3.4 Local Member Buckling Concepts

The cross sections of steel shapes tend to consist of an assembly of thin plates. When the cross section of a steel shape is subjected to large compressive stresses, the thin plates that make up the cross section may buckle before the full strength of the member is attained if the thin plates are too slender. When a cross sectional element fails in buckling, then the member capacity is reached. Consequently, local buckling becomes a limit state for the strength of steel shapes subjected to compressive stress. The figure below shows an example of flange local buckling. This member failed before the full strength of the member was realized because the slender flange plate buckled first.

- In the Euler equation the parameter **(L/r)** is known as the slenderness of the member. For a plate, the slenderness parameter is a function of the width/thickness **(b/t)** ratio, **λ**, of a slender plate cross sectional element.

- There are two different types of plate elements in a cross section: ***Stiffened*** and ***Unstiffened***.

Flange Local Buckling Example

- If a plate's edges are restrained against buckling, then the force required to buckle the plate increases. If one edge is restrained (i.e. "unstiffened" plate element) the force to cause out-of-plane buckling is less than that required to buckle a plate with two edges restrained against out-of-plane buckling (i.e. "stiffened" plate element). An intersecting plate at a plate edge adds a significant moment of inertia out of plane to the edge which prevents deflection at the attached edge. The figure below illustrates the modes of buckling for a stiffened and unstiffened plate elements.

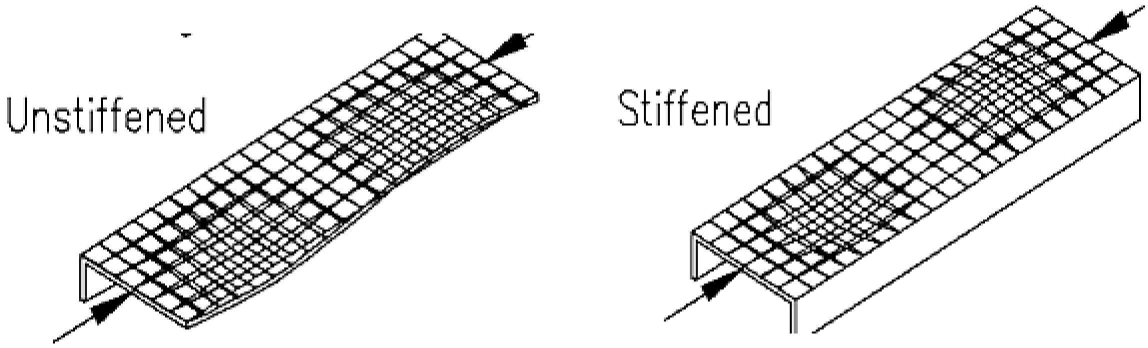
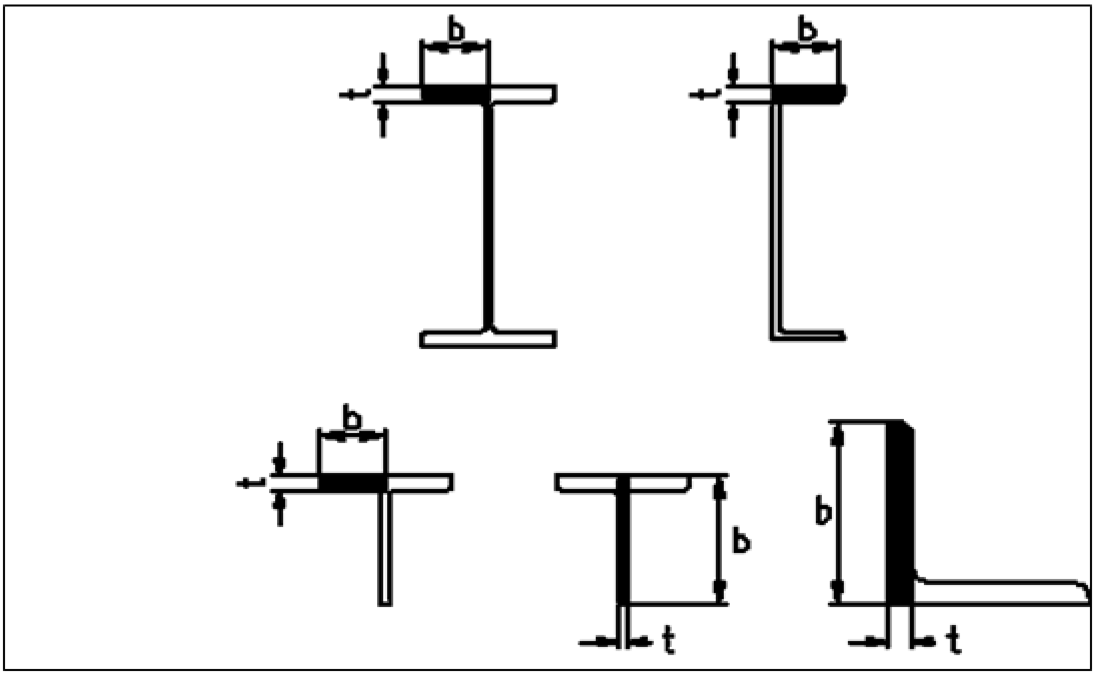


Plate Buckling Modes

- The figure below shows the *unstiffened elements* on some typical steel sections and the measurement of the element width, **b**, and thickness, **t**.



Unstiffened Elements

For example to prevent local buckling the plate slenderness, λ, should be less than limiting width-to-thickness ratio, λr, as following:

– Flange of I-, W- or T- shape: $\lambda_f = b_f/2t_f \leq \lambda_{rf} = 0.56\sqrt{E/F_y}$

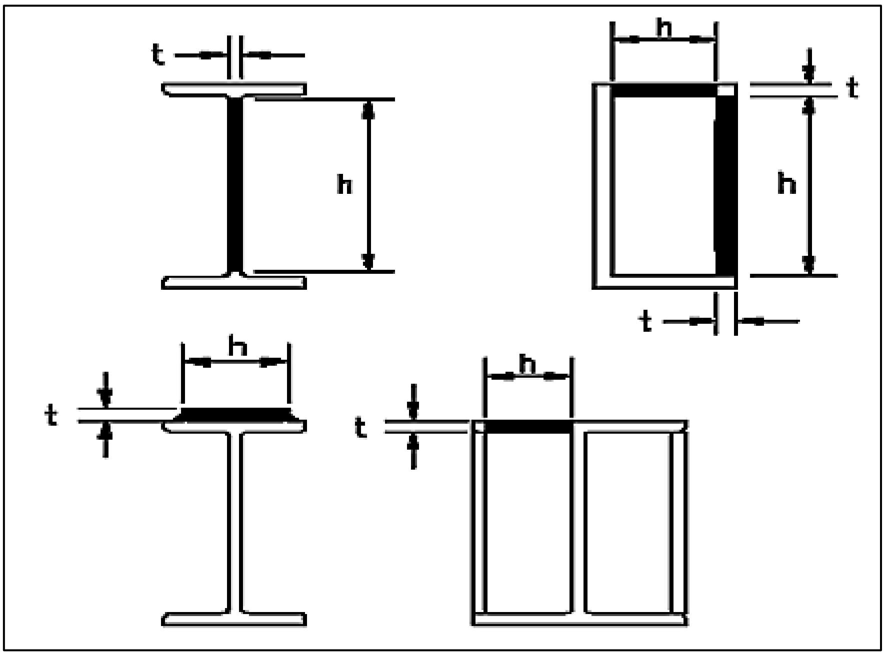
– Flange of C- shape: $\lambda_f = b_f/t_f \leq \lambda_{rf} = 0.56\sqrt{E/F_y}$

– Web of W- or C-shape: $\lambda_w = h/t_w \leq \lambda_{rw} = 0.75\sqrt{E/F_y}$

– Web of T- shape: $\lambda_w = d/t_w \leq \lambda_{rw} = 0.75\sqrt{E/F_y}$

– For single angle: $\lambda_a = b_{larger\ leg}/t \leq \lambda_{rw} = 0.45\sqrt{E/F_y}$

- The figure below shows the stiffened elements on some typical steel sections and the measurement of the element width, **h**, and thickness, **t**.



Stiffened Elements

For example to prevent local buckling for stiffness element:

– Web of W- or C-shape: $\lambda_w = h/t_w \le \lambda_{rw} = 1.49\sqrt{E/F_y}$

– Side of tube: $\lambda_{tube} = h/t \le \lambda_{r,\,tube} = 1.40\sqrt{E/F_y}$

- See table B4.1 p.16
- If $\lambda \le \lambda_r$, the shape is non-slender. Otherwise, the shape is slender.
- If the width-to-thickness ratio λ is greater than λ_r, ($\lambda > \lambda_r$) use the provisions of AISC E7 and compute a reduction factor Q. • Compute KL/r and F_e as usual.

- If $\dfrac{KL}{r} \le 4.71\sqrt{\dfrac{E}{QF_y}}$ or $\dfrac{QF_y}{F_e} \le 2.25$,

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y \qquad \text{(AISC Equation E7-2)}$$

- If $\dfrac{KL}{r} > 4.71\sqrt{\dfrac{E}{QF_y}}$ or $\dfrac{QF_y}{F_e} > 2.25$,

$$F_{cr} = 0.877F_e \qquad \text{(AISC Equation E7-3)}$$

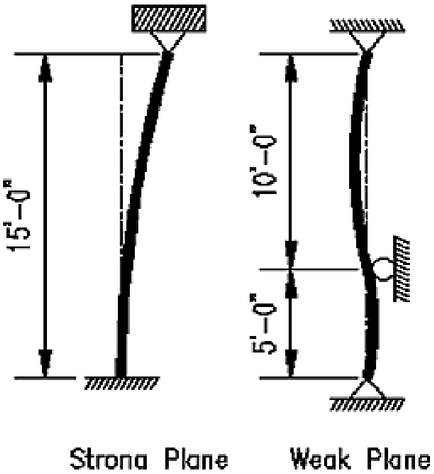
- The nominal strength is $P_n = F_{cr}A_g$ (AISC Equation E7-1)

The reduction factor Q is the product of two factorsQ_s for unstiffened elements and Q_a for stiffened elements.

- If the shape has no slender unstiffened elements, $Q_s = 1.0$.
- If the shape has no slender stiffened elements, $Q_a = 1.0$.

To calculate fQ_s for unstiffened elements and Q_a for stiffened elements see AISC E7-4 to E7-19. P40 to p.43

Example 3-6: A W8×35 Gr. 36 column is to be 15 ft long. In the strong plane, the column is part of an unbraced frame, one end is to be considered fixed and the other pinned. In the weak plane, the column is part of a braced frame, both ends are to be considered pinned and there is a lateral support provided 5 ft from one end.



Solution: - For W8×35

A =10.3 in.^{2} , r_x = 3.51 in., r_y = 2.03 in.,

L = 15 ft; L_x = 15 ft; L_{y1} = 5 ft; L_{y2} = 10 ft
$K_x L_x = 0.8*15 = 12$ ft
$K_y L_{y1} = 1*5 = 5$ ft;
$K_y L_{y2} = 1*10 = 10$ ft (control)

$$\frac{K_x L}{r_x} = \frac{12*12}{3.51} = 50.9$$

$$\frac{K_y L}{r_y} = \frac{10*12}{2.03} = 59.11 \text{.....Controls (largest KL/r)}$$

$$< 4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000ksi}{65ksi}} = 99.49$$

$$F_e = \frac{\pi^2(29{,}000\text{ ksi})}{59.11^2} = 82\text{ ksi}$$

$$F_{cr} = 0.658^{\left(36/82\right)}(36\text{ksi}) = 29.96\text{ksi}$$

$$P_n = (29.96\text{ ksi})(10.3\text{ in}^2) = 308.57\text{ kips}$$

$P_d = \phi_c P_n = 0.9*556.89 = 277.7$ kips

Or one can find ϕ_c F_{cr} from table 4-22 p. 4-319

- Local buckling checking

$\lambda_f = b_f/2t_f = 8.10 < \lambda_{rf} = 0.56\sqrt{E/F_y} = 11.8$ (unstiffener) O.K.

$\lambda_w = h/t_w = 20.5 < \lambda_{rw} = 1.49\sqrt{E/F_y} = 31.5$ (stiffener) O.K.

Neither flange local buckling nor web local buckling will precede member buckling. So, the design axial compressive strength of the column is 501.2.

Width-to-Thickness Ratios: Compression Elements in Members Subject to Axial Compression

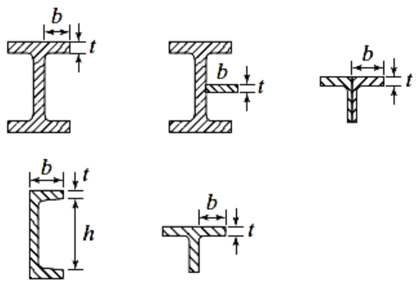
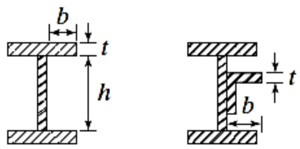
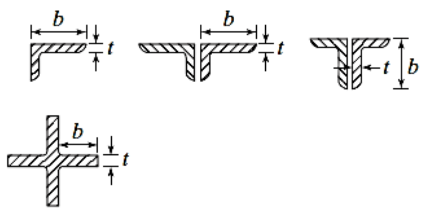
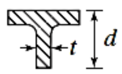
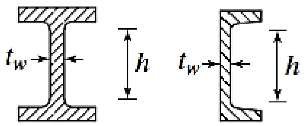
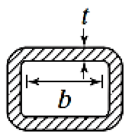
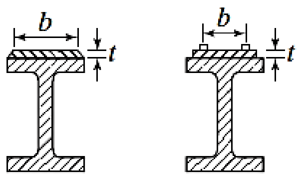
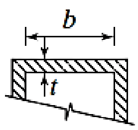
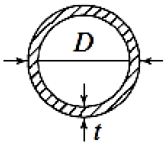
	Case	Description of Element	Width-to-Thickness Ratio	Limiting Width-to-Thickness Ratio λ_r (nonslender / slender)	Examples
Unstiffened Elements	1	Flanges of rolled I-shaped sections, plates projecting from rolled I-shaped sections, outstanding legs of pairs of angles connected with continuous contact, flanges of channels, and flanges of tees	b/t	$0.56\sqrt{\frac{E}{F_y}}$	
	2	Flanges of built-up I-shaped sections and plates or angle legs projecting from built-up I-shaped sections	b/t	$0.64\sqrt{\frac{k_c E}{F_y}}$ [a]	
	3	Legs of single angles, legs of double angles with separators, and all other unstiffened elements	b/t	$0.45\sqrt{\frac{E}{F_y}}$	
	4	Stems of tees	d/t	$0.75\sqrt{\frac{E}{F_y}}$	

TABLE 5.2 Continued

	Case	Description of Element	Width-to-Thickness Ratio	Limiting Width-to-Thickness Ratio λ_r (nonslender / slender)	Examples
Stiffened Elements	5	Webs of doubly symmetric I-shaped sections and channels	h/t_w	$1.49\sqrt{\frac{E}{F_y}}$	
	6	Walls of rectangular HSS and boxes of uniform thickness	b/t	$1.40\sqrt{\frac{E}{F_y}}$	
	7	Flange cover plates and diaphragm plates between lines of fasteners or welds	b/t	$1.40\sqrt{\frac{E}{F_y}}$	
	8	All other stiffened elements	b/t	$1.49\sqrt{\frac{E}{F_y}}$	
	9	Round HSS	D/t	$0.11\frac{E}{F_y}$	

Example 3-7: An **HSS 16 x 16 x ½** with $\mathbf{F_y}$ is used for an **18-ft**-long column with simple end supports.
(a) Determine $\mathbf{\emptyset_c P_n}$ with the appropriate AISC equations.
(b) Repeat part (a), using Table 4-4 in the AISC Manual.

Solution:

(a) Using an HSS

$$16 \times 16 \times \frac{1}{2}(A = 28.3 \text{ in}^2, t_{\text{wall}} = 0.465 \text{ in}, r_x = r_y = 6.31 \text{ in})$$

Calculate $\frac{b}{t}$ (AISC Table B4.1a, Case 6)

b is approximated as the tube size $-2 \times t_{\text{wall}}$

$$\frac{b}{t} = \frac{16 - 2(0.465)}{0.465} = 32.41 < 1.40\sqrt{\frac{E}{F_y}} = 1.40\sqrt{\frac{29{,}000}{46}}$$

$$= 35.15 \quad \therefore \text{ Section has no slender elements}$$

$\frac{b}{t}$ ratio also available from Table 1-12 of Manual

Calculate $\frac{KL}{r}$ and F_{cr}

$K = 1.0$

$$\left(\frac{KL}{r}\right)_x = \left(\frac{KL}{r}\right)_y = \frac{(1.0)(12 \times 18) \text{ in}}{6.31 \text{ in}} = 34.23$$

$$< 4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{46}} = 118.26$$

$\therefore$ Use AISC Equation E3-2 for F_{cr}

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{(\pi^2)(29{,}000)}{(34.23)^2} = 244.28 \text{ ksi}$$

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right]F_y = \left[0.658^{\frac{46}{244.28}}\right]46$$

$$= 42.51 \text{ ksi}$$

LRFD $\emptyset_c = 0.90$

$\emptyset_c F_{cr} = (0.90)(42.51) = 38.26$ ksi

$\emptyset_c P_n = \emptyset_c F_{cr} A = (38.26)(28.3)$

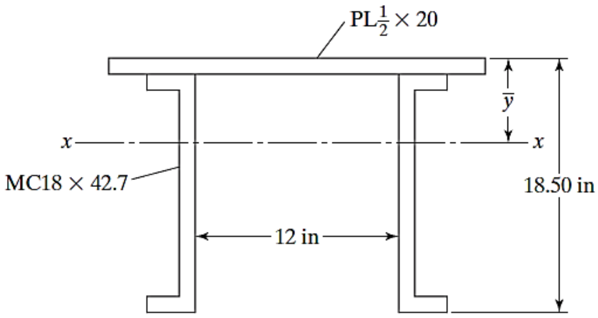
$= 1082$ k

(b) From the Manual, **Table 4-4**

$\phi_c P_n = 1080$ k

Example 3-8:

Determine the LRFD design strength $\phi_c P_n$ for the axially loaded column shown in the figure. If **KL = 19 ft** and **50-ksi** steel is used.



Solution

A =12.6 in^2, d=18.00 in,
I_x = 554 in^4, I_y =14.3 in^4, P.P(1-36)
$\bar{x}$ = 0.877 in from back of C)

$$A_g = (20)\left(\tfrac{1}{2}\right) + (2)(12.6) = 35.2 \text{ in}^2$$

$$\bar{y} \text{ from top} = \frac{(10)(0.25) + (2)(12.6)(9.50)}{35.2} = 6.87 \text{ in}$$

$$I_x = (2)(554) + (2)(12.6)(9.50 - 6.87)^2 + \left(\frac{1}{12}\right)(20)\left(\frac{1}{2}\right)^3 + (10)(6.87 - 0.25)^2$$
$$= 1721 \text{ in}^4$$

$$I_y = (2)(14.3) + (2)(12.6)(6.877)^2 + \left(\tfrac{1}{12}\right)\left(\tfrac{1}{2}\right)(20)^3 = 1554 \text{ in}^4$$

$$r_x = \sqrt{\frac{1721}{35.2}} = 6.99 \text{ in}$$

$$r_y = \sqrt{\frac{1554}{35.2}} = 6.64 \text{ in}$$

$$\left(\frac{KL}{r}\right)_x = \frac{(12)(19)}{6.99} = 32.62$$

$$\left(\frac{KL}{r}\right)_y = \frac{(12)(19)}{6.64} = 34.34 \leftarrow$$

From the Manual, Table 4-22, we read for $\frac{KL}{r} = 34.34$ that $\phi_c F_{cr} = 41.33$ ksi for 50 ksi steel.

$$\phi_c P_n = \phi_c F_{cr} A_g = (41.33)(35.2) = 1455 \text{ k}$$

Example 3-9:
Using F_y= 50 ksi select the lightest W14 available for the service column loads P_D = 130 k and P_L = 210 k. KL = 10 ft.
Solution

$$P_u = (1.2)(130\text{ k}) + (1.6)(210\text{ k}) = 492\text{ k}$$

$$\text{Assume } \frac{KL}{r} = 50$$

Using $F_y = 50$ ksi steel

$\phi_c F_{cr}$ from AISC Table 4-22 = 37.5 ksi

$$\text{A Reqd} = \frac{P_u}{\phi_c F_{cr}} = \frac{492\text{ k}}{37.5\text{ ksi}} = 13.12\text{ in}^2$$

Try W14 × 48 ($A = 14.1\text{ in}^2$, $r_x = 5.85$ in,
$r_y = 1.91$ in)

$$\left(\frac{KL}{r}\right)_y = \frac{(12\text{ in/ft})(10\text{ ft})}{1.91\text{ in}} = 62.83$$

$\phi_c F_{cr} = 33.75$ ksi from AISC Table 4-22

$$\phi_c P_n = (33.75\text{ ksi})(14.1\text{ in}^2)$$
$$= 476\text{ k} < 492\text{ k N.G.}$$

Try next larger section W14 × 53 ($A = 15.6\text{ in}^2$,
$r_y = 1.92$ in)

$$\left(\frac{KL}{r}\right)_y = \frac{(12\text{ in/ft})(10\text{ ft})}{1.92\text{ in}} = 62.5$$

$$\phi_c F_{cr} = 33.85\text{ ksi}$$

$$\phi_c P_n = (33.85\text{ ksi})(15.6\text{ in}^2)$$

$$= 528\text{ k} > 492\text{ k} \quad \textbf{OK}$$

Use W14 × 53.

Example 3-10:
Select the lightest available W12 section, using the LRFD for the following conditions: F_y = 50 ksi, P_D = 250 k, P_L = 400 k, K_xL_x = 26 ft and K_yL_y = 13 ft.
(a) By trial and error
(b) Using AISC tables

Solution

(a) Using trial and error to select a section, using the LRFD expressions, and then checking the section with the LRFD method.

$$P_u = (1.2)(250\text{ k}) + (1.6)(400\text{ k}) = 940\text{ k}$$

$$\text{Assume } \frac{KL}{r} = 50$$

$$\text{Using } F_y = 50\text{ ksi steel}$$
$$\phi_c F_{cr} = 37.5\text{ ksi (AISC Table 4-22)}$$

$$A\text{ Reqd} = \frac{940\text{ k}}{37.5\text{ ksi}} = 25.07\text{ in}^2$$

Try W12 × 87 ($A = 25.6\text{ in}^2$, $r_x = 5.38$ in, $r_y = 3.07$ in)

$$\left(\frac{KL}{r}\right)_x = \frac{(12\text{ in/ft})(26\text{ ft})}{5.38\text{ in}} = 57.99 \leftarrow \therefore \left(\frac{KL}{r}\right)_x \text{ controls}$$

$$\left(\frac{KL}{r}\right)_y = \frac{(12\text{ in/ft})(13\text{ ft})}{3.07\text{ in}} = 50.81$$

$$\phi_c F_{cr} = 35.2\text{ ksi (Table 4-22)}$$

$$\phi_c P_n = (35.2\text{ ksi})(25.6\text{ in}^2)$$

$$= 901\text{ k} < 940\text{ k } \textbf{N.G.}$$

A subsequent check of the next-larger W12 section, a W12 x 96, shows that it will work for the LRFD procedure.