University of Anbar
College of Engineering
Mechanical Engineering Dept.

# Fluid Mechanics

## Handout Lectures for Year Two
## Chapter Three/ Fluid Kinematics

**Course Tutor**

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# Chapter Three
# Fluid Kinematics

## 1.1 FUNDAMENTALS OF FLOW VISUALIZATION

While quantitative study of fluid dynamics requires advanced mathematics, much can be learned from flow visualization—the visual examination of flow field features. Flow visualization is useful not only in physical experiments (Fig. 3–1), but in numerical solutions as well [computational fluid dynamics (CFD)]. In fact, the very first thing an engineer using CFD does after obtaining a numerical solution is simulate some form of flow visualization, so that he or she can see the "whole picture" rather than merely a list of numbers and quantitative data. Why? Because the human mind is designed to rapidly process an incredible amount of visual information; as they say, a picture is worth a thousand words. There are many types of flow patterns that can be visualized, both physically (experimentally) and/or computationally.

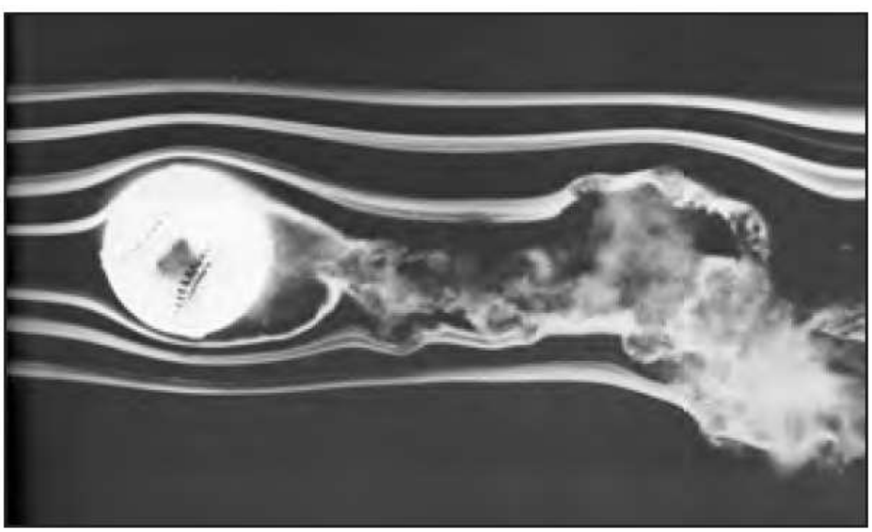


**Figure 3.1:** Spinning baseball.

### 1.2 Streamlines

A ***streamline*** is a curve that is everywhere tangent to the instantaneous local velocity vector.

Streamlines are useful as indicators of the instantaneous direction of fluid motion throughout the flow field. For example, regions of recirculating flow and separation of a fluid off of a solid wall are easily identified by the streamline pattern. Streamlines cannot be directly observed experimentally except in steady flow fields, in which they are coincident with pathlines and streaklines, to be discussed next. Mathematically, however, we can write a simple expression for a streamline based on its definition.

Consider an infinitesimal arc length $d\vec{r} = dx\vec{i} + dy\vec{j} + dz\vec{k}$ along streamline; $d\vec{r}$ must be parallel to the local velocity vector $\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$ by definition of the streamline. By simple geometric arguments using similar triangles, we know that the components of $d\vec{r}$ must be proportional to those of $\vec{V}$ (Fig. 3–2). Hence,

Equation for a Streamline:

$$\frac{dr}{V} = \frac{dr}{u} = \frac{dr}{v} = \frac{dr}{w} \qquad (3\text{–}1)$$

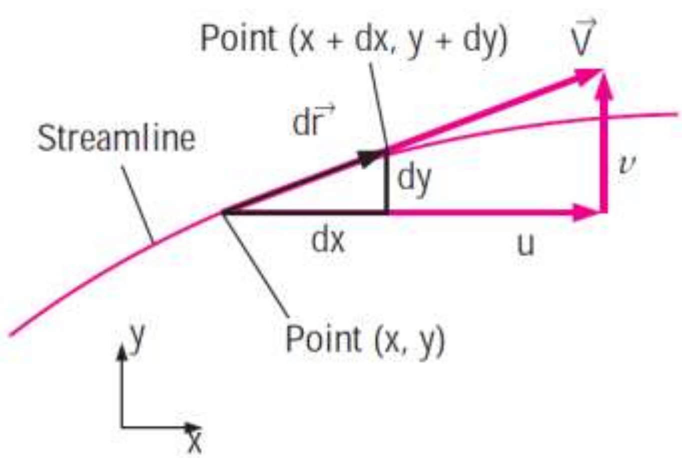



**Figure 3.2:** Two-dimensional flow in the *xy*-plane, arc length $d\vec{r}$= (d*x*, d*y*) along a streamline is everywhere tangent to the local instantaneous velocity vector $\vec{V}$ = (*u*, *v*).

where $dr$ is the magnitude of $d\vec{r}$ and $V$ is the speed, the magnitude of $\vec{V}$. Equation 3–1 is illustrated in two dimensions for simplicity in Fig. 3–2. For a known velocity field, we can integrate Eq. 3–1to obtain equations for the streamlines. In two dimensions, (x, y), (u, v), the following differential equation is obtained:

Streamline in the xy-plane: $\left(\frac{dy}{dx}\right)_{along\ a\ \text{Streamline}} = \frac{v}{u}$ (3–2)

In some simple cases, Eq. 3–2 may be solvable analytically; in the general case, it must be solved numerically. In either case, an arbitrary constant of integration appears, and the family of curves that satisfy Eq. 3–2 represents streamlines of the flow field.

**EXAMPLE 4–4 Streamlines in the *xy*-Plane—An Analytical Solution**

For the steady, incompressible, two-dimensional velocity field of Example 4–1, plot several streamlines in the right half of the flow ($x > 0$) and compare to the velocity vectors plotted in Fig. 4–4.

**SOLUTION** An analytical expression for streamlines is to be generated and plotted in the upper-right quadrant.

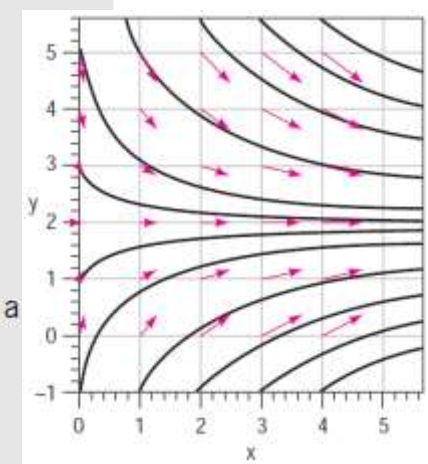
***Assumptions*** **1** The flow is steady and incompressible. **2** The flow is two-dimensional, implying no *z*-component of velocity and no variation of *u* or *v* with *z*.

***Analysis*** Equation 4–16 is applicable here; thus, along a streamline,

$$\frac{dy}{dx} = \frac{1.5 - 0.8y}{0.5 + 0.8x}$$

We solve this differential equation by separation of variables:

$$\frac{dy}{1.5 - 0.8y} = \frac{dx}{0.5 + 0.8x} \quad \rightarrow \quad \int \frac{dy}{1.5 - 0.8y} = \int \frac{dx}{0.5 + 0.8x}$$




After some algebra (which we leave to the reader), we solve for *y* as a tion of *x* along a streamline,

$$y = \frac{C}{0.8(0.5 + 0.8x)} + 1.875$$

where *C* is a constant of integration that can be set to various values in order to plot the streamlines. Several streamlines of the given flow field are shown in Fig. 4–17.

### 1.3 Pathlines

A ***pathline*** is the actual path traveled by an individual fluid particle over some time period.

Pathlines are the easiest of the flow patterns to understand. A pathline is a Lagrangian concept in that we simply follow the path of an individual fluid particle as it moves around in the flow field (Fig. 3–3). Thus, a pathline is the same as the fluid particle's material position vector ($x_{particle}(t)$, $y_{particle}(t)$, $z_{particle}(t)$), traced out over some finite time interval. In a physical experiment, you can imagine a tracer fluid particle that is marked somehow—either by color or brightness—such that it is easily distinguishable from surrounding fluid particles. Now imagine a camera with the shutter open for a certain time period, $t_{start} > t > t_{end}$, in which the particle's path is recorded; the resulting curve is called a pathline. An intriguing example is shown in Fig. 3–3 for the case of waves moving along the surface of water in a tank. Neutrally buoyant white **tracer particles** are suspended in the water, and a time-exposure photograph is taken for one complete wave period. The result is pathlines that are elliptical in shape, showing that fluid particles bob up and down and forward and backward, but return to their original position upon completion of one wave period; there is no net forward motion. You may have experienced something similar while bobbing up and down on ocean waves.

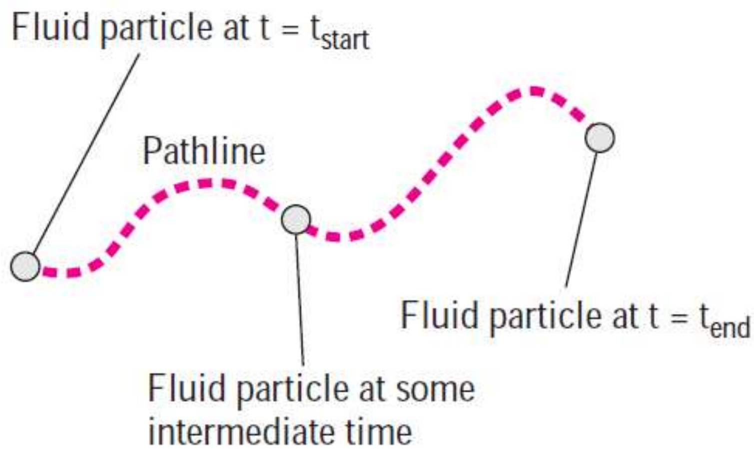



**Figure 3.3:** A pathline is formed by following the actual path of a fluid particle.