for two concentric cylinders of finite length: (a) outer cylinder to inner cylinder; (b) outer cylinder to itself.

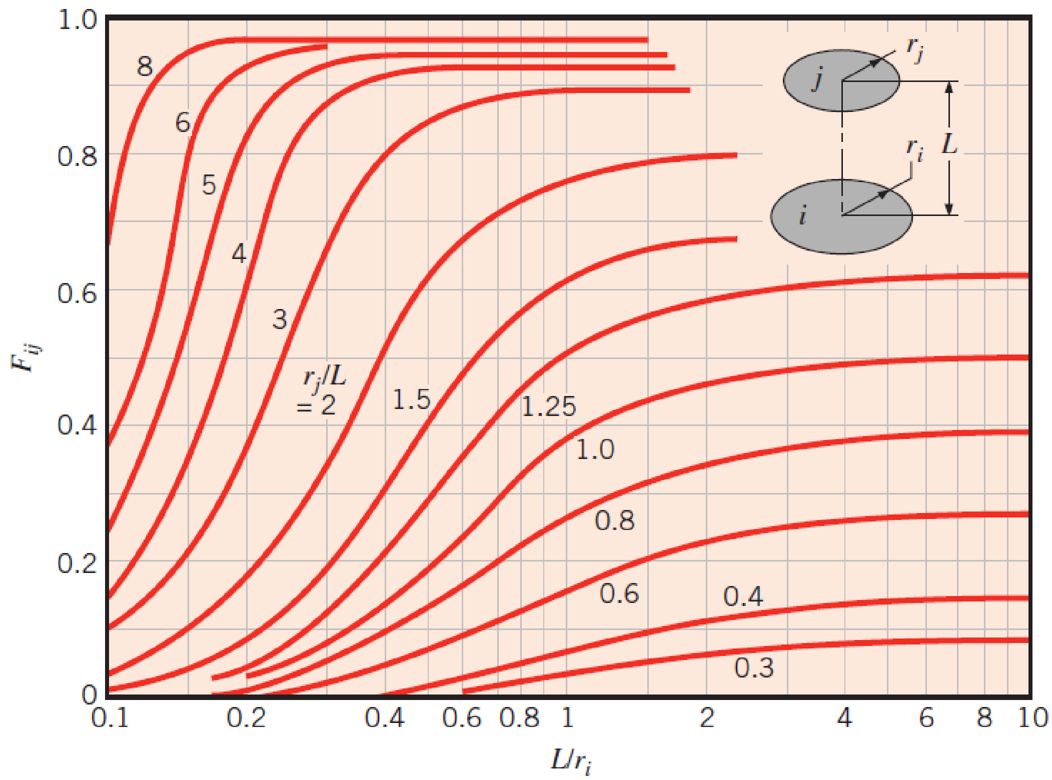


Figure 6.10: View factor for coaxial parallel disks.

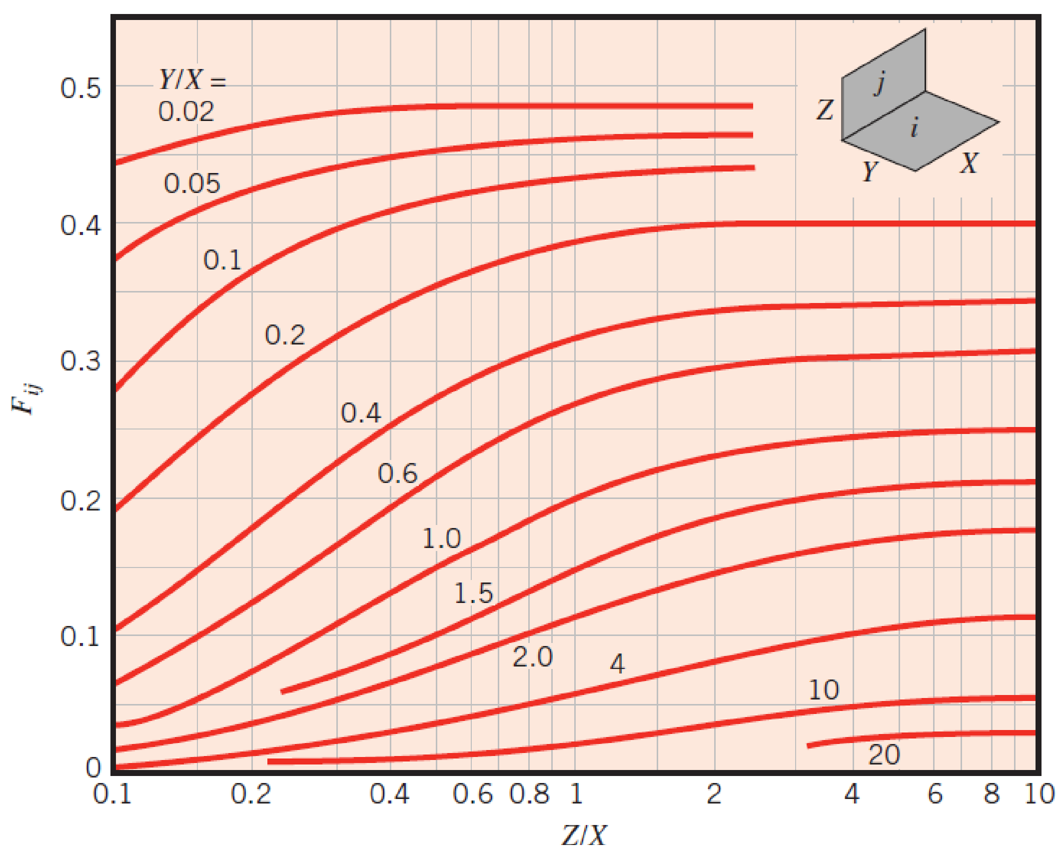


Figure 6.11: View factor for perpendicular rectangles with a common edge.

iii. The Superposition Rule

The first relation concerns the additive nature of the view factor for a subdivided surface and may be inferred from Figure 6.12. Considering radiation from surface *i* to surface *j*, which is divided into *n* components, it is evident that

$$F_{i(j)} = \sum_{k=1}^{n} F_{ik} \tag{6-17}$$

where the parentheses around a subscript indicate that it is a composite surface, in which case (*j*) is equivalent to (1, 2, . . . , *k*, . . . , *n*). This expression simply states that radiation reaching a composite surface is the sum of the radiation reaching its parts. Although it pertains to subdivision of the receiving surface, it may also be used to obtain the second view factor relation, which pertains to subdivision of the originating surface. Multiplying Equation 6.17 by A_i and applying the reciprocity relation, Equation 6.14, to each of the resulting terms, it follows that

$$A_j F_{(j)i} = \sum_{k=1}^{n} A_k F_{ki} \tag{6-18}$$

$$F_{(j)i} = \frac{\sum_{k=1}^{n} A_k F_{ki}}{\sum_{k=1}^{n} A_k} \tag{6-19}$$

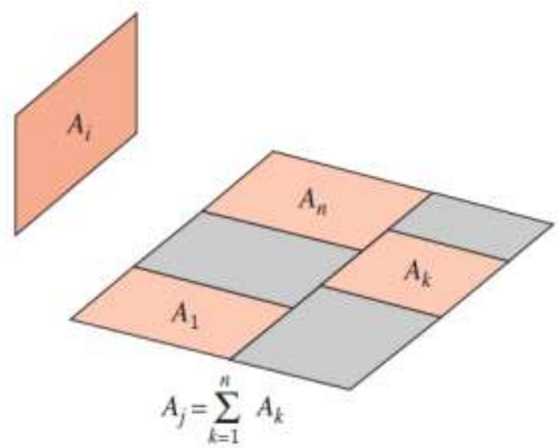
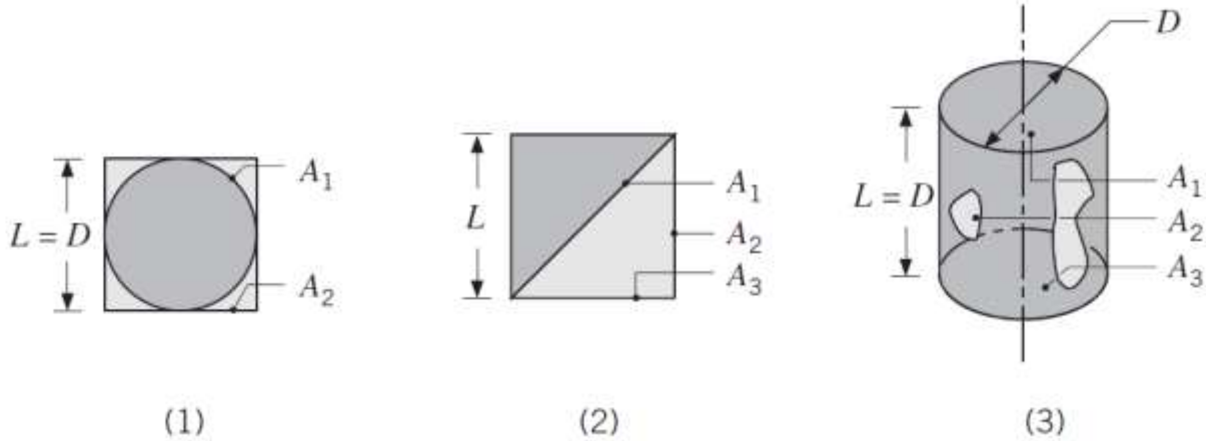


Figure 6.12: Areas used to illustrate view factor relations.

Equations 6.18 and 6.19 may be applied when the originating surface is composed of several parts.

Example 1: Determine the view factors F_{12} and F_{21} for the following geometries:



1. Sphere of diameter D inside a cubical box of length $L = D$.
2. One side of a diagonal partition within a long square duct.
3. End and side of a circular tube of equal length and diameter.

Solution:

1. Sphere within a cube:
   By inspection, $F_{12} = 1$
   By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\pi D^2}{6L^2} \times 1 = \frac{\pi}{6}$
2. Partition within a square duct:
   From summation rule, $F_{11} + F_{12} + F_{13} = 1$
   where $F_{11} = 0$
   By symmetry, $F_{12} = F_{13}$
   Hence $F_{12} = 0.50$
   By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\sqrt{2}L}{L} \times 0.5 = 0.71$
3. Circular tube:
   From Table 13.2 or Figure 13.5, with $(r_3/L) = 0.5$ and $(L/r_1) = 2$, $F_{13} = 0.172$
   From summation rule, $F_{11} + F_{12} + F_{13} = 1$
   or, with $F_{11} = 0$, $F_{12} = 1 - F_{13} = 0.828$
   From reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\pi D^2/4}{\pi DL} \times 0.828 = 0.207$

Example 2: Determine the view factors from the base of the pyramid shown in Figure 6.13 to each of its four side surfaces. The base of the pyramid is a square, and its side surfaces are isosceles triangles.

Solution: The base of the pyramid (surface 1) and its four side surfaces (surfaces 2, 3, 4, and 5) form a five-surface enclosure. The first thing we notice about this enclosure is its symmetry. The four side surfaces are symmetric about the base surface. Then, from the *symmetry rule*, we have

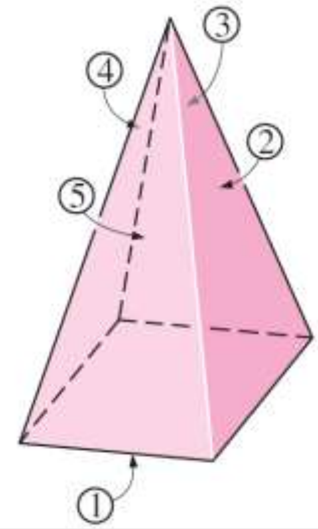


Figure 6.13: The pyramid considered in Example 2.

$$F_{12} = F_{13} = F_{14} = F_{15}$$

Also, the *summation rule* applied to surface 1 yields

$$\sum_{j=1}^{5} F_{1j} = F_{11} + F_{12} + F_{13} + F_{14} + F_{15} = 1$$

However, $F_{11} = 0$, since the base is a *flat* surface. Then the two relations above yield

$$F_{12} = F_{13} = F_{14} = F_{15} = \mathbf{0.25}$$

Example 3: Determine the view factor from any one side to any other side of the infinitely long triangular duct whose cross section is given in Figure 6.14.

Solution: The view factors associated with an infinitely long triangular duct are to be determined.

The widths of the sides of the triangular cross section of the duct are L_1, L_2, and L_3, and the surface areas corresponding to them are A_1, A_2, and A_3, respectively. Since the duct is infinitely long, the fraction of radiation leaving any surface that escapes through the ends of the duct is negligible. Therefore, the infinitely long duct can be considered to be a three-surface enclosure, N= 3.

This enclosure involves $N^2 = 3^2 = 9$ view factors, and we need to determine

$$\tfrac{1}{2}N(N-1) = \tfrac{1}{2} \times 3(3-1) = 3$$

of these view factors directly. Fortunately, we can determine all three of them by inspection to be

$$F_{11} = F_{22} = F_{33} = 0$$

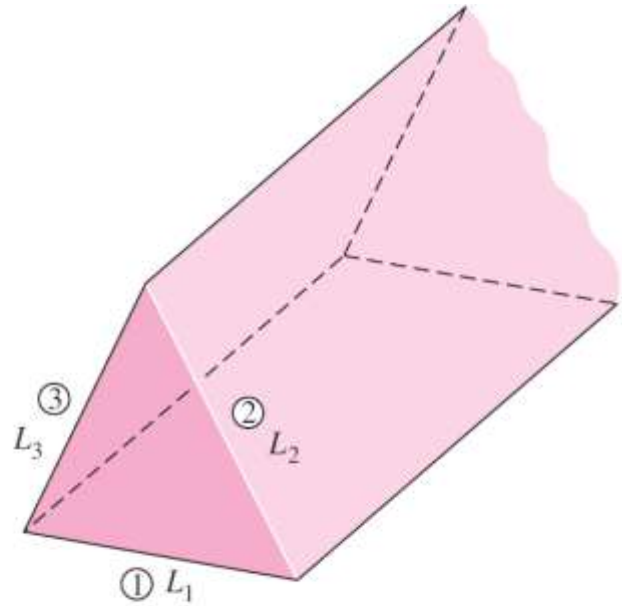


Figure 6.14: The infinitely long triangular duct considered in Example 3.

since all three surfaces are flat. The remaining six view factors can be determined by the application of the summation and reciprocity rules. Applying the summation rule to each of the three surfaces gives

$$F_{11} + F_{12} + F_{13} = 1$$
$$F_{21} + F_{22} + F_{23} = 1$$
$$F_{31} + F_{32} + F_{33} = 1$$

Noting that $F_{11} = F_{22} = F_{33} = 0$ and multiplying the first equation by A_1, the second by A_2, and the third by A_3 gives

$$A_1F_{12} + A_1F_{13} = A_1$$
$$A_2F_{21} + A_2F_{23} = A_2$$
$$A_3F_{31} + A_3F_{32} = A_3$$

Finally, applying the three reciprocity relations $A_1F_{12} = A_2F_{21}$, $A_1F_{13} = A_3F_{31}$, and $A_2F_{23} = A_3F_{32}$ gives

$$A_1F_{12} + A_1F_{13} = A_1$$
$$A_1F_{12} + A_2F_{23} = A_2$$
$$A_1F_{13} + A_2F_{23} = A_3$$

This is a set of three algebraic equations with three unknowns, which can be solved to obtain

$$F_{12} = \frac{A_1 + A_2 - A_3}{2A_1} = \frac{L_1 + L_2 - L_3}{2L_1}$$

$$F_{13} = \frac{A_1 + A_3 - A_2}{2A_1} = \frac{L_1 + L_3 - L_2}{2L_1}$$

$$F_{23} = \frac{A_2 + A_3 - A_1}{2A_2} = \frac{L_2 + L_3 - L_1}{2L_2}$$

6.3.2 Blackbody Radiation Exchange

In general, radiation may leave a surface due to both reflection and emission, and on reaching a second surface, experience reflection as well as absorption. However, matters are simplified for surfaces that may be approximated as blackbodies, since there is no reflection. Hence energy leaves only as a result of emission, and all incident radiation is absorbed. Consider radiation exchange between two black surfaces of arbitrary shape (Figure 6.15). Defining $q_{i\text{-}j}$ as the rate at which radiation *leaves* surface *i* and is *intercepted* by surface *j*, it follows that

$$q_{i\to j} = (A_i J_i)\, F_{i\to j} \qquad (6\text{-}20)$$

or, since *radiosity* equals *emissive power* for a black surface ($J_i = E_{bi}$),

$$q_{i\to j} = A_i E_{bi}\, F_{i\to j} \qquad (6\text{-}21)$$

Similarly,

$$q_{j\to i} = A_j E_{bj}\, F_{j\to i} \qquad (6\text{-}22)$$

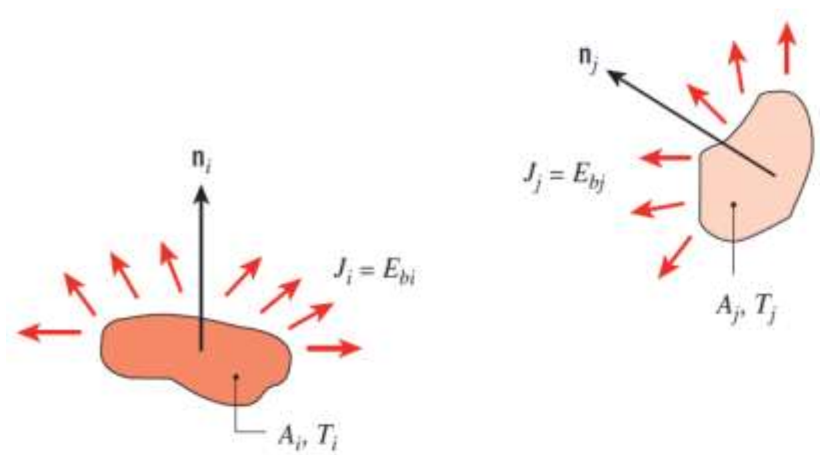


Figure 6.15: Radiation transfer between two surfaces that may be approximated as blackbodies.

The *net radiative exchange* between the two surfaces may then be defined as

$$q_{ij} = q_{i\to j} - q_{j\to i} \tag{6-23}$$

from which it follows that

$$q_{ij} = A_i E_{bi} F_{i\to j} - A_j E_{bj} F_{j\to i} \tag{6-24}$$

$$q_{ij} = A_i F_{i\to j}\sigma\left(T_i^4 - T_j^4\right) \tag{6-25}$$

Equation 6.25 provides the *net* rate at which radiation *leaves* surface *i* as a result of its interaction with *j*, which is equal to the *net* rate at which *j gains* radiation due to its interaction with *i*.

The foregoing result may also be used to evaluate the net radiation transfer from any surface in an *enclosure* of black surfaces. With N surfaces maintained at different temperatures, the net transfer of radiation from surface *i* is due to exchange with the remaining surfaces and may be expressed as

$$q_i = \sum_{j=1}^{N} A_i F_{i\to j}\sigma\left(T_i^4 - T_j^4\right) \tag{6-26}$$

The net radiative heat flux, $q_i'' = q_i/A_i$, was denoted as q_{rad}'' in Chapter 1. The subscript *rad* has been dropped here for convenience.

Example 4: A furnace cavity, which is in the form of a cylinder of 50mm diameter and 150mm length, is open at one end to large surroundings (see Figure 6.16) that are at 27°C. The bottom of the cavity is heated independently, as are three annular sections that comprise the sides of the cavity. All interior surfaces of the cavity may be approximated as blackbodies and are maintained at 1650°C. What is the required electrical power input to the bottom surface of the cavity? What is the electrical power to the top, middle, and bottom sections of the cavity sides? The backs of the electrically heated surfaces are well insulated.

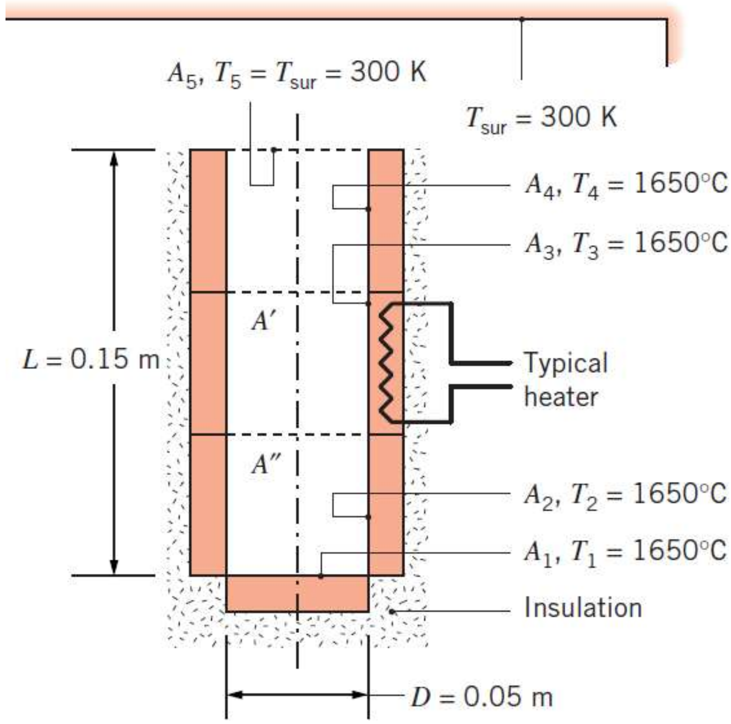


Figure 6.16: Temperature of furnace surfaces and surroundings.

Surface 1: $$q_1 = A_1F_{15}\sigma(T_1^4 - T_5^4) = A_5F_{51}\sigma(T_1^4 - T_5^4) \quad (1)$$

Surface 2: $$q_2 = A_2F_{25}\sigma(T_2^4 - T_5^4) = A_5F_{52}\sigma(T_2^4 - T_5^4) \quad (2)$$

Surface 3: $$q_3 = A_3F_{35}\sigma(T_3^4 - T_5^4) = A_5F_{53}\sigma(T_3^4 - T_5^4) \quad (3)$$

Surface 4: $$q_4 = A_4F_{45}\sigma(T_4^4 - T_5^4) = A_5F_{54}\sigma(T_4^4 - T_5^4) \quad (4)$$

We will determine the view factors by first defining two hypothetical surfaces A' and A'' as shown in the schematic. From Table 13.2 with $(r_i/L) = (r_j/L) = (0.025\text{ m}/0.150\text{ m}) = 0.167$, $F_{51} = 0.0263$. With $(r_i/L) = (r_j/L) = (0.025\text{ m}/0.100\text{ m}) = 0.25$, $F_{5A''} = 0.0557$ so that $F_{52} = F_{5A''} - F_{51} = 0.0557 - 0.0263 = 0.0294$. Likewise, with $(r_i/L) = (r_j/L) = (0.025\text{ m}/0.050\text{ m}) = 0.5$, $F_{5A'} = 0.172$ so that $F_{53} = F_{5A'} - F_{5A''} = 0.172 - 0.0557 = 0.1163$. Finally, $F_{54} = 1 - F_{5A'} = 1 - 0.172 = 0.828$. The electrical power delivered to each of the four furnace surfaces can now be determined by solving Equations 1 through 4 for the radiation loss from each surface with $A_5 = \pi D^2/4 = \pi \times (0.05\text{ m})^2/4 = 0.00196\text{ m}^2$.

$$q_1 = 0.00196\text{ m}^2 \times 0.0263 \times 5.67 \times 10^{-8}\text{ W/m}^2\cdot\text{K}^4 \times (1923\text{ K}^4 - 300\text{ K}^4) = 39.9\text{ W} \quad \triangleleft$$

$$q_2 = 0.00196\text{ m}^2 \times 0.0294 \times 5.67 \times 10^{-8}\text{ W/m}^2\cdot\text{K}^4 \times (1923\text{ K}^4 - 300\text{ K}^4) = 44.7\text{ W} \quad \triangleleft$$

$$q_3 = 0.00196\text{ m}^2 \times 0.1163 \times 5.67 \times 10^{-8}\text{ W/m}^2\cdot\text{K}^4 \times (1923\text{ K}^4 - 300\text{ K}^4) = 177\text{ W} \quad \triangleleft$$

$$q_4 = 0.00196\text{ m}^2 \times 0.828 \times 5.67 \times 10^{-8}\text{ W/m}^2\cdot\text{K}^4 \times (1923\text{ K}^4 - 300\text{ K}^4) = 1260\text{ W} \quad \triangleleft$$

6.3.3 Radiation Exchange between Opaque, Diffuse, Gray Surfaces in an Enclosure

In general, radiation may leave an opaque surface due to both *reflection* and *emission*, and on reaching a second opaque surface, experience reflection as well as absorption. In an enclosure, such as that of Figure 6.17.a, radiation may experience multiple reflections off all surfaces, with partial absorption occurring at each.

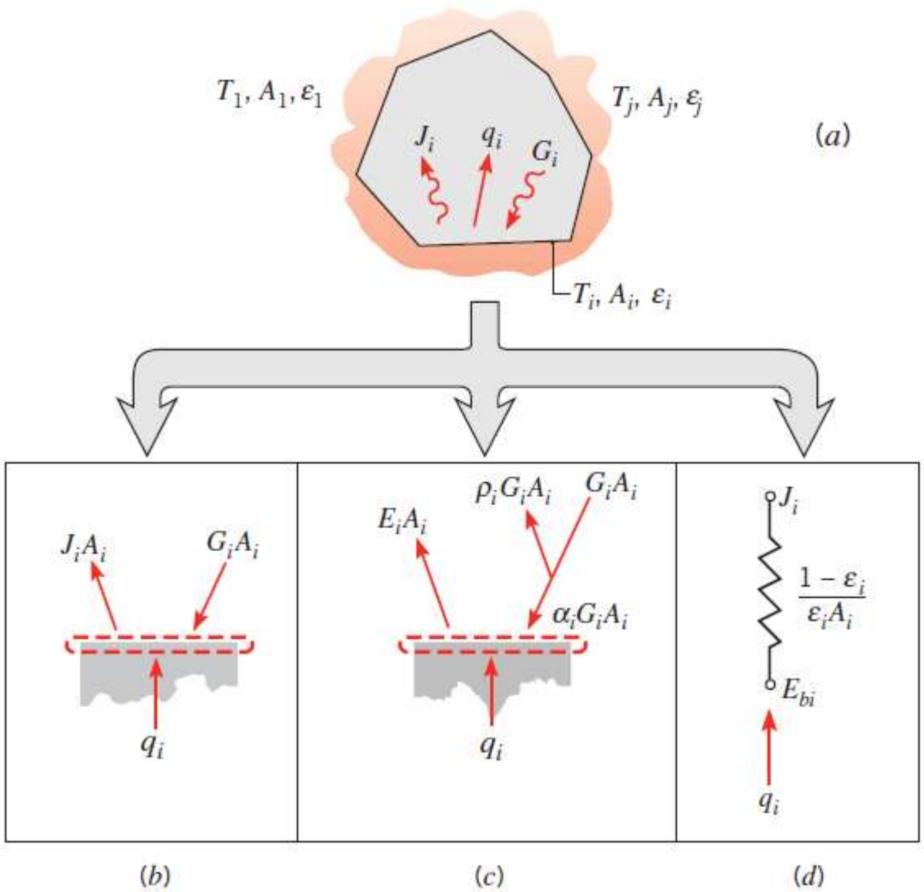


Figure 6.17: Radiation exchange in an enclosure of diffuse, gray surfaces with a nonparticipating medium. (a) Schematic of the enclosure. (b) Radiative balance. (c) Radiative balance. (d) Resistance representing net radiation transfer from a surface.

Analyzing radiation exchange in an enclosure may be simplified by making certain assumptions. Each surface of the enclosure is assumed to be *isothermal* and to be characterized by a *uniform radiosity* and a *uniform irradiation*. The surfaces are also assumed to be *opaque* (τ= 0) and to have emissivities, absorptivities, and reflectivities that are independent of direction (the surfaces are *diffuse*) *and* independent of wavelength (the surfaces are *gray*). Under these conditions the emissivity is equal to the absorptivity, $\varepsilon=\alpha$ (a form of Kirchhoff's law). Finally, the medium within the enclosure is taken to be *nonparticipating*. The problem is generally one in which either the temperature T_i or the net radiative heat flux q_i'' associated with each of the surfaces is known. The objective is to use this information to determine the unknown radiative heat fluxes and temperatures associated with each of the surfaces.

A. Net Radiation Exchange at a Surface

The term q_i, which is the *net* rate at which radiation *leaves* surface *i*, represents the net effect of radiative interactions occurring at the surface (Figure 6.17.b). It is the rate at which energy would have to be transferred to the surface by other means to maintain it at a constant temperature. It is equal to the difference between the surface radiosity and irradiation and may be expressed as

$$q_i = A_i(J_i - G_i) \quad (6\text{-}27)$$

Using the definition of the radiosity,

$$J_i = E_i + \rho_i G_i \quad (6\text{-}28)$$

The net radiative transfer from the surface may be expressed as

$$q_i = A_i(E_i - \alpha_i G_i) \quad (6\text{-}29)$$

where use has been made of the relationship $\alpha_i = 1 - \rho_i$ for an opaque surface. This relationship is illustrated in Figure 6.17.*c*. Noting that $E_i = \varepsilon_i E_{bi}$ and recognizing that $\rho_i = 1 - \alpha_i = 1 - \varepsilon_i$ for an opaque, diffuse, gray surface, the radiosity may also be expressed as

$$J_i = \varepsilon_i E_{bi} + (1 - \varepsilon_i)G_i \quad (6\text{-}30)$$

Solving for G_i and substituting into Equation 6.27, it follows that

$$q_i = A_i(J_i - \frac{J_i - \varepsilon_i E_{bi}}{1-\varepsilon_i}) \quad (6\text{-}31)$$

$$q_i = \frac{E_{bi} - J_i}{(1-\varepsilon_i)/\varepsilon_i A_i} \quad (6\text{-}32)$$

Equation 6.32 provides a convenient representation for the net radiative heat transfer rate from a surface. This transfer, which is represented in Figure 6.17.*d*, is associated with the driving potential (E_{bi} - J_i) and a *surface radiative resistance* of the form (1 - ε_i)/ ε_i A_i. Hence, if the emissive power that the surface would have if it were black exceeds its radiosity, there is net radiation heat transfer from the surface; if the inverse is true, the net transfer is to the surface.

It is sometimes the case that one of the surfaces is very large relative to the other surfaces under consideration. For example, the system might consist of multiple small surfaces in a large room. In this case, the area of the large surface is effectively infinite ($A_i \rightarrow \infty$), and we see that its surface radiative resistance, (1 - ε_i)/ ε_i A_i, is effectively zero, just as it would be for a black surface (ε_i =1). Hence, J_i = E_{bi}, and *a surface which is large relative to all other surfaces under consideration can be treated as if it were a blackbody*.

B. Radiation Exchange between Surfaces

To use Equation 6.32, the surface radiosity J_i must be known. To determine this quantity,it is necessary to consider radiation exchange between the surfaces of the enclosure. The irradiation of surface *i* can be evaluated from the radiosities of all the surfaces in the enclosure. In particular, from the definition of the view factor, it follows that the total rate at which radiation reaches surface *i* from all surfaces, including *i*, is

$$A_i G_i = \sum_{j=1}^{N} F_{ji} A_j J_j \quad (6\text{-}33)$$

or from the *reciprocity relation*, Equation 6.14,

$$A_i G_i = \sum_{j=1}^{N} F_{ij} A_i J_j \quad (6\text{-}34)$$

Canceling the area A_i and substituting into Equation 6.27 for G_i,

$$q_i = A_i(J_i - \sum_{j=1}^{N} F_{ij} J_j) \tag{6-35}$$

or, from the *summation rule*, Equation 6.15,

$$q_i = A_i(\sum_{j=1}^{N} F_{ij} J_i - \sum_{j=1}^{N} F_{ij} J_j) \tag{6-36}$$

Hence,

$$q_i = \sum_{j=1}^{N} F_{ij}\ A_i\big(J_i - J_j\big) = \sum_{j=1}^{N} q_{ij} \tag{6-37}$$

Radiation Network Approach

Equation 6.37equates the net rate of radiation transfer from surface *i*, *qi*, to the sum of components q_{ij} related to radiative exchange with the other surfaces. Each component may be represented by a *network element* for which (J_i - J_j) is the driving potential and $(A_iF_{ij})^{-1}$ is a *space* or *geometrical resistance* (Figure 6.18). Combining Equations 6.32 and 6.37, we then obtain

$$q_i = \sum_{j=1}^{N} \frac{(J_i - J_j)}{(A_iF_{ij})^{-1}} = \frac{E_{bi} - J_i}{(1-\varepsilon_i)/\varepsilon_i A_i} \tag{6-38}$$

As shown in Figure 6.18, this expression represents a radiation balance for the radiosity *node* associated with surface *i*. The rate of radiation transfer (current flow) to *i* through its surface resistance must equal the net rate of radiation transfer (current flows) from *i* to all other surfaces through the corresponding geometrical resistances.

Note that Equation 6.38 is especially useful when the surface temperature T_i (hence E_{bi}) is known. Although this situation is typical, it does not always apply. In particular, situations may arise for which the net radiation transfer rate at the surface q_i, rather than the temperature T_i, is known. In such cases the preferred form of the radiation balance is Equation 6.37, rearranged as

$$q_i = \sum_{j=1}^{N} \frac{(J_i - J_j)}{(A_iF_{ij})^{-1}} \tag{6-39}$$

Use of network representations was first suggested by Oppenheim. The network is built by first identifying nodes associated with the radiosities of each of the N surfaces of the enclosure. The method provides a useful tool for visualizing radiation exchange in the enclosure and, at least for simple enclosures, may be used as the basis for predicting this exchange.

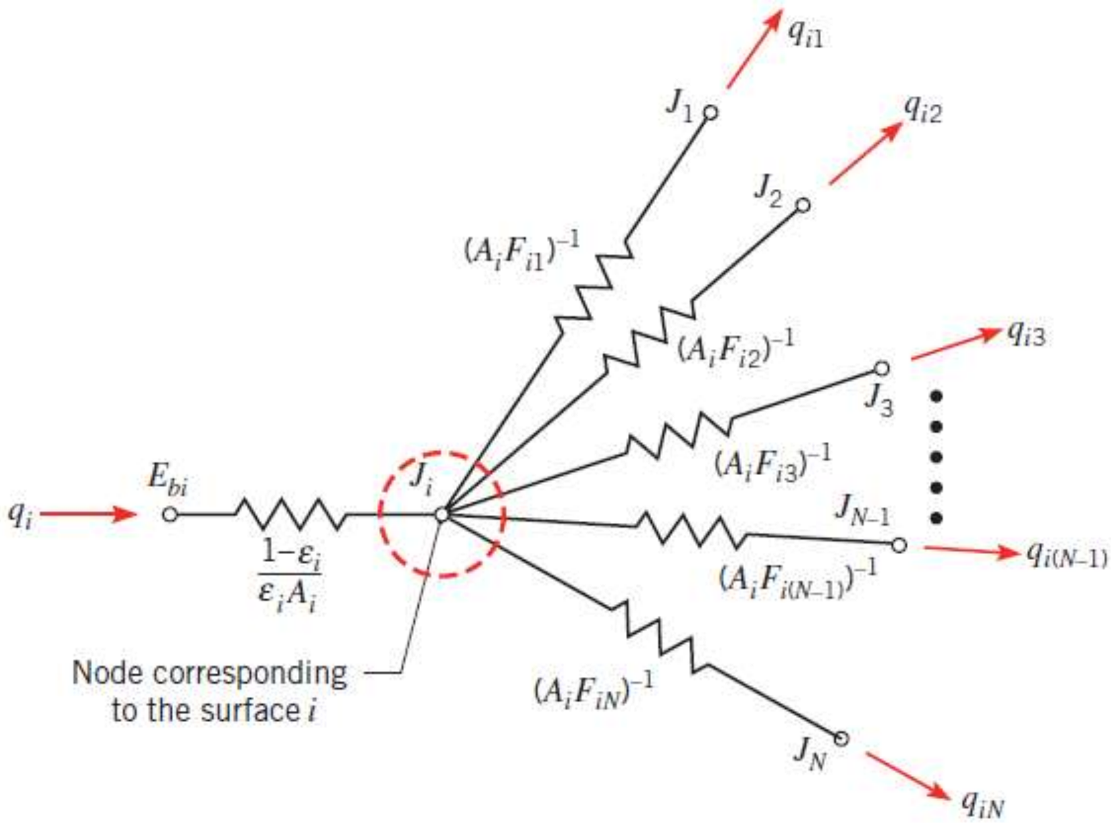
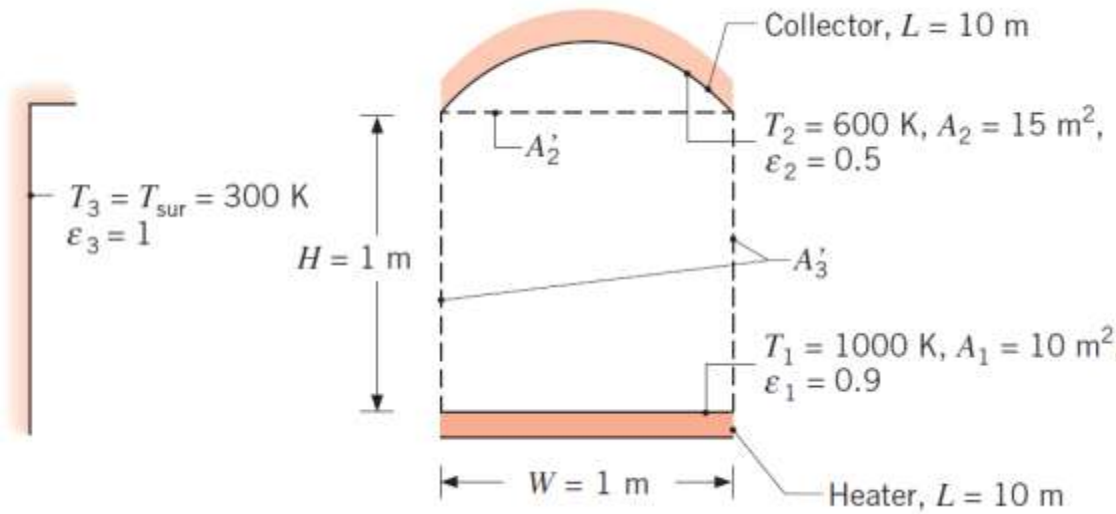
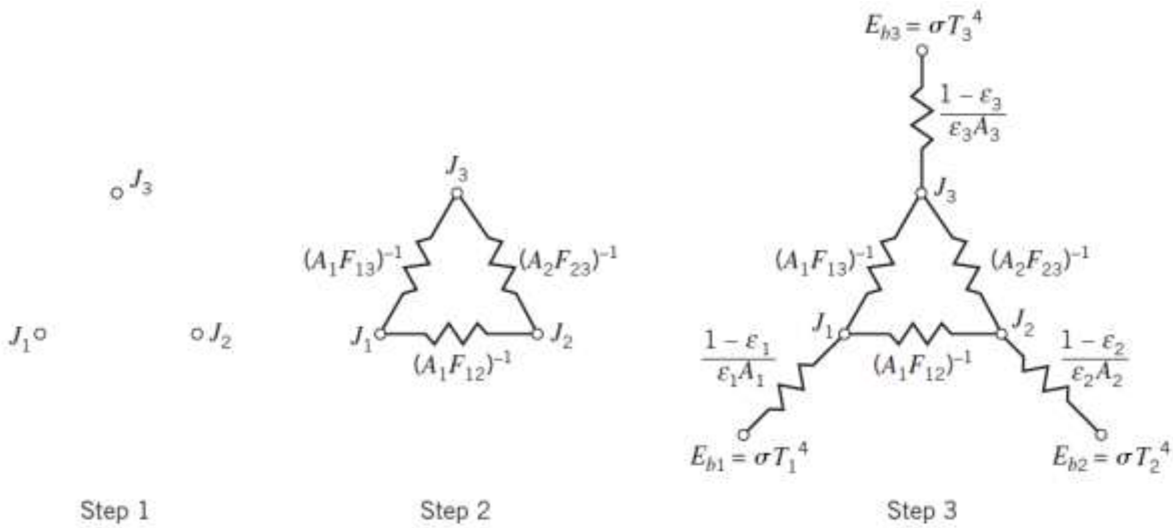


Figure 6.18: Network representation of radiative exchange between surface i and the remaining surfaces of an enclosure.

Example 4: In manufacturing, the special coating on a curved solar absorber surface of area A_2= 15 m^2 is cured by exposing it to an infrared heater of width W = 1 m. The absorber and heater are each of length L = 10 m and are separated by a distance of H = 1 m. The upper surface of the absorber and the lower surface of the heater are insulated. The heater is at T_1= 1000 K and has an emissivity of ε_1= 0.9, while the absorber is at T_2= 600 K and has an emissivity of ε_2 = 0.5. The system is in a large room whose walls are at 300 K. What is the net rate of heat transfer to the absorber surface?

Solution:





The radiation network is constructed by first identifying nodes associated with the radiosities of each surface, as shown in step 1 in the following schematic. Then each radiosity node is connected to each of the other radiosity nodes through the appropriate space resistance, as shown in step 2. We will treat the surroundings as having a large but unspecified area, which introduces difficulty in expressing the space resistances $(A_3F_{31})^{-1}$ and $(A_3F_{32})^{-1}$. Fortunately, from the reciprocity relation, we can replace A_3F_{31} with A_1F_{13} and A_3F_{32} with A_2F_{23}, which are more readily obtained. The final step is to connect the blackbody emissive powers associated with the temperature of each surface to the radiosity nodes, using the appropriate form of the surface resistance.

In this problem, the surface resistance associated with surface 3 is zero according to assumption 4; therefore, $J_3 = E_{b3} = \sigma T_3^4 = 459\ \text{W/m}^2$.

Summing currents at the J_1 node yields

$$\frac{\sigma T_1^4 - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - J_2}{1/A_1F_{12}} + \frac{J_1 - \sigma T_3^4}{1/A_1F_{13}} \tag{1}$$

while summing the currents at the J_2 node results in

$$\frac{\sigma T_2^4 - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} = \frac{J_2 - J_1}{1/A_1F_{12}} + \frac{J_2 - \sigma T_3^4}{1/A_2F_{23}} \tag{2}$$

The view factor F_{12} may be obtained by recognizing that $F_{12} = F_{12'}$, where A_2' is shown in the schematic as the rectangular base of the absorber surface. Then, from Figure 13.4 or Table 13.2, with $Y/L = 10/1 = 10$ and $X/L = 1/1 = 1$,

$$F_{12} = 0.39$$

From the summation rule, and recognizing that $F_{11} = 0$, it also follows that

$$F_{13} = 1 - F_{12} = 1 - 0.39 = 0.61$$

The last needed view factor is F_{23}. We recognize that, since radiation propagating from surface 2 to surface 3 must pass through the hypothetical surface A_2',

$$A_2F_{23} = A_2'F_{2'3}$$

and from symmetry $F_{2'3} = F_{13}$. Thus

$$F_{23} = \frac{A_2'}{A_2}F_{13} = \frac{10\ \text{m}^2}{15\ \text{m}^2} \times 0.61 = 0.41$$

We may now solve Equations 1 and 2 for J_1 and J_2. Recognizing that $E_{b1} = \sigma T_1^4 = 56{,}700\ \text{W/m}^2$ and canceling the area A_1, we can express Equation 1 as

$$\frac{56{,}700 - J_1}{(1-0.9)/0.9} = \frac{J_1 - J_2}{1/0.39} + \frac{J_1 - 459}{1/0.61}$$

or

$$-10J_1 + 0.39J_2 = -510{,}582 \tag{3}$$

Noting that $E_{b2} = \sigma T_2^4 = 7348\ \text{W/m}^2$ and dividing by the area A_2, we can express Equation 2 as

$$\frac{7348 - J_2}{(1-0.5)/0.5} = \frac{J_2 - J_1}{15\ \text{m}^2/(10\ \text{m}^2 \times 0.39)} + \frac{J_2 - 459}{1/0.41}$$

$$0.26J_1 - 1.67J_2 = -7536 \qquad (4)$$

Solving Equations 3 and 4 simultaneously yields $J_2 = 12{,}487\ \text{W/m}^2$.

An expression for the net rate of heat transfer *from* the absorber surface, q_2, may be written upon inspection of the radiation network and is

$$q_2 = \frac{\sigma T_2^4 - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2}$$

resulting in

$$q_2 = \frac{(7348 - 12{,}487)\text{W/m}^2}{(1-0.5)/(0.5 \times 15\ \text{m}^2)} = -77.1\ \text{kW}$$

Hence, the net heat transfer rate *to* the absorber is $q_{\text{net}} = -q_2 = 77.1\ \text{kW}$. ◁

C. The Two-Surface Enclosure

The simplest example of an enclosure is one involving two surfaces that exchange radiation only with each other. Such a two-surface enclosure is shown schematically in Figure 6.19.*a*. Since there are only two surfaces, the net rate of radiation transfer from surface 1, q_1, must equal the net rate of radiation transfer *to* surface 2, -q_2, and both quantities must equal the net rate at which radiation is exchanged between 1 and 2. Accordingly,

$$q_1 = -q_2 = q_{12} \qquad (6\text{-}40)$$

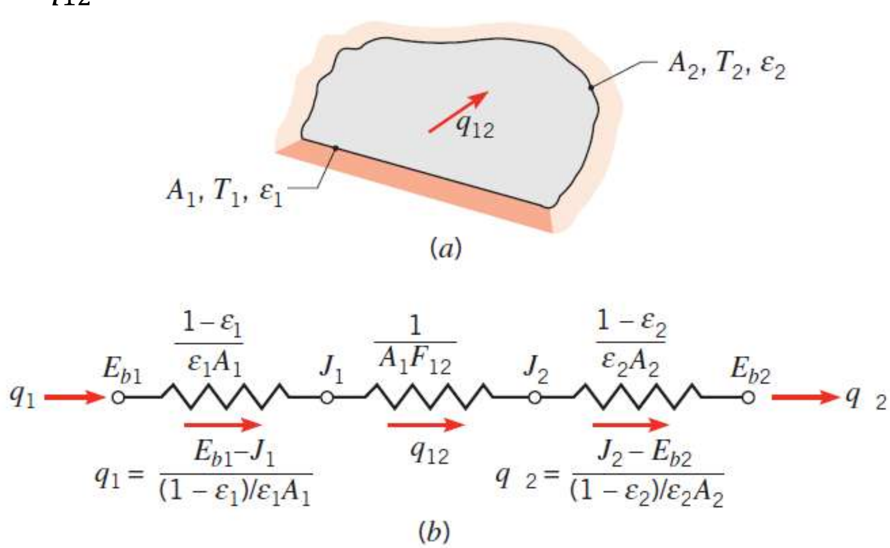


Figure 6.19: The two-surface enclosure. (a) Schematic. (b) Network representation.

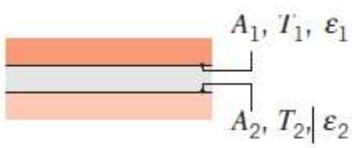
The radiation transfer rate may be determined by applying Equation 6.38 to surfaces 1 and 2 and solving the resulting two equations for J_1 and J_2. The results could then be used with Equation 6.32 to determine q_1 (or q_2). However, in this case the desired result is more readily obtained by working with the network representation of the enclosure shown in Figure 6.19.*b*. From Figure 6.19.b we see that the total resistance to radiation exchange between surfaces 1 and 2 is comprised of the two surface resistances and the geometrical resistance. Hence, substituting from Equation 12.32, the net radiation exchange between surfaces may be expressed as

$$q_{12} = q_1 = -q_2 = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}} \qquad (6\text{-}41)$$

The foregoing result may be used for any two isothermal diffuse, gray surfaces that *form an enclosure* and *are each characterized by uniform radiosity and irradiation.* Important special cases are summarized in Table 6.3.

Table 6.3: Special Diffuse, Gray, Two-Surface Enclosures

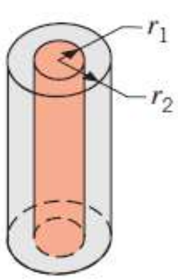
Large (Innite) Parallel Planes



$$A_1 = A_2 = A \qquad F_{12} = 1$$

$$q_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$$

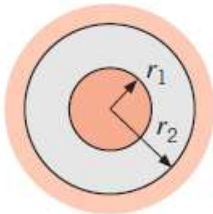
Long (Innite) Concentric Cylinders



$$\frac{A_1}{A_2} = \frac{r_1}{r_2} \qquad F_{12} = 1$$

$$q_{12} = \frac{\sigma A_1(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)}$$

Concentric Spheres

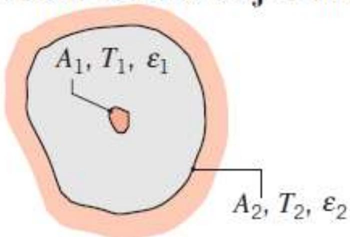


$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$$

$$F_{12} = 1$$

$$q_{12} = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)^2}$$

Small Convex Object in a Large Cavity



$$\frac{A_1}{A_2} \approx 0$$

$$F_{12} = 1$$

$$q_{12} = \sigma A_1 \varepsilon_1 (T_1^4 - T_2^4)$$

D. Radiation Shields

Radiation shields constructed from low emissivity (high reflectivity) materials can be used to reduce the net radiation transfer between two surfaces. Consider placing a radiation shield, surface 3, between the two large, parallel planes of Figure 6.20.*a*. Without the radiation shield, the net rate of radiation transfer between surfaces 1 and 2 is given by Table 6.3 for Large (Infinite) Parallel Planes. However, with the radiation shield, additional resistances are present, as shown in Figure 6.20.*b*, and the heat transfer rate is reduced. Note that the emissivity associated with one side of the shield ($\varepsilon_{3,1}$) may differ from that associated with the opposite side ($\varepsilon_{3,2}$) and the radiosities will always differ. Summing the resistances and recognizing that $F_{13} = F_{32} = 1$, it follows that

$$q_{12} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{1-\varepsilon_{3,1}}{\varepsilon_{3,1}} + \frac{1-\varepsilon_{3,2}}{\varepsilon_{3,2}}} \qquad (6\text{-}42)$$

Note that the resistances associated with the radiation shield become very large when the emissivities $\varepsilon_{3,1}$ and $\varepsilon_{3,2}$are very small.

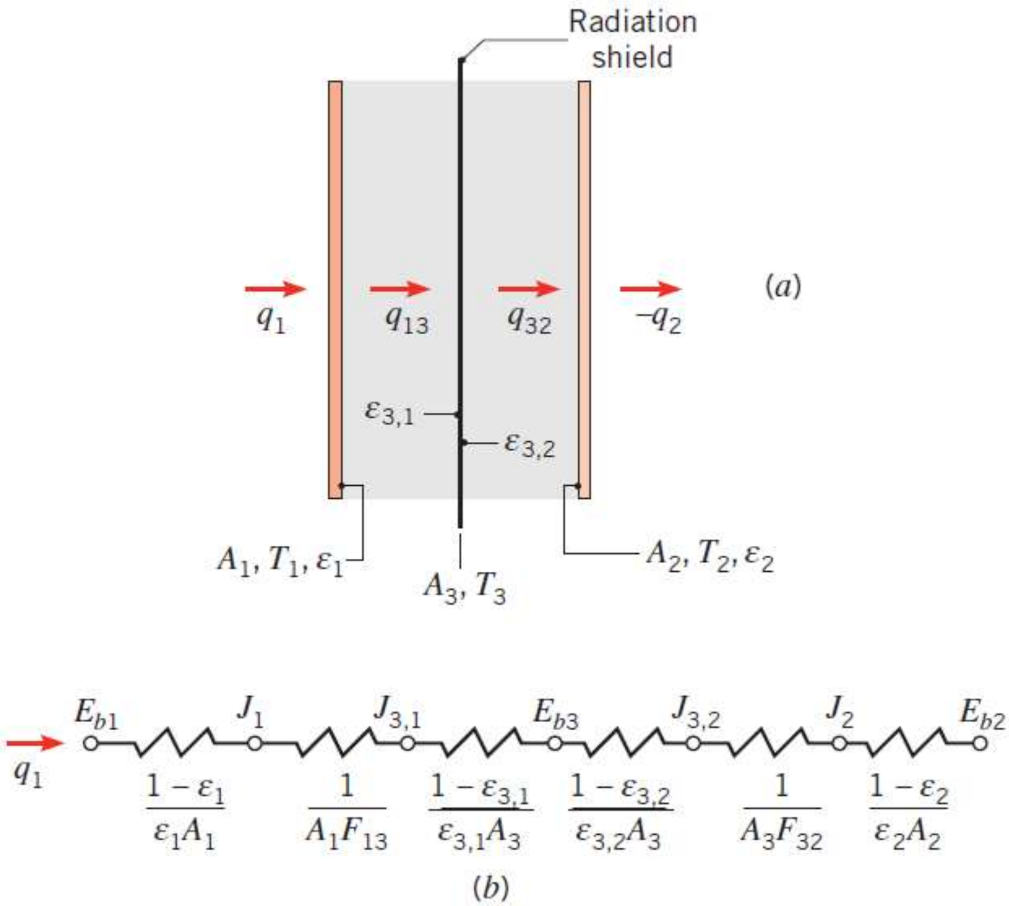


Figure 6.20: Radiation exchange between large parallel planes with a radiation shield. (a) Schematic. (b) Network representation.

Equation 6.42 may be used to determine the net heat transfer rate if T_1 and T_2 are known. From knowledge of q_{12} and the fact that $q_{12} = q_{13} = q_{32}$, the value of T_3 may then be determined by expressing Equation for Large (Infinite) Parallel Planes for q_{13} or q_{32}.

The foregoing procedure may readily be extended to problems involving multiple radiation shields. In the special case for which all the emissivities are equal, it may be shown that, with N shields,

$$(q_{12})_N = \frac{1}{N+1}(q_{12})_0 \qquad (6\text{-}43)$$

where $(q_{12})_0$ is the radiation transfer rate with no shields ($N = 0$).

Example 5:

A cryogenic fluid flows through a long tube of 20-mm diameter, the outer surface of which is diffuse and gray with $\varepsilon_1 = 0.02$ and $T_1 = 77$ K as shown in Figure 6.21. This tube is concentric with a larger tube of 50-mm diameter, the inner surface of which is diffuse and gray with $\varepsilon_2 = 0.05$ and $T_2 = 300$ K. The space between the surfaces is evacuated. Calculate the heat gain by the cryogenic fluid per unit length of tubes. If a thin radiation shield of 35-mm diameter and $\varepsilon_3 = 0.02$ (both sides) is inserted midway between the inner and outer surfaces, calculate the change (*percentage*) in heat gain per unit length of the tubes.

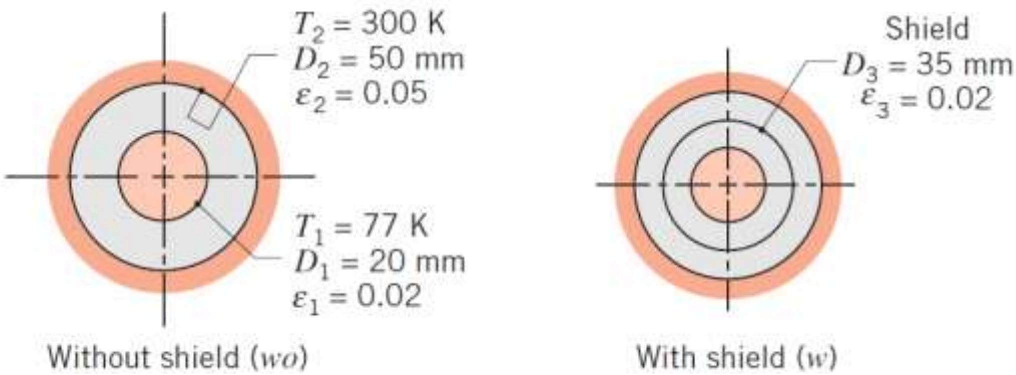


Figure 6.21: Concentric tube arrangement with diffuse, gray surfaces of different Emissivities and temperatures.

1. The network representation of the system without the shield is shown in Figure 6.19,

$$q = \frac{\sigma(\pi D_1 L)(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)}$$

Hence

$$q' = \frac{q}{L} = \frac{5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4\ (\pi \times 0.02\ \text{m})[(77\ \text{K})^4 - (300\ \text{K})^4]}{\dfrac{1}{0.02} + \dfrac{1-0.05}{0.05}\left(\dfrac{0.02\ \text{m}}{0.05\ \text{m}}\right)}$$

$$q' = -0.50\ \text{W/m}$$

2. The network representation of the system with the shield is shown in Figure 6.20, and the heat rate is now

$$q = \frac{E_{b1} - E_{b2}}{R_{\text{tot}}} = \frac{\sigma(T_1^4 - T_2^4)}{R_{\text{tot}}}$$

where

$$R_{\text{tot}} = \frac{1-\varepsilon_1}{\varepsilon_1(\pi D_1 L)} + \frac{1}{(\pi D_1 L)F_{13}} + 2\left[\frac{1-\varepsilon_3}{\varepsilon_3(\pi D_3 L)}\right] + \frac{1}{(\pi D_3 L)F_{32}} + \frac{1-\varepsilon}{\varepsilon_2(\pi D_2}$$

or

$$R_{\text{tot}} = \frac{1}{L}\left\{\frac{1-0.02}{0.02(\pi \times 0.02\text{ m})} + \frac{1}{(\pi \times 0.02\text{ m})1}\right.$$
$$+ 2\left[\frac{1-0.02}{0.02(\pi \times 0.035\text{ m})}\right] + \frac{1}{(\pi \times 0.035\text{ m})1} + \frac{1-0.05}{0.05(\pi \times 0.05\text{ n}}$$

$$R_{\text{tot}} = \frac{1}{L}(779.9 + 15.9 + 891.3 + 9.1 + 121.0) = \frac{1817}{L}\left(\frac{1}{\text{m}^2}\right)$$

Hence

$$q' = \frac{q}{L} = \frac{5.67 \times 10^{-8}\text{ W/m}^2\cdot\text{K}^4\,[(77\text{ K})^4 - (300\text{ K})^4]}{1817\text{ (1/m)}} = -0.25\text{ W/m}$$

The percentage change in the heat gain is then

$$\frac{q'_w - q'_{wo}}{q'_{wo}} \times 100 = \frac{(-0.25\text{ W/m}) - (-0.50\text{ W/m})}{-0.50\text{ W/m}} \times 100 = -50\%$$

E. The Reradiating Surface

The assumption of a *reradiating surface* is common to many industrial applications. This idealized surface is characterized by *zero* net radiation transfer ($qi = 0$). It is closely approached by real surfaces that are well insulated on one side *and* for which convection effects may be neglected on the opposite (radiating) side. With $qi = 0$, it follows from Equations 6.32, 6.38 and 6.27 that $G_i = J_i = E_{bi}$. Hence, if the radiosity of a reradiating surface is known, its temperature is readily determined. In an enclosure, the equilibrium temperature of a reradiating surface is determined by its interaction with the other surfaces, and it is *independent of the emissivity of the reradiating surface.*

A three-surface enclosure, for which the third surface, surface R, is reradiating, is shown in Figure 6.22.*a*, and the corresponding network is shown in Figure 6.22.*b*. Surface R is presumed to be well insulated, and convection effects are assumed to be negligible. Hence, with $q_R = 0$, the net radiation *transfer* from surface 1 must equal the net radiation

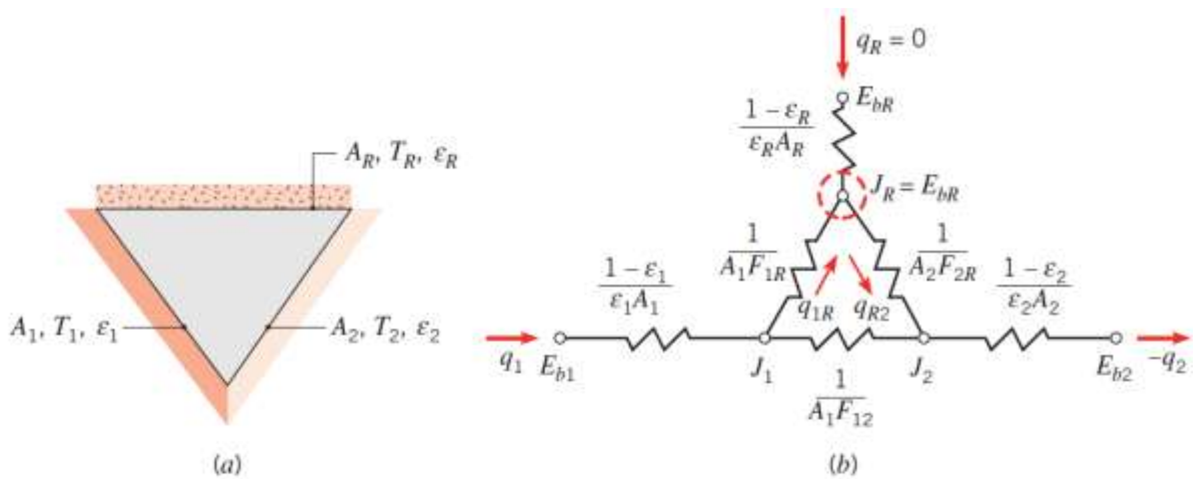


Figure 6.22: A three-surface enclosure with one surface reradiating. (*a*) Schematic. (*b*) Network representation.

transfer to surface 2. The network is a simple series–parallel arrangement, and from its analysis it is readily shown that

$$q_1 = -q_2 = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1F_{12} + [(1/A_1F_{1R}) + (1/A_2F_{2R})]^{-1}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}} \quad (6\text{-}44)$$

Knowing $q_1 = -q_2$, Equations 6.32 and 6.38 may be applied to surfaces 1 and 2 to determine their radiosities J_1 and J_2. Knowing J_1, J_2, and the geometrical resistances, the radiosity of the reradiating surface J_R may be determined from the radiation balance

$$\frac{J_1 - J_R}{(1/A_1F_{1R})} - \frac{J_R - J_2}{(1/A_2F_{2R})} = 0 \quad (6\text{-}45)$$

The temperature of the reradiating surface may then be determined from the requirement that $\sigma T_R^4 = J_R$.

Example 6:

A paint baking oven consists of a long, triangular duct in which a heated surface is maintained at 1200 K and another surface is insulated as shown in Figure 6.23. Painted panels, which are maintained at 500 K, occupy the third surface. The triangle is of width W =1 m on a side, and the heated and insulated surfaces have an emissivity of 0.8. The emissivity of the panels is 0.4. During steady-state operation, at what rate must energy be supplied to the heated side per unit length of the duct to maintain its temperature at 1200 K? What is the temperature of the insulated surface?

Solution:

1. The system may be modeled as a three-surface enclosure with one surface reradiating. The rate at which energy must be supplied to the heated surface may then be obtained from Equation 6.44:

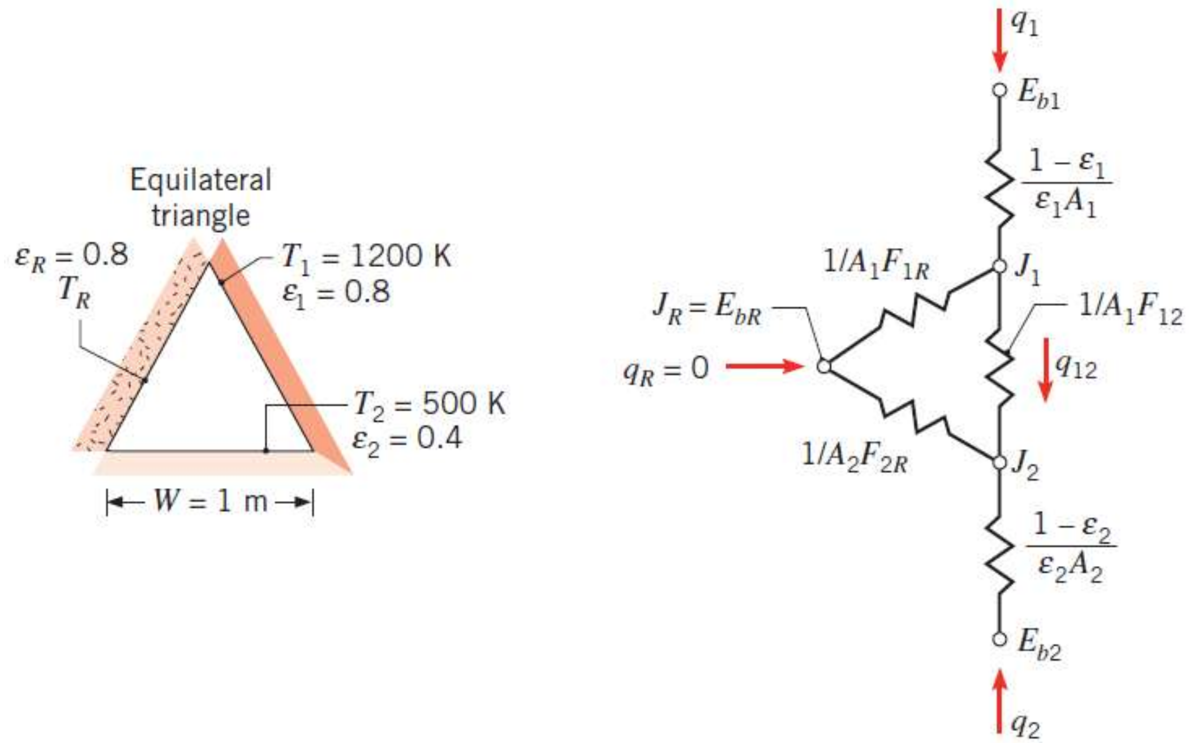


Figure 6.23: Surface properties of a long triangular duct that is insulated on one side and heated and cooled on the other sides

$$q_1 = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1F_{12} + [(1/A_1F_{1R}) + (1/A_2F_{2R})]^{-1}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

From symmetry, $F_{12} = F_{1R} = F_{2R} = 0.5$. Also, $A_1 = A_2 = W \cdot L$, where L is the duct length. Hence

$$q_1' = \frac{q_1}{L} = \frac{5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(1200^4 - 500^4)\,\text{K}^4}{\dfrac{1-0.8}{0.8 \times 1\,\text{m}} + \dfrac{1}{1\,\text{m} \times 0.5 + (2+2)^{-1}\,\text{m}} + \dfrac{1-0.4}{0.4 \times 1\,\text{m}}}$$

or

$$q_1' = 37\,\text{kW/m} = -q_2' \qquad \triangleleft$$

2. The temperature of the insulated surface may be obtained from the requirement that $J_R = E_{bR}$, where J_R may be obtained from Equation 6.44.

However, to use this expression J_1 and J_2 must be known. Applying the surface energy balance, Equation 6.32, to surfaces 1 and 2, it follows that

$$J_1 = E_{b1} - \frac{1-\varepsilon_1}{\varepsilon_1 W} q_1' = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 (1200 \text{ K})^4$$

$$- \frac{1-0.8}{0.8 \times 1 \text{ m}} \times 37{,}000 \text{ W/m} = 108{,}323 \text{ W/m}^2$$

$$J_2 = E_{b2} - \frac{1-\varepsilon_2}{\varepsilon_2 W} q_2' = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 (500 \text{ K})^4$$

$$- \frac{1-0.4}{0.4 \times 1 \text{ m}} (-37{,}000 \text{ W/m}) = 59{,}043 \text{ W/m}^2$$

From the energy balance for the reradiating surface, Equation 13.31, it follows that

$$\frac{108{,}323 - J_R}{\dfrac{1}{W \times L \times 0.5}} - \frac{J_R - 59{,}043}{\dfrac{1}{W \times L \times 0.5}} = 0$$

Hence

$$J_R = 83{,}683 \text{ W/m}^2 = E_{bR} = \sigma T_R^4$$

$$T_R = \left(\frac{83{,}683 \text{ W/m}^2}{5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4} \right)^{1/4} = 1102 \text{ K}$$

F. Multimode Heat Transfer

Thus far, radiation exchange in an enclosure has been considered under conditions for which conduction and convection could be neglected. However, in many applications, convection and/or conduction are comparable to radiation and must be considered in the heat transfer analysis.

Consider the general surface condition of Figure 6.24.*a*. In addition to exchanging energy by radiation with other surfaces of the enclosure, there may be external heat addition to the surface, as, for example, by electric heating, and heat transfer from the surface by convection and conduction. From a surface energy balance, it follows that

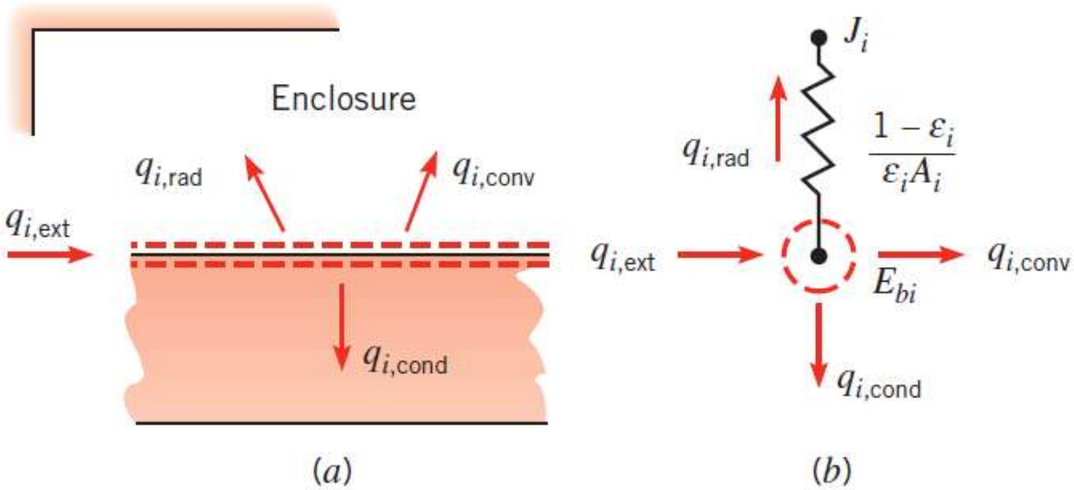


Figure 6.24: Multimode heat transfer from a surface in an enclosure. (*a*) Surface energy balance. (*b*) Circuit representation.

$$q_{i,\text{ext}} = q_{i,\text{rad}} + q_{i,\text{conv}} + q_{i,\text{cond}} \tag{6-46}$$

where $q_{i,\text{ext}}$, $q_{i,\text{cond}}$, and $q_{i,\text{conv}}$ represent current flows to or from the surface node. Note, however, that while $q_{i,\text{cond}}$ and $q_{i,\text{conv}}$ are proportional to temperature differences, $q_{i,\text{rad}}$ is proportional to the difference between temperatures raised to the fourth power. Conditions are simplified if the back of the surface is insulated, in which case $q_{i,\text{cond}} = 0$. Moreover, if there is no external heating and convection is negligible, the surface is reradiating.