University of Anbar
College of Engineering
Mechanical Engineering Dept.

Fluid Mechanics

Handout Lectures for Year Two
Chapter Five/ Flow i n pipes

Course Tutor

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Chapter Five
Flow i n pipes

5.1. LAMINAR AND TURBULENT FLOWS

The flow regime in the first case is said to be ***laminar***, characterized by smooth streamlines and highly ordered motion, and ***turbulent*** in the second case, where it is characterized by velocity fluctuations and highly disordered motion. The ***transition*** from laminar to turbulent flow does not occur suddenly; rather, it occurs over some region in which the flow fluctuates between laminar and turbulent flows before it becomes fully turbulent. Most flows encountered in practice are turbulent. Laminar flow is encountered when highly viscous fluids such as oils flow in small pipes or narrow passages as shown in Figure 5.1.

We can verify the existence of these laminar, transitional, and turbulent flow regimes by injecting some dye streaks into the flow in a glass pipe, as the British engineer ***Osborne Reynolds*** (1842–1912) did over a century ago. We observe that the dye streak forms a ***straight and smooth line at low velocities*** when the flow is laminar (we may see some blurring because of molecular diffusion), has bursts of ***fluctuations in the transitional regime***, and ***zigzags rapidly and randomly*** when the flow becomes fully turbulent. These zigzags and the dispersion of the dye are indicative of the fluctuations in the main flow and the rapid mixing of fluid particles from adjacent layers.

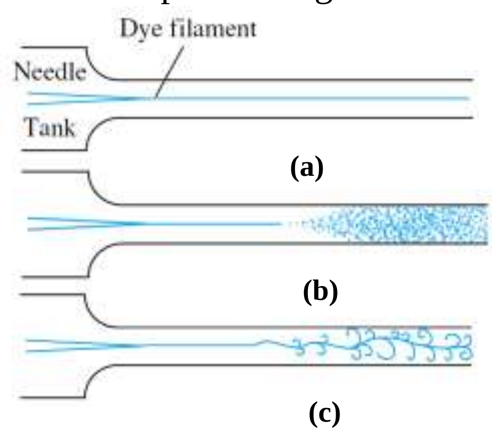


Figure 5.1: Spinning Reynolds' sketches of pipe-flow transition: (a) low-speed, laminar flow; (b) high-speed, turbulent flow; (c) spark photograph of condition (b).

5.2. Reynolds Number

After exhaustive experiments in the 1880s, Osborne Reynolds discovered that the flow regime depends mainly on the ratio of ***inertial forces* to *viscous forces*** in the fluid. This ratio is called the ***Reynolds number*** and is expressed for internal flow in a circular pipe as,

$$\mathrm{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{avg} D}{\nu} = \frac{\rho V_{avg} D}{\mu}$$

where V_{avg}= average flow velocity (m/s), D= characteristic length of the geometry (diameter in this case, in m), and $\upsilon = \mu/\rho$= kinematic viscosity of the fluid (m^2/s). Note that the Reynolds number is a ***dimensionless*** quantity. Also, kinematic viscosity has the unit m^2/s, and can be viewed as ***viscous diffusivity*** or ***diffusivity for momentum***.

The Reynolds number at which the flow becomes turbulent is called the ***critical Reynolds number,*** Re_{cr}. The value of the critical Reynolds number is different for different geometries and flow conditions. For internal flow in a circular pipe, the generally accepted value of the critical Reynolds number is Re_{cr}= 2300.

For flow through noncircular pipes, the Reynolds number is based on the hydraulic diameter D_h defined as (Figure 5.2),

Hydraulic diameter: $$D_h = \frac{4A_c}{p}$$

where A_c is the cross-sectional area of the pipe and p is its wetted perimeter. The hydraulic diameter is defined such that it reduces to ordinary diameter D for circular pipes,

Circular pipes: $$D_h = \frac{4A_c}{p} = \frac{4(\pi D^2/4)}{\pi D} = D$$

Square duct: $$D_h = \frac{4a^2}{4a} = a$$

Rectangular duct: $$D_h = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$$

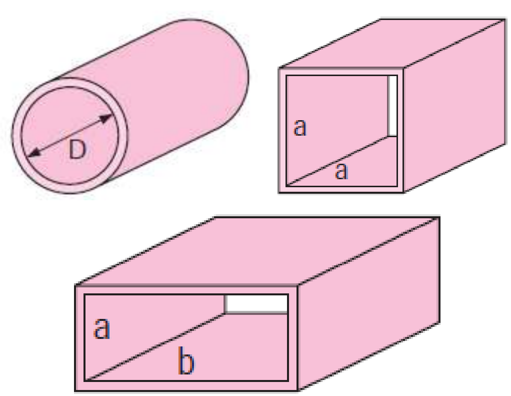


Figure 5.2

Under most practical conditions, the flow in a circular pipe is laminar for Re ≤ 2300, turbulent for Re ≥ 4000, and transitional in between. That is,

$$\begin{aligned} Re &\lesssim 2300 && \text{laminar flow} \\ 2300 \lesssim Re &\lesssim 4000 && \text{transitional flow} \\ Re &\gtrsim 4000 && \text{turbulent flow} \end{aligned}$$

5.3. LAMINAR FLOW IN PIPES

We mentioned in Section 5.2. that flow in pipes is laminar for Re ≤ 2300, and that the flow is fully developed if the pipe is sufficiently long (relative to the entry length) so that the entrance effects are negligible.

In fully developed laminar flow, each fluid particle moves at a constant axial velocity along a streamline and the velocity profile $u(r)$ remains unchanged in the flow direction. There is no motion in the radial direction, and thus the velocity component in the direction normal to flow is everywhere *zero*. There is no acceleration since the flow is steady and fully developed.

Now consider a ring-shaped differential volume element of radius *r*, thickness *dr*, and length *dx* oriented coaxially with the pipe, as shown in Figure 5.3. The volume element involves only pressure and viscous effects and thus the pressure and shear forces must balance each other. The pressure force acting on a submerged plane surface is the product of the pressure at the centroid of the surface and the surface area. A force balance on the volume element in the flow direction gives

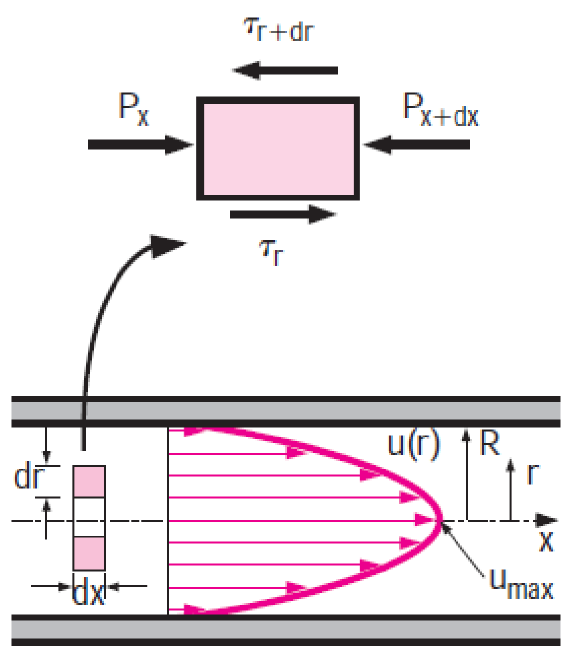


Figure 5.3: Free-body diagram of a ring-shaped differential fluid element of radius *r*, thickness *dr*, and length *dx* oriented coaxially with a horizontal pipe in fully developed laminar flow.

$$(2\pi r\,dr\,P)_x - (2\pi r\,dr\,P)_{x+dx} + (2\pi r\,dx\,\tau)_r - (2\pi r\,dx\,\tau)_{r+dr} = 0$$

which indicates that in fully developed flow in a horizontal pipe, the viscous and pressure forces balance each other. Dividing by $2\pi dr dx$ and rearranging,

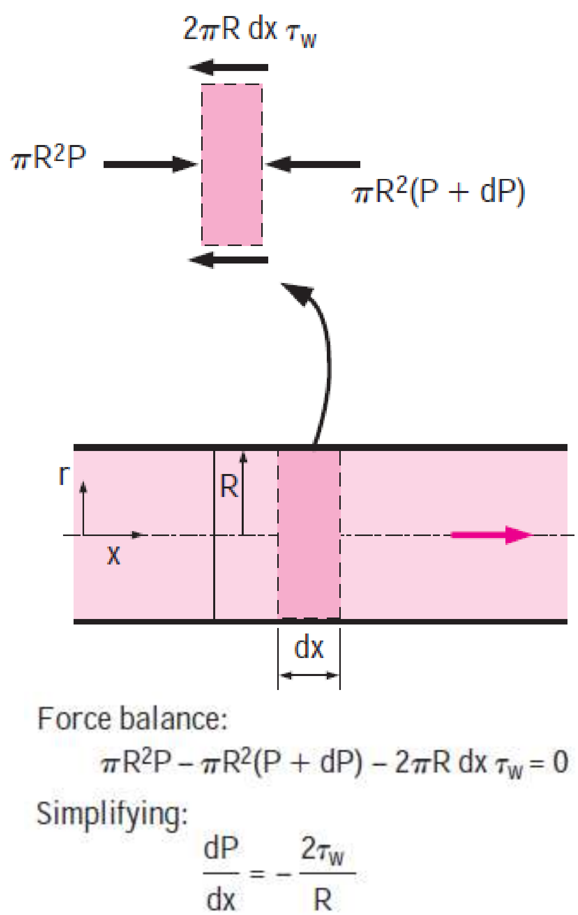
$$r\frac{P_{x+dx} - P_x}{dx} + \frac{(r\tau)_{r+dr} - (r\tau)_r}{dr} = 0$$

Taking the limit as dr, $dx \to 0$ gives

$$r\frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0$$

Substituting $\tau = -\mu(du/dr)$ and taking μ= constant gives the desired equation,

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx}$$



The quantity du/dr is negative in pipe flow, and the negative sign is included to obtain positive values for t. (Or, $du/dr = -\,du/dy$ since $y = R - r$.) The left side of above Equation is a function of r, and the right side is a function of x. The equality must hold for any value of r and x, and an equality of the form $f(r) = g(x)$ can be satisfied only if both $f(r)$ and $g(x)$ are equal to the same constant. Thus we conclude that dP/dx = constant. This can be verified by writing a force balance on a volume element of radius R and thickness dx (a slice of the pipe), which gives

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

Here τ_w is constant since the viscosity and the velocity profile are constants in the fully developed region. Therefore, dP/dx= constant.

by rearranging and integrating it twice to give

$$u(r) = \frac{1}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2$$

The velocity profile u(r) is obtained by applying the boundary conditions $\partial u/\partial r = 0$ at r = 0 (because of symmetry about the centerline) and u = 0 at r = R (the no-slip condition at the pipe surface). We get

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$$

Therefore, the velocity profile in fully developed laminar flow in a pipe is parabolic with a maximum at the centerline and minimum (*zero*) at the pipe wall. Also, the axial velocity u is positive for any r, and thus the axial pressure gradient dP/dx must be negative (i.e., pressure must decrease in the flow direction because of viscous effects).

$$V_{avg} = \frac{2}{R^2}\int_0^R u(r) r\, dr = \frac{-2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right) r\, dr = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)$$

Combining the last two equations, the velocity profile is rewritten as

$$u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$

This is a convenient form for the velocity profile since V_{avg} can be determined easily from the flow rate information. The maximum velocity occurs at the centerline and is determined from the velocity profile equation (equation above) by substituting $r = 0$,

$$u_{max} = 2V_{avg}$$

Therefore, the average velocity in fully developed laminar pipe flow is one half of the maximum velocity.

5.4. Pressure Drop and Head Loss

A quantity of interest in the analysis of pipe flow is the pressure drop (P since it is directly related to the power requirements of the fan or pump to maintain flow. We note that dP/dx= constant, and integrating from x= x_1 where the pressure is P_1 to x= x_1 + L where the pressure is P_2 gives

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L}$$

Substituting above equation into the V_{avg} expression, the pressure drop can be expressed as,

Laminar flow: $$\Delta P = P_1 - P_2 = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2}$$

In fluid flow, ΔP is used to designate pressure drop, and thus it is P_1 & P_2. A pressure drop due to viscous effects represents an irreversible pressure loss, and it is called **pressure loss** ΔP_L to emphasize that it is a *loss* (just like the head loss h_L, which is proportional to it). Therefore, the drop of pressure from P_1 to P_2 in this case is due entirely to viscous effects, and above equation represents the pressure loss ΔP_L when a fluid of viscosity m flows through a pipe of constant diameter *D* and length *L* at average velocity V_{avg}.

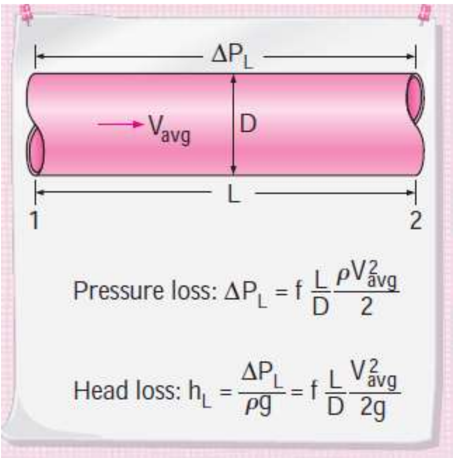
In practice, it is found convenient to express the pressure loss for all types of fully developed internal flows (laminar or turbulent flows, circular or noncircular pipes, smooth or rough surfaces, horizontal or inclined pipes).

Pressure loss: $$\Delta P_L = f\frac{L}{D}\frac{\rho V_{avg}^2}{2}$$

where $\rho V^2_{avg}/2$ is the *dynamic pressure*

f is the **Darcy friction factor,** $$f = \frac{8\tau_w}{\rho V_{avg}^2}$$

It is also called the **Darcy–Weisbach friction factor**,



It should not be confused with the friction coefficient C_f [also called the Fanning friction factor] which is defined as $C_f = 2\tau_w/(rV^2_{avg}) = f/4$.

Solving for *f* gives the friction factor for fully developed laminar flow in a circular pipe,

Circular pipe, laminar: $$f = \frac{64\mu}{\rho D V_{avg}} = \frac{64}{Re}$$

This equation shows that in laminar flow, the friction factor is a function of the Reynolds number only and is independent of the roughness of the pipe surface.

Head loss: $$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V^2_{avg}}{2g}$$

Once the pressure loss (or head loss) is known, the required pumping power to overcome the pressure loss is determined from

$$\dot{W}_{pump,\,L} = \dot{V}\,\Delta P_L = \dot{V}\rho g h_L = \dot{m} g h_L$$

where V is the volume flow rate and ṁ is the mass flow rate.

Example:

Water properties (ρ= 62.42 lbm/ft^3 and μ=1.038×10^{-3} lbm/ft .s) is flowing through a 0.12 in (= 0.010 ft) diameter 30 ft long horizontal pipe steadily at an average velocity of 3.0 ft/s (see Figure 5.4). Determine (**a**) the head loss, (**b**) the pressure drop, and (**c**) the pumping power requirement to overcome this pressure drop.

Solution:

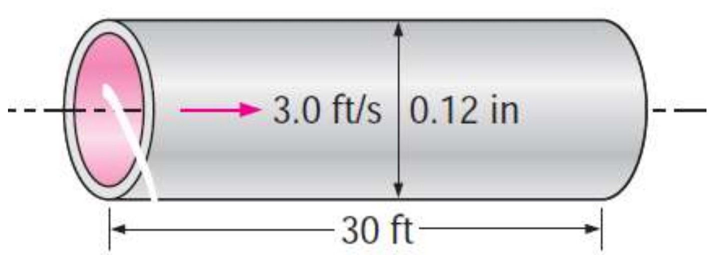


Figure 5.4: Schematic for above Example.

(**a**) First we need to determine the flow regime. The Reynolds number is

$$Re = \frac{\rho V_{avg} D}{\mu} = \frac{(62.42 \text{ lbm/ft}^3)(3 \text{ ft/s})(0.01 \text{ ft})}{1.038 \times 10^{-3} \text{ lbm/ft} \cdot \text{s}} = 1803$$

which is less than 2300. Therefore, the flow is laminar. Then the friction factor and the head loss become

$$f = \frac{64}{Re} = \frac{64}{1803} = 0.0355$$

$$h_L = f \frac{L}{D} \frac{V_{avg}^2}{2g} = 0.0355 \frac{30 \text{ ft}}{0.01 \text{ ft}} \frac{(3 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} = \mathbf{14.9 \text{ ft}}$$

(**b**) Noting that the pipe is horizontal and its diameter is constant, the pressure drop in the pipe is due entirely to the frictional losses and is equivalent to the pressure loss,

$$\Delta P = \Delta P_L = f \frac{L}{D} \frac{\rho V_{avg}^2}{2} = 0.0355 \frac{30 \text{ ft}}{0.01 \text{ ft}} \frac{(62.42 \text{ lbm/ft}^3)(3 \text{ ft/s})^2}{2} \left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right)$$

$$= \mathbf{929 \text{ lbf/ft}^2 = 6.45 \text{ psi}}$$

(**c**) The volume flow rate and the pumping power requirements are

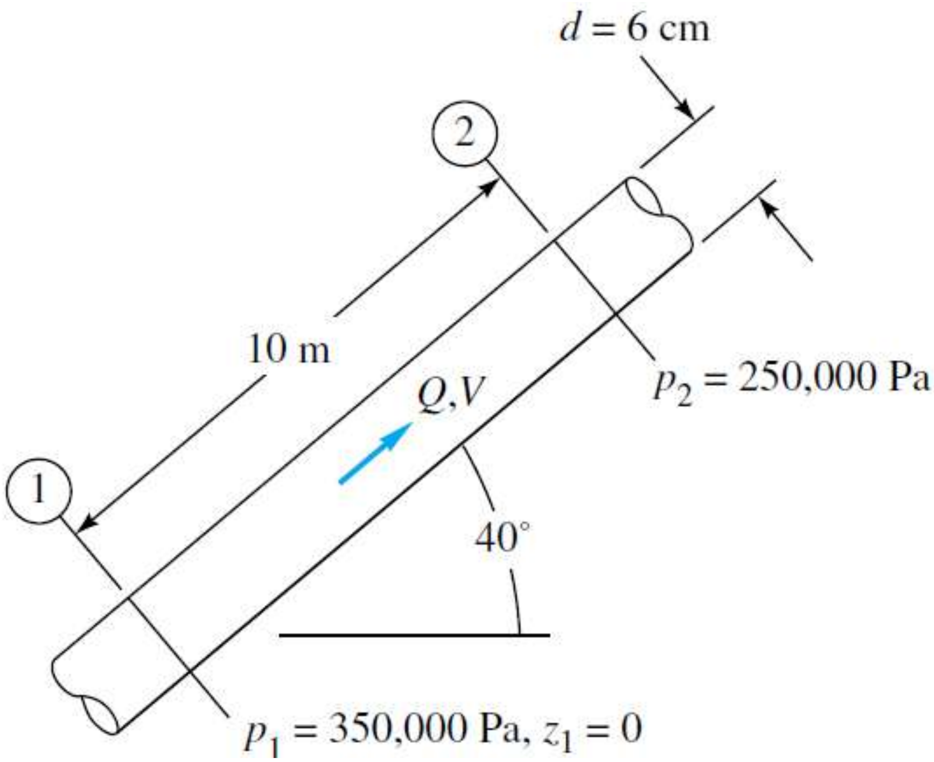
$$\dot{V} = V_{avg} A_c = V_{avg}(\pi D^2/4) = (3 \text{ ft/s})[\pi (0.01 \text{ ft})^2/4] = 0.000236 \text{ ft}^3\text{/s}$$

$$\dot{W}_{pump} = \dot{V} \Delta P = (0.000236 \text{ ft}^3\text{/s})(929 \text{ lbf/ft}^2) \left(\frac{1 \text{ W}}{0.737 \text{ lbf} \cdot \text{ft/s}} \right) = \mathbf{0.30 \text{ W}}$$

Example:

An oil with ρ= 900 kg/m^3 and υ= 0.0002 m^2/s flows upward through an inclined pipe as shown in Figure below. The pressure and elevation are known at sections 1 and 2, 10 m apart. Assuming steady laminar flow, (a) verify that the flow is up, (b) compute h_f between 1 and 2, and compute (c) volume flow rate, (d) Velocity, and (e) Reynolds number. Is the flow really laminar?

Solution:



For later use, calculate

$$\mu = \rho\nu = (900\ \text{kg/m}^3)(0.0002\ \text{m}^2/\text{s}) = 0.18\ \text{kg/(m} \cdot \text{s)}$$

$$z_2 = \Delta L \sin 40° = (10\ \text{m})(0.643) = 6.43\ \text{m}$$

The flow goes in the direction of falling HGL; therefore compute the hydraulic grade-line height at each section

$$\text{HGL}_1 = z_1 + \frac{p_1}{\rho g} = 0 + \frac{350{,}000}{900(9.807)} = 39.65\ \text{m}$$

$$\text{HGL}_2 = z_2 + \frac{p_2}{\rho g} = 6.43 + \frac{250{,}000}{900(9.807)} = 34.75\ \text{m}$$

The HGL is lower at section 2; hence the flow is from 1 to 2 as assumed. *Ans. (a)*

The head loss is the change in HGL:

$$h_f = \text{HGL}_1 - \text{HGL}_2 = 39.65\ \text{m} - 34.75\ \text{m} = 4.9\ \text{m} \qquad \textit{Ans. (b)}$$

Half the length of the pipe is quite a large head loss.

We can compute Q from the various laminar-flow formulas, notably Eq. (6.47)

We can compute Q from the various laminar-flow formulas, notably Eq. (6.47)

$$Q = \frac{\pi \rho g d^4 h_f}{128 \mu L} = \frac{\pi(900)(9.807)(0.06)^4(4.9)}{128(0.18)(10)} = 0.0076 \text{ m}^3/\text{s} \qquad \textit{Ans. (c)}$$

Divide Q by the pipe area to get the average velocity

$$V = \frac{Q}{\pi R^2} = \frac{0.0076}{\pi(0.03)^2} = 2.7 \text{ m/s} \qquad \textit{Ans. (d)}$$

With V known, the Reynolds number is

$$\text{Re}_d = \frac{Vd}{\nu} = \frac{2.7(0.06)}{0.0002} = 810 \qquad \textit{Ans. (e)}$$

This is well below the transition value Re= 2300, and so we are fairly certain the flow is laminar.