

University of Anbar
College of Engineering
Mechanical Engineering Dept.



Fluid Mechanics-II

(ME 2305)

Handout Lectures for Year Two
Chapter One/ Viscous Flow in Ducts

Course Tutor

Assist. Prof. Dr. Waleed M. Abed

Chapter One

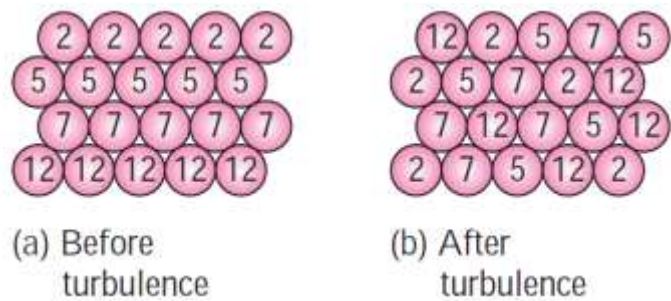
Viscous Flow in Ducts

1.1. TURBULENT FLOW IN PIPES

Most flows encountered in engineering practice are turbulent, and thus it is important to understand how turbulence affects wall shear stress. However, turbulent flow is a complex mechanism dominated by fluctuations, and despite tremendous amounts of work done in this area by researchers, the theory of turbulent flow remains largely undeveloped. Therefore, we must rely on experiments and the empirical or semi-empirical correlations developed for various situations.

Turbulent flow is characterized by random and a rapid fluctuation of swirling regions of fluid, called eddies, throughout the flow. These fluctuations provide an additional mechanism for momentum and energy transfer. In laminar flow, fluid particles flow in an orderly manner along pathlines, and momentum and energy are transferred across streamlines by molecular diffusion. In turbulent flow, the swirling eddies transport mass, momentum, and energy to other regions of flow much more rapidly than molecular diffusion, greatly enhancing mass, momentum, and heat transfer. As a result, turbulent flow is associated with much higher values of friction, heat transfer, and mass transfer coefficients (see Figure 1.1).

Figure 1.1: The intense mixing in turbulent flow brings fluid particles at different momentums into close contact and thus enhances momentum transfer.



Even when the average flow is steady, the eddy motion in turbulent flow causes significant fluctuations in the values of velocity, temperature, pressure, and even density (in compressible flow). Figure 1.2 shows the variation of the instantaneous velocity component u with time at a specified location, as can be measured with a hot-wire anemometer probe or other sensitive device. We observe that the instantaneous values of the velocity fluctuate about an average value, which suggests that the velocity can be expressed as the sum of an average value \bar{u} and a fluctuating component u' ,

$$u = \bar{u} + u' \quad \dots\dots 1.1$$

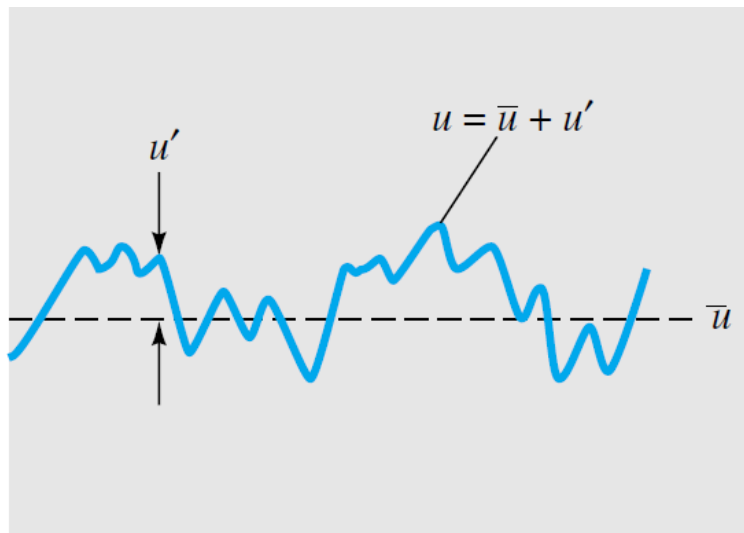


Figure 1.2: Fluctuations of the velocity component u with time at a specified location in turbulent flow.

1.2. Turbulent Shear Stress

It is convenient to think of the turbulent shear stress as consisting of two parts: the *laminar component*, which accounts for the friction between layers in the flow direction (expressed as $\tau_{lam} = -\mu \frac{d\bar{u}}{dr}$), and the *turbulent component*, which accounts for the friction between the fluctuating fluid particles and the fluid body

(denoted as τ_{turb}) and is related to the fluctuation components of velocity). Then the total shear stress in turbulent flow can be expressed as

$$\tau_{total} = \tau_{lam} + \tau_{turb} \quad \dots\dots 1.2$$

The typical average velocity profile and relative magnitudes of laminar and turbulent components of shear stress for turbulent flow in a pipe are given in Figure 3.1.

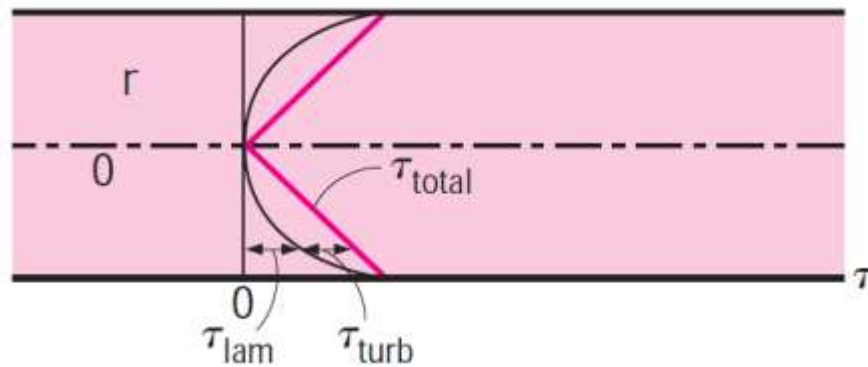


Figure 1.3: The velocity profile and the variation of shear stress with radial distance for turbulent flow in a pipe.

In many of the simpler turbulence models, turbulent shear stress is expressed in an analogous manner as suggested by the French mathematician *Joseph Boussinesq* (1842–1929) in 1877 as

$$\tau_{turb} = -\mu_t \frac{d\bar{u}}{dr} \quad \text{or} \quad \tau_{turb} = -\mu_t \frac{d\bar{u}}{dy} \quad \dots\dots 1.3$$

where μ_t is the eddy viscosity or turbulent viscosity, which accounts for momentum transport by turbulent eddies. Then the total shear stress can be expressed conveniently as

$$\tau_{total} = \tau_{lam} + \tau_{turb} = \mu \frac{d\bar{u}}{dy} + \mu_t \frac{d\bar{u}}{dy} = (\mu + \mu_t) \frac{d\bar{u}}{dy} \quad \dots\dots 1.4$$

$$\tau_{total} = \rho(\nu + \nu_t) \frac{d\bar{u}}{dy} \quad \dots\dots 1.5$$

where $\nu_t = \mu_t/\rho$ is the *kinematic eddy viscosity* or *kinematic turbulent viscosity* (also called the *eddy diffusivity of momentum*). The concept of eddy viscosity is

very appealing, but it is of no practical use unless its value can be determined. In other words, eddy viscosity must be modeled as a function of the average flow variables; we call this *eddy viscosity closure*. For example, in the early 1900s, the German engineer *L. Prandtl* introduced the concept of ***mixing length*** (l_m), which is related to the average size of the eddies that are primarily responsible for mixing, and expressed the turbulent shear stress as

$$\tau_{turb} = -\mu_t \frac{d\bar{u}}{dy} = \rho l_m^2 \left(\frac{d\bar{u}}{dy} \right)^2 \quad \dots\dots\dots 1.6$$

1.3. Turbulent Velocity Profile

Unlike laminar flow, the expressions for the velocity profile in a turbulent flow are based on both analysis and measurements, and thus they are semi-empirical in nature with constants determined from experimental data. Consider fully-developed turbulent flow in a pipe, and let u denote the time-averaged velocity in the axial direction.

Typical velocity profiles for fully developed laminar and turbulent flows are given in Figure 1.4. Note that the velocity profile is parabolic in laminar flow but is much fuller in turbulent flow, with a sharp drop near the pipe wall. Turbulent flow along a wall can be considered to consist of four regions, characterized by the distance from the wall. The very thin layer next to the wall where viscous effects are dominant is the ***viscous*** (or ***laminar*** or ***linear*** or ***wall***) sublayer. The velocity profile in this layer is very nearly ***linear***, and the flow is streamlined. Next to the viscous sublayer is the ***buffer layer***, in which turbulent effects are becoming significant, but the flow is still dominated by viscous effects. Above the buffer layer is the ***overlap*** (or ***transition***) ***layer***, also called the ***inertial sublayer***, in which the turbulent effects are much more significant, but still not dominant. Above that

is the *outer* (or *turbulent*) *layer* in the remaining part of the flow in which turbulent effects dominate over molecular diffusion (viscous) effects.

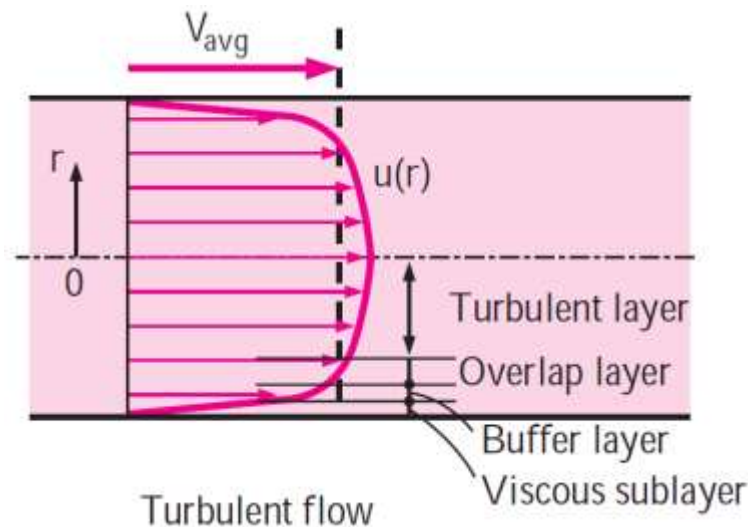


Figure 1.4: The velocity profile in fully developed pipe flow is parabolic in laminar flow, but much fuller in turbulent flow.

Then the velocity gradient in the viscous sublayer remains nearly constant at $du/dy = \tau_w/\mu$, and the wall shear stress can be expressed as

$$\tau_w = \mu \frac{u}{y} = \rho \nu \frac{u}{y} \quad \text{or} \quad \frac{\tau_w}{\rho} = \nu \frac{u}{y} \quad \dots 1.7$$

where y is the distance from the wall (note that $y = R - r$ for a circular pipe). The quantity τ_w/ρ is frequently encountered in the analysis of turbulent velocity profiles. The square root of τ_w/ρ has the dimensions of velocity, and thus it is convenient to view it as a fictitious velocity called the *friction velocity* expressed

as $u^* = \sqrt{\tau_w/\rho}$. Substituting this into Eq. 1.7, the velocity profile in the viscous

sublayer can be expressed in dimensionless form as

Viscous sublayer:
$$\frac{u}{u^*} = \frac{y u^*}{\nu}$$

This equation is known as the law of the wall, and it is found to satisfactorily correlate with experimental data for smooth surfaces for $0 \leq \frac{yu^*}{\nu} \leq 5$. Therefore, the thickness of the viscous sublayer is roughly

Thickness of viscous sublayer:
$$y = \delta_{\text{sublayer}} = \frac{5\nu}{u_*} = \frac{25\nu}{u_\delta}$$

where u_δ is the flow velocity at the edge of the viscous sublayer, which is closely related to the average velocity in a pipe. The quantity $\frac{\nu}{u^*}$ has dimensions of length and is called the viscous length; it is used to nondimensionalize the distance y from the surface. In boundary layer analysis, it is convenient to work with nondimensionalized distance and nondimensionalized velocity defined as

Nondimensionalized variables:
$$y^+ = \frac{yu_*}{\nu} \quad \text{and} \quad u^+ = \frac{u}{u_*}$$

Note that the friction velocity u^* is used to nondimensionalize both y and u , and y^+ resembles the Reynolds number expression.

Dimensional analysis indicates and the experiments confirm that the velocity in the overlap layer is proportional to the logarithm of distance, and the velocity profile can be expressed as

The logarithmic law:
$$\frac{u}{u_*} = \frac{1}{\kappa} \ln \frac{yu_*}{\nu} + B \quad \dots\dots 1.8$$

where k and B are constants whose values are determined experimentally to be about 0.40 and 5.0, respectively. Equation 1.8 is known as the logarithmic law. Substituting the values of the constants, the velocity profile is determined to be

Overlap layer:
$$\frac{u}{u_*} = 2.5 \ln \frac{yu_*}{\nu} + 5.0 \quad \text{or} \quad u^+ = 2.5 \ln y^+ + 5.0$$

A good approximation for the outer turbulent layer of pipe flow can be obtained by evaluating the constant B in Eq. 1.8 from the requirement that maximum velocity in a pipe occurs at the centerline where $r=0$. Solving for B from Eq. 1.8 by setting $y = R$ & $r = R$ and $u = u_{\max}$, and substituting it back into Eq. 1.8 together with $k = 0.4$ gives

Outer turbulent layer:
$$\frac{u_{\max} - u}{u_*} = 2.5 \ln \frac{R}{R - r}$$

The deviation of velocity from the centerline value $u_{\max} - u$ is called the *velocity defect*, and the above equation is called the *velocity defect law*.

1.4. The Moody Chart

The friction factor in fully developed turbulent pipe flow depends on the Reynolds number and the *relative roughness* (ε/D), which is the ratio of the mean height of roughness of the pipe, to the pipe diameter. The functional form of this dependence cannot be obtained from a theoretical analysis, and all available results are obtained from painstaking experiments using artificially roughened surfaces (usually by gluing sand grains of a known size on the inner surfaces of the pipes). Most such experiments were conducted by *Prandtl's student J. Nikuradse* in 1933, followed by the works of others. The friction factor was calculated from the measurements of the flow rate and the pressure drop.

The experimental results obtained are presented in tabular, graphical, and functional forms obtained by curve-fitting experimental data. In 1939, *Cyril F. Colebrook* (1910–1997) combined the available data for transition and turbulent flow in smooth as well as rough pipes into the following implicit relation known as the *Colebrook equation*:

Turbulent flow:
$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re} \sqrt{f}} \right) \dots 1.9$$

We note that the logarithm in Eq. 1.9 is a base 10 rather than a natural logarithm. In 1942, the American engineer *Hunter Rouse* (1906–1996) verified *Colebrook’s equation* and produced a graphical plot of f as a function of Re and the product $Re\sqrt{f}$. He also presented the laminar flow relation and a table of commercial pipe roughness. Two years later, **Lewis F. Moody** (1880–1953) redrew Rouse’s diagram into the form commonly used today. The now famous **Moody chart** is given in the appendix as Figure 1.5. It presents the Darcy friction factor for pipe flow as a function of the Reynolds number and ε/D over a wide range. It is probably one of the most widely accepted and used charts in engineering. Although it is developed for circular pipes, it can also be used for noncircular pipes by replacing the diameter by the hydraulic diameter. An approximate explicit relation for f was given by *S. E. Haaland* in 1983 as

$$\frac{1}{\sqrt{f}} \cong -1.8 \log \left[\frac{6.9}{Re} + \left(\frac{\varepsilon/D}{3.7} \right)^{1.11} \right] \quad \dots\dots\dots 1.10$$

The results obtained from this relation are within 2% of those obtained from the **Colebrook equation**. Equivalent roughness values for some commercial pipes are given in Table 1.1 as well as on the Moody chart.

Table 1.1: Equivalent roughness values for new commercial pipes.

Material	Roughness, ε	
	ft	mm
Glass, plastic	0 (smooth)	
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

We make the following observations from the Moody chart:

- ✓ For laminar flow, the friction factor decreases with increasing Reynolds number, and it is independent of surface roughness.
- ✓ The friction factor is a minimum for a smooth pipe (but still not zero because of the no-slip condition) and increases with roughness. The *Colebrook equation* in this case ($\varepsilon = 0$) reduces to the *Prandtl equation* expressed as $1/\sqrt{f} = 2.0 \log(\text{Re}\sqrt{f}) - 0.8$
- ✓ The transition region from the laminar to turbulent regime ($2300 < \text{Re} < 4000$) is indicated by the shaded area in the Moody chart. The flow in this region may be laminar or turbulent, depending on flow disturbances, or it may alternate between laminar and turbulent, and thus the friction factor may also alternate between the values for laminar and turbulent flow. The data in this range are the least reliable. At small relative roughnesses, the friction factor increases in the transition region and approaches the value for smooth pipes.
- ✓ At very large Reynolds numbers (to the right of the dashed line on the chart) the friction factor curves corresponding to specified relative roughness curves are nearly horizontal, and thus the friction factors are independent of the Reynolds number. The flow in that region is called *fully rough turbulent flow* or just *fully rough flow* because the thickness of the viscous sublayer decreases with increasing Reynolds number, and it becomes so thin that it is negligibly small compared to the surface roughness height. The viscous effects in this case are produced in the main flow primarily by the protruding roughness elements, and the contribution of the laminar sublayer is negligible. The Colebrook equation in the *fully rough zone* ($\text{Re} \rightarrow \infty$) reduces to the *von Kármán equation* expressed as which is explicit in f .

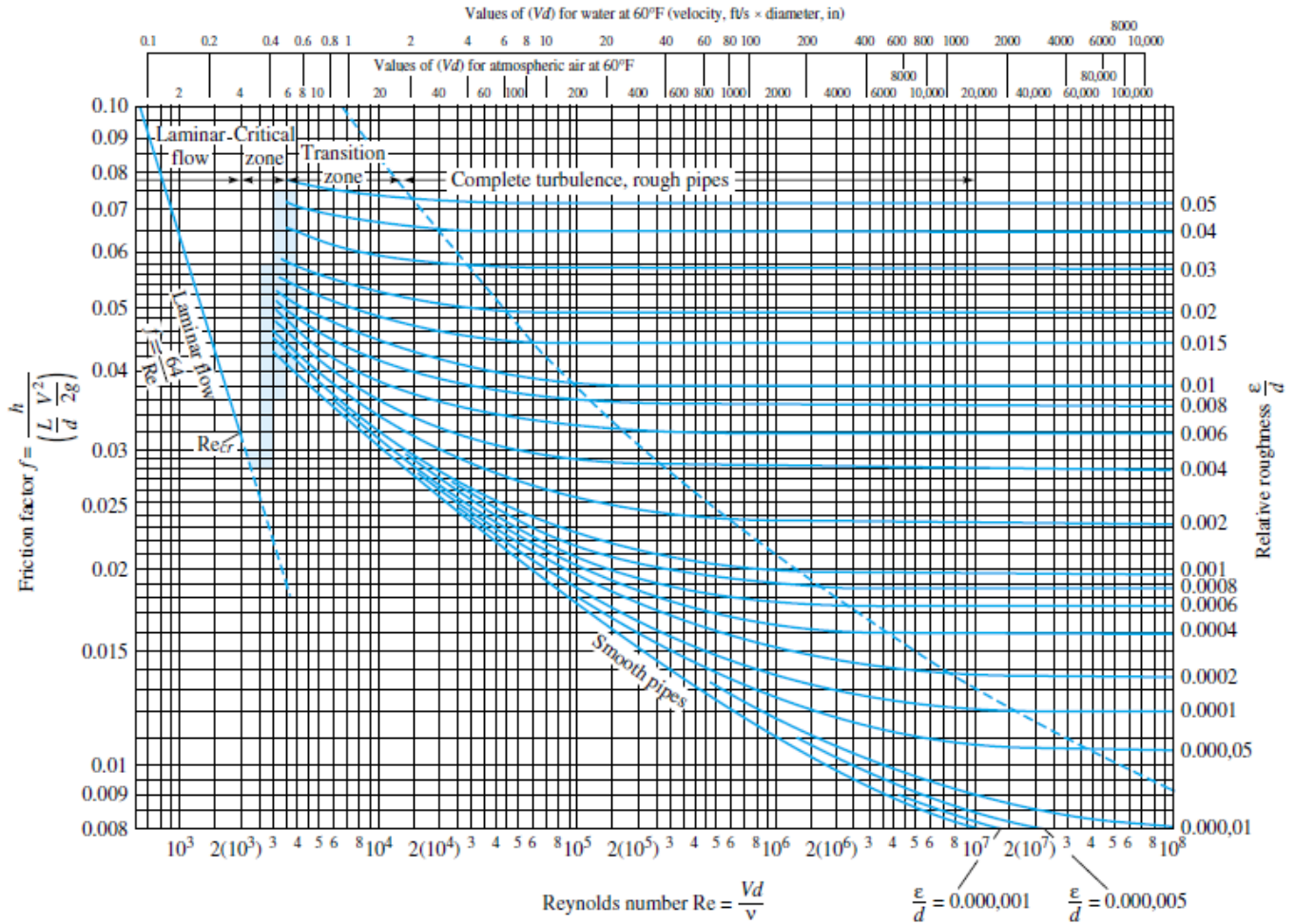


Figure 1.5: The Moody chart for the friction factor for fully developed flow.

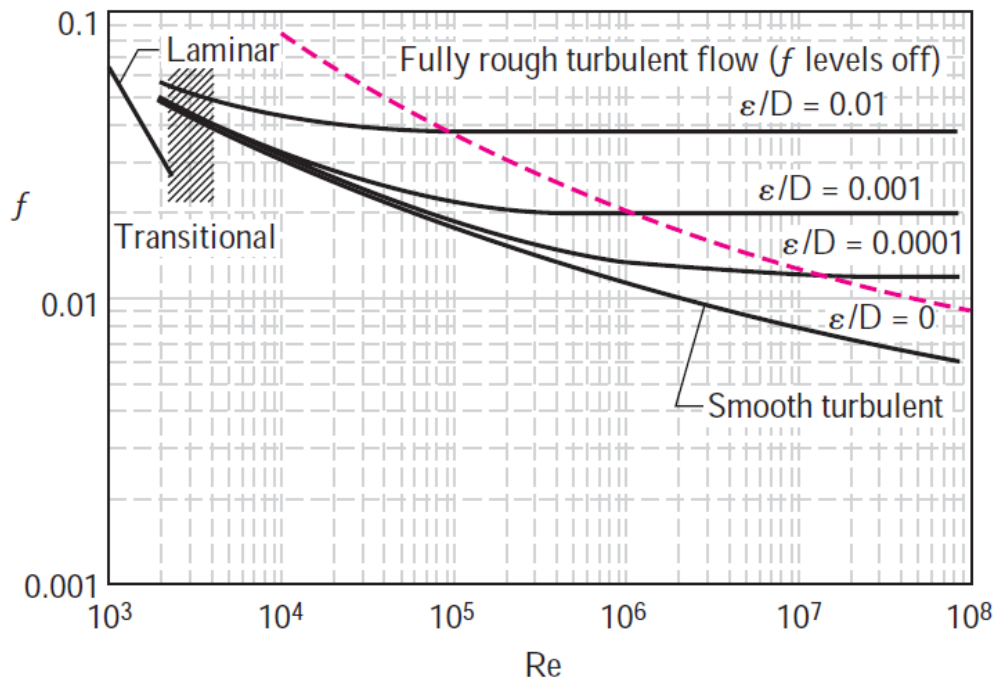


Figure 1.6: At very large Reynolds numbers, the friction factor curves on the Moody chart are nearly horizontal, and thus the friction factors are independent of the Reynolds number.

1.5. Types of Fluid Flow Problems

In the design and analysis of piping systems that involve the use of the Moody chart (or the *Colebrook equation*), we usually encounter three types of problems (the fluid and the roughness of the pipe are assumed to be specified in all cases).

1. Determining the **pressure drop** (or head loss) when the pipe length and diameter are given for a specified flow rate (or velocity)
2. Determining the **flow rate** when the pipe length and diameter are given for a specified pressure drop (or head loss)
3. Determining the **pipe diameter** when the pipe length and flow rate are given for a specified pressure drop (or head loss)

Problems of the *first type* are straightforward and can be solved directly by using the Moody chart. Problems of the *second type* and *third type* are commonly encountered in engineering design (in the selection of pipe diameter, for example, that minimizes the sum of the construction and pumping costs), but the use of the Moody chart with such problems requires an iterative approach unless an equation solver is used.

In problems of the *second type*, the diameter is given but the flow rate is unknown. A good guess for the friction factor in that case is obtained from the completely turbulent flow region for the given roughness. This is true for large Reynolds numbers, which is often the case in practice. Once the flow rate is obtained, the friction factor can be corrected using the Moody chart or the Colebrook equation, and the process is repeated until the solution converges. (Typically only a few iterations are required for convergence to three or four digits of precision.)

In problems of the *third type*, the diameter is not known and thus the Reynolds number and the relative roughness cannot be calculated. Therefore, we start calculations by assuming a pipe diameter. The pressure drop calculated for the assumed diameter is then compared to the specified pressure drop, and calculations are repeated with another pipe diameter in an iterative fashion until convergence.

To avoid tedious iterations in head loss, flow rate, and diameter calculations, *Swamee* and *Jain* proposed the following explicit relations in 1976 that are accurate to within 2% of the Moody chart:

$$h_L = 1.07 \frac{\dot{V}^2 L}{gD^5} \left\{ \ln \left[\frac{\epsilon}{3.7D} + 4.62 \left(\frac{\nu D}{\dot{V}} \right)^{0.9} \right] \right\}^{-2} \quad \begin{array}{l} 10^{-6} < \epsilon/D < 10^{-2} \\ 3000 < Re < 3 \times 10^8 \end{array}$$

$$\dot{V} = -0.965 \left(\frac{gD^5 h_L}{L} \right)^{0.5} \ln \left[\frac{\epsilon}{3.7D} + \left(\frac{3.17 \nu^2 L}{gD^3 h_L} \right)^{0.5} \right] \quad Re > 2000$$

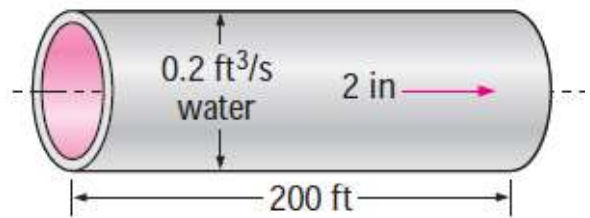
$$D = 0.66 \left[\epsilon^{1.25} \left(\frac{L \dot{V}^2}{g h_L} \right)^{4.75} + \nu \dot{V}^{9.4} \left(\frac{L}{g h_L} \right)^{5.2} \right]^{0.04} \quad \begin{array}{l} 10^{-6} < \epsilon/D < 10^{-2} \\ 5000 < Re < 3 \times 10^8 \end{array}$$

Examples:

Example 1:

Water ($\rho = 62.36 \text{ lbf/ft}^3$ and $\mu = 7.536 \times 10^{-4} \text{ lbf/ft}\cdot\text{s}$) is flowing steadily in a 2 in diameter horizontal pipe made of stainless steel at a rate of $0.2 \text{ ft}^3/\text{s}$ (see Figure below). Determine the pressure drop, the head loss, and the required pumping power input for flow over a 200 ft long section of the pipe.

Solution: We recognize this as a problem of the first type, since flow rate, pipe length, and pipe diameter are known. First we calculate the average velocity and the Reynolds number to determine the flow regime:



$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.2 \text{ ft}^3/\text{s}}{\pi (2/12 \text{ ft})^2/4} = 9.17 \text{ ft/s}$$

$$Re = \frac{\rho V D}{\mu} = \frac{(62.36 \text{ lbf/ft}^3)(9.17 \text{ ft/s})(2/12 \text{ ft})}{7.536 \times 10^{-4} \text{ lbf/ft}\cdot\text{s}} = 126,400$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is calculated using Table 1.1.

$$\epsilon/D = \frac{0.000007 \text{ ft}}{2/12 \text{ ft}} = 0.000042$$

The friction factor corresponding to this relative roughness and the Reynolds number can simply be determined from the *Moody chart*. To avoid any reading error, we determine f from the *Colebrook equation*:

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{0.000042}{3.7} + \frac{2.51}{126,400\sqrt{f}}\right)$$

Using an equation solver or an iterative scheme, the friction factor is determined to be $f = 0.0174$. Then the pressure drop (which is equivalent to pressure loss in this case), head loss, and the required power input become

$$\begin{aligned} \Delta P = \Delta P_L &= f \frac{L}{D} \frac{\rho V^2}{2} = 0.0174 \frac{200 \text{ ft}}{2/12 \text{ ft}} \frac{(62.36 \text{ lbf/ft}^3)(9.17 \text{ ft/s})^2}{2} \left(\frac{1 \text{ lbf}}{32.2 \text{ lbf} \cdot \text{ft/s}^2}\right) \\ &= \mathbf{1700 \text{ lbf/ft}^2} = \mathbf{11.8 \text{ psi}} \end{aligned}$$

$$h_L = \frac{\Delta P_L}{\rho g} = f \frac{L}{D} \frac{V^2}{2g} = 0.0174 \frac{200 \text{ ft}}{2/12 \text{ ft}} \frac{(9.17 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} = \mathbf{27.3 \text{ ft}}$$

$$\dot{W}_{\text{pump}} = \dot{V} \Delta P = (0.2 \text{ ft}^3/\text{s})(1700 \text{ lbf/ft}^2) \left(\frac{1 \text{ W}}{0.737 \text{ lbf} \cdot \text{ft/s}}\right) = \mathbf{461 \text{ W}}$$

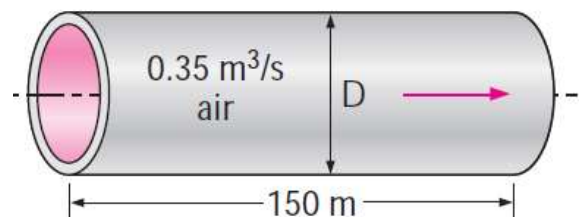
Example 2:

Heated air at 35°C is to be transported in a 150 m long circular plastic duct at a rate of 0.35 m³/s (see Figure below). If the head loss in the pipe is not to exceed 20 m, determine the minimum diameter of the duct.

Solution:

The density, dynamic viscosity and kinematic viscosity of air at 35°C are $\rho = 1.145 \text{ kg/m}^3$, $\mu = 1.895 \times 10^{-5} \text{ kg/m} \cdot \text{s}$, and $\nu = 1.655 \times 10^{-5} \text{ m}^2/\text{s}$.

the friction factor, and the head loss relations can be expressed as (D is in m, V is in m/s, and Re and f are dimensionless)



$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.35 \text{ m}^3/\text{s}}{\pi D^2/4}$$

$$Re = \frac{VD}{\nu} = \frac{VD}{1.655 \times 10^{-5} \text{ m}^2/\text{s}}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) = -2.0 \log\left(\frac{2.51}{Re\sqrt{f}}\right)$$

$$h_L = f \frac{L V^2}{D 2g} \quad \rightarrow \quad 20 = f \frac{150 \text{ m}}{D} \frac{V^2}{2(9.81 \text{ m/s}^2)}$$

The roughness is approximately zero for a plastic pipe (Table 1.1). Therefore, this is a set of four equations in four unknowns, and solving them with an equation solver such as EES gives

$$D = 0.267 \text{ m}, \quad f = 0.0180, \quad V = 6.24 \text{ m/s}, \quad \text{and} \quad Re = 100,800$$

Therefore, the diameter of the duct should be more than 26.7 cm if the head loss is not to exceed 20 m. Note that $Re > 4000$, and thus the turbulent flow assumption is verified.

The diameter can also be determined directly from the third *Swamee–Jain* formula to be

$$D = 0.66 \left[\varepsilon^{1.25} \left(\frac{L \dot{V}^2}{gh_L} \right)^{4.75} + \nu \dot{V}^{9.4} \left(\frac{L}{gh_L} \right)^{5.2} \right]^{0.04}$$

$$= 0.66 \left[0 + (1.655 \times 10^{-5} \text{ m}^2/\text{s})(0.35 \text{ m}^3/\text{s})^{9.4} \left(\frac{150 \text{ m}}{(9.81 \text{ m/s}^2)(20 \text{ m})} \right)^{5.2} \right]^{0.04}$$

$$= 0.271 \text{ m}$$

Example:

Liquid ammonia at -20°C is flowing through a 30 m long section of a 5 mm diameter copper tube at a rate of 0.15 kg/s. Determine the pressure drop, the head loss, and the pumping power required to overcome the frictional losses in the tube.

Solution:

The density and dynamic viscosity of liquid ammonia at -20°C are $\rho = 665.1 \text{ kg/m}^3$ and $\mu = 2.361 \times 10^{-4} \text{ kg/m}\cdot\text{s}$. The roughness of copper tubing is $1.5 \times 10^{-6} \text{ m}$.

First we calculate the average velocity and the Reynolds number to determine the flow regime:

$$V = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho(\pi D^2 / 4)} = \frac{0.15 \text{ kg/s}}{(665.1 \text{ kg/m}^3)[\pi(0.005 \text{ m})^2 / 4]} = 11.49 \text{ m/s}$$

$$Re = \frac{\rho V D}{\mu} = \frac{(665.1 \text{ kg/m}^3)(11.49 \text{ m/s})(0.005 \text{ m})}{2.361 \times 10^{-4} \text{ kg/m}\cdot\text{s}} = 1.618 \times 10^5$$

which is greater than $Re > 4000$. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon / D = \frac{1.5 \times 10^{-6} \text{ m}}{0.005 \text{ m}} = 3 \times 10^{-4}$$

The friction factor can be determined from the *Moody chart*, but to avoid the reading error, we determine it from the *Colebrook equation* using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{Re \sqrt{f}} \right) \rightarrow \frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{3 \times 10^{-4}}{3.7} + \frac{2.51}{1.618 \times 10^5 \sqrt{f}} \right)$$

It gives $f = 0.01819$. Then the pressure drop, the head loss, and the useful pumping power required become

$$\begin{aligned} \Delta P = \Delta P_L &= f \frac{L}{D} \frac{\rho V^2}{2} \\ &= 0.01819 \frac{30 \text{ m}}{0.005 \text{ m}} \frac{(665.1 \text{ kg/m}^3)(11.49 \text{ m/s})^2}{2} \left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2} \right) \left(\frac{1 \text{ kPa}}{1 \text{ kN/m}^2} \right) = 4792 \text{ kPa} \end{aligned}$$

$$h_L = \frac{\Delta P_L}{\rho g} = f \frac{L}{D} \frac{V^2}{2g} = 0.01819 \frac{30 \text{ m}}{0.005 \text{ m}} \frac{(11.49 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = \mathbf{734 \text{ m}}$$

$$\dot{W}_{\text{pump}} = \dot{V} \Delta P = \frac{\dot{m} \Delta P}{\rho} = \frac{(0.15 \text{ kg/s})(4792 \text{ kPa})}{665.1 \text{ kg/m}^3} \left(\frac{1 \text{ kW}}{1 \text{ kPa}\cdot\text{m}^3/\text{s}} \right) = \mathbf{1.08 \text{ kW}}$$