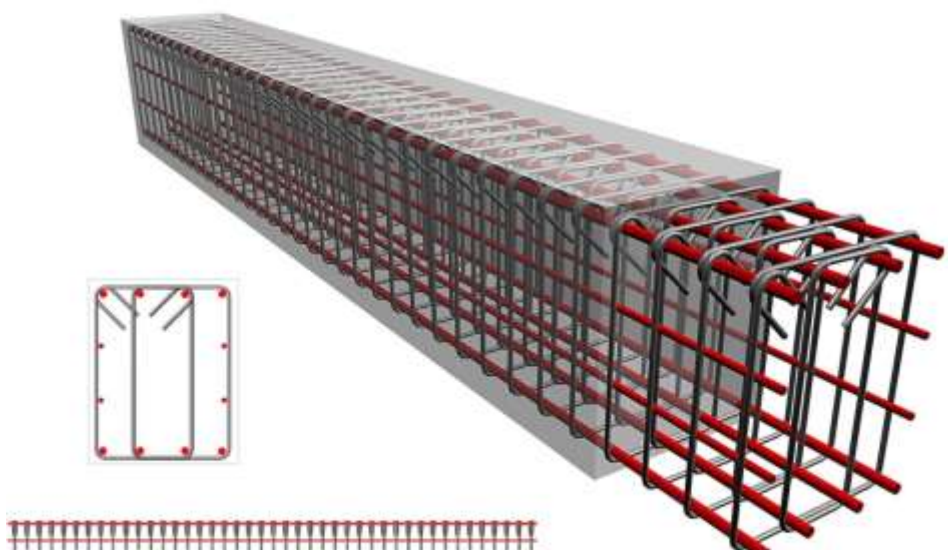
Design of Doubly Reinforced Rectangular Beam



In design of singly reinforced beams (ρ) is be taken equal to (ρ_{max}) to insure tension failure. When the cross-section of beam is limited because of Architectur reasons or service reasons and its resistance strength is not enough to withstand Applied Moment. In this case, the solution is by adding compression steel instead of an equaivalent tensile steel to keep the Neutral Axis (N.A.) in the same position in the case of ($\rho = \rho_{max}$) to ensure tensile failure.

To calculate the steel reinforcement of both, tension and compression the next procedure must be do.

1- Calculate the design moment from structural analysis.
2- Find (ρ_{max}) from equation or table (P3).
3- Find (ρ) value from equation or table (P4), and if $\rho \leq \rho_{max}$ then the section is singly beam designed as singly reinforced beam, or the s is doubly and it will be designed as the next steps.
4- Find maximum design moment (Mu_{max}) which will be genrate by Maximum allowd Steel reinforcement area. (As_{max}) and here will be call it (As_1), and we will Call Mu_{max} (Mu_1). for this case $\phi = 0.483 + 83.3\,\varepsilon_t = 0.816$

$$A_{s1} = \rho_{max} \cdot b \cdot d$$

We can use $\rho = \rho_t \Longrightarrow$ to ensure $\phi = 0.9$

$$a = \frac{As_1\, f_y}{0.85\, f'_c\, b}$$

$$M_{u1} = \phi M_n = \phi A_{s1} f_y \left(d - \frac{a}{2}\right)$$

5- Calculate design moment which withstand compression Steel (A'_s) and the equivelant tensile Steel reinforcement and the design moment must equal to :-

$$M_u = M_{u1} + M_{u2}$$

$$M_{u2} = M_u - M_{u1}$$

where :- M_u = design moment results from Str. analysis
M_{u1} = design moment results from tension reinforcement steel and concrete compressive
M_{u2} = design moment results from compression steel reinforcement and the equivelant tensile steel reinforcement.

6- Calculate compression steel stress.

$$c = a / \beta_1$$

$$\varepsilon'_s = \frac{c - d'}{c} \varepsilon_u$$

$$f'_s = E_s \varepsilon'_s = 600 \frac{c - d'}{c} \leq f_y$$

7- Calculate compression steel area from equilebrium eq.

$$A'_s = \frac{M_{u2}}{\phi f'_s (d - d')}$$

8- Calculate equivelant tensile steel area (balanced by comp. Steel)

$$A_{s2} f_y = A'_s f'_s \quad \therefore \quad A_{s2} = \frac{A'_s f'_s}{f_y}$$

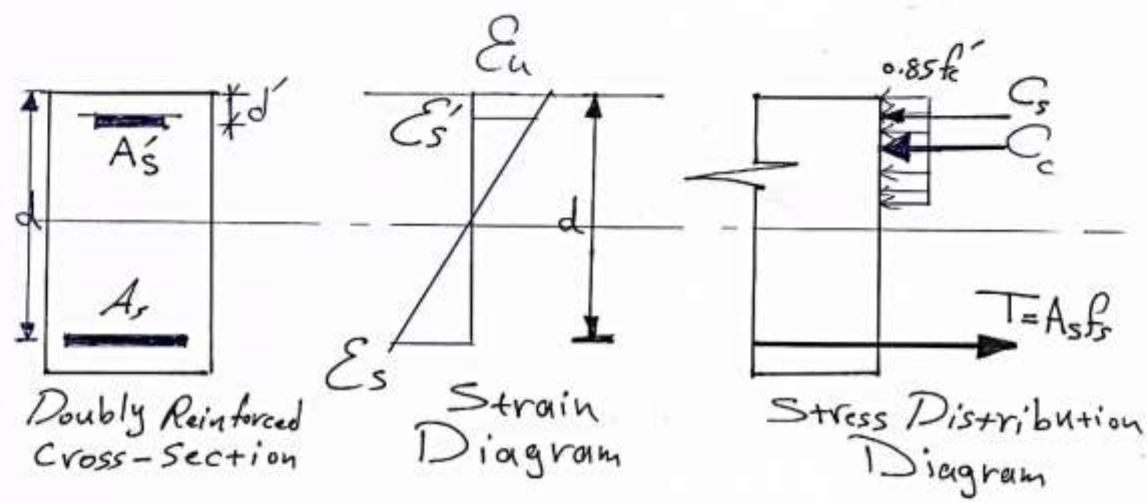
9- Find total area of tensile steel

$$A_s = A_{s1} + A_{s2}$$

10- Chose the diameter of steel reinforcement bar and find the number of these bars, then check the distances among bars according to ACI-Code requirements.

Analysis and Design of Doubly Reinforced Rectangular Beams

If a beam cross section is limited because of architectural or other considerations, it may happen that the concrete can not develop the compression force required to resist the given bending moment. In this case, reinforcement is added in the compression zone, resulting in a so-called doubly reinforced beam, i.e., one with compression as well as tension reinforcement.



A_s' : area of steel reinforcement for compression

$$\rho' = \frac{A_s'}{bd}$$

ρ_{cy}' : minimum tensile reinforcement ratio that will ensure yielding of the compression steel at failure.

$$\rho_{cy}' = 0.85\beta_1 \frac{600}{600-f_y} \cdot \frac{f_c'}{f_y} \cdot \frac{d'}{d} + \rho'$$

if $\rho \geq \rho_{cy}'$ $\Rightarrow$ Compression Steel yield at failure.

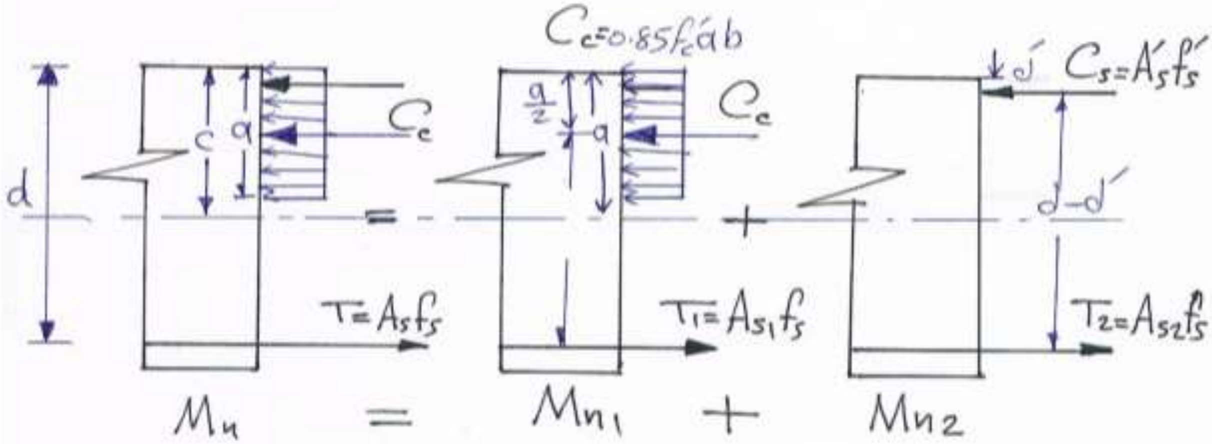
if $\rho < \rho_{cy}'$ $\Rightarrow$ Compression Steel Stress will not reach (f_y) at failure.

Compression reinforcement may be added according to another consideration like:-

- Decreasing the deflection resulting from creep.
- Fixing the shear reinforcement.
- Resistance of tensile force resulting from changing of moment.

Analysis of Doubly Reinforced Rectangular Beams:-

a- Tension and Compression Steel Both at Yield Stress:-



$$A'_s f_y = A_{s2} f_y \qquad \therefore A'_s = A_{s2}$$

$$A_{s1} = A_s - A_{s2} \qquad \therefore A_{s1} = A_s - A'_s$$

from force equilibrium:

$$A_{s1} f_y = 0.85 f'_c a b \Longrightarrow a = \frac{A_{s1} f_y}{0.85 f'_c b}$$

$$a = \frac{(A_s - A'_s) f_y}{0.85 f'_c b}$$

$$M_n = M_{n1} + M_{n2} = A_{s1} f_y \left(d - \frac{a}{2}\right) + A_{s2} f_y (d - d')$$

$$M_n = M_{n1} + M_{n2} = 0.85 f'_c a b \left(d - \frac{a}{2}\right) + A'_s f_y (d - d')$$

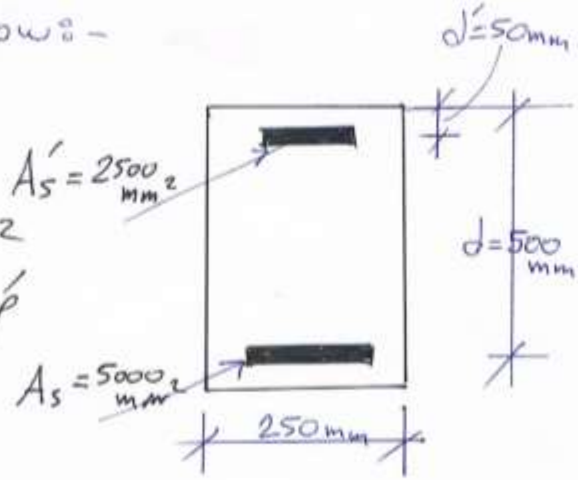
ρ'_b : balanced reinforcement ratio for doubly reinforced beam

$$\rho'_b = \rho_b + \rho'$$

ρ_b : balanced reinforcement ratio for corresponding singly reinforced beam

$$\rho'_{max} = \rho_{max} + \rho'$$

Ex. :- Find the nominal moment for the cross section of doubly rectangular reinforced concrete beam shown in the figure below:-



$f_y = 350\,MPa$, $f'_c = 30\,MPa$

Solution

$$\rho = \frac{5000}{250 \times 500} = 0.04, \quad \rho' = \frac{2500}{500 \times 250} = 0.02$$

$$\rho'_{cy} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{d'}{d} \cdot \frac{600}{600 - f_y} + \rho'$$

$$\rho'_{cy} = 0.0349$$

$$\rho_b = 0.85 \beta_1 \frac{f'_c}{f_y} \cdot \frac{600}{600 + f_y}$$

$$\rho_b = 0.039$$

$\rho = 0.04 > \rho'_{cy} = 0.0349$ ∴ both tension & compression steel at yield stress

$$\rho'_b = \rho_b + \rho' = 0.059$$

$$\rho = 0.04 < \rho'_b, \quad a = \frac{(5000 - 2500) \times 350}{0.85 \times 30 \times 250} = 137.254\ mm$$

$$\therefore M_n = 2500 \times 350 \left(500 - \frac{137.254}{2}\right) + 2500 \times 350 (500 - 50)$$

$M_n = 771.2 \times 10^6 \ N.mm$
$= 771.2 \ kN.m$

b- Compression Steel below Yield Stress

$\rho < \rho'_{cy} \longrightarrow$ Compression steel will not reach f_y

$$\rho'_b = 0.85\beta_1 \frac{f'_c}{f_y} \frac{600}{600+f_y} + \rho' \frac{f'_s}{f_y}$$

$$\rho'_b = \rho_b + \rho' \frac{f'_s}{f_y}$$

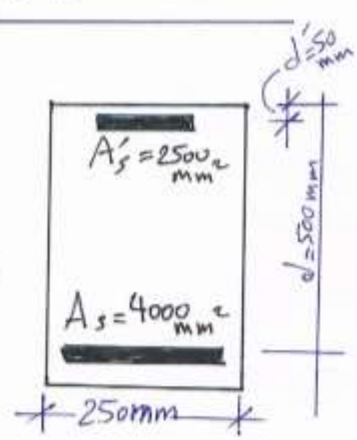
$\rho \leq \rho'_b \Longrightarrow$ Tension steel will yield

- Find (C) $\quad k_1 = \dfrac{A_s f_y - 600A'_s}{0.85\beta_1 f'_c b}, \quad k_2 = \dfrac{600 A'_s d'}{0.85\beta_1 f'_c b}$

$$C = \frac{k_1 + \sqrt{k_1^2 + 4k_2}}{2}, \quad f'_s = \frac{c-d}{c}$$

- Find $M_n = 0.85 f'_c a b (d - \frac{a}{2}) + A'_s f'_s (d - d')$

E.X. :- Find the nominal moment for cross-section of doubly rect. reinforced concrete beam shown in the figure, if, $f'_c = 30MPa, f_y = 350MPa$



Solution:-

$$\rho = \frac{4000}{250\times500} = 0.032 \quad , \quad \rho' = \frac{A'_s}{250\times500} = 0.02$$

$$\rho_b = 0.85\,\beta\,\frac{f'_c}{f_y}\,\frac{600}{600+f_y} = 0.85^2 \cdot \frac{30}{350} \cdot \frac{600}{600+350}$$

$$\rho_b = 0.039$$

$$\rho'_{cy} = 0.85\,\beta_1\,\frac{600}{600-f_y}\,\frac{f'_c}{f_y}\,\frac{d'}{d} + \rho' = 0.0349$$

$$\rho = 0.032 < \rho'_{cy} = 0.0349$$

$\therefore$ Compression Steel will not reach f_y

Check Tension Steel :-

$(\rho'_b) = ?$

find $f'_s \Rightarrow f'_s = 600 - (600+f_y)\frac{d'}{d}$

$$= 600 - (600+350)*\frac{50}{500} = 505\,MPa > f_y = 350\,MPa$$

$\therefore f'_s = 350\,MPa$

$$\rho'_b = \rho_b + \rho' \cdot \frac{f'_s}{f_y} = 0.039 * \frac{1}{1} + 0.02 = 0.059$$

$\because \rho < \rho'_b \therefore$ Tension Steel reach f_y (yield)

Find (c) value

$$K_1 = \frac{A_s f_y - 600 A'_s}{0.85\,\beta_1 f'_c\, b} = \frac{4000(350) - 600*(2500)}{(0.85)^2 \cdot 30 \cdot 250} = -18.45$$

$$K_2 = \frac{600 A'_s d'}{0.85\,\beta_1 f'_c\, b} = \frac{600*2500*50}{(0.85)^2 \cdot 30 \cdot 250} = 13840.83$$

$$C = \frac{k_1 + \sqrt{k_1^2 + 4k_2}}{2} = \frac{-18.45 + \sqrt{(-18.45)^2 + 4 \times 13840.83}}{2} = 108.8 \text{ mm}$$

$$a = \beta_1 c = 0.85 * 108.8 = 92.5 \text{ mm}$$

$$f_s' = \frac{c - d'}{c} 600 = \frac{(108.8 - 50)}{108.8} * 600 = 324.3 \text{ MPa}$$

$$M_n = 0.85 f_c' a b \left(d - \frac{a}{2}\right) + A_s' f_s' (d - d')$$

$$= 0.85 \times 30 \times 92.5 \times 250 \left(500 - \frac{92.5}{2}\right) + 2500 \times 324.3 (500 - 50)$$

$$M_n = 632.408 * 10^6 \text{ N·mm}$$

$$= 632.4 \text{ kN·m}$$

C- Tensile steel below the yield stress

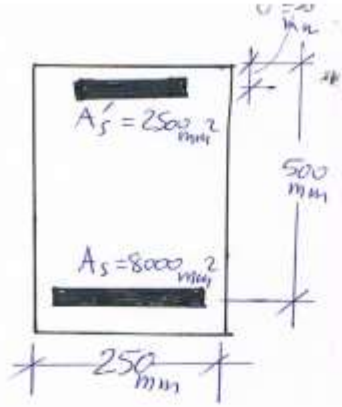
In this case $\rho > \rho_b'$ then we must find (C) by the following equation:-

$$A_s * \frac{(d-c)}{c} 600 = 0.85 \beta_1 f_c' C b + A_s' * 600 * \frac{(c - d')}{c}$$

Then find f_s', f_s

$$M_n = 0.85 f_c' a b \left(d - \frac{a}{2}\right) + A_s' f_s' (d - d')$$

EX:- Find the nominal moment for cross-section of doubly rectangular reinforced beam shown below, for the following data: $f'_c = 30\ MPa$, $f_y = 350\ MPa$



Solution:-

$$\rho = \frac{8000}{250 \times 500} = 0.064 \quad , \quad \rho' = \frac{A'_s}{bd} = 0.02$$

$$\rho_b = 0.039 \quad , \quad \rho'_{cy} = 0.0349$$

$$\rho'_b = \rho_b + \rho' = 0.059$$

$\rho = 0.064 > \rho'_b = 0.059$ ∴ The failure will be compression failure

$\rho = 0.064 > \rho'_{cy}$ ∴ Compression steel will reach f_y i.e. $f'_s = f_y$

Find (C):-

$$A_s \frac{(d-C)}{C} (600) = 0.85 \beta_1 f'_c C b + A'_s f_y$$

$$8000 \cdot \frac{(500-C)}{C} \cdot (600) = (0.85)^2 * 30 * C * 250 + 2500 * 350$$

$C = 323mm$ (الحل بواسطة طريقة الدستور)

$$f_s = 600\left(\frac{500-C}{C}\right) = 600\left(\frac{500-323}{328}\right) = 328.8\ MPa$$

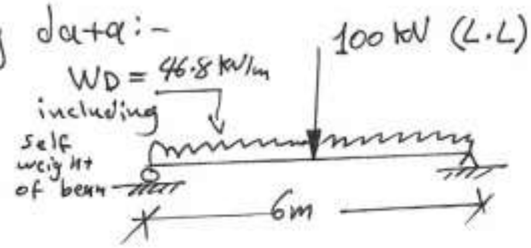
$$a = \beta_1 C = 0.85 * 323 = 274.6\ mm$$

$$M_n = 0.85 f'_c a\, b\left(d - \frac{a}{2}\right) + A'_s f_y (d - d')$$

$$M_n = 1028.7\ kN.m$$

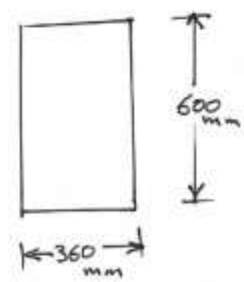
Design:-

Ex.:- For a simply supported beam shown in the Fig. shown below, Find the area of Steel & its details for the following data:-



$f_y = 400$ Mpa , $f'_c = 20$ MPa

Note:- If there is need for compression steel use $d' = 65$ mm.



Solution:- Assume 2 layers of steel Reinf.

* $d = h - 100$
  $= 500$ mm

* $P_u = 100 * 1.6 = 160$ kN

* $W_u = 46.8 * 1.2 = 56.16$ kN/m

• $M_u = \frac{P_u * L}{4} + \frac{W_u * L^2}{8}$

$= 160 * \frac{6}{4} + 56.16 * \frac{(6)^2}{8} = 492.72$ kN.m

• from Table (r3) $\rho_{max} = 0.0155$

• $R = \frac{M_u}{\phi b d^2}$, $m = \frac{f_y}{0.85 f'_c}$

$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{f_y}}\right)$

$m = \frac{400}{0.85 * 20} = 23.5$, $R = \frac{492.72 * 10^6}{0.9 * 360 * (500)^2} = 6.08$

$\rho = \frac{1}{23.5}\left(1 - \sqrt{1 - \frac{2 * 23.5 * 6.08}{400}}\right) = 0.0198$

$\because \rho > \rho_{max} \Longrightarrow$ Design the beam as a Doubly Reinforced Beam.

- Find A_{s1}

$A_{s1} = \rho_{max}\, b\, d = 0.155 * 360 * 500 = 2790\ mm^2$

$a = \frac{A_{s1} f_y}{0.85 f'_c b} = \frac{2790 * 400}{0.85 * 20 * 360} = 182\ mm$

$M_{u1} = \phi M_{n1} = \phi A_{s1} f_y (d - \frac{a}{2})$

$\because \rho > \rho_{max} \longrightarrow \therefore \rho > \rho_t$

$\therefore \phi < 0.9$

$\phi = 0.483 + 83.3 \varepsilon_t = 0.483 + 83.3 * 0.004$
$= 0.816$

$\therefore M_{u1} = 0.816 * 2790 * 400 [500 - \frac{182}{2}] * 10^{-6} = 372.5\ kN.m$

- Find M_{u2}

$M_{u2} = M_u - M_{u1} = 492.72 - 372.5 = 120\ kN.m$

- Calculate the compressive Steel Reinf. Stress.

$C = a/\beta_1 = \frac{182}{0.85} \approx 214\ mm$

$$f'_s = \frac{c-d'}{c}\,600 = \left(\frac{214-65}{65}\right) * 600 = 418 \text{ MPa} > f_y = 400 \text{ MPa}$$

- Finding area of compressive steel reinf.

$$A'_s = \frac{M_{u2}}{\phi f'_s (d-d')} = \frac{120.22 * 10^6}{0.816 * 400 (500-65)} = 847 \text{ mm}^2$$

- Area of tension steel insted of comp. steel
  (مساحة حديد السحب الذي يقابل حديد الانضغاط)

$$A_{s2} = A'_s = 847 \text{ mm}^2$$

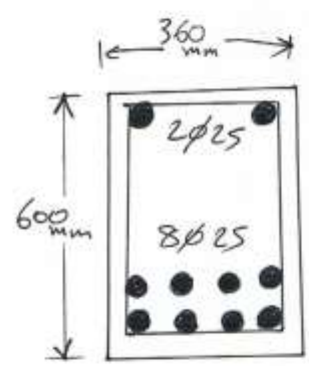
- Total Tension Steel Reinforcement

$$A_s = A_{s1} + A_{s2} = 2790 + 847 = 3637 \text{ mm}^2$$

- Use $\phi 25$ → No of bars $= \frac{3637}{\frac{\pi}{4} * (25)^2} = 7.4$

∴ Use $8 \phi 25$

$$S = \frac{360-100-4*25}{3} = 53 \text{ mm} > 25 \text{ mm} \quad \therefore \text{ o.k.}$$



for Steel Reinf. in Comp Zone

$A'_s = 847$ No of bars $= \frac{847}{491} = 1.725$

∴ Use $2 \phi 25$

.: A rectangular beam has width 250mm & Effective depth 460mm. $f_y = 300$ MPa, $f'_c = 20$ MPa. What is the maximum moment that can be utilized in design, according to the ACI Code,

...en: a- $A_s = 2000\ mm^2$ b- $A_s = 5160\ mm^2$

...tion :-

$$\rho_b = 0.85\beta_1 \frac{f'_c}{f_y} \frac{600}{600+f_y} = (0.85)^2 \times \frac{20}{300} \times \frac{600}{600+300}$$

$\rho_b = 0.032$ or from Table (A-3) Page 350

$$\rho = \frac{A_s}{bd} = \frac{2000}{250 * 460} = 0.0174 < \rho_b = 0.032$$

∴ The section is underreinforced

To calculate ϕ value we must find ρ_t

$$\rho_t = 0.85\beta_1 \frac{f'_c}{f_y} \frac{0.003}{0.003 + \varepsilon_t} = (0.85)^2 * \frac{20}{300} * \frac{0.003}{0.003+0.005}$$

$\rho_t = 0.018$ or from Table (A-3), Page 350

$\rho = 0.0174 < 0.0180$ ∴ $\phi = 0.9$

$M_u = \phi M_n$

$$M_u = \phi \rho b d^2 f_y \left[1 - 0.59 \frac{\rho f_y}{f'_c}\right]$$

$$M_u = 0.9 * 0.0174 * 250 * 460^2 * 300 * \left[1 - \frac{0.59 * 0.0174 * 300}{20}\right]$$

$= 210,253,958$ N.mm

or

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{2000 * 300}{0.85 * 20 * 250} = 141.176 \text{ mm}$$

$$M_u = \phi A_s f_y \left(d - \frac{a}{2}\right) = 0.9 * 2000 * 300 \left(460 - \frac{141.176}{2}\right)$$

$$= 210,282,480 \text{ N.mm}$$

$$\simeq 210.282 \text{ kN.m}$$

or $$M_u = 0.85 * \phi f'_c a b \left(d - \frac{a}{2}\right)$$

$$= 0.85 * 0.9 * 20 * 141.176 * 250 \left(460 - \frac{141.176}{2}\right)$$

$$= 210,281,779 \text{ N.mm} \simeq 210.281 \text{ kN.m}$$

b- $A_s = 5160 \text{ mm}^2$ $\quad \rho = \frac{A_s}{bd} = \frac{5160}{250 * 460} = 0.04487 \simeq 0.045$

$\rho_b = 0.032$ $\quad \rho = 0.045 > \rho_b = 0.032$

∴ The beam is over reinforced

1- find $m = \frac{600}{0.85 \beta_1 f'_c} = \frac{600}{0.85 * 0.85 * 20} = 41.522$

$\rho * m = 0.045 * 41.522 = 1.869$

$$k_u = \sqrt{\left(\frac{\rho m}{2}\right)^2 + \rho m} - \frac{\rho m}{2}$$

$$= \sqrt{\left(\frac{1.869}{2}\right)^2 + 1.869} - \frac{1.869}{2} = 0.721$$

3- Find c

$C = k_u d = 0.721 * 460 = 331.66\ mm$

$\simeq 331.7\ mm$

4- Find a $\quad a = 0.85 C = 0.85 * 331.7 = 281.945\ mm$

($0.85 = \beta_1$)

$a \simeq 282\ mm$

5- Find M_n $\quad M_n = 0.85 f'_c * a * b(d - \frac{a}{2})$

$= 0.85 * 20 * 282 * 250 * (460 - \frac{282}{2})$

$= 382,321,500\ N.mm$

$= 382.321 \times 10^6\ N.mm = 382.321\ kN.m$