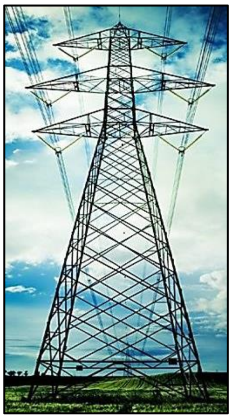
CHAPTER FOUR

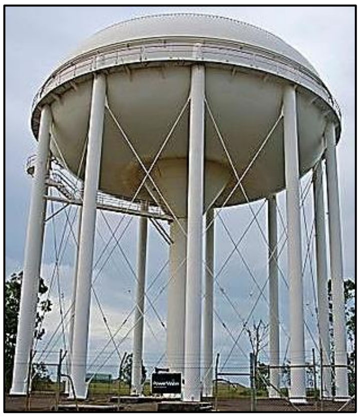
COMPRESSION MEMBERS

4.1 Overview

There are several types of compression members, the column being the best known. Among the other types are the top chords of trusses and various bracing members. In addition, many other members have compression in some of their parts. These include the compression flanges of rolled beams and built-up beam sections, and members that are subjected simultaneously to bending and compressive loads. Columns are usually thought of as being straight vertical members whose lengths are considerably greater than their thicknesses. Compression member: is a structural member which carries pure axial compression loads like compression members in:
Generally the used in:

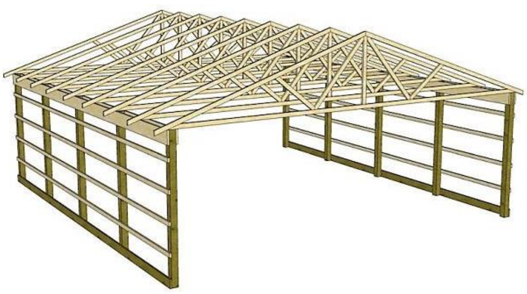
1- ***Column Supports & Towers***





Columns as supports & compressive member in towers

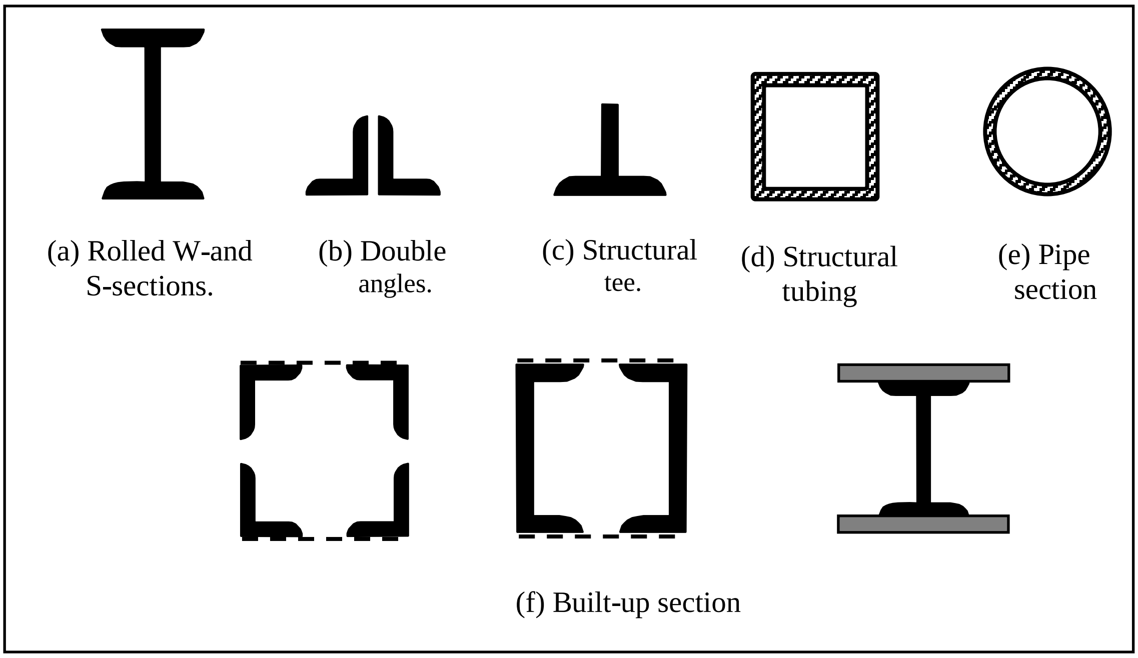
2- ***Trusses & Bridges***




Compressive member in truss

3- ***Columns in building frames***

Steel shapes, which are used as compression members, are shown in the figure below.



Steel shapes used as compression members

The stress in the column cross-section is given by:

$$f = P/A$$

Where,
f is compressive stress which is assumed to be uniform over the entire cross-section,
P is the magnitude of load,
A is the cross-sectional area normal to the load.
If the applied load increased slowly, it will ultimately reach a value P_{cr} that will cause buckling of the column, P_{cr} is called the critical buckling load of the column, i.e. $P > P_{cr}$ lead to buckling.

4.2 Elastic Flexural Buckling of a Pin-Ended Column

The deflection at distance **z** is denoted by **u**
Moment equilibrium about **A** in the buckling state gives:

$M - P.u = 0.0 \; .... \; M = P.u$

$M = EI\Phi = -EI\,(d^2u/d^2z) = P.u$

$EI\,(d^2u/d^2z) + P.u = 0.0$

$d^2u/d^2z + (P/EI).\,u = 0.0$

$d^2u/d^2z + \alpha^2.\,u = 0.0 \quad ...... \; \alpha^2 = P/EI$

$u = A \sin \alpha z + B \cos \alpha z$

$u = 0.0 \quad @\; z = 0.0 \quad .... \; B = 0.0$

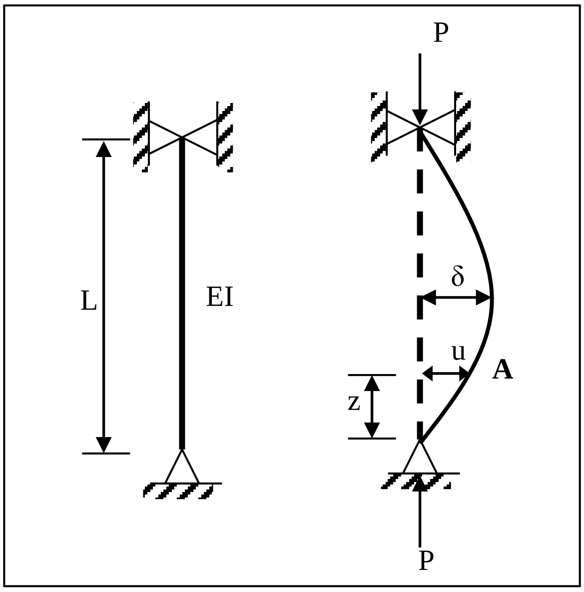
$u = 0.0 \quad @\; z = L \; .... \; A \neq 0.0$

then $\sin \alpha L = 0.0 \quad \alpha L = n\pi \quad ...\alpha = n\pi/L$

Where n = 1, 2, 3 ….

Then $P = P_{crn} = (n\pi/L)^2\, EI$

Thus, the Euler load of a pin – ended column is: $P_E = P_{cr1} = (\pi/L)^2\, EI$



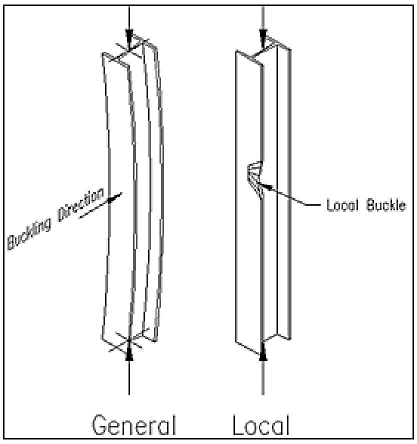
Pin – ended column under axial load

4.3 Buckling Basics

There are two main modes of buckling failure that may be experienced by
steel members: Overall (or general) buckling and local buckling. The Swiss mathematician Leonhard Euler developed an equation that predicts the critical buckling load P_{cr}, for a straight pinned end column. The equation is:

$$P_{cr} = \pi^2 EI/L^2$$

Where, I = moment of inertia about axis of buckling.
This equation to be valid:



- The member must be elastic
- Its ends must be free to rotate but translate laterally

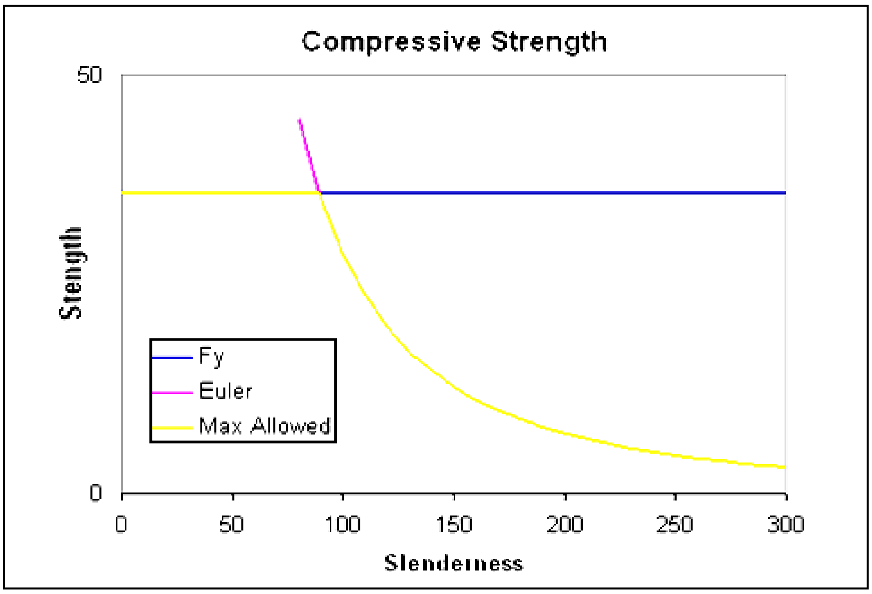
Dividing by the area of the element, we get an equation for the critical buckling stress:

$$\sigma_{cr} = \pi^2 E/(L/r)^2$$

Where the member cross sectional dependent term **(L/r)** is referred to as the "**slenderness**" of the member.

$$\sigma_{max} = \text{minimum}[\pi^2 E/(L/r)^2, F_y]$$

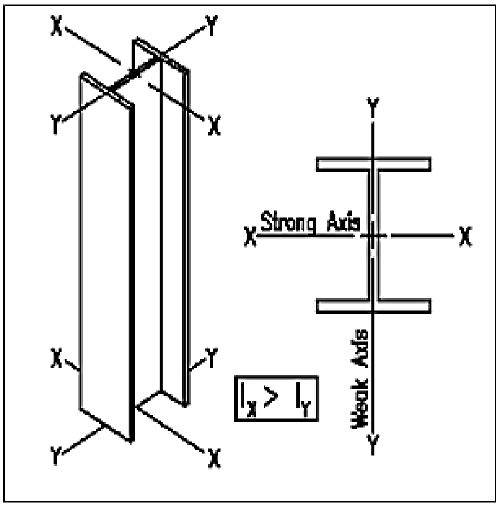
This relationship is graphed in the figure below



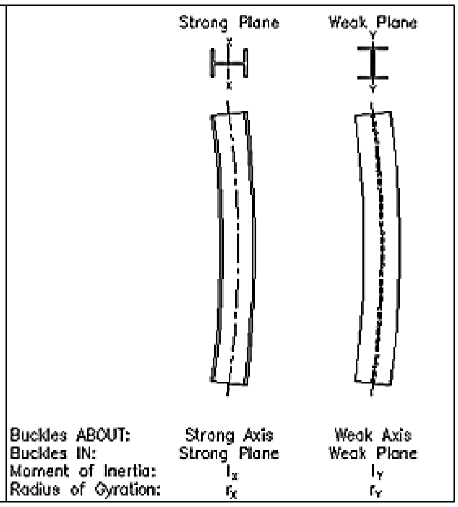
Theoretical Maximum Compressive Stress

4.4 General Member Buckling Concepts

The figure below illustrates the principle axes of a typical wide flange compression member. Other members shapes can be similarly drawn. With the exception of circular (pipe) sections, all the available shapes have a readily identifiable set of principle axes. Buckling is a two dimensional (planar) event. In other words it happens **IN** a **PLANE** that is perpendicular to the **AXIS** that it happens **ABOUT**.



Compression Member Principle Axis



Compression Member Principle Planes

3.4.1 Effective Length Coefficients and End Support Conditions

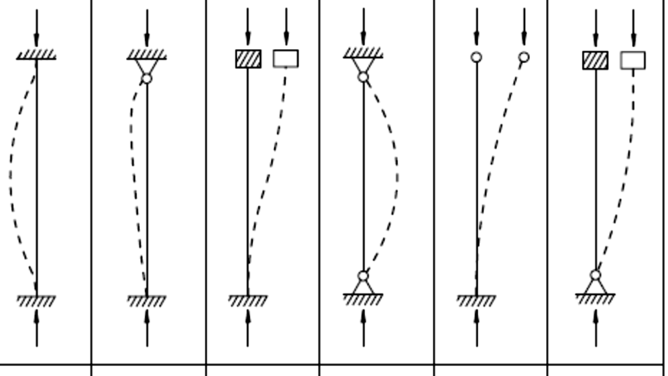
Theoretically, end supports are either pinned or fixed. In reality they can be designed to be pinned or rigid and may actually fall somewhere in between truly pinned or fixed. The support conditions will have an impact on the effective length, L_e. Effective length, L_e, of a compression member is the distance between where inflection points (Inflection point is a location of zero moment) are on a compression member. Effective length can be expressed as:

$$L_e = K\,L \quad \ldots\ldots \quad P_e = \pi^2 EI/(KL)^2$$

Where K is an effective length coefficient, **L** is the actual length of the compression member in the plane of buckling & P_e is elastic flexural buckling load in column. Different end conditions give different lengths for equivalent half-sine wave as shown in the figure below.

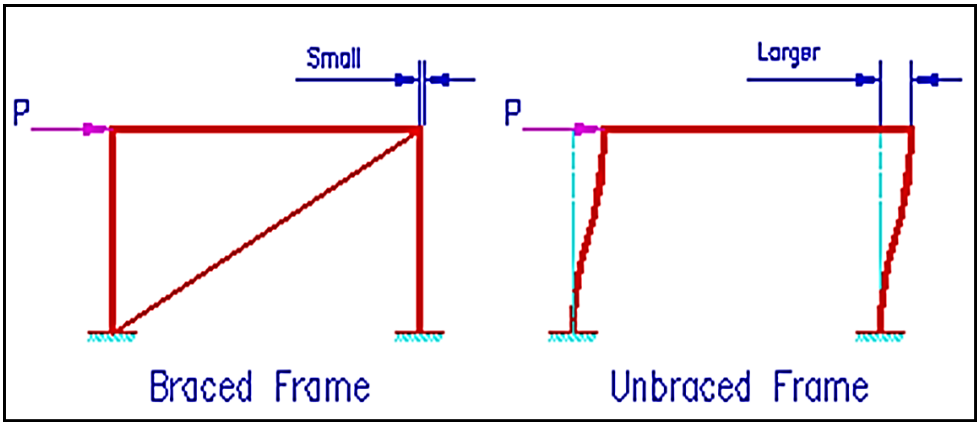
- The theoretical values of effective length coefficients assume that joints are completely fixed against rotation or totally free to rotate. Reality is usually somewhere in between. This affects the value of K.
- Table C-C2.2 is presented in LRFDM p. 240 to predicted the both theoritical and recommended design value of K of isolated column and its depended on support condition.

Table C-2.
Effective Length Factors (*K*) for Columns

Buckled shape of column is shown by dashed line	(a)	(b)	(c)	(d)	(e)	(f)
						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10	2.0
End condition code		Rotation fixed and translation fixed Rotation free and translation fixed Rotation fixed and translation free Rotation free and translation free				

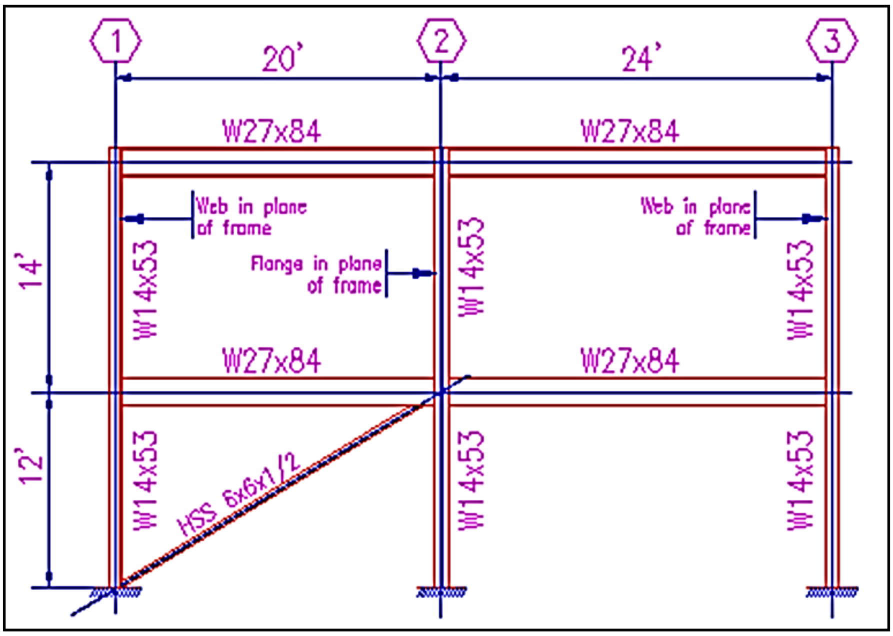
An theoretical effective length coefficient (K) values for different supports conditions for isolated column

- In the building, the cases with no joint translation are considered to be "braced frames" since some kind of bracing between the two levels is necessary to prevent lateral movement under shearing loads. The other cases are referred to as being "unbraced frames".



Braced vs. Unbraced Frame

- The figure below shows that different lengths of the same column can have different effective length coefficients in the same plane of buckling. Consider everything in the plane of buckling. Upper portion of this frame is **UNBRACED**. Lower portion of this frame is **BRACED**.



- Note that all the columns shown have an out-of-plane direction that must also be considered as well. Each direction will have totally independent lateral support and end conditions. It is highly recommended that you to draw both elevations of the column so that you can clearly see the conditions that apply to each column. Also note that the columns may have different laterally unsupported lengths in each direction as well.

- Two Charts are presented in LRFDM p. 241 to predicted the value of K of column in frames. One for braced frames (sidesway inhibited) and one for unbraced frames (sidesway uninhibited). To use these charts you must determine the rotational stiffness, **G**, of each joint in the plane of buckling being considered.

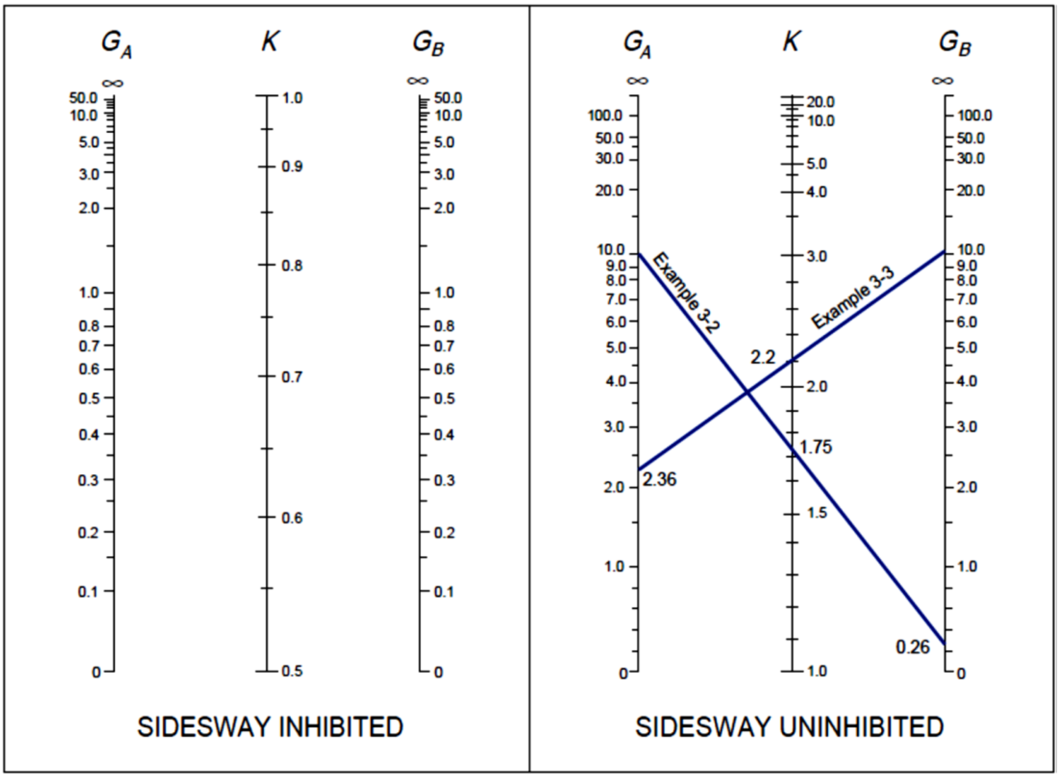
$$G = \frac{\sum(I_c/L_c)}{\sum(I_g/L_g)}$$

Where:

Σ indicates a summation of all member rigidity connected to that joint and lying on the plane in which buckling of column is considered.

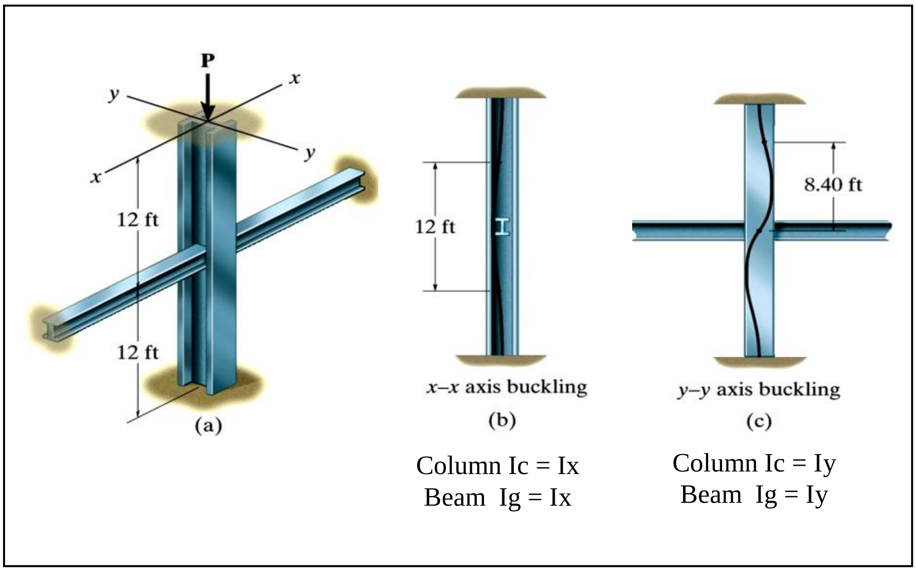
I_c & I_g moment of inertia of column & girder section, respectively.

L_c & L_g unsupported length of column & girder section, respectively.



Alignment charts for effective length of columns in continuous frames. The subscripts A and B refer to the joints at the two ends of the column section being considered.

The figure shows a typical framed joint, and Effective lengths in different directions



Example 3-1*:* Determine the buckling strength of **W12*50** column. Its length **20'**, the minor (weak) axis of buckling pinned at both ends, while major (strong) axis of buckling pinned at one end and fixed at the other end. **E = 29 ksi**.

Solution:

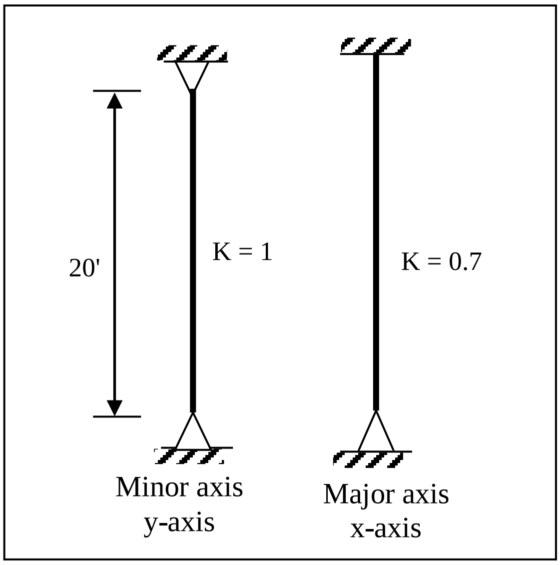
Note: for W-section x-axis is the strong Axis, while y-axis is the weak one.

P 1-39 $I_x = 394\ in^4$, $I_y = 56.3\ in^4$

The buckling Euler strength

$$P_e = \pi^2 EI/(KL^2)$$

The value of K of isolated column for different end condition can be predicted from table C-2.2 in LRFDM p. 240.



$P_{e\ (x\text{-}x)} = \pi^2(29000)*394/(0.8*20*12)^2$
$= 3035.8$ kips

$P_{e\ (y\text{-}y)} = \pi^2(29000)*56.3/(1*20*12)^2$
$= 279.8$ kips

Example 3-2

(a) A W10 × 22 is used as a 15-ft long pin-connected column. Using the Euler expression, determine the column's critical or buckling load. Assume that the steel has a proportional limit of 36 ksi.

(b) Repeat part (a) if the length is changed to 8 ft.

Solution

(a) Using a 15-ft long W10 × 22 ($A = 6.49 \text{ in}^2, r_x = 4.27 \text{ in}, r_y = 1.33 \text{ in}$)

Minimum $r = r_y = 1.33$ in

$$\frac{L}{r} = \frac{(12 \text{ in/ft})(15 \text{ ft})}{1.33 \text{ in}} = 135.34$$

$$\text{Elastic or buckling stress } F_e = \frac{(\pi^2)(29 \times 10^3 \text{ ksi})}{(135.34)^2}$$

$$= 15.63 \text{ ksi} < \text{the proportional limit of 36 ksi}$$

OK column is in elastic range

Elastic or buckling load $= (15.63 \text{ ksi})(6.49 \text{ in}^2) = 101.4$ k

(b) Using an 8-ft long W10 × 22,

$$\frac{L}{r} = \frac{(12 \text{ in/ft})(8 \text{ ft})}{1.33 \text{ in}} = 72.18$$

$$\text{Elastic or buckling stress } F_e = \frac{(\pi^2)(29 \times 10^3 \text{ ksi})}{(72.18)^2} = 54.94 \text{ ksi} > 36 \text{ ksi}$$

∴ column is in inelastic range and Euler equation is not applicable.

3.4.2 Long Columns

The Euler formula predicts very well the strength of long columns where the axial buckling stress remains below the proportional limit. Such columns will buckle *elastically*.

3.4.3 Short Columns

For very short columns, the failure stress will equal the yield stress and no buckling will occur.

3.4.4 Intermediate Columns

For intermediate columns, some of the fibers will reach the yield stress and some will not. The members will fail by both yielding and buckling, and their behavior is said to be *inelastic*. Most columns fall into this range. (For the Euler formula to be applicable to such columns, it would have to be modified according to the reduced modulus concept or the tangent modulus concept to account for the presence of residual stresses.)

3.5 Column Formulas

The AISC Specification provides one equation (the Euler equation) for long columns with elastic buckling and an empirical parabolic equation for short and intermediate columns. With these equations, a flexural buckling stress, F_{cr}, is determined for a compression member. Once this stress is computed for a particular member, it is multiplied by the cross-sectional area of the member to obtain its nominal strength P_n.

P_n is the nominal compressive strength of the member is computed by the following equation :

$$nP_n = F_{cr}\,A_g$$

$$P_d = \emptyset_c P_n = \emptyset_c F_{cr}\,A_g \quad = \text{LRFD compression strength } (\boldsymbol{\emptyset_c = 0.9})$$

Where:

- F_{cr} is the critical flexural buckling stress.
- A_g is the gross cross sectional area of the member.

The criteria for selecting which formula to use is based on either the slenderness ratio for the member or the relationship between the Euler buckling stress and the yield stress of the material. The selection can be stated as:

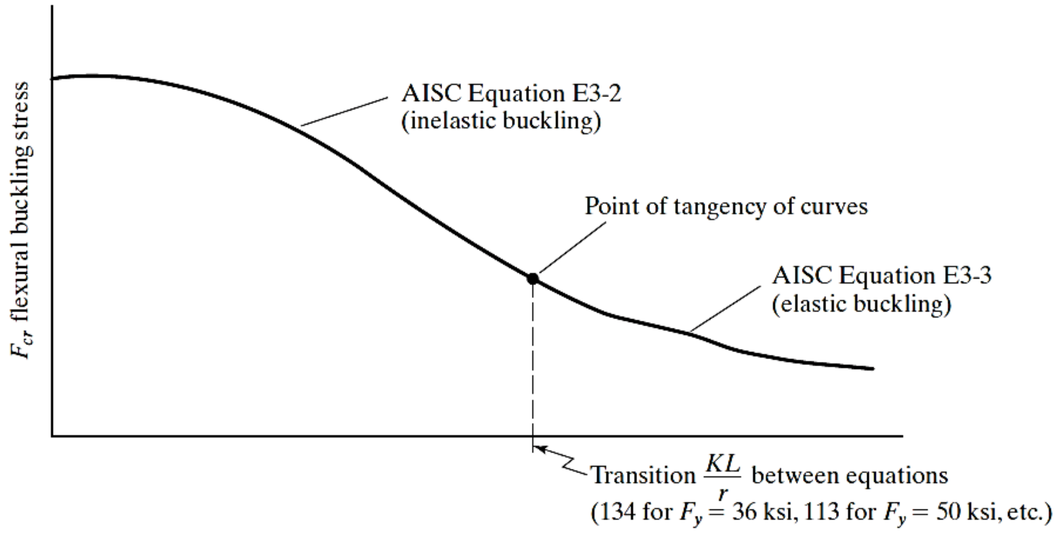
- If $KL/r \leq 4.71*\sqrt{\frac{E}{F_y}}$ then $F_{cr} = [0.658^{\,Fy/Fe}]\,F_y$
- If $KL/r > 4.71*\sqrt{\frac{E}{F_y}}$ then $F_{cr} = 0.877\,F_e$

In these expressions, $\mathbf{F_e}$ is the elastic critical buckling stress—that is, the Euler stress—calculated with the effective length of the column **KL**.

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

Note: The **AISC** Manual provides computed values of critical stresses $\boldsymbol{\emptyset_c F_{cr}}$ in their **Table 4-22 PP (4-318)**. The values are given for practical **KL/r** values (**0** to **200**) and for steels with **Fy = 36, 42, 46,** and **50 ksi.**

These equations are represented graphically in the figure below



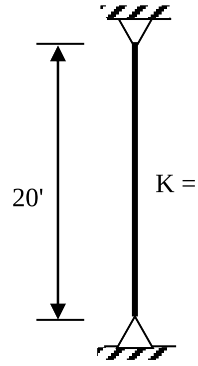
AISC column curve.

Example 3-3:

Determine the design strength of W14*74 column. Its length 20', it's pinned at both ends. E = 29 ksi.

Solution:

$A_g = 21.8\ in^2$, $r_x = 6.04\ in^4$, $r_y = 2.48\ in^4$, $f_y = 36$ ksi



$$\frac{K_y L}{r_y} = \frac{1*20*12}{2.48} = 96.77.......(control)$$

$$\frac{K_x L}{r_x} = \frac{1*20*12}{6.04} = 39.73$$

$$\max.\frac{KL}{r} = 96.77 < 200......ok$$

$$4.71\sqrt{\frac{F_y}{E}} = 4.71\sqrt{\frac{50}{29000}} = 113$$

$$\frac{KL}{r} = 96.77 < 113.....\text{use AISC Equation E3 - 2. p33}$$

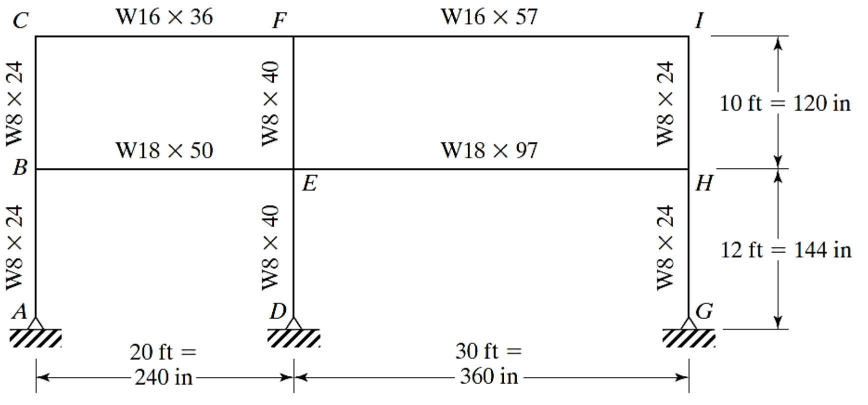
$F_e = Л^2E/(KL/r)^2 = 30.56$ ksi

$F_{cr} = [0.658^{Fy/Fe}]\ F_y = 25.21$ ksi

$P_d = \phi_c\ F_{cr}\ A_g = 0.9\ (25.21)(21.8) = 495$ kips

Example 3-4:

Determine the effective length factor for each of the columns of the frame shown in the figure, if the frame is not braced against sidesway.



Solution.

Stiffness factors: *E* is assumed to be 29,000 ksi for all members and is therefore neglected in the equation to calculate *G*.

	Member	Shape	I	L	I/L
Columns	AB	W8 × 24	82.7	144	0.574
	BC	W8 × 24	82.7	120	0.689
	DE	W8 × 40	146	144	1.014
	EF	W8 × 40	146	120	1.217
	GH	W8 × 24	82.7	144	0.574
	HI	W8 × 24	82.7	120	0.689
Girders	BE	W18 × 50	800	240	3.333
	CF	W16 × 36	448	240	1.867
	EH	W18 × 97	1750	360	4.861
	FI	W16 × 57	758	360	2.106

G factors for each joint:

Joint	$\Sigma(I_c/L_c)/\Sigma(I_g/L_g)$	G
A	Pinned Column, $G = 10$	10.0
B	$\frac{0.574 + 0.689}{3.333}$	0.379
C	$\frac{0.689}{1.867}$	0.369
D	Pinned Column, $G = 10$	10.0
E	$\frac{1.014 + 1.217}{(3.333 + 4.861)}$	0.272
F	$\frac{1.217}{(1.867 + 2.106)}$	0.306
G	Pinned Column, $G = 10$	10.0
H	$\frac{0.574 + 0.689}{4.861}$	0.260
I	$\frac{0.689}{2.106}$	0.327

Column **K** factors from the chart

Column	G_A	G_B	K
AB	10.0	0.379	1.76
BC	0.379	0.369	1.12
DE	10.0	0.272	1.74
EF	0.272	0.306	1.10
GH	10.0	0.260	1.73
HI	0.260	0.327	1.10